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Endogenous Union Formation

Pablo Ruiz-Verdú
Stanford University

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Stanford Institute for Economic Policy Research
Stanford University
Stanford, CA 94305
(650) 725-1874

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Labor Markets Under Endogenous Union Formation *

Pablo Ruiz-Verdú†


Abstract

This paper develops a theory of the labor market in which both union status and the wages of union and nonunion firms are endogenously determined. This brings together two classes of research –on the wage effects of unions and on the incidence of unionism– that have been largely separate in previous work. The implications of this unified theory diverge from current conventional wisdom. If employers set wages taking into account their effect on workers’ incentives to organize, union density and the union wage gap are negatively correlated. In turn, union organization takes place only when it is less costly than dissuading workers from organizing, which happens if unions increase total rents, or if there is asymmetric information. Against conventional wisdom, organization is more likely in firms with lower than average rents, and average industry rents, by themselves, are uninformative to predict union density or the wage gap. The theory reconciles the time series behavior of union density and the wage gap in the U.S., and it is the first to provide an explanation for the cross-sectional distribution of these variables. JEL: J51, J31, J41, D82.

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†Address: Department of Economics. Stanford University. Stanford, CA 94305-6072. E-mail: pablorv@stanford.edu.
1 Introduction

In the 1980s and 1990s, union density in the U.S. private sector has steadily declined, falling from 22% in 1980 to less than 10% in 1998. At the same time, the union wage premium\(^1\) for private sector workers has remained untrended.\(^2\) Existing research has problems accounting for these trends, mainly because the study of the incidence of unionism, and of the wage effects of trade unions have followed separate paths. This paper develops a model of the labor market in which union status and the wages of union and nonunion firms are \textit{jointly} determined, and thus brings these two strands of research together. The implications of this unified model diverge in striking ways from current conventional wisdom, and are consistent with available evidence.

According to the basic textbook model, the incidence of unionism is determined as the equilibrium outcome of a market for union services. \textit{Given} the expected wage gains from organization, union fees are set so as to equalize demand for and supply of union services.\(^3\) The expected wage gains from unionization determine the location of the demand curve, and thus the equilibrium union membership: higher expected wage gains shift the demand curve to the right, and therefore increase equilibrium union membership. Expected wage gains, in turn, are assumed to be determined mainly by product market conditions: large profits or a relatively inelastic product demand allow unions to extract higher wages without driving firms bankrupt. However, the determination of the expected union wage premium is left \textit{outside} of the model.

More recently, a series of articles has emphasized the effect of expected wage gains on union organization.\(^4\) The effect of expected wage gains on union organization is determined as the equilibrium outcome of a market for union services. \textit{Given} the expected wage gains from organization, union fees are set so as to equalize demand for and supply of union services.\(^3\) The expected wage gains from unionization determine the location of the demand curve, and thus the equilibrium union membership: higher expected wage gains shift the demand curve to the right, and therefore increase equilibrium union membership. Expected wage gains, in turn, are assumed to be determined mainly by product market conditions: large profits or a relatively inelastic product demand allow unions to extract higher wages without driving firms bankrupt. However, the determination of the expected union wage premium is left \textit{outside} of the model.

\(^{1}\)Union density is defined as the proportion of workers who are members of a trade union, or who are covered by a collective bargained agreement. By union wage premium or gap, I will refer to the estimate of the difference between the wage of a union worker, and an otherwise identical nonunion worker.

\(^{2}\)A similar pattern can be observed in Great Britain after the labor market reforms of the 1980s and early 1990s. Moreover, at least in the U.S. case, the constancy of the wage gap may be due in part to a compositional effect, as the share of employment of industries with high wage premia has declined. Union wage gaps by industry seem to have increased for most industries, at least for part of the period. See Linneman and Wachter (1986), and Linneman, Wachter, and Carter (1990).

\(^{3}\)See, for example, Ehrenberg and Smith (1997), or, for a more thorough analysis, Hirsch and Addison (1986), chapter 2. Starting with Abowd and Farber (1982), several papers have extended this basic framework to account for the possibility that union fees do not adjust sufficiently to bring about equilibrium, so that there are queues for union jobs. See Farber (1983), and Farber (1990).
union wage gains on employers’ incentives to resist unionization. According to this line of research, a high premium induces employers to oppose organization attempts, driving the costs of organization up, and leading to an upward shift of the supply for union services. Therefore, the effect of an increase in expected wage gains on equilibrium union membership depends on the relative sizes of the shifts of the demand and supply curves. But, even if this view recognizes the role of employers in the determination of union membership, it still takes expected wage gains as given, and does not consider the possibility that employers may try to affect unionization by decreasing those wage gains. Only Rosen (1969) and Dickens (1986) provide models in which nonunion employers set wages so as to affect workers’ incentives to organize. However, their models do not explain the incidence of unionism, namely, why certain firms become organized while other remain nonunion.

With respect to the effects of unionism, a large theoretical literature has studied wage and employment determination under collective bargaining. However, this literature takes union status as given, and generally assumes nonunion employers behave competitively. As a result, it ignores the restrictions that the strategic behavior of nonunion employers imposes on the equilibrium effects of unions: in those firms where union effects are potentially large, employers will find it profitable to resist unionization. Moreover, to the extent that wage setting in the nonunion sector departs from the competitive benchmark, these models are not well-suited to explain the relative effects of unions.

If the impact of unions could be easily estimated, the above drawbacks would be of little concern. However, the main problem for the vast empirical literature on union effects is precisely the fact that the union status of firms is not randomly assigned, but is likely to be related to characteristics unobservable to the econometrician. Therefore, conventional estimates of those effects entangle the true impact of unions with that of unmeasured differences between union and nonunion firms or workers. After reviewing the empirical literature on the union-nonunion wage gap, H. G. Lewis argues:

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5Lazear (1983) is an exception. Following the seminal work in Rosen (1969), in his analysis, union membership and the wage premium are both determined in equilibrium. However, he implicitly assumes the existence of an industry-wide union, and reduces the role of employers to either accept the wage demands of the union, or resist organization at a certain cost. Employers are assumed not to play any active role in wage setting.

6For a review of this literature, see Pencavel (1991), or Booth (1995).

7See Lewis (1986), Hirsch (1991), and Flanagan (1999) for reviews of this literature.
“after surveying both simultaneous equations and panel wage gap estimates, I have come empty-handed in my search for the sign and numerical size of the omitted-variable or selectivity bias [stemming from the determination of union status] in Micro, OLS, CS (cross-section) gap estimates” (Lewis (1986), p.5).

In this paper, I develop a theory of the labor market in which the union status of firms and the wages of union and nonunion firms are jointly determined as equilibrium outcomes. The theory formalizes the insight, overlooked by most theoretical work, that firms and workers predate unions: At the outset, employers deal with individual workers, and it is only if employees believe that they can gain from organizing that collective bargaining will take place.

According to the theory, workers decide whether to organize based on the wage offered by the firm and the expected costs and benefits of organization. Management, on its part, does not take wages as given, but sets them taking into account the effect of the proposed wage on workers’ incentives to organize. As a result, and under standard assumptions, no unions form in equilibrium unless they increase the joint surplus accruing to management and workers by an amount that exceeds the costs of organizing the union: As long as the presence of a union does not sufficiently increase the quasi-rents generated by workers, it is always in the interest of the firm to set wages high enough so as to buy off the workers to ensure they do not attempt to organize. Put differently, the demand-supply model described above is of little help to explain the determination of union membership: given the “price” of unionization, employers set wages so as to make the expected wage gain from organization equal to its price.

This result implies that, in the benchmark model of complete information (section 2), unionization is always efficient. In the central part of the paper (sections 3 and 4), I extend the theory to investigate how asymmetric information between employers and workers affects wage determination, and whether it may lead to inefficient union formation. Employers are assumed to be better informed than their workers about the rents to be divided. This implies that wage offers acquire a new role as a signal of the value of the relationship between the firm and its workers: when offered a wage, workers form beliefs about the firm’s rents, and decide whether to organize

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8 Others are less pessimistic. See, for example, Robinson (1989).
9 This assumption accords with what we know of the behavior of nonunion employers under the threat of organization. See Foulkes (1980), and Rees and Schultz (1970).
10 Asymmetric information has also been proposed as an explanation of strikes. For a review of asymmetric information models of strikes, see Kennan and Wilson (1993)
based on those beliefs.

With asymmetric information, unionization may occur even if it reduces total rents. Surprisingly, although conventional wisdom would lead us to expect that unions will form in more profitable firms, it is those firms with lower rents that have the greater probability of becoming unionized. Such firms accept union organization as their only way to credibly signal to workers that their rents are low.

At odds with conventional views, the theory implies that the union-nonunion wage gap does not depend upon product market power. A relationship between the two will only arise when product market power is correlated with the direct determinants of the wage gap, namely the costs of union organization to workers, the uncertainty over the firm’s rents, and the efficiency cost of unions. In particular, the union-nonunion wage gap will be larger the higher the costs of organization. Moreover, union density and the wage gap will be negatively correlated.

Several applications of the theory are presented in section 5. First, it explains the somewhat puzzling trends of unionism in the U.S. and Great Britain, characterized by a steadily declining union density and untrended wage gaps. It also provides a parsimonious explanation for the cross-sectional variation of union density and the union wage premium in the U.S. with respect to a variety of factors, and for the fact that unions in countries with lower union densities seem to be able to extract higher wage premia. Finally the model is applied to evaluate conventional estimates of union effects. The prediction of the theory that unionization is more likely in firms with lower unobservable rents, implies that estimates of effects on profits will tend to be upwards biased, and estimates of wage gaps downward biased, as firms with low rents will be overrepresented among union firms.

2 Union (Non) Formation under Complete Information

In this section, I propose a model to represent, in the simplest form possible, the process of wage determination when workers can organize to influence the terms of the contract. In the model, there is a firm with a fixed number of available jobs, and a pool of identical potential workers, who interact over two periods.

