

A Purchasing Power Parity Paradox*

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Abstract

The Balassa-Samuelson hypothesis is considered the most important structural model of long-run deviations from purchasing power parity (PPP). I present a simple model that builds on the Balassa-Samuelson hypothesis and in which, paradoxically, PPP necessarily holds in the long run. In this model real exchange rates converge because productivity levels converge. The model’s predictions are tested empirically for 24 OECD countries in the period 1950-92. Overall, the analysis supports the prediction that productivity convergence is accompanied by real exchange rate convergence.

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1 Introduction

This paper attempts to marry the purchasing power parity (PPP) and growth (in particular “convergence club”) literatures. It raises and tests a very simple hypothesis: that convergence in levels of per-capita income within a group of countries implies convergence towards absolute PPP between them. The motivation for this exercise is mainly empirical and it relies on two stylized facts. The first is that at any given point in time rich countries tend to have higher same currency prices relative to poor ones. The second stylized fact – the existence of convergence clubs - is that within specific groups of countries and during specific periods of time, an inverse relationship can be found between a country’s initial level of per-capita income and its rate of growth of per capita income over time. Combining these two stylized facts, we can expect to find a third one: within a convergence club there should be convergence towards absolute PPP.

At the heart of the paper’s analysis is the venerable Balassa-Samuelson (BS) hypothesis. The BS hypothesis, advanced almost 40 years ago, is considered the most important structural model of long-run deviations from PPP. The novelty of this paper’s approach is that it shows that ultimately the BS hypothesis does not necessarily imply that PPP does not hold in the long run. I present a model that augments the BS framework with a technological diffusion mechanism to yield convergence in both per capita incomes and in real exchange rates across countries over time.

Section 2 positions the paper within the existing literature and motivates the theoretical and empirical analysis. Section 3 presents the theoretical model. Empirical testing of the model’s predictions is carried out in section 4. Section 5 concludes.
2 The existing literature

PPP and structural determinants of the real exchange rate

PPP is a simple empirical proposition that, once converted into a common currency, national price levels should be equal. The building block of PPP is the law of one price, which says that goods market arbitrage will equate prices of traded goods across countries. If all goods are traded and if weights of all goods in aggregate price indices are identical across countries, aggregate price levels will be equated. Thus according to PPP, the real exchange rate, which can be defined as the ratio of two countries’ price levels, expressed in a common currency, should be equal to unity, for all pairs of countries and at all times.

Since data on absolute price levels across countries is hard to obtain, empirical testing has concentrated on relative PPP, which says that the real exchange rate should be constant, or at least stationary, over time. In recent years there is a growing consensus in the empirical literature that (relative) PPP tends to hold in the long run, i.e. that real exchange rates are stationary with a very slow rate of mean reversion. In many cases half-lives of deviations from relative PPP are estimated to be in the range of 3 to 5 years.\(^1\)

On the other hand a well established stylized fact is that when all countries’ price levels are translated to a common currency at prevailing nominal exchange rates, rich countries tend to have higher price levels than poor ones.\(^2\) This inverse relationship between a country’s relative per-capita income level and its real exchange rate obviously

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contradicts absolute PPP. A few theoretical explanations for this deviation from absolute
PPP have been offered, most of them focus on the distinction between tradable and non-
tradable goods. The most prominent among these explanations is the BS hypothesis. A
generalized formal model that captures the BS argument is presented in section 3. Here I
will concentrate on its basic features. Balassa and Samuelson argued that productivity
tends to be higher in rich countries than in poor ones and that this tendency is more
pronounced in the tradable than in the non-tradable goods sector. Assuming perfect
international capital mobility, wage equalization across sectors in each country and the
law of one price for tradable goods, this pattern of productivity differentials between
sectors and across countries will lead to higher non-tradable goods prices in rich
countries. Assuming identical shares of tradable and non-tradable goods in the overall
price indices across countries, this means that rich countries will tend to have higher price
levels and appreciated real exchange rates relative to poor ones. Moreover, even if the
productivity differential between two countries is identical in both sectors, the real
exchange rate in the rich one will still be appreciated if non-tradables are more labor
intensive than tradables. This is the Baumol-Bowen hypothesis that is also formally
analyzed in section 3.

PPP is not complete. For recent challenges see Engel (2000). For potential biases in the empirical
See Bergstranal (1991). The original references are Balassa (1964) and Samuelson (1964). Discussions of the model can be
The validity of the last assumption is contested in the empirical literature. Barriers to trade (tariff and non-tariff), transportation costs, imperfect competition and other factors drive a wedge between prices of traded goods in different countries. The first major attack on the empirical validity of the Law of One
Price is Isard (1977). The literature on this issue was surveyed recently by Goldberg and Knetter (1997). The other assumptions underlying the BS argument could also, of course, be subject to criticism.
A second, related, theory that also predicts that rich countries will have appreciated real exchange rates is due to Bhagwati (1984) and others. It focuses on differences in endowments of capital and labor across countries. Rich countries tend to have higher capital-labor ratios than poor countries (because of imperfect capital mobility) and thus higher marginal product of labor and higher wages.\(^6\) Assuming again that non-tradables (largely services) are labor-intensive relative to tradables (largely commodities), non-tradables will tend to be cheaper in poor countries than in rich ones.

