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The Market for Reputations as an Incentive Mechanism

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Abstract

Can an active market for firm reputations provide their owners with incentives to exert effort even when these owners have finite horizons? This paper shows that rational agents exert effort even at the end of their finite active career to reap the benefits of selling their firm's good reputation. The market for reputations will always be active in equilibrium if shifts of firm ownership are not observable. Furthermore, market forces equalize the incentives of young and old agents, implying that age does not affect effort. As a result, the anonymous separation of "entity" (firm) from "identity" (owner) may be socially beneficial: if shifts of ownership become public information then the market for reputations can collapse, causing some incentives to disappear. It is also shown that a market for reputations cannot perfectly sort good agents from bad ones. JEL C70, D82, L14.

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1 Introduction

The dynamic effects of current performance on future payoffs are central to the economics of reputation and career concerns. Fama (1980) argued that the managerial labor market will alleviate the moral hazard problem and discipline managers to work, since future wages will depend on past performance. Holmstrom (1982) showed that this argument has some qualifications: such career concerns may be too strong early in one’s career, and will disappear as the end of a career approaches, implying that the moral hazard problem still has bite. In another framework of dynamic games, Kreps et. al. (1982) show that in situations where an agent’s characteristics are unknown to others (incomplete information), the agent will engage in strategic behavior that imitates “good” types. Thus, agents will create a reputation of being good, and other agents will rationally infer this equilibrium behavior. In this setup too, however, incentives to be good disappear as agents approach the end of the game.

If the concerns to maintain a reputation can be extended to terminal periods by using an indicator that survives beyond an agent’s active career, then this problem might be mitigated. Indeed, there is a fundamental difference between an individual’s reputation and a firm’s reputation: a firm’s reputation is a tradeable asset. If a reputation is acquired under a firm’s name, or entity, and it is separated from the identity of the firm’s owner, then incentives might survive throughout the owner’s career. This paper investigates the conditions needed to guarantee that long-term incentives are provided through an active market for reputations.1

This idea has been formulated by Kreps (1990), who demonstrates that reputation can become a tradeable asset – and provide incentives – even when agents live for only one period. The argument is simple and appealing: an agent will be trusted (by a client), and in turn earn a premium, only if he acquires the good name of his predecessor, and will be able to sell his own good name if and only if he himself honors the trust of the client. If the loss from not being able to sell a good name outweighs the benefits from abusive behavior, then agents will have incentives to honor trust, and cooperative behavior is sustained in equilibrium.

The appealing feature of Kreps’s equilibrium is that short lived agents become “ageless” in the sense that do not really face a terminal period. The theory is fragile, however, because of multiple equilibria. In particular, there are many

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1Another mechanism that can mitigate end-of-career effects may be the legacy of a family name. This paper does not address such issues but rather concentrates on a market approach without dynastic families.
equilibria in which the firm is not bought, its name has no value, and thus no incentives are provided. These “bad” equilibria are no more or less likely than the “good” reputational equilibrium. Furthermore, the good equilibrium is supported by clients coordinating their punishments in response to previous poor behavior of the firm’s owner. The extent to which their reaction is rational depends on past actions that are irrelevant to future fundamentals, since past behavior has no direct link to future behavior but is indirectly linked through the players’ strategies. Indeed, if one restricts beliefs and actions to depend only on payoff-relevant information, then only the non-reputational equilibrium survives. Second, this theory is mute with respect to how reputations arise and become valuable assets. In particular, there is no account of how a firm’s reputation, represented by the value of its name, may increase (or decrease) in value as is commonly observed in reality. This is too is due to the fact that client beliefs are not tied down, but are rather assigned arbitrarily.

This paper considers a model in which agents supply services to clients for two periods and then retire. The demographics of agents is given by an overlapping generations (OLG) model, while clients live for only one period. Agents fall into three categories: some are inherently good, some are inherently bad, and some can choose how good to be at a private cost. Clients do not know the agent’s type and thus are exposed to both moral hazard (hidden action) and adverse selection (hidden information). Furthermore, clients receive services from firms, while the identities of the agents running the firms are not known to the clients. This central assumption creates a separation of entity from identity, the entity, which is captured by the intangible name of a firm, can be traded across agents without clients being aware of an actual transfer. Of course, in equilibrium clients are aware of this trade and form rational expectations regarding the composition of agents behind the different classes of names.

The paper’s first main insight provides a rationale for an active market for firm reputations which is the separation of entity from identity. The driving force is that if clients cannot observe trade in names, and if they also believe that names are not traded, then good histories must be attributed to good agents. This implies that good histories command a premium over no histories, which in turn causes good names to have value and to be traded.