In the first period, the firm offers a wage $w^0$ to the pool of potential workers, who act as wage takers and who can obtain a wage $\overline{w}$ outside the firm. If no worker accepts work at that wage, the firm closes for the period
and obtains zero revenues. Otherwise, the jobs are filled and employees acquire firm-specific skills through on-the-job learning. The skills acquired imply that second period revenues net of all other costs from employing a worker, $v$, are higher than those that could be obtained by hiring an alternative worker that period, $\bar{v}$. Without loss of generality, I assume $w = 0$ and $\bar{v} = 0$.

At the beginning of the second period, the firm offers a wage $w$ to its employees, who then have three options: they can reject the wage and leave the firm, they can accept to work at the offered wage, or they can establish a union to renegotiate the wage. Note that workers are assumed not to have any effective bargaining power if they attempt to renegotiate the wage individually, although the presence of specific skills would, in principle, confer a certain amount of bargaining power to individual workers. This is just a simplifying assumption: the analysis below applies to all cases in which workers’ bargaining is greater if they bargain collectively.

The actual process of union organization is left unmodelled. This process will involve several activities, like gathering information about organization, informing other workers, or meeting and discussing the issue with co-workers and management. Moreover, it may expose workers to unfair dismissals or other actions taken by employers to resist organization. In the model, it is assumed that all these activities can be summarized by a single variable, $e$, the costs to workers of establishing a union. That is, workers know that they can establish a union if they are willing to spend $e$ in organizing.

The process of union-management bargaining is also left unmodelled. However this process takes place, workers and employers know that if a union formed, the wage would be renegotiated to $0 < w^u \leq v^u$, where $v^u$ stands for total rents if the union forms.

Finally, I do not model the way in which unionization affects firm performance. Unionization may increase or reduce productivity, affect input choices by increasing wages, reduce rents if firms spend resources in resisting the organization attempt, or affect the set of self-enforceable contracts available to management and workers, to name a few possibilities. There-

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11. Even in the absence of specific skills, firing and hiring costs may create a wedge between the value to the firm of an existing employee and that of a new worker.

12. The requirement that the firm offer the same wage to all workers is not restrictive. Nothing substantial would change in the model if we relaxed this assumption, except that union avoidance would become cheaper. Moreover, if the union can also discriminate among its members, Dickens (1986) shows that the least costly way for the firm to prevent organization is to pay all workers the same wage.

13. For a theoretical analysis of the conditions determining the relative advantage of individual versus collective bargaining, see Stole and Zwiebel (1996), and Segal (1999).
fore, \( v^u \) will generally differ from \( v \). Again, no matter how \( v^u \) is determined, both workers and employers do know \( v^u \).

The advantage of this reduced form approach is its generality, especially important given the lack of an agreed upon model to represent the processes of union organization and collective bargaining. Different models may yield different predictions about \( e, w^u, \) or \( v^u \), yet the analysis below remains unchanged.

Figure 1, in which \( o \) stands for workers’ organization decision, depicts the model’s time-line.

![Figure 1](image)

Now, note that employers can always guarantee that the firm remains nonunion. If they offer a wage \( w \), such that \( w > w^u - e \), workers will have no incentive to organize. They also know that offering \( w < w^u - e \) will trigger organization. Therefore, unions will only form in those firms in which organization is less costly to the firm than union avoidance. Proposition 1 shows that, under complete information, this can never be the case unless unions increase rents.

**Proposition 1 (The Union Paradox)** Let \( v^u - e \leq v \). Then, at the unique subgame perfect equilibrium of the game the firm offers \( w^u - e \), workers accept the offer, and no collective action takes place.

**Proof.** In the second period, if the firm offered \( w < w^u - e \), workers would organize with certainty, and the firm’s payoff would be \( \pi^u = v^u - w^u \). By offering \( w = w^u - e + \epsilon \), where \( \epsilon > 0 \), the firm ensures that workers do not organize, and thus its payoff is \( \pi = v - w^u + e - \epsilon \). Clearly, if \( v^u \leq v + e \) for \( \epsilon \) small enough, \( \pi > \pi^u \). In equilibrium, workers’ strategy cannot involve a positive probability of organization for \( w = w^u - e \), as the same argument would apply. Therefore, in equilibrium, workers will accept with probability one any \( w \geq w^u - e \). If that is the case, and workers’ outside option is zero in both periods, then it is optimal for workers to accept any \( w^0 \geq -(w^u - e) \), and thus for the firm to offer \( w^0 = -(w^u - e) \) in the first period. As before, any strategy implying a positive probability of rejection
in the first period could not be part of an equilibrium profile. Therefore, the equilibrium strategies are: (i) the firm offers wage \( w^0 = -(w^u - e) \) in the first period \(^{14}\), and \( w = w^u - e \) in the second period; (ii) workers accept any \( w^0 \) such that \( w^0 \geq -(w^u - e) \), and any \( w \) such that \( w \geq w^u - e \). If \( w < w^u - e \), workers establish a union. Q.E.D.

According to Proposition 1, no unions will appear in equilibrium, unless they are socially efficient. Therefore, if unions are thought to be merely redistributive \( (v^u = v) \), or if they actually reduce the rents accruing to the firm \( (v^u < v) \), either by negatively altering the firm’s production function, or by distorting its input choice, then it appears to be a puzzle that unions form at all. By analogy with the well-known Hicks’s Paradox for the case of strikes, Proposition 1 is labeled “The Union Paradox”: as in the case of strikes, which consume resources yet have only redistributive purposes, inefficient union organization should not be observed.

However, if unions increase rents sufficiently (that is, if \( v^u - e > v \)), then it is optimal for management not to prevent organization. Note that this does not imply that management prefers a situation in which unions can form to one in which union organization is not possible; unless the positive effect of unions is substantial, profits will be lower in unionized firms than in nonunion firms paying the alternative wage. The problem is that, in the presence of the threat of unionization, the firm cannot remain nonunion unless it pays high enough wages. If unions increase rents, it may then be optimal to bring in the union and pay union wages, since the larger rents may more than compensate for the increased wage bill. The following corollary states this result.

**Corollary 1** Union organization will take place if and only if \( v^u > v + e \).

In a sense, Corollary 1 can be regarded as a special case of the Coase Theorem: inefficient institutions will not arise as long as the agents involved are able to bargain together and effectively implement their bargain. What the model shows is that given the way labor markets are assumed to work, the conditions are met that guarantee that efficiency is indeed achieved.

Corollary 1 has novel implications for the relationship between union formation and variables, like product market power, which have traditionally been held to explain it. According to the corollary, the probability with

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\(^{14}\)In the model, I am assuming that the wage that a worker can obtain if she does not accept the firm’s wage is zero for both periods. Of course, if there were more firms in the industry, the reservation wage would be determined as yet another equilibrium variable. In that case, and depending on the relation between labor demand and supply, workers may be able to extract part of the rents.
which a firm becomes unionized is determined by the costs of union organization and the impact of unions on total rents. Therefore, as long as there are enough rents to make unionization viable (that is, as long as $w^u > e$), product market power will encourage unionization only if unions are more likely to be efficient where market power is greater. In the absence of empirical or theoretical support for the latter association, there are no a priori reasons to expect product market power and organization to be related.

The consequences of Corollary 1 for the union wage gap have a similar flavor. The wage gap is equal to

$$E(w^u|v^u - e > v) - E(w^u - e|v^u - e \leq v),$$

where the expectations are taken over the distribution of firms. Therefore it is determined by $e$ and the difference between the wages that would be negotiated in those firms in which unions increase rents significantly, and in those in which they do not do so. Again, unless there are reasons to believe that, say, firms in more concentrated industries are more likely to benefit from unionization, there is no justification in the claim that the union wage gap will be higher in industries where firms have greater market power.

Another implication of Proposition 1 concerns the interpretation of union density as a measure of union power, where union power is understood as the degree to which unions are able to extract rents if they form. If unions do not increase rents, union formation is an event off the equilibrium path. Therefore, union density would be a good measure of union power only if it were true that those industries where the union threat is greater are also characterized by a larger departure from the complete information model. This is not necessarily the case. For example, suppose that unions formed due to agents’ mistakes when playing the game described in the above section. If there is a lot at stake (say, because $v - v^u$ is very large), management may be more careful not to make mistakes. We could then see the counterintuitive result of larger union density in sectors where the effects of unionization are smaller.

Finally, note that Corollary 1 has clear policy consequences: it implies that the optimal industrial relations policy should aim at setting $e$ as low as possible, while at the same time leaving the decision to engage in collective bargaining to firms and workers. In the following sections, I study

\footnote{Note that Corollary 1 implies that the union wage gap will not be meaningfully defined if we want to account for all workplace characteristics, as two identical firms will never have different union status. Having made this caveat, one may still want to measure the wage gap across individuals.}
whether the efficiency result of Corollary 1 still holds when the assumption of complete information is dropped, and, therefore, whether, in the presence of market failures, it is still optimal to facilitate organization as much as possible.

3 Union Formation under Asymmetric Information

The presence of asymmetric information appears to be the most likely reason why management and workers may fail to reach an efficient outcome. Therefore, in this and the following sections, I study how asymmetric information affects wage setting, and whether it may lead to inefficient unionization. Since we are interested in determining whether inefficient collective bargaining can take place, I will assume, unless otherwise noted, that unions do not increase quasi-rents:

Assumption \( v^u \geq v^u \).

Along the lines of the incomplete information literature on strikes, I assume that the source of private information lies in the quasi-rents generated by a skilled worker \( v \). As in that literature, management learns how large these quasi-rents are, while workers do not have access to that information. Indeed, it seems reasonable to assume that management can form a much better assessment of future revenues than workers themselves. After all, the role of management is, to a large extent, to predict what revenues would follow from different courses of action.

The model works as follows. As in the complete information case, there are two periods. In the first period, all firms and workers are identical, as management has not yet learned what quasi-rents a skilled worker would generate, and workers have not yet acquired firm-specific skills. To simplify, I assume that firms can be of only two types, high-value \( H \) and low-value \( L \) firms, with \( v_H > v_L \) and \( v_H^u > v_L^u \). The commonly known distribution of types is given by \( q \), the probability that a firm is high-value. At the beginning of the first period, management offers a wage \( w^0 \), and workers decide whether to accept the wage and join the firm. After workers join the firm, management learns the firm’s type, and workers accumulate firm-specific human capital.