A third explanation is demand oriented.\(^7\) It suggests that, assuming non-homothetic tastes, price levels are higher in countries with higher per capita income because non-tradables are luxuries in consumption while tradables are necessities. Countries with higher real per-capita income will exhibit, in equilibrium, stronger demand for non-tradables relative to tradables, raising their relative price.\(^8\)

The cross-section evidence on the relationship between relative incomes and real exchange rates tends to raise the question: does it have a time-series counterpart? The literature answers this question only indirectly, mainly by empirically testing variants of the BS hypothesis. In the time-series context the BS proposition is that the real exchange rate of country i relative to country j will tend to appreciate when i’s rate of growth of productivity relative to j’s is faster in the tradable than in the non-tradable sector. The results coming from this line of research lend some support to the BS hypothesis:

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\(^6\) This argument assumes that factor endowment differences between rich and poor countries are sufficiently great that factor-price equalization does not hold.


\(^8\) Of course, factors other than national income (including its supply and demand elements) influence real exchange rates across countries. Some of the variables suggested in the literature as determinants of the real exchange rate are the share of government expenditures (and specifically military expenditures) in GDP, terms of trade, foreign aid, relative abundance in mineral resources and others. See: Bergsrand (1992), Clague (1986), Connolly and Devreux (1992), Devreux and Connolly (1996), Edwards and van
differential rates of growth of productivity across sectors and countries do, in certain cases, influence real exchange rates behavior.\(^9\)

The findings of the two lines of research presented above are to some extent contradictory. Those of the “structural” literature say that we should expect real exchange rates to follow productivity growth differentials and other real, structural, factors. Thus long run trends of appreciation or depreciation of the real exchange rate are possible. On the other hand the findings of the “non-structural” PPP literature, which uses price and nominal exchange rate data only, say that we should not expect any long-run trend in the real exchange rates. A natural environment for examining the validity of the competing claims of the two schools is one in which different countries have very different growth experiences over time. In such an environment one might expect real exchange rates to have changed most dramatically over time. One such environment is a convergence club.

**Convergence clubs**

Two main concepts of convergence appear in the growth literature. They are termed $\beta$-convergence and $\sigma$-convergence. We say that there is absolute $\beta$-convergence if poor economies tend to grow faster than rich ones. The concept of $\sigma$-convergence can be defined as follows: a group of economies are converging in the sense of $\sigma$ if the dispersion of their real per-capita income levels tends to decrease over time. The two

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\(^9\) There is a large body of empirical literature analyzing the behavior of real exchange rate over time using the differential productivity approach. In many cases this variable is augmented by other explanatory variables such as government expenditures and the terms of trade. See for example Amano and van Norden (1995), Bahmani-Oskooee and Rhee (1996), Canzoneri et. al. (1999), Chinn (1997, 2000), Chinn and Johnston (1997), De Gregorio, Giovannini and Krueger (1994), De Gregorio, Giovannini and Wolf (1994), De Gregorio and Wolf (1994), Dutton and Strauss (1997), Edison and Klovland (1987), Engel
concepts are, of course, related: $\beta$-convergence is a necessary, but not a sufficient condition for $\sigma$-convergence.\textsuperscript{10} Real exchange rate levels are heavily influenced by monetary factors, especially in the short and the medium runs, and thus are prone to be much more volatile than per-capita income levels. Because of this fact it would be much harder for us to detect $\sigma$-convergence in our data. Therefore the focus of this paper will be on $\beta$-convergence.

One of the key predictions of the neoclassical growth model is that the growth rate of an economy will be positively related to the distance that separates it from its own steady state. This is the concept known in the growth literature as conditional $\beta$-convergence. To facilitate the distinction, the concept of $\beta$-convergence discussed above is sometimes called absolute convergence. The conditional and the absolute convergence hypotheses coincide only if all the economies have the same steady state. In order to test the hypothesis of unconditional convergence we need a group of countries that are expected to have the same steady state. A set of such countries would then tend to form a “convergence club”: a group of countries in which initially poor economies tend to grow faster than the rich ones and thus catch-up or converge.