The second main insight is that the market for names which is supported by the presence of adverse selection, can alleviate the problems associated with moral hazard despite the finite life horizon of agents. The reputation concerns provide incentives for opportunistic agents throughout their career: young agents are concerned with
their future income, while old agents are concerned with the value of their firm's name. It is shown that the incentives provided by these two reputational concerns are quantitatively the same; good names (associated with good histories) are scarce and the price of a good name will capture all the benefits from having one. This implies that the market for names causes agents to behave as if they have no terminal period since they internalize the future value of their good name. Thus, the "ageless" feature of Kreps's exogenously chosen equilibrium arises endogenously in the present model.

The two main insights described above crucially rest on the assumption that clients cannot observe trade of names. If clients could observe trade then they can believe that only incompetent agents would buy a name rather than build their own reputations. These beliefs would prevent the market for names from being active, which in turn would pose no challenge to the pessimistic beliefs of clients. Hence, a corollary of the above is that providing clients with the missing information of name trades will, in many cases, be harmful because old agents lose the incentives provided by the market for names. This exacerbates the moral hazard problem and reduces social surplus.

The analysis also reveals a third insight: the market for names cannot clearly separate between good and bad agents. In particular, there is no equilibrium in which good agents fully separate themselves by buying successful names; every equilibrium must have some bad agents buying these names as well. Intuitively, if only good agents buy successful names then clients do not update beliefs downward when these names perform poorly. Hence, bad agents will value successful names more than good agents because their alternative option of starting their own successful name is bleak. A direct implication of this result is that the model generates sensible reputation dynamics: reputations increase after good performance and decrease after bad performance, a crucial characteristic for reputations to provide incentives.

An important difference between this paper and most other models of firm reputation is that it uses a general competitive equilibrium approach. This is key in deriving two of the main results in this paper: that young and old agents face the same incentives, and that the market for names cannot fully separate the types of agents. The partial equilibrium analysis used in repeated game models is not rich enough to identify these results. In particular, such models do not have an economic link between the prices clients pay for services, and the prices agents pay for names. This link is needed to identify the results above: the price of a name is directly affected by the types of agents who buy it, and the scarcity of successful names causes
its price to capture its reputational value.

There is a small literature that models firm reputation as a tradeable asset. As discussed earlier, Kreps (1990) shows that the firm as an entity can outlive the agents in it, and demonstrates an equilibrium that provides short lived agents with incentives as if they lived forever. In Tadelis (1999) an OLG model similar to the one in this paper is presented, but it ignores moral hazard and only considers adverse selection. The simple adverse selection model generates trade in all equilibria, and a similar no-separation result. The lack of moral hazard, however, disables the model in Tadelis (1999) from addressing the important issues of incentive provision and welfare analysis which are central to this paper. Mailath and Samuelson (2000) consider a different model in which an infinitely lived firm provides a service for clients, and the observed quality is a noisy signal of the firm’s actions (imperfect monitoring). They show that if the firm type changes over time without clients being aware of this change, then high quality equilibria exist even when strategies are restricted to be Markov Perfect. Their emphasis, however, is not on the forces that create an active market for names. In fact, since they use a repeated game, there is an equilibrium in which bad behavior prevails and reputations are meaningless.\footnote{A similar idea was introduced earlier by Cremer (1986). Cremer considers an $n$-player prisoner's dilemma where agents live for $n$ periods in an OLG manner. He shows that despite the fact that agents have finite lifetimes, once they belong to an “organization” – a social norm – that is an infinite entity, then a large amount of cooperation can be supported.}

Another paper that combines moral hazard and adverse selection to generate interesting reputation dynamics is Diamond (1989). In his model, however, reputations belong to individuals and are not traded.

The paper is organized as follows: after the model is set up in Section 2, Section 3 analyzes a benchmark model without trade of names. Section 4 analyzes the equilibria of a simple two period model, and section 5 analyzes the incentive and welfare properties of the market for names. Section 6 generalizes to the infinite horizon steady-state analysis; Section 7 demonstrates the no-full-separation result; and Section 8 offers some concluding remarks and discussions.

\footnote{They also show that if type changes are endogenous then bad types are likely to value a very good reputation more than good types. Yet, to generate this result in a partial equilibrium framework they exogenously assume that good types have a better outside option if they do not buy a reputation. In the model presented here this arises endogenously.}
2 The Economy

Consider a simple model of economic activity where in each period a risk neutral client (or buyer) employs a risk neutral agent (or seller) to provide a service for that period only. The service provided generates an outcome that is either a success (e.g., high quality) or a failure (low quality). For simplicity assume that all clients are homogeneous so that a successful outcome yields a return of 1, while a failure yields a return of 0.