In the second period, management makes a new wage offer to the employed workers \( w \), who, not knowing the firm’s type, do not know the wage
that would result from unionization. However, both workers and management have a common assessment of what union wage would result for each possible level of quasi-rents. Based on the distribution of types and the wage offer made by the firm, workers update their beliefs about the firm’s type, and decide whether to accept the wage offer or to organize a union.

In case a union is formed, management and the union bargain over the wage. The result of that bargaining is \( w_u^H \) in high-value firms, and \( w_u^L \) in low-value firms, where \( w_u^H \geq w_u^L \) (Assumption \((W_u)\)). That is, either union officials are able to assess the firm’s rents better than individual workers (which is a safe assumption to make: union officials have the time and skills to assess firm performance that individual workers lack), or, in the bargaining process, some information is revealed.\(^\text{16}\)

Figure 2 depicts the asymmetric information model.

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**Figure 2.**

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\[ v_i - w, w \]

\[ v_i^u - w_i^u, w_i^u - e \]

Assumption \((W_u)\) implies that the cost of unionization in terms of wages is higher for high value firms than for low-value firms. The following assumption states that these costs are still higher for high-value firms once the effect of unions on rents is also taken into account. That is, even if the effect of unions on rents is higher in low-value firms, this effect is not larger than the wage effect.

**Assumption Single Crossing (SC).** \( v_H - v_i^H + w_i^H > v_L - v_i^L + w_i^L \).

Assumption \((SC)\) thus states that the foregone rents if organization occurs are greater for high-value firms than for low-value firms. The assumption is labeled Single Crossing, along the lines of the literature on games with incomplete information, since it implies that isoprofit lines of high- and low-value firms can cross only once. In economic terms, the single crossing condition means that high-value firms are willing to pay more to reduce the

\(^\text{16}\)It is difficult to conceive a bargaining model that would yield the opposite outcome, namely \( w_u^H < w_u^L \).
probability of organization than low-value firms.\footnote{To see this, let $o$ be the probability of organization. Then, if a firm offers a wage $w$, and faces probability of organization $o$, its profits are $\Pi = o(v^u - w^u) + (1 - o)(v_i - w)$, so that a straightforward application of the Implicit Function Theorem implies that the slope of an isoprofit line in $(w, o)$ space at a given point $(w, o)$ is $-\frac{1-o}{v^u - v^u}$. Assumption (Single Crossing) thus implies that high-value firms are willing to pay more to avoid a marginal increase in $o$ than low-value firms.} Unless otherwise noticed, the single crossing condition will be assumed to hold throughout the rest of the paper.

How does the presence of incomplete information alter the results of section 2? First note that, if no information is credibly communicated to workers, they would require a wage equal to the expected wage from unionization in order not to organize. If $q$ or $w^u_H - w^u_L$ is high enough, this expected wage will be high, making it very costly for low-value firms to avoid organization. Low-value firms will try to signal their low value to workers but they cannot credibly do so while at the same time avoiding organization, as in that case it would clearly be optimal for high-value firms to imitate them. Therefore, if the ex ante expected wage from organization is high enough, the only alternative for low-value firms is to trigger organization, thus revealing their type to the union. High-value firms will still find it profitable to avoid the risk of unionization, so that only low-value firms become unionized. If the ex ante expected union wage is not too high, however, low-value firms may avoid organization with positive probability by paying the same wage offered by high-value firms. In this case, if workers use mixed strategies, unionization can occur in high-value firms if the probability with which workers organize is low enough. However, it will still be true that the probability of organization is higher in low-value firms. The analysis below formalizes this argument.

Let $\rho : \mathbb{R}_+ \to [0, 1]$ represent the worker’s beliefs after observing a wage offer, so that $\rho(x) = \Pr(v = v_H | w = x)$ is the probability the worker assigns to the firm being high-value, given that a wage $x$ has been offered.

Let $o : \mathbb{R}_+ \to [0, 1]$ be the behavioral strategy of the worker in the second period, where $o(w) \in [0, 1]$ specifies the probability with which the worker will organize given that the wage $w$ was offered.

Let $P_i$ be the behavioral strategy (a probability measure over wages) of a type-$i$ firm in the second period, and let $S_i$ be the support of $P_i$.\footnote{Note that $P_i$ is defined over the Borel sets of $\mathbb{R}_+$. However, abusing notation, I will write $P(w)$ to express $P(\{w\})$ when $w$ is played with positive probability.}

\begin{lemma}
In any Perfect Bayesian Equilibrium (PBE), high-value firms do not become unionized with certainty. If workers play pure strategies on the
Proof. First notice that in any equilibrium it must be the case that for $w > w_H^u - e$, $o(w) = 0$. Thus, if a high-value firm offers $w' = w_H^u - e + \varepsilon$, for $\varepsilon > 0$, its payoff is $v_H^u - w_H^u + e - \varepsilon$, which, for $\varepsilon$ small enough, will always be higher than $v_H^u - w_H^u$.

Now, suppose that there exists an equilibrium in which the worker plays pure strategies, and high-value firms face a positive probability of organization. Then, there is some wage in $S_H$, for which $o(w) = 1$, so that the equilibrium payoff for a high-value firm is $v_H^u - w_H^u$. As shown in the previous paragraph, that wage cannot be optimal. Q.E.D.

Therefore, high-value firms always find it more profitable to avoid organization altogether rather than offering some wage for which organization would occur with certainty. The next proposition shows that there indeed exist equilibria in which high-value firms avoid organization, and low-value firms are organized with positive probability, or even with certainty. Note that, in what follows, I will discuss only the second period subgame. In the first period the wage will be set so as to make workers indifferent between the job and alternative employment.

**Proposition 2** i. There exist fully revealing PBE in which low-value firms get organized with probability

$$u_{Ls} \in \left\lfloor \frac{w_H^u - w_L^u}{(w_H^u - w_L^u) + v_H^u - v_L^u + e}, \min\{1, \frac{w_H^u - w_L^u}{v_L^u - v_L^u + e}\} \right\rfloor,$$

and high-value firms prevent organization with certainty by offering a wage $w_H = w_H^u - e$.

ii. No other fully revealing PBE exist.

**Proof.** (All omitted proofs can be found in the appendix.)

Assumption (SC) guarantees that there exist equilibria in which different types of firms sort themselves out fully. Since high-value firms are more willing to pay to avoid organization, separating equilibria always exist.

However, if the costs of unionization to low-value firms are relatively high, and the ex ante expected wage is not excessive, low-value firms might find it optimal to avoid organization altogether, as the next proposition shows.
Condition (1)

\[ v_L - (qw_H + (1 - q)w_L^H - e) \geq v_L^u - w_L^u \]  \quad (1)

**Proposition 3** If condition (1) holds, then there exist pooling equilibria in which both types of firms avoid organization with certainty, and the wage is

\[ w_p \in (qw_H^u + (1 - q)w_L^H - e, \min \{v_L^u - v_L^u + w_L^H, w_H^H - e\}) . \]

If condition (1) does not hold, then no pooling equilibria exist in which unionization is avoided with certainty.

Condition (1) simply states that low-value firms prefer to pay the ex ante expected wage than to become unionized with certainty. If it holds, there exists a continuum of pooling equilibrium wages that lead to no organization.

However, if we restrict beliefs to be nondecreasing in the wage offer, all equilibria with \( w_p > qw_H^u + (1 - q)w_L^H - e \) are eliminated (as they would require \( \rho(w) > q \) for \( w < w_p \)), and the only pooling wage is \( w_p = qw_H^u + (1 - q)w_L^H - e \).

As the next proposition shows, however, there are equilibria in which high-value firms become unionized with positive probability. If organization happens with a small probability for a certain wage, and that wage is low enough, it may pay high-value firms to offer that wage and risk organization. As we shall see, though, it will still be true that the probability of unionization in high-value firms is not higher than in low-value firms.

**Proposition 4** If Condition (1) holds, there exist pooling and semi-separating PBE in which both types of firms face a positive probability of organization. If Condition (1) does not hold, no pooling or semi-separating equilibria exist.\(^{20}\)

If condition (1) holds with equality, there exists a unique wage \( w_p = qw_H^u + (1 - q)w_L^H - e \) such that, if both types of firms offer \( w_p \), organization may occur with positive probability. If it holds strictly, low-value firms may offer other wages with positive probability. In the latter case, the

\(^{19}\)Such an equilibrium would be supported by beliefs that find the wage offer unininformative for wages below the a priori expected benefits from organization. For example, the following beliefs would support the pooling equilibrium: \( \rho(w) = q \) for \( w \leq qw_H^u + (1 - q)w_L^H - e \), and \( \rho(w) = 1 \) for \( w > qw_H^u + (1 - q)w_L^H - e \).

\(^{20}\)Although the result is not proven in this paper, all the PBE in this section also satisfy the usual refinements (they are sequential equilibria, trembling-hand equilibria, and satisfy the intuitive criterion.)
probability of organization for wages other than $w_p$ must be higher than $o(w_p)$, as, otherwise, it would be optimal for high-value firms to imitate low-value ones. Therefore, we can obtain a weaker version of Proposition 2 for the case of pooling equilibria.

**Corollary 2** In any PBE in which high-value firms are organized with positive probability, the probability of unionization in high-value firms is

$$u_H = o(w_p) \leq \frac{(1 - \rho(w_p))(w_H^u - w_L^u)}{(1 - \rho(w_p))(w_H^u - w_L^u) + v_H - v_H^u + e}.$$  

In low-value firms the probability of organization is $u_L = o(w_p)$ if condition (1) holds with equality, and it is

$$u_L = o(w_p) + (1 - o(w_p))(\rho(w_p) - q)\frac{(w_H^u - w_L^u)}{v_L - v_L^u + e},$$  

if (1) is strict.