By testing for a negative relationship between average annual rates of growth and initial levels of income, Baumol (1986) concluded that industrial countries appear to belong to one convergence club, middle income countries to a separate, only moderately converging club, and that low income countries actually diverged over time.\textsuperscript{11} Following this analysis a large body of literature has tackled the empirical question of growth and

\textsuperscript{10} For discussion of these concepts see Galor (1996) and Sala-i-Martin (1996).
\textsuperscript{11} A similar early analysis of convergence is Abramovitz (1986).
convergence. The consensus that emerged in this literature suggested an absence of
global convergence to a single steady state. Yet the existence of a convergence club
among the top end of the income spectrum, specifically among OECD countries, is a
stylized fact of the empirical growth literature.\textsuperscript{12}

\textsuperscript{12} See Barro and Sala-i-Martin (1992, 1995), and Sala-i-Martin (1996). For convergence among OECD
countries see Dowric and Nguyen (1989).
3 Model

This section presents a simple theoretical model that consists of three basic elements: (1) the BS model in its cross-section dimension; (2) the BS model in its time-series dimension; (3) a simple technological diffusion mechanism. The combination of these three elements yields the result of simultaneous dual convergence: in levels of per capita income and in real exchange rates. In other words under the assumptions of the model absolute PPP necessarily holds in the long run.

I will assume a small open economy that takes the world real rate of interest as given. Two goods are produced in the economy, traded and non-traded, according to the following constant returns to scale Cobb-Douglas production functions:

\[ Y_t^T = \Theta_t^T (L_t^T)^\alpha (K_t^T)^{1-\alpha} \]  
(3.1)

\[ Y_t^N = \Theta_t^N (L_t^N)^\beta (K_t^N)^{1-\beta} \]  
(3.2)

where \( Y_t^i, \Theta_t^i, L_t^i, K_t^i \) are output, productivity (or technology) level, labor input and capital input at time \( t \) in sector \( i \) (\( i \) being \( T \) or \( N \)), while \( \alpha \) and \( \beta \) are production function parameters. Taking first order conditions with respect to capital and labor for each sector yields the following equations:

\[ R_t = (1-\alpha)\Theta_t^T (k_t^T)^{-\alpha} \]  
(3.3)

\[ W_t = \alpha\Theta_t^T (k_t^T)^{1-\alpha} \]  
(3.4)

\[ R_t = P_t (1-\beta)\Theta_t^N (k_t^N)^{-\beta} \]  
(3.5)

\[ W_t = P_t \beta\Theta_t^N (k_t^N)^{1-\beta} \]  
(3.6)

where \( R_t \) and \( W_t \) are the world real rate of interest and domestic wage rate, respectively, \( k_t^i \) is capital per worker in sector \( i \) and \( P_t \) is the relative price of non-traded goods (i.e.
\( P_t = P_t^N / P_t^T \). Solving for the relative price of non-traded goods we get the following expression:

\[
P_t = \left[ \frac{\alpha (1 - \alpha) \alpha^{1-\alpha}}{\beta (1 - \beta) \beta} \right]^{\frac{\beta}{\alpha}} R_t^{\beta} (\theta_j^N)^{-\frac{\beta}{\alpha}} (\theta_i^T ) = \gamma R_t^{\beta} (\theta_j^N)^{-\frac{\beta}{\alpha}} (\theta_i^T )^{\frac{\beta}{\alpha}}
\]

(3.7)

Note that if \( \alpha = \beta \) then \( P_t = \frac{\theta_i^T}{\theta_j^T} \).

I will now define national price level in country i and time t, \( NPL_{i,t} \), as a weighted average of traded and non-traded goods prices. For simplicity I will assume that the weights are constant and equal across time and countries.\(^{13}\) Therefore for every i and t we have:

\[
NPL_{i,t} = \left( \frac{P_t^N}{P_t^T} \right)^{\delta} \left( \frac{P_t^T}{P_t^T} \right)^{1-\delta}
\]

(3.8)

where \( P_t^N \) and \( P_t^T \) are the prices of the non-traded and traded goods, respectively. The real exchange rate of country i relative to country j at time t is then defined as follows:

\[
Q_{i,j,t} = \frac{NPL_{j,t}}{NPL_{i,t}} = \left( \frac{P_t^N}{P_t^T} \right)^{\delta} \left( \frac{P_t^T}{P_t^T} \right)^{1-\delta}
\]

(3.9)

Assuming that purchasing power parity holds for traded goods, i.e. \( P_t^T = P_{j,t}^T \), recalling the definition of the relative price of non-traded goods and using (3.7) we get:

\[
Q_{i,j,t} = \left[ \frac{(\theta_j^T / \theta_i^T)^{\frac{\beta}{\alpha}}} {(\theta_j^N / \theta_i^N)} \right]^\delta
\]

(3.10)

\(^{13}\) This is the usual assumption made in the literature and is based on a Cobb-Douglas utility function with a unit elasticity of substitution.
If \( j \) is a richer country than \( i \), we would expect productivity levels in \( j \) to be higher than in \( i \). Moreover, we would expect the productivity ratio to be higher in the tradable sector. According to the BS hypothesis this difference in productivity ratios is what causes the real exchange rate of \( i \) to be depreciated relative to \( j \). Formally: if 
\[
\frac{\Theta^T_{j,t}}{\Theta^T_{i,t}} > \frac{\Theta^N_{j,t}}{\Theta^N_{i,t}} \quad \text{and} \quad \alpha = \beta \]
then \( Q_{i,j,t} > 1 \). On the other hand, if 
\[
\frac{\Theta^T_{j,t}}{\Theta^T_{i,t}} = \frac{\Theta^N_{j,t}}{\Theta^N_{i,t}} > 1 \quad \text{and} \quad \alpha < \beta \]
then we still get \( Q_{i,j,t} > 1 \), as Baumol and Bowen have argued.