Clients face both adverse selection (hidden information) and moral hazard (hidden action): there are different types of agents who differ in their probability of success, and clients cannot distinguish between them. In particular, there are three types of agents: good agents, or $G$-types, in proportion $\gamma$, bad agents, or $B$-types, in proportion $\beta$, and opportunistic agents, or $O$-types, in proportion $1 - \beta - \gamma$. $G$-types succeed with probability $P_G \in (0,1)$, and $B$-types succeed with zero probability. Finally, an $O$-type can choose his probability of success by exerting effort $e \in [0, 1]$ at a private cost $c(e)$, where his probability of success is given by $P_O(e) = eP_G$. Moral hazard is captured by assuming that $d(e) > 0$, that is, without some form of incentives the opportunistic type would always choose to exert no effort and act as a bad type. For convenience, assume that $c''(\cdot) < 0$ which ensures a unique solution to the agent’s problem, and normalize $c(0) = 0$.

Assume that agents are active in the economy for two periods, after which they cease to produce and leave for retirement, so that wealth is valuable after the productive lifetime terminates. Agents will enter and exit the economy in an overlapping-generations (OLG) fashion, in which the total size of the population, and the distribution of types of agents is constant over time. In contrast, clients live for only one period and can observe the firms’ (names’) track records for assessment of their types. Furthermore, clients are anonymous and cannot contract among themselves.

Each agent in this economy runs his own firm, which is represented by a name, and it is assumed that no two firms can share the same name. An agent has two alternative choices at the beginning of his lifetime: he can either choose a new name to represent his firm (which implies that he will have no track record) or he can buy a

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4It might seem that $\tilde{P}_B = 0$ may have strong effects, but it will become clear from the analysis that having $\tilde{P}_B > 0$ will not change the qualitative results. If $\tilde{P}_B = 0$ the opportunistic type’s probability of success should be redefined as $P_O(e) = eP_G + (1 - e)\tilde{P}_B$ to maintain the same structure.

5As in Tadelis (1999), and similar to Diamond (1989), the implication of this assumption is that a firm’s reputation, summarized by its past performance, is the only intertemporal linkage.
name from an agent who is about to retire, thus inheriting the track record associated with that name. The value of a firm’s service is determined by the perfect observation of that firm’s past performance.\(^6\)

It is assumed that there is a continuum of clients and agents, and the price of supplying a service is determined competitively. To simplify, assume that the clients are on the long side of the market. That is, the measure of the continuum of clients is larger than that of the agents so that competition causes each client to pay her full surplus when transacting with an agent. This also implies that there will be full employment of the agents in the economy.

**Assumption A1:** *Compensation cannot be based on the transaction’s outcome.*

That is, problems of verifiability prevent the parties from writing outcome-contingent contracts because courts cannot distinguish success from failure. This implies that each client who employs an agent will pay up-front for the expected value of the service supplied.

**Assumption A2:** *Shifts of name-ownership are not observable by clients.*

This implies that a client employing a firm with a history cannot determine whether the current agent running the firm is himself responsible for that history, or whether he has just bought it. Thus, the actual identity of the agent who provides the service is separated from the firm’s entity, that is, the name. This turns out to be a key property that introduces important noise into the economy: the impact of the current owner on the firm’s past performance is uncertain. This extreme assumption can be weakened to accommodate a situation in which only part of the population is oblivious to changes of ownership, in which case the qualitative results would carry over.\(^7\)

**Assumption A3:** *At the beginning of each period every active agent can either choose to retain his past name or unobservably change it.*

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\(^6\)Note that perfectly observable histories are not necessary. It would suffice to have a well defined summary statistic to drive the results of this paper.

\(^7\)Such an environment can be considered as a reduced form of a more elaborate model in which clients are heterogeneous and have costs of learning if a firm changed hands or not, and in equilibrium a proportion of the population with high costs chooses to remain ignorant and rely on reputations.
This assumption is symmetric to assumption A2. Once the agent chooses a new name, then his past performance is erased and he can just as well be an agent that has now arrived into the economy with a clean record.8

Assumption A4: With probability \( \varepsilon > 0 \) an agent cannot change his name.

This assumption eliminates some “unreasonable belief” equilibria that can arise when all agents can change their name costlessly, and no names are traded.9

Note that if there are no \( O \)-types then \( \gamma + \beta = 1 \) and the model reduces to that of Tadelis (1999). The economic activities of each period are illustrated in the following time line:

![Time line for each period](image)

**Figure 1:** The time line for each period.

3 Benchmark: No Reputation Markets

This section outlines a simple benchmark model that will help relate the model to the repeated games literature and the career concerns. It will be convenient to consider the model described in the previous section, but restrict attention to two periods, each period having the sequence of events depicted in Figure 1 above. To capture the essence of an OLG economy there will be three generations. Generation 0 lives for the first period, generation 1 lives in both periods, and generation 2 lives in the second period only. Furthermore, the size of these one period generations are equal

8An agent can also abandon his past name and then buy a name from another agent. In all the equilibria of the model presented it turns out that agents who wish to abandon their past are indifferent between choosing a new name or buying a name. Therefore, I will assume that agents who wish to erase their past will just choose a new name.