Therefore, the probability of organization is not higher in high-value firms than in low-value firms.

Proposition 2 and Corollary 2 together imply that the probability of organization is never higher in high-value firms than in low-value firms. This is a surprising result given the widely held view that unionization will be more likely in those firms where there are more rents for the union to appropriate. In the next section, I will discuss this result in greater detail.

4 Implications of the Asymmetric Information Model of Union Formation

4.1 Probability of Organization and Union Density

For given parameter values, the probability of organization will be determined by the probability with which organization can take place in different types of equilibrium, and the types of equilibrium that can obtain given those parameter values. As shown in the previous section, there are two possible regimes, depending on whether condition (1) holds.

1. **Full Separation.** This regime obtains when condition (1) does not hold, that is, when $v_L - v_L^u + e < q(w_H^u - w_L^u)$. In this case, the only possible equilibria involve full separation. Therefore, according to Proposition 2, firms
avoid organization with certainty if they are high-value, and get organized with probability

\[ u_{Ls} \in \left[ \frac{w_H^u - w_L^u}{(w_H^u - w_L^u) + v_H - v_H + e}, \min\{1, \frac{w_H^u - w_L^u}{v_L - v_L + e}\} \right] \]

if they are low-value. The ex ante probability of organization is thus

\[ u_s \in \left[ \frac{(1 - q)(w_H^u - w_L^u)}{(w_H^u - w_L^u) + v_H - v_H + e}, \min\{(1 - q), \frac{(1 - q)(w_H^u - w_L^u)}{v_L - v_L + e}\} \right] \]

2. Multiple Equilibria. This regime obtains when condition (1) holds, that is, when \( v_L - v_H^u + e \geq q(w_H^u - w_L^u) \), so that all types of equilibria are possible. The ex ante probability of organization in a separating equilibrium is as described above, and, in a semi-separating (or pooling, if \( \rho(w_p) = q \)) equilibrium, it is

\[ u_p = q u_{Hp} + (1 - q) u_{Lp} = o(w_p) + (1 - o(w_p)) (\rho(w_p) - q) \frac{(w_H^u - w_L^u)}{v_L - v_L^u + e} \]

where the last equality follows from Corollary 2.

The following proposition summarizes how the probability of organization changes as different parameter values change. Note, however, that, due to the multiplicity of equilibria, comparative statics results are weaker than in contexts in which equilibria are unique. By saying that the probability of organization does not decrease when a certain parameter \( x \) increases (from \( x \) to \( x' \)) we mean that if there are equilibria that exist under \( x' \) and not under \( x \), these equilibria involve a higher probability of organization than any equilibrium under \( x \). Similarly, if there are equilibria that exist under \( x \), but not under \( x' \), these equilibria involve a lower probability of organization than any equilibrium under \( x' \).

**Proposition 5** Let

\[ v_H - v_H^u - (v_L - v_L^u) > -(1 - q)(w_H^u - w_L^u) \quad (2) \]

Then, the probability of organization is:

i. nondecreasing in \( (w_H^u - w_L^u) \).

ii. nonincreasing in \( e, (v_L - v_H^u), (v_H - v_H^u) \)

---

Footnote: For a formal definition of the concepts of nonincreasing and nondecreasing in this context, see the proof of the corollary in the appendix.
Condition (2) is simply an strengthening of assumption *Single Crossing*. In case unions decrease the rents of low-value firms more than those of high-value ones, condition (2) requires that the differential effect be bounded. The bound depends on the difference between types (as expressed by $(w^U_H - w^L_L)$), so that if high- and low-value firms are very different, the differential effect of unions on rents is also allowed to be large.

To see how parameter changes affect the probability of organization, suppose that a legislative change makes organization more costly for workers (increases $e$). Such an increase reduces workers’ net benefits from organization, making it worthwhile for a smaller set of wages. If this reduction is large enough, low-value firms may not find it optimal to signal their type anymore, and instead imitate high-value firms, thus avoiding organization with positive probability. This could make some firms switch from the fully separating equilibrium to a pooling equilibrium, leading to a lower probability of organization.

But the same change in $e$ will also reduce the bounds for the probability of organization in both pooling and separating equilibria. The reason for this is that an increase in $e$ increases profits from sure avoidance one-to-one (that is, every dollar by which $e$ increases, is a dollar saved if the firm avoids organization with certainty by offering $(w^U_H - e)$), while expected profits if organization takes place with positive probability go up only by the increase in $e$ times the probability of avoidance. Since profits from sure avoidance increase more than those from uncertain avoidance, the probability of organization in equilibria with positive probability of organization must decrease for firms not to deviate and ensure that organization does not take place.

All other variables in Proposition 5 affect the probability of organization in a similar fashion.

An increase in the probability of a firm being high-value (an increase in $q$) would, however, have an ambiguous effect on the probability of organization. On the one hand, it would reduce the likelihood that any given sort of equilibrium results in organization, as low-value firms are more likely to be organized. On the other hand, an increase in $q$ could render it too expensive for low-value firms to avoid organization, which could lead to an increase in the probability of organization, as pooling equilibria are no longer viable. For $q$ large enough, however, an increase in $q$ will be unambiguously associated with a decrease in the probability of organization (as either all firms fall under regime 1, or condition (1) holds for all $q$), so that as $q$ tends to 1, the incomplete information model converges to the complete information model. Although this convergence is not necessary when $q$ goes
to zero, it will also take place for plausible beliefs.\textsuperscript{22}

To summarize, the changes that would lead, other things equal, to an increase in the probability of organization are:

- A reduction in the costs of union organization.
- A decrease in the efficiency cost of unions.
- An increase in the spread of the distribution of union negotiated wages.

For example, if an increase in the bargaining power of unions has the effect of increasing $w^U_H$ more than $w^U_L$, union density would increase.

An increase in the probability of organization will certainly increase the number of firms at which collective bargaining takes place. It is reasonable to expect that it will also lead to an increase in union density (i.e. the percentage of workers who are union members), but this is not necessarily the case. For example, if high-value firms’ share of employment is large, a change that leads to full separation may increase the ex ante probability of organization, but, by ensuring that high-value firms remain nonunion, it may decrease union density. Similarly, employment growth will increase the percentage of firms (or jobs) that are not susceptible of being unionized in that period (in the language of the model, it will increase the proportion of firms that are at the first stage of the game, when there are no quasi-rents to divide.) In what follows, I will assume that the results for the probability of organization extend to union density, but these caveats should be borne in mind.

4.2 Unionization and Rents\textsuperscript{23}

It is a common view that the larger the rents to be divided, the more likely it is for unions to form (e.g. Hirsch (1991), p. 60). From workers’ perspective, it seems obvious that the larger the potential gains from organization, the larger should be the demand for unions. This has generally been taken to

\textsuperscript{22}That is, beliefs that assign a high enough probability to a firm being low-value for wages above $w^U_L - e$. For example, if, in the spirit of trembling-hand perfection, we allowed firms to make mistakes, and we imposed the additional constraint that a high-value firm is not much more likely than a low-value firm to make a mistake to wages slightly above $w^U_L - e$, then as $q \to 0$, low-value firms would be able to avoid organization with certainty by offering $w^U_L - e + \varepsilon$, for small $\varepsilon > 0$, so that the only equilibria would be pooling equilibria with no organization.

\textsuperscript{23}By rents I am actually referring to quasi-rents, that is, the amount in excess of the value to the firm of an alternative worker.
imply that unionization will be more likely where rents are higher. If firms did not act at all to prevent organization, or if their actions were unrelated to their rents, this view would hold true. The problem is that where rents are larger so are the losses to the firm if workers organize, and greater will be the efforts of management to prevent union organization.

Some authors have acknowledged that employers’ behavior is also a determinant of organization, but they have considered mostly employers’ responses to union organization attempts.24 Instead of focusing on the behavior of employers once unionization is attempted, the model in this paper looks into the actions that employers can and will take to prevent organization to be attempted at all. And the predictions are clear cut: the probability of organization conditional on being low-value is weakly higher than if the firm is high value. That is, against the common view described above, the probability of unionization within the relevant (i.e. otherwise identical) group of firms is negatively related to the rents to be divided between the firm and its employees.

The model is, however, ambiguous about the relationship between unionization and average rents in a given industry, except for the case in which rents are so low so as to make unionization always unprofitable, as shown in the following corollary.

**Corollary 3** If

\[ qw_H^u + (1 - q)w_L^u - e < 0 \]  

then no organization will take place in equilibrium. All firms will offer \( w = 0 \), which will be accepted by workers.

**Proof.** If condition (3) holds, then it is never optimal for workers to unionize in low-value firms. But then it is always optimal for high-value firms to imitate low-value firms, and only a pooling equilibrium can obtain. Condition (3) guarantees that organization never happens if that is the case. Q.E.D.

If condition (3) holds initially, an increase in average rents that makes unionization profitable in expectation will increase the probability of organization. However, the impact of further increases in average rents will depend on the way these increases come about: changes that lead to a widening of the difference between union negotiated wages \( (w_H^u - w_L^u) \) will lead, other things equal, to an increase in union density; changes leading to an increase

in the negative effects of unionism (by increasing \(v - v^h\)) will lead, other things equal, to a decrease in union density. That is, what matters is the shape of the distribution of unobservable rents within an industry; knowledge of average rents, by itself, does not us to make predictions about unionization, except in the cases covered by Corollary 3. As it was discussed above, an increase in \(q\) would have an ambiguous effect on union density.

4.3 Profitability

An established fact about unions is that they reduce the profitability of firms.\(^{25}\) Under Assumption \((v^u)\), the model above yields the same prediction. However, as it will be discussed in a latter section, observed union effects on profits will overestimate the true impact of unions, as organization is more likely in firms with lower rents.

To ascertain the true effect of unions on profits, we would like to take a nonunion firm, and without any other change in the environment, transform it into a union firm. The consequent change in profits would be the union effect. If two identical firms end up with a different union status, because of a different realization of the same mixed strategy equilibrium, the model offers such an ideal experiment. In that case, we can calculate the effect of organization on profits directly.