So far the discussion has concentrated on comparing price level across countries at a given point in time. To see the time-series counterparts of these results we take time derivatives of (3.10) to yield:

\[
\frac{\dot{Q}_{i,j,t}}{Q_{i,j,t}} = \delta \left[ \frac{\beta}{\alpha} \left( \frac{\Theta^T_{j,t}}{\Theta^T_{i,t}} - \frac{\Theta^N_{j,t}}{\Theta^N_{i,t}} \right) - \left( \frac{\Theta^N_{j,t}}{\Theta^T_{i,t}} - \frac{\Theta^N_{i,t}}{\Theta^N_{j,t}} \right) \right] = \delta \left( a^T_{i,j,t} - \frac{\beta}{\alpha} a^N_{i,j,t} \right) \tag{3.11}
\]
where \( a^T_{i,j,t} \) and \( a^N_{i,j,t} \) are the differential productivity growth rates between countries \( i \) and \( j \) in the tradable and the non-tradable sectors, respectively. Equation (3.11) says that, assuming \( \alpha = \beta \), if the differential productivity growth is higher in the tradable sector than in the non-tradable sector, country \( i \) will experience a real appreciation over time relative to country \( j \). If \( \alpha < \beta \), then even a balanced differential productivity growth will lead to real appreciation.

The novel element in our theoretical framework comes from augmenting it by an exogenous technological diffusion mechanism.\(^{14}\) I will assume the following equations for the rate of growth of productivity in the two sectors:

\(^{14}\) A similar specification of exogenous technological diffusion mechanism can be found in Dowrick and Nguyen (1989) and Bernard and Jones (1996a, 1996b).
\[
\frac{\dot{\Theta}_{L,t}^f}{\Theta_{L,t}^f} = \phi^f + \left(\frac{\dot{\Theta}_{L,t}^f}{\Theta_{L,t}^f} - 1\right)
\] (3.12)

\[
\frac{\dot{\Theta}_{L,t}^n}{\Theta_{L,t}^n} = \phi^n + \left(\frac{\dot{\Theta}_{L,t}^n}{\Theta_{L,t}^n} - 1\right)
\] (3.13)

where \(\phi^f\) and \(\phi^n\) are positive productivity growth parameters, while \(\Theta_{L,t}^f\) and \(\Theta_{L,t}^n\) are the time \(t\) productivity levels in the economy with the highest productivity level (I will assume that the same country has the productivity leadership in the two sectors).

Plugging (3.12) and (3.13) into (3.11) yields:

\[
\frac{\dot{Q}_{i,L,t}}{Q_{i,L,t}} = \delta \left[ \frac{\beta}{\alpha} \left( 1 - \frac{\Theta_{L,t}^f}{\Theta_{L,t}^i} \right) - \left( 1 - \frac{\Theta_{L,t}^n}{\Theta_{L,t}^i} \right) \right]
\] (3.14)

Thus if \(\alpha=\beta\) country \(i\) will experience real appreciation relative to the productivity leader as long as it is closer (in ratio terms) to the leader in non-tradable sector productivity than in tradable sector productivity. When \(\alpha<\beta\) country \(i\) will experience real appreciation even if the productivity ratio between itself and the leader is the same across the two sectors. If we make the simplifying assumption that non-tradable productivity is constant and equal across time and countries, then real appreciation will result from tradable sector productivity catch-up.

The model presented above implies that over time countries are catching up to the leader, not only in terms of productivity, but also in terms of wages, capital-labor ratios, per capita incomes and real exchange rates. Dual convergence is thus a result of technological catch-up.
4 Empirical analysis

Does the data exhibit the pattern of dual convergence predicted by the model? In this section I will attempt to show that to a large extent the answer to this question is positive. Within the group of OECD countries almost all had in 1950 a low level of per-capita GDP and a depreciated real exchange rate relative to the U.S.; subsequently the vast majority of these countries grew faster than the U.S. and experienced real exchange rate appreciation. Thus convergence in per-capita GDP levels was accompanied by convergence towards absolute PPP.