9It is important to note that if both A3 and A4 are dropped (i.e., histories are completely “sticky”) then all the results in the paper will carry through. Thus, the justification for this rather ad-hoc assumption is by thinking of both assumptions A3 and A4 as capturing a more realistic process of name-changing
to the size of the two period generation. Thus, this economy will always consist of a proportion $\gamma$ of $G$-types, a proportion $\beta$ of $B$-types, and a proportion $1 - \gamma - \beta$ of $O$-types as described in section 2 above. To simplify notation assume that each generation of agents is of measure one, so that the total measure of agents is 2. This convention will be adopted throughout the paper. The time line of this two period economy is described in Figure 2 below:

$$
\begin{array}{c c c}
\text{t = 0} & \text{t = 1} & \text{t = 2} \\
\text{Generation 0:} & \boxed{} & \boxed{} \\
\text{Generation 1:} & \boxed{} & \boxed{} & \boxed{} \\
\text{Generation 2:} & \boxed{} & \boxed{} & \boxed{} \\
\end{array}
$$

Figure 2: A two period economy.

The following assumption is maintained in this benchmark section, and only in this section:

**Assumption A5:** Names cannot be traded but can unobservably and costlessly be changed by any continuing agent.

This implies that all agents of generation 2 will have new names (no histories) at the beginning of period 2. Agents of generation 1 will have the choice of sticking to their history or changing it, and agents of generation 0 will retire, and their names will disappear with them.¹⁰

This simple model can be solved by a dynamic Rational Expectation Equilibrium (REE) as follows. At $t = 0$ all agents have no history, and the wage $w_0$ will depend on clients’ beliefs with respect to the effort level of the $O$-types. At $t = 1$ there will be three wages depending on whether an agent had a past success ($S$), a past failure ($F$), or a new name ($N$). Denote these wages by $w_1(S)$, $w_1(F)$ and $w_1(N)$ respectively.

Two things are obvious. First, all agents who failed in the first period will be better off changing their name and “mingling” with the new agents. This is easily verified using Bayes rule and follows from the fact that having a past failure will generate beliefs that are worse than those generated from having no history. Thus, one can restrict attention to equilibria in which only $S$ and $N$ histories are observed.

¹⁰Note that this assumption contradicts A4 where I assumed that a proportion $\varepsilon$ of agents cannot change their names. The next section will clarify the role of A4. Furthermore, later in the paper the limit case of $\varepsilon = 0$ is considered so that comparisons to this section are meaningful.
in the second period. Second, $O$-types will choose $e = 0$ in the second period, and thus, $w_1(S)$ and $w_1(N)$ only depend on the clients’ beliefs about the likelihood of a $G$-type having such a history.

To save on notation, assume without loss of generality that second period income is not discounted when considering the lifetime earnings of an agent. Thus, given the equilibrium wages the expected utilities of the good and bad types respectively at $t = 0$ are,

$$ Eu_G = u_0 + P_G w_1(S) + (1 - P_G) w_1(N) , $$
$$ Eu_B = u_0 + w_1(N) . $$

The expected utility of the opportunistic type depends on his choice of effort $e$. Attention is hereon restricted to symmetric equilibria in which all the $O$-types choose the same effort level $e \in [0, 1]$. Thus, given the equilibrium wages the expected utility of the $O$-type at $t = 0$ is,

$$ Eu_O = u_0 + e P_G w_1(S) + (1 - e P_G) w_1(N) - c(e) . $$

The $O$-type’s choice of effort will affect second period wages, and these in turn feedback into the incentives of $O$-types. Thus, an equilibrium will be characterized by the tuple $(u_0, w_1(S), w_1(N), e)$ such that $e$ is a best response given $w_1(S)$ and $w_1(N)$, and these wages are correct given rational expectations on $e$. As mentioned earlier, what determines second period wages are the correct beliefs of a $G$-type behind any history, and these are calculated by applying Bayes rule. In particular, given an effort level $e \in [0, 1]$ equilibrium beliefs imply that,

$$ \Pr\{G|S\} = \frac{\gamma P_G}{\gamma P_G + (1 - \gamma - \beta) e P_G} , \tag{1} $$

and,

$$ \Pr\{G|N\} = \frac{\gamma (1 - P_G) + \gamma}{\gamma (1 - P_G) + (1 - \gamma - \beta) (1 - e P_G) + \beta + 1} = \frac{2 \gamma - \gamma P_G}{2 - \gamma P_G - (1 - \gamma - \beta) e P_G} . \tag{2} $$

That is, a firm with a past success is generated by $G$-types and $O$-types from generation 1 who succeeded, which accounts for (1) above. A firm with a new name is generated by all new (generation 2) agents, and by all the agents from generation
1 who failed (and then changed their name), which accounts for (2) above. Given the equilibrium beliefs in (1) and (2), equilibrium wages are calculated by the equation
\[ w_1(h) = \Pr\{G|h\} \cdot P, \]
since clients are heterogeneous, and they value successes at 1 and failures at 0.