Let \(\pi_H^u\) and \(\pi_H^p\) be the profits a high-value firm obtains if unionized, and if it remains nonunion, respectively, and let \(\pi_L^u\) and \(\pi_L^p\) be defined equally for a low-value firm. Consider the case in which a pooling equilibrium obtains. Then, \(\pi_H^u = v_H^u - w_H^u\), \(\pi_L^u = v_L^u - w_L^u\), \(\pi_H^p = v_H - w_p\), and \(\pi_L^p = v_L - w_p\). Thus, under assumptions \((v^u)\) and \((W)\), \(\pi_H^u > \pi_H^p\), and \(\pi_L^u > \pi_L^p\), so that unions do decrease profits if they do not have a positive effect in productivity.\(^{26}\) Of course, this is a truism, as it was assumed that unions do not increase rents. In firms where organization increases rents, the impact of unions on profits cannot be established a priori.

However, as it will be discussed in a later section, even if Assumption \((v^u)\) holds, so that unionization decreases profits, the measurement of this

\(^{25}\)In summarizing the results from the empirical literature on the issue, Hirsch states: “The conclusion that unionization is associated with lower profitability holds for studies using industries, firms, or lines of business as the unit of observation; for models where the profitability measures are industry price-cost margins, firm rates of return to capital or sales, Tobin’s q or other market value measures, or stock market value changes in response to union “events”; for simultaneous equation as well as single equation models; and regardless of the time period under study.” (Hirsch (1991), p. 36.)

\(^{26}\)The same qualitative result holds for all other equilibria in which mixed strategies are played on the equilibrium path.
effect will be problematic.

The relationship between union density and industry profitability depends on the causes underlying the change in both variables. It can be shown that average profitability will be negatively correlated with union density if the cause of the change in both variables is a change in $e$. Increases in the cost of organization will tend to reduce union density, and to increase profits, both by increasing the profits of nonunion firms, and by allowing more firms to remain nonunion. If the cause of the movement in union density is a change in the distribution of unobservable variables, the sign of the correlation between union density and the wage gap cannot be established without knowledge of how the distribution changes.\footnote{Note that the above predictions refer to the correlation between the share of quasi-rents \textit{generated by workers} that accrues to the firm, and union density. However, profits may have other components that are not specific to the relationship between the firm and its workers, that is, a firm may obtain positive profits by employing alternative workers. The predictions in this section will apply to total profits as long as there does not exist a negative correlation between specific (to the relationship between the firm and its workers) and nonspecific profits, or the former are sufficiently larger than the latter.}

### 4.4 Union-Nonunion Wage Gap

The union wage gap can be defined in several ways, depending on the variables that are held fixed when comparing union and nonunion firms or workers. In this section, I will analyze the determinants of the \textit{true} effect of unions on compensation, that is, the difference between expected union and nonunion wages at a given firm. This wage gap can be defined ex post (after uncertainty is realized) or ex ante. The ex post wage gap is given by:

$$WG_i = E_i(w | U_i = 1) - E_i(w | U_i = 0),$$

where the expectations are over the wages that could obtain at a firm of type $i$, and $U_i$ is a dummy variable such that $U_i = 1$ when firm $i$ is organized. Note, however, that $WG$ may not be well defined if workers play pure strategies on the equilibrium path, so that, for a given type of firm, only one value for $U$ is possible.

The ex ante wage gap, or, equivalently, the average ex post wage gap among ex ante identical firms is:

$$\overline{WG} = qWG_H + (1 - q)WG_L$$

The value of these wage gaps will depend on the type of equilibrium that obtains:
1. Separating Equilibrium. In this case, \( W_G^H \) is not well defined, so that \( W_G \) is not well defined either. If workers play mixed strategies on the equilibrium path, \( W_G^L = e \).

2. Pooling Equilibrium. If the equilibrium involves a positive probability of organization:

\[
W_G^H = w_H^u - (qw_H^u + (1-q)w_L^u) + e = e + (1-q)(w_H^u - w_L^u) \\
W_G^L = w_L^u - (qw_H^u + (1-q)w_L^u) + e = e - q(w_H^u - w_L^u) \\
W_G = e.
\]

3. Semi-separating Equilibrium. In this case,

\[
W_G^H = w_H^u - (\rho(w_p)w_H^u + (1-\rho(w_p))w_L^u) + e = e + (\rho(w_p))(w_H^u - w_L^u) \\
W_G^L = e - \rho(w_p)(w_H^u - w_L^u)/(\rho(w_p)(1-q)(v_L - v_L^u + e) - (\rho(w_p) - q)(w_H^u - w_L^u))
\]

Therefore

\[
W_G = e + q(1 - \rho(w_p))(w_H^u - w_L^u)/[1 - (1-q)\rho(w_p)(v_L - v_L^u + e) - (\rho(w_p) - q)(w_H^u - w_L^u)] \leq e
\]

**Determinants of the Wage Gap**

**Costs of organization** As the expressions above show, both the ex ante and ex post wage gaps are mainly determined by \( e \), so that increases in \( e \) translate into increases in the wage gap. The model thus suggests that changes in the true wage gap should be traced back to changes in the costs of organization. Other variables are not likely to have a sizeable impact on the wage gap.

**Rents** In the introduction, it was pointed out that, once the behavior of employers is taken into account, the relationship between the size of the rents to be divided and the wage gap is not evident a priori. The model predicts that, in fact, no relationship exists between these two variables once we control for the real determinants of the wage gap. That is, the knowledge
that a firm has higher rents than another one, does not allow us, by itself, to make any prediction about the relative sizes of the wage gaps.

Although for two ex ante identical firms, the ex post wage gap will be higher in the one with higher unobservable rents ($WG_H \geq WG_L$ in any pooling or semi-separating equilibrium,) the relationship between rents and the wage gap breaks down once we compare firms that are not identical ex ante. To see this, consider the following example:

**Example.** There are two groups of firms, $A$ and $B$, whose rents are unobservable to workers and the researcher. However, it is known that the distribution of these rents is such that: $v^A_L > v^B_H$, $v^uA_L > v^uB_H$, $q^A$, $q^B$. That is, firms in group $A$ always have higher rents than firms in group $B$.

Now suppose that a pooling equilibrium with positive probability of organization obtains in both groups, and that

$$(1 - q^B)(w^uB_H - w^uB_L) > (1 - q^A)(w^uH_A - w^uA_L),$$

and

$q^B(w^uB_H - w^uB_L) < q^A(w^uA_H - w^uA_L).$

Then, \(^{28}\)

$$WG^B_H = e + (1 - q^B)(w^uB_H - w^uB_L) > WG^A_H = e + (1 - q^A)(w^uB_H - w^uH_A),$$

$$WG^B_L = e - q^B(w^uB_H - w^uB_L) > WG^A_L = e - q^A(w^uA_H - w^uA_L),$$

and

$$WG_A = WG_B = e$$

As the example above shows, the wage gap in a firm with higher rents can be higher than that in a firm with lower rents ($WG^B_H > WG^B_L$), or lower ($WG^A_H < WG^B_H$). Moreover, even if firms in group $A$ are known to have always higher rents than firms in group $B$, the ex ante wage gap is not greater in the former group. In fact, the relationship between the ex ante wage gap and average rents for different groups of firms cannot be determined without knowledge of the distribution of rents within groups. Although, in general, the ex ante wage gap (when well defined) is fully pinned down by $e$, if semi-separating equilibria obtain, it can be shown that the ex ante wage gap can go up, down or stay constant as average rents increase. Specifically, if changes in average rents lead to a higher efficiency cost of unions ($v^u_L - v^u_L$) or a lower difference between negotiated wages ($w^uH_H - w^uL_L$), the average wage gap may increase.

\(^{28}\)It is easy to check that the two inequalities are consistent.
In summary, the location of the distribution of rents that can obtain at a given firm is not informative to predict the wage gap (ex ante or ex post.) What matters is the shape of that distribution, and, for the ex post wage gap, the location of a firm within it.

**Union Density and Wage Gap** The main determinant of the union wage gap is the cost of organization to workers, \( e \), which has an opposite effect on union density: increases in \( e \) will lead to a higher wage gap and lower union density. Moreover, those features of the distribution of second-period variables that would lead to lower union density (a high value of \((v_L - v_L^u)\), or a low value of \((w_H^u - w_L^u)\)) would also lead to a higher wage gap. Therefore, the model predicts that union density and the union wage gap will tend to be negatively correlated. This implication is discussed in greater detail in the following sections.

## 5 Applications

### 5.1 Recent trends in U.S. and British Unionism

In the 1980s and 1990s, both the U.S. and Britain experienced a large decline in union density. According to the conventional demand-supply view discussed in the introduction, a common explanation for this decline is that it is a reflection of either an erosion of unions’ bargaining power, or a reduction in the rents that the union can attempt to appropriate, mainly due to changes in the competitive environment. Either case should shift the demand for unions to the left, leading to a smaller union wage premium. However, as Figure 3 shows, in the U.S. the union wage premium seems to have remained constant, or even trended upwards.\(^{29}\) A similar pattern can be observed in the U.K.\(^{30}\) Despite the fact that unions represent a much smaller proportion of the labor force, they seem to be able to guarantee the same wage gains for their members as in the past.

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\(^{29}\)Figure 3 plots union density and the union wage gap for private sector, nonagricultural workers in the U.S. for the period 1973-1981,1983-1998. The wage differentials are calculated in Hirsch and Schumacher (2000) from CPS data, excluding workers with earnings imputed by the Census. See Hirsch and Schumacher (2000) for a discussion of the dataset, the controls used in the wage equations, and the reasons to exclude allocated earners.