The database used in the empirical analysis is the Penn World Tables (PWT). The PWT displays a set of national accounts economic time series. Its expenditure entries are denominated in a common set of prices in a common currency so that real quantity comparisons can be made, both between countries and over time. In its latest version, which will be used in the paper, there are 152 countries and 43 years (1950-1992). The analysis here focuses on 24 OECD countries.\footnote{The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, West Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, The Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, United Kingdom and the United States. These include the twenty original (1961) members plus four members that joined the OECD by 1973. Current OECD members that joined only recently, namely the Czech republic, Hungary, Korea, Mexico and Poland, are excluded from the analysis.}

In this paper the variable relative per-capita GDP, Y, is defined as real GDP per-capita, in current international prices, relative to the U.S. The variable y is the natural logarithm of Y. The real exchange rate, Q, is defined as 100/P where P is the price level of GDP (%) \[(\text{PPP of GDP relative to the U.S. dollar})/(\text{exchange rate relative to the U.S. dollar})\]. The variable q is the natural logarithm of Q. For extended discussion of the variables and the data set see Heston and Summers (1991). Figure 1 is a scatter diagram
of all (986) observations on relative per-capita GDP (Y) and real exchange rate (Q) for the 23 countries during 1950-92.\textsuperscript{16} Visual inspection seems to indicate a strong negative relationship between relative per-capita GDP and real exchange rate.

Figure 2 plots the time series of relative per-capita GDP and real exchange rate for the twenty-three countries. For the majority of the countries a negative relationship between the two variables is again apparent. A striking feature of the graphs displayed in Figure 2 is that during the 1980s most countries experienced first a substantial real depreciation and then a large real appreciation relative to the U.S. Dollar. This fact points to the important influence of other variables, in this case mainly monetary, on the behavior of real exchange rates.

The next stage in our analysis is more formal. We will perform a procedure to test for unit roots in the individual time series, assuming that the data generating process is unknown. The procedure, which is outlined in Enders (1995), has four steps. In the first step we estimate the least restrictive model, which contains both a constant and a time trend: \( \Delta y_i = a_0 + \gamma y_{i-1} + \alpha t + \sum_{i=2}^{n} \beta \Delta y_{i-i-1} + \varepsilon \). We then use the \( \tau_\tau \) statistic to test the null hypothesis \( \gamma = 0 \). If the null hypothesis of a unit root is rejected we conclude that the series does not contain a unit root. Otherwise we go to step 2. In this step we have to determine whether too many deterministic regressors were included in Step 1. We thus test for the significance of the trend term under the null of a unit root (i.e. we use the \( \tau_{\beta_t} \) statistic to test the significance of \( \alpha_t \)). If the trend is not significant we proceed to Step 3. Otherwise we retest for the presence of a unit root using the standardized normal

\textsuperscript{16} Greece has data only for the period 1950-91; Portugal for the period 1950-90.
distribution \( (z) \). If the null of a unit root is rejected we conclude that the series does not contain a unit root. Otherwise we conclude that it does.

In Step 3 we run the regression without the trend. We test for the presence of a unit root using the \( \tau_\mu \) statistic. If the null is rejected we conclude that the series does not contain a unit root. Otherwise we test for the significance of the constant using the \( \tau_{\text{eq}} \) statistic. If the drift is not significant proceed to Step 4. Otherwise we test for the presence of a unit root using the standardized normal distribution. If the null hypothesis of a unit root is rejected, we conclude that the series does not contain a unit root. Otherwise we conclude that it does. In Step 4 we run the regression without the trend and the constant. We use the \( \tau \) statistic to test for the presence of a unit root. If the null hypothesis of a unit root is rejected we conclude that the series does not contain a unit root. Otherwise we conclude that it does. We first apply the procedure to the real exchange rate series \( (q) \). Results are presented in Table 1.