Incentives for \(O\)-types are given by the wage differential \(\Delta w \equiv w_1(S) - w_1(N)\), and the expected utility of \(O\)-types given above can be rewritten as,
\[ E_u = w_0 + eP_G \Delta w + w_1(N) - c(e). \]
Given \(\Delta w\), and the convexity of \(c(\cdot)\), the effort level of \(O\)-types is given by the first order condition (FOC),
\[ P_G \Delta w = c'(e). \]
It is easy to see that in equilibrium the level of effort will be sub-optimal, which follows from the fact that optimal effort must solve \(P_G \geq c'(e)\).\(^{11}\) It is also easy to see that \(\Delta w\) is decreasing in \(e\) (this follows from (1) and (2) above), so that if \(O\)-types are choosing higher effort in equilibrium, then the wage differential is smaller. Thus we can calculate the highest wage differential when all \(O\)-types behave as \(B\)-types, and the lowest wage differential when all \(O\)-types behave as \(G\)-types. Let
\[ \Delta w_B = \frac{2(1 - \gamma)P_G}{2 - \gamma P_G}, \quad \text{and} \quad \Delta w_G = \frac{2 \gamma \beta P_G}{(1 - \beta)(2 - P_G + \beta P_G)}, \]
as the bad and good wage differentials respectively. The following result (as well as all the formal results of the paper) is proved in the appendix:

**Proposition 1:** There is a unique equilibrium of the two-period model with no reputation markets. If \(P_G \Delta w_B \leq c'(0)\) then \(e = 0\) in equilibrium. If \(P_G \Delta w_G \geq c'(1)\) then \(e = 1\) in equilibrium. If \(P_G \Delta w_B > c'(0)\) and \(P_G \Delta w_G < c'(1)\) then \(e \in (0,1)\) in equilibrium.

The intuition for this result captures the essence of career concerns. If opportunistic agents are not expected to exert any effort in the first period, then first period success must be attributed to good types, and the wage differential is greatest. This could be part of an equilibrium only if the marginal cost of effort in the first period is too high for the opportunistic types to exert any effort, which is given by the condition \(P_G \Delta w_B \leq c'(0)\). Therefore, under this conditions, future career concerns cannot

\(^{11}\) This follows immediately from maximizing social surplus: \(eP_G - c(e)\), \(e \in [0,1]\). Since in equilibrium \(0 < w(N) < w(S) < 1\), we have that \(\Delta w < 1\) and sub-optimal effort arises in equilibrium.
provide incentives for opportunistic types to exert effort. Similarly, when all $O$-types choose to behave as $G$-types in the first period, then the wage differential is smallest. This could be part of an equilibrium only if the marginal cost of effort in the first period is low enough for the opportunistic types to exert $e = 1$, which is given by the condition $P_G \Delta w_B \geq c'(0)$. Therefore, under this conditions, future career concerns provide incentives for opportunistic types to exert full effort. If both these conditions are violated, then the $O$-types must exert some effort in equilibrium, and since $c'(e)$ increases in $e$, and $\Delta w$ decreases in $e$, there must be a unique equilibrium.\footnote{One can think of a discrete choice model in which $O$-types can be good ($e = 1$) or bad ($e = 0$), so that $e \in (0, 1)$ can be considered as a mixed strategy equilibria. This specification would be similar in essence to the equilibria in finitely repeated games with incomplete information as demonstrated in Kreps et. al. (1982).}

To summarize, career concerns may help solve the moral hazard problem for early stages of an agent's career, but disappear towards the end of his productive life. The presence of adverse selection is the driving force that gives agents incentives to distinguish themselves from the bad types and try to imitate the good types.\footnote{In Holmstrom's (1982) seminal paper there is no adverse selection but rather symmetric uncertainty about the agents' types. In a dynamic rational expectations equilibrium both the agents and the market learn about the agents' types. It is really client uncertainty about characteristics that provide incentives, and not the private information that agents have.} This will provide a useful benchmark in analyzing how the market for reputations can help to solve the moral hazard problem for later stages of an agent's career.

4 The Market for Reputations

This section describes the economic forces that cause reputations – captured by the firms’ names – to be traded in equilibrium. To distinguish these forces from those that play a role in infinitely lived economies, I begin by investigating the two period model that captures the spirit of the OLG structure as shown in Figure 2. This shows that the results of this paper are independent of the length of the economy’s horizon, as long as the flavor of overlapping generations is maintained together with the assumptions presented in Section 2.