\(^{30}\)See Blanchflower (1997), Andrews, Bell, and Upward (1998), Stewart (1991), Stewart (1995). The studies reviewed, using different datasets and methods, do not agree on the sign and size of the change in the wage gap. One can, however, conclude that no significant decline in the wage gap has taken place.
As discussed in the introduction, some authors have recognized the role played by employers in the determination of the union status of firms, and have advanced an explanation of the decline in union density precisely based on the existence of high wage premia. According to this view, it was the high union-nonunion wage gaps of the late 1970s in the U.S. what induced employers to adopt a more aggressive stance towards unions. This would have shifted the supply curve of unionism upward, in turn leading to a lower union density. This view can therefore rationalize the existence of high wage premia simultaneous with declining union density, but it has one main problem. Namely, it implies that employers’ efforts are able to decrease the capacity of unions to organize, but cannot alter the latter’s bargaining power, and that the ability of unionized workers to obtain high wages has not been affected by the declining union density. One would expect, on the contrary, that the mere presence of a larger nonunion sector would curtail that ability.

The model in this paper suggests a different and more plausible explanation for the apparent puzzle of high wage gaps and declining union density. In the model, changes in the costs of union organization have opposite effects on the probability of organization and the union wage premium, so that union density and the wage gap will tend to be negatively correlated. Therefore, if the costs of union organization have increased, because, say, the legal or social climate is less congenial to unions (or firms have moved to areas where this is the case), or employers have improved their union-busting technology, we may expect union density to decline and the wage gap to grow. Although we do not observe a clear negative correlation in the aggregate, at least in the U.S. such correlation does seem to hold when looking at individual industries: the untrended aggregate wage gap would be the effect of the increase of relative employment in sectors with a lower union wage premium.

\[31\] See Freeman and Medoff (1984), Freeman (1986), Freeman and Kleiner (1990), Blanchflower and Freeman (1992), Blanchflower (1997). Linneman, Wachter, and Carter (1990) provides an alternative explanation that also takes the high wage gap as the cause behind the decline in union density. In this case, union density is supposed to fall as a high wage gap moves union employers up along the labor demand curve.

\[32\] Freeman (1986) finds a negative correlation between the aggregate wage gap and new unionization after controlling for a few factors. I believe, though, that his results should be taken only as suggestive, due to the paucity of his dataset.

\[33\] See Linneman and Wachter (1986), Linneman, Wachter, and Carter (1990). The latter paper finds a negative coefficient for the union wage gap in union density equations for all major industries, except construction and government, for the years 1973–86. However, Allen (1988) offers an explanation for the decline in union density in construction, based
Moreover, data from the U.S. from 1930-1974, although less reliable than more recent micro-data, also points in the direction of a negative correlation between union density and the wage gap, or at least of an absence of a positive correlation. In particular, those periods in which the greatest increases in union density took place were accompanied by lower union wage gaps than periods of union decline or moderate growth. 34

5.2 The Pattern of Union Density and Union Wage Premia

The theory in this paper also offers an explanation for the cross-sectional pattern of union density and the union wage premium in the U.S., a pattern that seems not to have received much attention by the theoretical literature.

Wage Gap across Regions

Union density in the U.S. South is lower than in other regions. However, Lewis (1986) finds a higher wage gap in the South after surveying a myriad of studies that include regional dummies.35

Lewis also finds a larger premium (of about five percentage points)36 for workers who reside outside metropolitan areas. Those workers are characterized by a lower union density than residents of metropolitan areas.

Wage Gap across Sectors

Union density in the private sector has suffered an uninterrupted decline since 1973, while union density in the public sector has increased from about 23% in 1973 to 37.5% in 1998 (although it has stayed roughly constant since the early 1980s.)37 Again, we find a higher wage gap in the private than in the public sector.38

Establishment Size

on a reduction of the positive productivity effects of unions, consistent with the model in this paper. Note that a different explanation for the negative relationship between union density and the wage gap based on compositional effects is possible. A reduction of the expected wage gain from organization at each firm, which would reduce union density, need not be accompanied by a reduction of the average wage gain among those firms that become organized, if a large proportion of firms is near the margin. That is, those firms that would have been characterized by a low wage gain do not become organized, thus raising the average wage gain among those that do become organized.

35Lewis (1986), pp. 131-134.
36Ibid., pp. 134-135.
37Private sector membership has monotonically decreased as well since 1979, while public sector membership has more or less steadily advanced from three million workers to almost seven million in the same period.
At least in manufacturing, the wage gap seems to decrease as firm or establishment size increases,\(^3\) while union density is positively correlated with firm size.

Therefore, the cross-sectional pattern of wage premia suggests that there is a negative correlation between the wage gap and union density. This pattern is difficult to account for by conventional views, but is readily explained by the theory presented in this paper. Note that I have not presented any result on how the wage gap varies with the extent of unionism in an industry, which would be a more direct test of the theory. The reason is that results seem to be extremely sensitive to the choice of variable measuring the extent of unionism, and to pick up the effect of omitted variables more than effects of unionism. For a discussion, see Lewis (1986).

The same pattern seems to hold when comparing different countries, even if we limit the comparison to relatively similar countries like the U.S., Canada and the U.K.. Especially interesting is the case of the U.S. and Canada, given their relative similarity (also in what respects to collective bargaining regulation.) The higher union density in Canada is not accompanied by higher wage premia.

5.3 Selection Biases in the Estimation of Union Effects
Perhaps the main problem of the empirical literature on union effects is the fact that the union status of firms and workers is not randomly selected, but it is likely to be related to characteristics unobservable to the econometrician. The resulting estimation biases make the interpretation of OLS coefficients problematic. Different attempts have been made to account for these biases, and the success of these attempts is a matter of contention.\(^4\)

According to widely accepted views of unionism, unions are more likely to organize firms with higher pre-union rents. Therefore, it has been argued, the estimated effects of unions on profitability may be biased toward zero: the true negative effect of unions on profitability would be larger than estimated as unions would choose to organize only high-value firms (see Hirsch (1991), p. 37, p. 60, Voos and Mishel (1986).) The implications of the asymmetric information model challenge the interpretation of these studies: since the probability of organization is higher in those firms with lower un-

\(^3\)Lewis (1986)

\(^4\)The problems associated with the alternatives to OLS proposed to account for selection bias have led many researchers to favor standard OLS estimation (Lewis (1986), Freeman and I. Medoff (1982)). Others, like Robinson (1989) have a more positive view of alternative estimation methods.
observable rents, conventional estimates will overestimate the magnitude of the impact of unions on profits.

A similar problem arises in the estimation of union effects on wages. Since low-value firms are overrepresented in the group of union firms, the union-nonunion wage gap will be underestimated by conventional OLS estimation. That is,

$$WG = qWG_H + (1-q)WG_L \geq E(w| U_i = 1, i \in J) - E(w| U_i = 0, i \in J),$$

where $J$ is a group of ex ante identical firms.

Note, however, that usually reported industry wage premia (where industry is defined according to some industry classification system) may be averaging over both firms where unionization is possible, and firms where it is not so (as characterized by Corollary 3.) In this case, the industry wage gap will overestimate the average wage gap of the firms where organization is possible due to this composition effect. The sign of the bias in the estimation of the true wage gap will depend on the relative magnitude of this positive bias, and the negative bias considered above. More importantly, there will be other sources of bias stemming from differences across workers, if workers’ unobservable characteristics are correlated with the likelihood that they work at a unionized establishment. In fact, this source of bias is the one that has received most attention in the literature.\(^{41}\)

5.4 Policy Implications

As discussed above, unionization cannot reduce total rents under symmetric information, and any barrier to union organization can only be welfare reducing. If informational asymmetries do not play a major role determining unionization, industrial relations policy should aim at facilitating union organization, while ensuring that the decision to unionize is made in a decentralized way. More precisely, since policies aimed at altering $e$ may have other efficiency consequences, $e$ should be reduced until the expected marginal benefit of a reduction in $e$ equals its marginal cost.

However, in the presence of asymmetric information, lowering the costs of organization may reduce total rents, as unionization may occur in firms where it is not efficient. In this case, setting the optimal $e$ requires trading off inefficient union formation and inefficient union avoidance.

Note that, even if unions always reduced rents, the effect of changes in $e$ on total surplus is ambiguous. Increases in $e$ will reduce total surplus by

\(^{41}\)See, for example, Robinson (1989), Card (1996).
increasing the costs incurred by workers in those firms that become unionized. On the other hand, a higher $e$ will lead to a lower probability of organization, which may lead to a higher total surplus. Only when $e$ is large enough, so that further increases make organization always unprofitable for workers, is the effect of a rise in $e$ to unambiguously increase total surplus (which is then fully appropriated by firms.) Therefore, if we do not know whether unions increase or decrease rents, for low values of $e$, increases in $e$ are likely to reduce total surplus. As $e$ grows, the effect of further increases will depend on the likelihood that unions increase rents. If rent-enhancing unions are rare, increases in $e$ will tend to increase rents.

An important implication of the model is that, in any case, both types of losses are limited by the profit maximizing behavior of nonunion firms. In those firms where unions would have a very negative effect on firm performance, the profit maximizing policy for employers is to set wages high enough to avoid unionization. Similarly, if unions would increase rents by a large amount, it will be profit-maximizing to bring them in, as trying to avoid organization would require setting too high wages.

These limits on the efficiency effects of unions may explain the fact that, although the distributional impact of unions is well documented, empirical research is not conclusive about the effect of unions on efficiency.\footnote{At the micro level, a majority of studies finds that unions increase productivity. On the other hand, unions have also been shown to reduce investment in R&D, and to lead to lower productivity growth. For a brief review, see Freeman (2000). One should bear in mind, however, that the theory in this paper casts doubts on the interpretation of these empirical results. At the macroeconomic level, there is no clear relation between different industrial relations systems and macroeconomic performance. See Flanagan (1999).}

The asymmetric information model suggests, however, that efficiency gains can be unambiguously obtained by reducing the informational asymmetries between firms and workers. Measures such as compulsory worker representation on boards of directors (even if lacking any decision rights,) or information disclosure requirements have the potential to reduce the efficiency effects of asymmetric information.

\section{Concluding Remarks}

There are two main questions in the study of unionized labor markets: What determines the incidence of unionism, and what are the effects of unions on both union and nonunion firms?