**Table 1: Unit-root tests for the real exchange rate \( (q) \)**

<table>
<thead>
<tr>
<th>Country</th>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
<th>Step 4</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>NEW ZEALAND</td>
<td>-4.4321***</td>
<td>1</td>
<td></td>
<td></td>
<td>Stationary</td>
</tr>
<tr>
<td>U.K.</td>
<td>-4.3845***</td>
<td>2</td>
<td></td>
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<td></td>
<td></td>
<td>Stationary</td>
</tr>
<tr>
<td>GREECE</td>
<td>3.6051**</td>
<td>2</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>FINLAND</td>
<td>-3.5892**</td>
<td>4</td>
<td></td>
<td></td>
<td>Stationary</td>
</tr>
<tr>
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<td>-3.5608**</td>
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<td></td>
<td></td>
<td>Stationary</td>
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<td></td>
<td></td>
<td>Stationary</td>
</tr>
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<td>Stationary</td>
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<td></td>
<td></td>
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</tr>
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<td>-3.0851**</td>
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</tr>
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<td>-2.9585**</td>
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<td>-2.9584**</td>
<td>***</td>
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<td>-2.7555*</td>
<td>***</td>
<td>Stationary</td>
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<td>-3.1078</td>
<td>1</td>
<td>-2.7438*</td>
<td>***</td>
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<td>DENMARK</td>
<td>-2.9860</td>
<td>4</td>
<td>-2.6973*</td>
<td>***</td>
<td>Stationary</td>
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<tr>
<td>W. GERMANY</td>
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<td>-2.6171*</td>
<td>***</td>
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</tr>
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<td>Country</td>
<td>$t$-t</td>
<td>LL</td>
<td>$t$-t-(\tau)</td>
<td>$t$-t</td>
<td>$t$-t-(\mu)</td>
</tr>
<tr>
<td>--------------</td>
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<td>-------</td>
<td>-----------------</td>
<td>-------</td>
<td>----------------</td>
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<td>AUSTRALIA</td>
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<td>0</td>
<td></td>
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<tr>
<td>IRELAND</td>
<td>-3.8857**</td>
<td>3</td>
<td></td>
<td></td>
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<tr>
<td>NEW ZEALAND</td>
<td>-3.6550**</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LUXEMBOURG</td>
<td>-3.3227*</td>
<td>4</td>
<td></td>
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<tr>
<td>PORTUGAL</td>
<td>-3.0881</td>
<td>3</td>
<td>3.1383***</td>
<td>***</td>
<td></td>
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<td>-2.9098</td>
<td>1</td>
<td>2.3914*</td>
<td>***</td>
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<td>3</td>
<td>-0.3058</td>
<td>-3.6724***</td>
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<td>-3.4365**</td>
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<td>NETHERLANDS</td>
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<td>1.0270</td>
<td>-3.1880**</td>
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</tr>
<tr>
<td>FRANCE</td>
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<td>3</td>
<td>0.3859</td>
<td>-3.1847**</td>
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</tr>
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<td>GREECE</td>
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<td>3</td>
<td>-0.1274</td>
<td>-3.0158**</td>
<td>2</td>
</tr>
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<td>JAPAN</td>
<td>-1.5325</td>
<td>3</td>
<td>0.8810</td>
<td>-2.8249*</td>
<td>4</td>
</tr>
<tr>
<td>ITALY</td>
<td>-2.3157</td>
<td>1</td>
<td>1.6258</td>
<td>-2.7845*</td>
<td>3</td>
</tr>
<tr>
<td>SPAIN</td>
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<td>3</td>
<td>1.2162</td>
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</tr>
<tr>
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<td>1.1095</td>
<td>-2.2844*</td>
<td>2</td>
</tr>
<tr>
<td>BELGIUM</td>
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<td>2.2897</td>
<td>-1.0967</td>
<td>3</td>
</tr>
<tr>
<td>FINLAND</td>
<td>-2.6067</td>
<td>3</td>
<td>2.2092</td>
<td>-2.2965</td>
<td>3</td>
</tr>
<tr>
<td>NORWAY</td>
<td>-2.1077</td>
<td>4</td>
<td>1.8763</td>
<td>-1.3899</td>
<td>1</td>
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<td>U.K.</td>
<td>-2.5678</td>
<td>0</td>
<td>1.8063</td>
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<td>1</td>
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<tr>
<td>CANADA</td>
<td>-2.2826</td>
<td>2</td>
<td>2.2072</td>
<td>-1.0677</td>
<td>4</td>
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</table>

As Table 1 shows, application of the procedure to the real exchange rate (q) series leads to rejection of the null hypothesis and to the conclusion that in all cases the series does not contain a unit root. We next apply the procedure to the relative per-capita GDP (y) series. Results are presented in Table 2:
As Table 2 shows, application of the procedure to the relative per-capita GDP series leads to rejection of the null and to the conclusion that in all cases this series too does not contain a unit root.

So far the formal analysis concentrated on the statistical properties of the series. The more interesting questions, of course, concern the economics behind them. The first issue that we have to address is the speed of reversion, which is captured in the parameter $\gamma$. When the regression includes a constant only this parameter captures the speed of mean reversion. In the context of the real exchange rate literature it measures the speed of reversion to relative PPP. When the regression includes both a constant and a time trend this parameter captures the speed of reversion to a trend. Table 3 contains data on the average reversion parameter in the unit-root tests performed above for the twenty-three individual time series.

Table 3: The reversion parameter, $\gamma$

<table>
<thead>
<tr>
<th></th>
<th>Real exchange rate – $q$</th>
<th>Relative GDP per capita – $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With constant and time trend</td>
<td>With constant only</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>IHL</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Average</td>
<td>-0.3369</td>
<td>1.6871</td>
</tr>
</tbody>
</table>

Source: Penn World Tables and author’s calculations. IHL stands for “implied half life”.

The results presented in the table are quite interesting. First we note the similarity in the magnitude of the average $\gamma$ coefficient for the regressions with a constant and a trend: -0.3669 for the real exchange rate series and –0.3239 for the relative per-capita GDP series. Second we note the similarity in the magnitude of the average $\gamma$ for the
regressions with only a constant: -0.1434 for the real exchange rate series and –0.1454 for the relative per-capita GDP series.