The solution concept will again be Rational Expectations Equilibrium (REE), but departing from the analysis of the previous section, trade of names is allowed. As before, clients will pay firms up-front for their services given their expected benefit, which is determined by the correct beliefs about the composition of types, and the
actions of $O$-types. To simplify, attention is restricted to symmetric equilibria in which all the $O$-types of the same generation choose the same level of effort.

At date $t = 0$ (the beginning of the first period) agents from generations 0 and 1 choose names for their firms. Since no prior information is available to the clients, they will pay the same wage to all firms, which must equal the expected benefit from hiring a firm. This expected benefit depends on the behavior of $O$-types in the first period. At date $t = 1$ there is more going on. There will be two kinds of firms: some firms will have a past history while others will not. In turn, firms with a past history of success can either be operated by a $G$-type or $O$-type who succeeded and lived on to the second period (recall that $P_B = 0$), or by a new agent who bought the name from a good agent who retired.

An equilibrium of the two-period economy will be characterized by the wages that agents (firms) will be paid at $t = 0$, by the strategies of $O$-types of each generation in every period, and by prices in two markets at $t = 1$: the wages clients will pay for hiring firms with different track records and the prices agents will pay for names with different track records. Rational expectations imply that these prices correctly predict the probabilities of success for each of the corresponding histories. Since only past histories are observable then two distinct names with the same history should generate the same expectations for future success at $t = 1$. For this reason let $S$ denote any name at $t = 1$ with a past success, $F$ with a past failure, and $N$ a name with no past. For simplicity only allow retiring agents to possibly sell their name, and only new agents to possibly buy names at $t = 1$. Also, restrict attention to equilibria in which only $S$ names are traded, so that all agents who fail in the first period will change their name. The equilibrium wages of firms at $t = 1$ will be denoted by $w_1(h), h \in \{S, F, N\}$, and the equilibrium price of $S$ names at $t = 1$ will be denoted $v(S)$. Also, let $Pr\{S|h\}$ denote the conditional probability of a firm succeeding given that its history is $h \in \{S, F, N\}$.

Before proceeding with the complete characterization of equilibria, it is instructive to show that markets for names will be active in all equilibria.

**Proposition 2:** $S$ names will be traded in all equilibria.

To see why this is true note first that there are good types in every generation,

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14One can construct equilibria in which both $F$ and $S$ names are traded. The main point here is that $S$ names will always be traded as Proposition 1 below states. Also, equilibria can be constructed in which mid-life agents sell and buy names, but due to the indifference result established in Lemma 1 below, these equilibria are payoff equivalent to the set of equilibria identified in the paper.
so that there is always a supply of $S$ names at the beginning of the second period (from the good types who succeeded and then retire). Therefore, if there is no trade of such names then it must be that having a past success is worthless to agents at the beginning of the second period. This implies that opportunistic agents in the first period have no incentive to exert effort because it generates no future benefits. But then, if good names are not traded, then successful histories must belong to good types of generation 1 who continue to the second period. This would create expectations of success, which means that new agents will be willing to buy these names, and disguise themselves as good types, since trading names is not observable.

With full observability of ownership shifts this need not be true: assigning beliefs to clients that only bad types buy names will support equilibria with no trade of names.\footnote{If a small proportion of the clients observe name trading, these clients’ beliefs must correspond to the actual buyers of names. This will lower the value of names, but if the proportion of informed clients is not too large then proposition 2 still holds. Note that with symmetric uncertainty regarding types (agents do not know their ability parameter but can improve output as in Holmstrom 1982), Proposition 1 will still hold. The intuition is exactly the same. This is discussed further in Tadelis (1999).} This shows that the lack of information regarding transfers of name ownership is a driving force that guarantees an active market for names. This was first demonstrated in Tadelis (1999) in a pure adverse selection framework, and proposition 1 generalizes this result to a more realistic model in which moral hazard is present as well. Whether or not active name trading is socially beneficial needs to be determined by considering the effects of such markets on the incentives of $O$-types in equilibrium.

The following two results are helpful to characterize equilibria of the two period model:

**Lemma 1:** In any equilibrium, all new agents will be indifferent between buying a $S$ name and not buying one, and the price of a $S$ name is $v(S) = w_1(S) - w_1(N)$.

This follows because in any equilibrium $w_1(h)$ depends only on clients’ beliefs, which do not depend on the outcome of the second period. Since this is the last period then all types have the same benefit from buying a $S$ name. Since the supply of $S$ names is less than (measure) 1, and the potential demand is 1 (the measure of new agents) then the price of a $S$ name must be set to cause indifference, which is the only way to clear the market.
Proposition 3: In any equilibrium, all first period O-types have identical incentives and thus choose the same effort level.