In this paper, I have shown that the two questions cannot be answered in isolation. Inasmuch as the expected effects of unionization affect the behav-
ior of nonunion employers, and thus influence what firms become unionized, employers’ actions constrain the impact that unions can have in equilibrium.

To account for this interrelation, I have developed a theory of the labor market in which the incidence of unionism and the wages in union and nonunion firms are simultaneously determined in equilibrium. This theory shows how, once the behavior of nonunion employers is factored in, some common views of the determination of the union status of firms and the effects of unions on firm performance may be misguided.

First, variables usually assumed to influence union formation, such as firms’ technology or market power, play no direct role in determining whether firms become unionized or the size of the wage gap. Only if they are correlated with the costs of union organization, the extent of uncertainty, or the efficiency cost of unions would those variables have a statistical correlation with union density or the wage gap, though no causal relation.

Second, one should not expect a positive correlation between union density and the union wage gap. Instead, they will tend to be negatively correlated, as changes in the costs of union organization have opposite effects on both variables.

Finally, asymmetric information makes unionization negatively, rather than positively, related to rents. If there is uncertainty over rents, those firms with lower unobservable rents will be the ones that find it most expensive to avoid unions, and therefore will have a greater probability of becoming unionized. The observable component of rents will not affect the probability of union organization.

The theory provides a parsimonious explanation for a variety of empirical findings, some of which could not be accounted for by conventional views of unionism. In particular, it helps explain the joint behavior of union density and the union wage gap, both cross-sectionally and over time.

But, apart from its positive implications, the theory has important normative implications. In the benchmark case of complete information, it shows how unionization cannot reduce total rents, and that barriers to union organization can only preclude the formation of rent-enhancing unions. However, if there are informational problems, lowering the costs of organization may reduce total rents, as unionization may occur in firms where it is not efficient. In this case, there is a trade-off between inefficient union formation and inefficient union avoidance. Importantly, both types of losses are limited by the profit maximizing behavior of nonunion firms, and these limits on the efficiency effects of unions may explain the fact that, although the distributional impact of unions is well documented, empirical research is not conclusive about the effect of unions on efficiency.
An important implication of the asymmetric information model is that the endogeneity of the union status of firms may be an even more severe problem for the estimation of union effects than usually considered, as the variables likely to determine union status are both inherently unobservable for the econometrician, and intimately related to firm performance. In this context, a solid theoretical understanding of union formation becomes essential.

The theory presented in this paper suggests several directions for further research. On the theoretical side, the model needs to be extended to account for employment and investment determination. Further, other market failures, apart from asymmetric information, should be studied as potential sources of inefficient union formation. If other deviations from the benchmark model are likely to lead to inefficiencies, it is necessary to study how these inefficiencies come about and how they could be addressed. Bargaining or organizational spill-overs, and the presence of intertemporal commitment problems call for further research. The theoretical framework can then be applied to evaluate the efficiency and distributional effects of different wage-setting institutions and human resources policies.

Finally, the theory developed here provides a starting point for revisiting the empirical work studying the effects of unions. It generates a series of new testable hypotheses, and provides a much needed theoretical foundation for the specification of empirical models.
Appendix

Proof of Proposition 2. 1) At a separating equilibrium, \( \rho(w) = 1 \) for every \( w \in S_H \). Thus, any \( w \in S_H \) such that \( w < w_H^u - \epsilon \) cannot be optimal, as shown in Lemma 1. Similarly, \( w > w_H^u - \epsilon \) cannot be optimal either, as it is always better to offer \( w - \epsilon > w_H^u - \epsilon \). Therefore, it has to be the case that \( S_H = \{ w_H^u - \epsilon \} \), and \( o(w_H^u - \epsilon) = 0 \). If \( o(w_H^u - \epsilon) > 0 \), it would be better to offer \( w_H^u - \epsilon + \epsilon \), for \( \epsilon > 0 \) small enough.

2) \( \forall w < w_H^u - \epsilon \), \( \rho(w) \) \( w_H^u + (1 - \rho(w)) w_L^u - \epsilon \geq w \). Otherwise \( w_H^u - \epsilon \) would not be optimal for the high-value firm. In particular, if \( w \in S_L \), \( w_L^u - \epsilon \geq w \), \( \rho(w) > 0 \), and \( o(w)(v_H^u - w_H^u) + (1 - o(w))(v_H - w) \leq v_H - w_H^u + \epsilon \), for the high-value firm to not imitate the low-value one.

3) Now, given 1) and 2), for a low-value firm not to deviate and imitate a high-value firm, it has to be the case that, for any \( w \in S_L \), \( o(w)(v_H^u - w_H^u) + (1 - o(w))(v_L - w) \geq v_L - w_H^u + \epsilon \). For \( w < v_L^u - \epsilon \), \( o(w) = 1 \), so that if \( w < v_L^u - \epsilon \) and \( w \in S_L \), it must be the case that \( v_L^u - w_H^u \geq v_L - w_H^u + \epsilon \). If \( w = v_L^u - \epsilon \), workers may randomize, but if they do, the above inequality requires \( o(w) \leq \frac{v_L^u - w_H^u}{v_L - w_H^u + \epsilon} \) for low-value firms not to imitate high-value ones. Note that, if \( v_L^u - w_H^u \geq v_L - w_H^u + \epsilon \), then \( o(w) \) can be 1 for \( w \in S_L \).

2), and 3) thus imply that, if workers play mixed strategies on the equilibrium path

\[
\frac{w_H^u - w_L^u}{v_H - v_L^u + \epsilon + w_H^u - w_L^u} \leq o(w_H^u - \epsilon) \leq \frac{w_H^u - w_L^u}{v_L - v_L^u + \epsilon}
\]

, which can hold only if Assumption (SC) holds.

Now, Assumption (SC) is also sufficient for the existence of separating equilibria, as we can always find beliefs that support such equilibria. For example, if \( \rho(w) = 1 \) for all \( w < w_H \) such that \( w \notin S_L \), then the above equilibrium can be supported, and part i. is proven.

Part ii. follows from 1), as no fully separating separating equilibrium can exist in which high-value firms get organized with positive probability. Q.E.D.

Proof of Proposition 3. Lemma 1 implies that in any (fully) pooling equilibrium in which the worker plays pure strategies on the equilibrium path, both types of firms play pure strategies, that is both types of firms offer the same wage \( w_p \). Thus equilibrium payoffs are \( \Pi_L = v_L - w_p \), and \( \Pi_H = v_H - w_p \). To be an equilibrium with no organization, it has to be the case that \( w_p \geq qw_H^u + (1 - q) w_L^u - \epsilon \), and \( o(w_p) = 0 \), and there is no \( w < w_p \), such that \( o(w) = 0 \), i.e. for \( w < w_p \), \( w \leq \rho(w) w_H^u + (1 - \rho(w)) w_L^u - \epsilon \). For \( w_p \) to be an equilibrium, it has to be the case that no firm type would prefer organization, that is, \( v_H - w_p \geq v_H^u - w_H^u \), and \( v_L - w_p \geq v_L^u - w_L^u \). By Assumption (SC), we just need to ensure that \( v_L - w_p \geq v_L^u - w_L^u \) holds. Hence, \( v_L - v_L^u + w_L^u \geq w_p \geq qw_H^u + (1 - q) w_L^u - \epsilon \), which is only possible if \( v_L - v_L^u + w_L^u \geq qw_H^u + (1 - q) w_L^u - \epsilon \) or \( v_L - v_L^u + \epsilon \geq q (w_H^u - w_L^u) \). Note that \( w_p \leq w_H^u - \epsilon \), so that \( w_p \in \{ qw_H^u + (1 - q) w_L^u - \epsilon, \min \{ v_L - v_L^u + w_L^u, w_H^u - \epsilon \} \} \).

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Thus (1) is a necessary condition for the existence of a pooling equilibrium. To see that it is also sufficient, let \( \rho(w) = 1 \) for all \( w \neq w_p \). With these beliefs, it is clear that no type would want to deviate from \( w_p \). Q.E.D.

**Proof of Proposition 4.** First note that such equilibria cannot be fully separating. To see this, assume that there exists a wage \( w \in S_H \) such that \( w \notin S_L \), so that \( \rho(w) = 143 \). Then, if \( w < w_H^u - e \), \( o(w) = 1 \), and \( \Pi_H = v_H^u - w_H^u \). But this cannot be optimal as a wage \( w_H^u - e + \varepsilon \) would guarantee higher profits. If \( w = w_H^u - e \), the same argument would apply for \( \Pi_H = o(w)(v_H^u - w_H^u) + (1 - o(w))(v_H - w_H^u + e) \). Lastly, any \( w_H > w_H^u - e \) cannot be optimal. Thus, not only \( S_H \cap S_L \neq \emptyset \), but we also know that \( P_H(w \in S_H \text{ such that } w \notin S_L) = 0 \).

Now, for any \( w, w' \in S_H \), the following conditions must hold:

i. For workers to play \( o(w) \in [0, 1) \):

\[
w \geq \rho(w) w_H^u + (1 - \rho(w)) w_L^u - e \tag{4}
\]

ii. For low-value firms not to trigger certain organization: \( \Pi_L(w) \geq v_L^u - w_L^u \).