The most important observation in Table 3 is this: there is a large difference between the average $\gamma$ coefficients for the real exchange rate series between the regressions with a constant and a time trend and those with only a constant.\(^{17}\) As mentioned earlier, in essence the regression with only a constant tries to capture reversion to relative PPP. The half-life of deviation from relative PPP implied by the value of the $\gamma$ coefficient is 4.48 years. This figure fits well with the “consensus” estimate of half-lives of deviations from relative PPP discussed in Rogoff (1996): three to five years.\(^{18}\) The fact that deviations are so persistent led Rogoff (1996) to coin the term “The Purchasing Power Parity Puzzle”. But as shown above, when one includes a trend in the regressions the implied half-life drops to only 1.69 years. A similar result was obtained by Taylor (2001a) for a group of twenty countries in the post World War II years. The question then is this: what explains the trend in the real exchange rate? The answer proposed here is, of course, the process of dual convergence.

Table 4 provides evidence on the behavior on relative per-capita GDP and the real exchange rate over time for all twenty-three countries. The column labeled $Y_0$ shows the initial, 1950, relative per-capita GDP. The next column shows the trend-change in the natural logarithm of relative per-capita GDP, $y$, which was obtained by an OLS regression of the natural logarithm of relative per-capita GDP on a constant and a linear

\(^{17}\) There is, of course, a corresponding difference in the relative per-capita GDP series figures, but here we concentrate on the real exchange rate series.

\(^{18}\) See also Obstfeld and Rogoff (2001)
time trend. The next two columns show the initial real exchange rate \( (Q_0) \) and the trend-change in the natural logarithm of the real exchange rate, \( q \). The last two columns attempt to estimate the long-run relationship between \( y \) and \( q \). The penultimate column shows the slope coefficient from an OLS regression of \( y \) on a constant and \( q \). The last column shows the intercept coefficient from that regression.

**Table 4: Relative per-capita GDP and real exchange rate behavior**

<table>
<thead>
<tr>
<th>Country</th>
<th>( y_0 )</th>
<th>Trend in ( y )</th>
<th>( Q_0 )</th>
<th>Trend in ( q )</th>
<th>Regression of ( y ) on ( q ) and ( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Slope</td>
</tr>
<tr>
<td>Australia</td>
<td>0.8332</td>
<td>0.0019***</td>
<td>1.5657</td>
<td>0.0107***</td>
<td>-2.3911***</td>
</tr>
<tr>
<td>Austria</td>
<td>0.3251</td>
<td>0.0177***</td>
<td>1.6450</td>
<td>-0.0192***</td>
<td>-0.9032***</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.5071</td>
<td>0.0102***</td>
<td>1.2718</td>
<td>-0.0098***</td>
<td>-1.0609***</td>
</tr>
<tr>
<td>Canada</td>
<td>0.7058</td>
<td>0.0006***</td>
<td>1.0346</td>
<td>0.0026***</td>
<td>0.2533***</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.5915</td>
<td>0.0179***</td>
<td>1.5610</td>
<td>-0.0212***</td>
<td>-1.8969***</td>
</tr>
<tr>
<td>Finland</td>
<td>0.3875</td>
<td>0.0147***</td>
<td>1.2089</td>
<td>-0.0118***</td>
<td>-0.6994***</td>
</tr>
<tr>
<td>France</td>
<td>0.4573</td>
<td>0.0126***</td>
<td>1.2560</td>
<td>-0.0071***</td>
<td>-0.4599***</td>
</tr>
<tr>
<td>W. Germany</td>
<td>0.3776</td>
<td>0.0144***</td>
<td>1.4196</td>
<td>-0.0165***</td>
<td>-0.9149***</td>
</tr>
<tr>
<td>Greece</td>
<td>0.1557</td>
<td>0.0242***</td>
<td>0.9843</td>
<td>0.0035*</td>
<td>0.1123</td>
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<tr>
<td>Iceland</td>
<td>0.3975</td>
<td>0.0189***</td>
<td>0.7336</td>
<td>0.0028</td>
<td>0.0790</td>
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<tr>
<td>Ireland</td>
<td>0.3041</td>
<td>0.0130***</td>
<td>1.6810</td>
<td>-0.0130***</td>
<td>-0.9489***</td>
</tr>
<tr>
<td>Italy</td>
<td>0.3152</td>
<td>0.0182***</td>
<td>1.6431</td>
<td>-0.0145***</td>
<td>-0.7067***</td>
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<tr>
<td>Japan</td>
<td>0.1620</td>
<td>0.0382***</td>
<td>2.2800</td>
<td>-0.0295***</td>
<td>-0.7066***</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>0.6707</td>
<td>0.0031***</td>
<td>1.3875</td>
<td>-0.0126***</td>
<td>-0.6246*</td>
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<tr>
<td>Netherlands</td>
<td>0.5134</td>
<td>0.0092***</td>
<td>1.8793</td>
<td>-0.0212***</td>
<td>-2.0049***</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.8091</td>
<td>-0.0029***</td>
<td>1.4255</td>
<td>-0.0031**</td>
<td>0.5403**</td>
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<td>Norway</td>
<td>0.4772</td>
<td>0.0130***</td>
<td>1.4269</td>
<td>-0.0181***</td>
<td>1.1460***</td>
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<tr>
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<td>0.1421</td>
<td>0.0231***</td>
<td>1.6767</td>
<td>-0.0026</td>
<td>-0.1659**</td>
</tr>
<tr>
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<td>0.2312</td>
<td>0.0192***</td>
<td>1.8165</td>
<td>-0.0157***</td>
<td>-0.6253***</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.6597</td>
<td>0.0042***</td>
<td>1.3963</td>
<td>-0.0148***</td>
<td>-1.9434***</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.7635</td>
<td>0.0034***</td>
<td>1.5161</td>
<td>-0.0026***</td>
<td>-1.9621***</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.1264</td>
<td>0.0076***</td>
<td>1.4545</td>
<td>0.0163***</td>
<td>0.9957***</td>
</tr>
<tr>
<td>U.K.</td>
<td>0.5889</td>
<td>0.0035***</td>
<td>1.5635</td>
<td>-0.0104***</td>
<td>-2.2066***</td>
</tr>
<tr>
<td>Average</td>
<td>0.4561</td>
<td>0.0124</td>
<td>1.4708</td>
<td>-0.0110</td>
<td>-0.8428</td>
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</table>