The intuition is simple: since the price of a name will be \( v(S) = w_1(S) - w_1(N) = \Delta w \) then the wage differential that O-types of generation 1 face when they are young (in the first period) is equal to the sales premium that O-types from generation 0 get from producing a successful name and selling it. Thus, the incentives provided to young O-types from generation 1 are identical to those provided to "old" O-types of generation 0. This implies that one can restrict attention to equilibria in which all O-types choose the same level of effort \( e \in [0,1] \) in the first period, regardless of the generation they belong to.

This result indicates that the incentives provided by career concerns (the wage premium) are identical to those created by the market for names. This is striking because it implies that a market equilibrium analysis shows that wages and name prices are tightly connected, which in turn implies that incentives to work are independent of an agent's future horizon. This was generated by a model in which agents are active for only two periods, but the economic intuition behind the result seems to be quite general. Namely, the general conjecture is that a history generates value because it creates an expected sequence of wages to its owner, and an expectation of selling the realized future. If valuable histories are scarce, then the price they command should equal their value. This, in turn means that regardless of the owner's age, he is internalizing this future sequence of values.

Continuing with the equilibrium analysis, the only equilibrium parameter that affects the first period wage is the (correct) beliefs clients have about the actions of O-types in the first period. The first period wage must satisfy,

\[
w_0 = [\gamma + (1 - \gamma - \beta)e] P_G,
\]

which follows because G-types (in proportion \( \gamma \)) succeed with probability \( P_G \), O-types (in proportion \( 1 - \gamma - \beta \)) will succeed with probability \( eP_G \), and the clients' value from success and failure are 1 and 0 respectively.

In any REE clients must have correct beliefs about the composition of new agents who buy names at \( t = 1 \). Since all O-types will choose \( e = 0 \) in the second period, the population of generation 2 agents can be redefined into good and bad where the bad are of measure \( 1 - \gamma \). Let \( \mu \) (respectively \( \rho \)) denote the proportion of good (respectively bad) types who buy \( S \) names at \( t = 1 \). An equilibrium for the two period model will be a tuple \( \langle \mu, \rho, w_0, w_1(S), w_1(F), w_1(N), v(S), e \rangle \). Note that the
wages firms charge clients, and the prices new agents are willing to pay for names will be generated by the correct beliefs about \((\mu, \rho, e)\), so that a triplet \((\mu, \rho, e)\) will in fact uniquely determine the other equilibria parameters.

In equilibrium \((\mu, \rho, e)\) must satisfy the market clearing condition,
\[
\gamma P_G + (1 - \gamma - \beta)eP_G = \mu \gamma + \rho (1 - \gamma) \, .
\] (3)
which guarantees that the supply of \(S\) names (the left hand side of (3)) is equal to the demand (the right hand side of (3)). Recall that clients will pay their full expected surplus up-front, so in equilibrium it must be that for all \(h\), \(w_1(h) = \Pr\{S|h\} = \Pr\{G|h\} \cdot P_G\). Given \((\mu, \rho, e)\) that satisfy (3) above, the probabilities are determined by Bayes Rule as follows,
\[
\Pr\{G|S\} = \frac{\gamma P_G + \mu \gamma}{\gamma P_G + (1 - \gamma - \beta)eP_G + \mu \gamma + \rho (1 - \gamma)}
\] (4)
and,
\[
\Pr\{G|N\} = \frac{\gamma (1 - \varepsilon) (1 - P_G) + (1 - \mu) \gamma}{2 \gamma P_G + \gamma \mu - \varepsilon \gamma (1 - P_G)}
\] (5)
where the second equality in both equations follows from market clearing and some simple algebra. The correct beliefs about \(\mu\) and \(\rho\) will determine \(w_1(h)\) for all \(h\) according to Bayes updating as described in (4) and (5) above. For ease of calculations restrict attention to the limiting case where \(\varepsilon = 0\), which yields (from (5) above):
\[
\Pr\{G|N\} = \frac{2 \gamma - \gamma P_G - \mu \gamma}{2 - 2 \gamma P_G - 2 e P_G (1 - \gamma - \beta)} \, .
\]
This has no qualitative affect on any of the results. The following proposition characterizes the set of equilibria in which only \(S\) names are traded.

**Proposition 4:** There exist \(\hat{\varepsilon} < \delta\) so that \((\mu, \rho, e)\) is an equilibrium if and only if the following three hold:

(i) \(\mu \in [\hat{\varepsilon}, \delta]\)

(ii) \((\mu, \rho, e)\) satisfy market clearing

(iii) \(\dot{c}(e) = \Delta w P_G\)
Proposition 3 implies that there is a continuum of equilibria with respect to prices: the interval \([\delta, \bar{\delta}]\) is non-empty, and any \(\mu \in [\delta, \bar{\delta}]\) can be supported in equilibrium. This basically follows from the indifference result established in Lemma 5, that is, as long as the price for names reflects the wage differential that the name generates, agents will be indifferent. In particular, the equilibrium price for \(S\) names is increasing in \(\mu\) so that \(\mu = \bar{\delta}\) supports the lowest price equilibrium. In this equilibrium many bad types of generation 2 are buying the names so that the price of a \(S\) name is either zero (there are enough good types in the second period so that having no history is not too harmful), or it is positive (there are too few good types so that even when all \(S\) names are bought by bad types, this is still better than having no history). Any equilibrium with \(\mu > \delta\) commands a positive price for \(S\) names. When \(\mu = \pi\) the price of an \(S\) name is the highest possible, which is when the largest possible number of new good types are buying these names without violating market clearing. The effect of the market for names on the incentives of opportunistic agents in the first period is investigated in the next section.