That is, from (4),

\[
o(w)(v_L^u - w_L^u) + (1 - o(w)) (v_L - \rho(w) w_H^u - (1 - \rho(w)) w_L^u + e) \geq v_L^u - w_L^u
\]

or

\[
v_L - v_L^u + e \geq \rho(w)(w_H^u - w_L^u) \tag{5}
\]

iii. For high-value firms not to avoid organization with certainty, \( \Pi_H(w) \geq v_H - w_H^u + e \), or

\[
o(w)(v_H^u - w_H^u) + (1 - o(w))(v_H - \rho(w) w_H^u - (1 - \rho(w)) w_L^u + e) \geq v_H - w_H^u + e
\]

Which can be rewritten as

\[
\frac{(1 - \rho(w))(w_H^u - w_L^u)}{(1 - \rho(w))(w_H^u - w_L^u) + (v_H - v_H^u + e)} \geq o(w) \tag{6}
\]

Note that (6) guarantees that \( \Pi_H(w) \geq v_H^u - w_H^u \), as it is always better for high-value firms to avoid unionization with certainty than to face unionization with certainty, and, given Assumption (SC), that \( \Pi_L(w) \geq v_L - w_L^u + e \).

iv. Now, suppose that there are at least two wages in \( S_H \): \( w \) and \( w' \). Then, for a high-value firm to randomize between \( w \) and \( w' \), it must be the case that:

\[
o(w)(v_H^u - w_H^u) + (1 - o(w))(v_H - w) = o(w')(v_H^u - w_H^u) + (1 - o(w'))(v_H - w'), \tag{7}
\]

43 This would occur for almost any \( w \in S_H \) such that \( w \notin S_L \). That is the set of \( w \in S_H \) such that \( w \notin S_L \) and \( \rho(w) < 1 \) has measure zero with respect to \( P_H \). In general, although I will not explicitly state it, if \( P_H \) is a continuous distribution, the following results will hold a.e. and not for every possible \( w \).
and

$$o(w) (v_u^L - w_u^L) + (1 - o(w)) (v_L - w) =$$
$$o(w') (v_u^L - w_u^L) + (1 - o(w')) (v_L - w')$$

(8)

The previous two equations form a system of linear equations with $o(w)$ and $o(w')$ as unknowns. It can be easily checked that the two equations are linearly independent, so that the system has a unique solution. Since $o(w) = 1$ and $o(w') = 1$ is a solution of the system, there are no other values of $o(w)$ and $o(w')$ that satisfy (7) and (8). Therefore, high-value firms must play pure strategies, that is, $S_H$ has just one element, $w_p$, and $\rho(w_p) \geq q$. This makes (1) necessary for (5) to hold for all $w \in S_L$.

If (1) holds with equality, $S_L$ must also be a singleton, as, if low-value firms played $w_p$ with probability less than one, $\rho(w_p) > q$, so that inequality (5) would not hold. If (1) holds strictly, then there are equilibria in which low-value firms randomize over other wages that lead to sure organization as well. If inequality (5) is strict, this is not possible.) For randomization to be optimal for low-value firms:

$$o(w_u^L - e)(v_u^L - w_u^L) + (1 - o(w_u^L - e))(v_L - w_u^L + e) = o(w_p)(v_u^L - w_p^u) + (1 - o(w_p))(v_L - w_p),$$

or

$$o(w_u^L - e) = o(w_p) + \rho(w_H^u - w_p^u) (v_L - v_u^L + e),$$

(9)

while (9) together with Assumption Single Crossing ensure that high-value firms do not want to offer $w_H^u - e$.

Now, since (4), (5) and (6) (and (9) if low-value firms randomize) are not inconsistent, it suffices to find beliefs that would support equilibrium wages satisfying these three inequalities. Let $\rho(w_p) = q$ (so that both types offer the same wage with certainty,) and $\rho(w) = 1$ for any $w \neq w_p$. Then, if $v_L - v_u^L + e \geq q(w_H^u - w_p^u)$ holds, no low-value firm would like to deviate to a wage less than $w_H^u - e$. If $o(w)$ is chosen small enough so as to make (6) hold, then high-value firms have no incentives to deviate, and low-value firms have no incentives to deviate to $w_H^u - e$.

Proof of Corollary 2. If (1) holds with equality, then low-value firms play $w_p$ with probability one, so that the probability of organization is $o(w_p)$ for both low-value and high-value firms, where, as shown above,

$$o(w_p) \leq \frac{(1 - \rho(w_p))(w_H^u - w_p^u)}{(1 - \rho(w_p))(w_H^u - w_p^u) + v_H - v_L^u + e}.$$
of low-value firms it is

\[
\begin{align*}
u_L &= P_L(w_p)\omega(w_p) + (1 - P_L(w_p))\omega(w_L - e) = \\
&= \frac{q(1 - \rho(w_p))}{\rho(w_p)}\omega(w_p) + \frac{\rho(w_p) - q}{\rho(w_p)(1 - q)}\omega(w_L - e) = \\
&= \omega(w_p) + (1 - \omega(w_p))(\rho(w_p) - q)\frac{(w_H^u - w_L^u)}{(1 - q)(v_L - v_L^u + e)}, \quad (10)
\end{align*}
\]

where the second equality follows from the fact that

\[
\rho(w_p) = \frac{q}{q + (1 - q)\rho_L(w_p)},
\]

and the third from equation (9), derived in the proof of Proposition 4.

Now, since \( \rho(w_p) \geq q \), \( u_H \geq u_L \), so that the probability of organization in low-value firms is higher than in high-value firms. \( Q.E.D. \)

### Comparative Statics

Let \( E(x) \) be the set of second stage equilibrium strategy profiles for a given value of parameter \( x \), and let \( U(x) \) be the set of equilibrium values of the probability of organization:

\[
U(x) = \{ qo(w_H) + (1 - q)\int_{S_L} o(w)dP_L : (o(.), P_L, w_H) \in E(x) \}
\]

**Definition** The set \( S \) is as high as the set \( T \) (in the Strong Set Order), \( S \geq S_T \), if and only if (i) each \( x \in S \setminus T \) is greater than each \( y \in T \) and (ii) each \( x' \in T \setminus S \) is less than each \( y' \in S \).

**Definition** A set-valued function \( F : \mathbb{R} \rightarrow 2^\mathbb{R} \) is nondecreasing (nonincreasing) if for \( x > y \), \( F(x) \geq S F(y) \) (\( F(y) \geq S F(x) \)).

**Proof of Proposition 5.** I will prove the result for \( e \). The proof for all other variables is straightforward, once the result for \( e \) is established.

1. If \( v_L - v_L^u + e < q(w_H^u - w_L^u) \), then only separating equilibria obtain. Therefore, from Proposition 2,

\[
U_1(e) = \left[ \frac{(1 - q)(w_H^u - w_L^u)}{w_H^u - w_L^u + v_H - v_L^u + e}, (1 - q) \right]. \quad (11)
\]

2. If \( e \) is such that \( v_L - v_L^u + e < (w_H^u - w_L^u) \), and \( v_L - v_L^u + e > q(w_H^u - w_L^u) \), then \( U(e) = U_{\text{pooling}}(e) \cup U_{\text{separating}}(e) \), where \( U_y(e) \) is the set of values that the probability of organization can take at an equilibrium of type \( y \) (fully
separating, or (partially) pooling.) Now, for a given pooling equilibrium, the probability of organization is

\[ u = qo(w_p) + (1 - q)u_L = o(w_p) + (1 - o(w_p))(\rho(w_p) - q)\frac{(w_H^u - w_L^u)}{v_L - v_L^u + e}, \]

where the second equality comes from equation (10).

Let \( u(\rho) \) be the supremum probability of organization for a fixed \( \rho(w_p) \), i.e. for a fixed \( P_L(.) \). Then, from condition (6) in Proposition 4, and, after some algebraic manipulation:

\[ u(\rho) = (w_H^u - w_L^u)(1 - \rho(w_p))(v_L - v_L^u) + (1 - q)e + (\rho(w_p) - q)(v_H - v_H^u) \]

\[ \frac{(v_L - v_L^u + e)((1 - \rho(w_p))(w_H^u - w_L^u) + v_H - v_H^u + e)}{v_L - v_L^u + e}, \]

which is increasing in \( \rho(w_p) \), given condition (2). Therefore, the supremum of \( u(\rho) \) takes place at the supremum of the values of \( \rho \) compatible with low-value firms not triggering certain organization (\( p \)). If \( v_L - v_L^u + e < (w_H^u - w_L^u) \), then \( p \) is given by \( v_L - p w_H^u - (1 - p)w_L^u + e = v_L^u - w_L^u \), so that \( p = \frac{v_L - v_L^u + e}{w_H^u - w_L^u} \). Therefore,

\[ u(p) = 1 - \frac{q(w_H^u - w_L^u)}{v_L - v_L^u + e}(w_H^u - w_L^u + v_H - v_H^u) \]

It follows that \( U_{pooling}(e) = [0, u(p)] \). As before,

\[ U_{separating}(e) = \left[ \frac{(1 - q)(w_H^u - w_L^u)}{(w_H^u - w_L^u) + v_H - v_H^u + e}, (1 - q) \right]. \]

Now, it can be shown that \( \frac{(1 - q)(w_H^u - w_L^u)}{(w_H^u - w_L^u) + v_H - v_H^u + e} \leq u(p) \leq (1 - q) \), so that

\[ U(2)(e) = U_{pooling}(e) \cup U_{separating}(e) = [0, 1 - q]. \]

3. If \( e \) is such that \( v_L - v_L^u + e > w_H^u - w_L^u \), then \( p = 1 \), so that, from (12),

\[ u(1) = \frac{(1 - q)(w_H^u - w_L^u)}{v_L - v_L^u + e}, \]

Therefore

\[ U_{pooling}(e) = [0, \frac{(1 - q)(w_H^u - w_L^u)}{v_L - v_L^u + e}], \]

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and, from Proposition 2,

\[
U_{\text{separating}}(e) = \left[ \frac{(1 - q)(w^u_H - w^v_H)}{(w^u_H - w^v_L) + v_H - v^u_H + e}, \frac{(1 - q)(w^u_H - w^v_L)}{v_L - v^u_L + e} \right].
\]

Thus

\[
U_3(e) = \left[ 0, \frac{(1 - q)(w^u_H - w^v_L)}{v_L - v^u_L + e} \right]
\]

Note that if sets \(A = [a, \bar{a}]\) and \(B = [b, \bar{b}]\) are intervals, \(A \geq_S B\) if and only if \(a \geq b\), and \(\bar{a} \geq \bar{b}\).

Now, take \(e', e\), such that \(e' > e\). Then, it follows from expressions (11), (13), and (14), that \(U(e) \geq_S U(e')\). \(Q.E.D.\)
References


Figure 3. Union-nonunion wage gap among U.S. private sector nonagricultural workers. 1973-1998.