Source: Penn World Tables and author’s calculations. Notes: *, **, *** denote, respectively, significance at the 10%, 5%, 1% levels in the relevant tests.

Did the OECD countries indeed form a convergence club? The data presented in Table 4 shows that they did: all countries started out in 1950 with a lower per-capita GDP than the U.S. and all, except for New Zealand, grew faster than the U.S. during the following four decades. What happened to the real exchange rate of those countries

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19 Natural logarithm of the original series is used in order to preserve consistency with the tests performed above.
during this period? There is statistically strong evidence for the hypothesis of initial depreciated real exchange rate and subsequent real appreciation in eighteen out of twenty-three cases.\textsuperscript{21} The second to last column shows that in those eighteen cases there is evidence for a long run inverse relationship between relative per-capita GDP and the real exchange rate.\textsuperscript{22} The evidence presented in the last column lead to the conclusion that although the real exchange rates are converging, they are not converging to unity as implied by the model. If absolute PPP holds when relative per-capita GDP is unity, then the intercept coefficient in our regression should be equal to zero. But in practice it is significantly different from zero in nineteen cases. On average, the implied deviations from absolute PPP are in the order of thirteen percent, i.e. if all countries had the same per-capita GDP as the U.S. they would have, on average, prices that are thirteen percent higher than those in the U.S. Nevertheless, Table 4 establishes that there a remarkable amount of support for the dual $\beta$-convergence hypothesis.

\textsuperscript{20} The results from such regression are not spurious as Table 1 and 2 show that both variables are stationary.

\textsuperscript{21} Canada and Turkey started out with a depreciated real exchange rate but exhibited a statistically very significant trend of real depreciation. Greece and Iceland started out with an appreciated currency but neither showed a very significant trend of either depreciation or appreciation. New Zealand, despite the fact that it had a lower per-capita GDP and a depreciated real exchange rate in 1950, showed a statistically very significant trend of growth divergence and real exchange rate convergence.

\textsuperscript{22} Statistical evidence against the dual convergence hypothesis is established in only three cases: Canada, New Zealand and Turkey.
5 Conclusion

The paper has shown that within a specific group of countries and over a specific period of time, convergence in levels of per-capita GDP was accompanied to a large extent by a convergence in real exchange rates. To the best of my knowledge this paper is the first to raise and test this hypothesis of dual convergence. An important message emerging form the exercise performed here is that real factors should be at the center of real exchange rate analysis. Differences in technologies, endowments and tastes should always be taken into account.

An interesting implication of the paper concerns the question of undervaluation and overvaluation of currencies. The results presented above suggest that in order to properly analyze the question of real undervaluation or overvaluation in the dynamic sense we need to know the fundamental determinants of the steady state levels of per-capita GDP of the countries involved and their current positions relative to these steady states. This obviously makes the calculations extremely complicated.

The paper raises another interesting question. Is it possible to find a similar relationship between convergence in levels of per-capita GDP and relative price levels across other sets of countries and regions? Research in economic growth has shown that a pattern of absolute convergence in levels of per-capita income characterizes earlier historic periods (such as the late nineteenth century) and geographic and political units (such as U.S. states). It would be interesting to examine whether in those cases, although possibly somewhat different from the theoretical perspective, convergence in relative prices can be detected.
Bibliography

• Micossi, Stefano and Gian Maria Milesi-Ferretti, 1994, “Real Exchange Rates and the Prices of Nontradable Goods,” IMF Working Paper No. 94/19 (February)
Figure 1: Relative GDP per capita and real exchange rate
OECD, 1950-92
Figure 2: Relative GDP per capita and real exchange rate in OECD countries