5 Incentive Effects: Two Periods

As Proposition 2 implies, if a regulator can make the identity of owners transparent, then pessimistic beliefs will cause the market for names to shut down, resulting in an economy with no trade of names. This economy was analyzed in section 3, and provides a useful benchmark to see how the market for names affects the \(O\)-types’ equilibrium effort level. By comparing the two models, with and without a market for names, one can evaluate the welfare implications of a regulator’s actions with respect to imposing disclosure of identities.

The market for names will have two effects: first, for the agents of generation 0, it provides a potential incentive to exert effort in their terminal period. Second, the composition of name-buyers from generation 2 will affect the incentives for the agents of generation 1 in their initial period, thus affecting their career concerns. However, these incentives will vary with the wage differential, which in turn depends on the equilibrium in the economy. As Proposition 3 shows, there is a continuum of equilibria, which means that such a comparison is not straightforward.
5.1 Extreme Equilibria: Lower Bounds

It is instructive to consider the effect of the market for names on the parameters for extreme equilibria identified in proposition 1, where either \( e = 1 \) or \( e = 0 \) are the equilibrium effort level. Note, however, that wages depend on \( \mu \), which in turn implies that the good and bad wage differentials also depend on \( \mu \). More precisely,

\[
\Delta w_B (\mu) = w_1 (S|e = 0) - w_1 (N|e = 0) = \frac{P_G + \mu - P_G (2\gamma - \gamma P_G - \mu \gamma)}{2} = \frac{P_G + \mu - 2\gamma P_G}{2(1 - \gamma P_G)},
\]

and,

\[
\Delta w_G (\mu) = w_1 (S|e = 1) - w_1 (N|e = 1) = \frac{\gamma P_G + \mu \gamma - P_G (2\gamma - \gamma P_G - \mu \gamma)}{2(1 - \beta)} = \frac{2\gamma P_G + \gamma \mu - \gamma P_G}{2(1 - \beta)(1 - P_G + \beta P_G)}.
\]

It is easy to check that \( \Delta w_G (\mu) \) and \( \Delta w_B (\mu) \) are increasing in \( \mu \). This implies that the “lower” the equilibrium (lower \( \mu \) the worse are the potential incentives in the economy with trade of names. However, it is possible to establish the “lower bound” wage differentials for the lowest equilibrium of the economy:

**Proposition 5:** (i) If \( \gamma < \frac{1}{2} \) and \( c'(0) < \Delta w_B (0) P_G = \frac{P_G (1 - 2\gamma)}{2(1 - \gamma P_G)} \) then in the first period O-types must choose \( e > 0 \) in all equilibria.

(ii) If \( \beta > \frac{1}{2} \) and \( c'(1) < \Delta w_G (0) = \frac{\gamma P_G (2\beta - 1)}{2(1 - \beta)(1 - P_G + \beta P_G)} \) then in the first period O-types must choose \( e = 1 \) in all equilibria.

The intuition for part (i) is as follows: when expectations are that O-types choose no effort, then the model reduces to one of two types – good and bad. If there is a minority of good types (\( \gamma < \frac{1}{2} \)) then in all equilibria the price of a name is positive. Therefore, if the marginal cost of effort is low enough at \( e = 0 \) then an O-type will deviate from the expectations and choose some positive effort level. Similarly for part (ii), when expectations are that all O-types choose \( e = 1 \), and if there is a majority of bad types (\( \beta > \frac{1}{2} \)) then in all equilibria the price of a name is positive, and if the
marginal cost of effort is low enough at \( e = 1 \) then all \( O \)-types will indeed choose \( e = 1 \).

Propositions 3 and 4 imply that when there are too many good types, that is, \( \gamma \geq \frac{1}{2} \), then the worse equilibrium for incentives has \( \delta > 0 \), which in turn causes \( \Delta w_B(\delta) = 0 \). In this case there are no incentives for either young (generation 1) or old (generation 0) agents to exert effort in the first period, independent of the cost function \( c(\cdot) \). Thus, there exist parameter values for which the worse equilibrium in the market for names is welfare reducing compared to the case where no names are traded. That is, not only does the market for names fail to create incentives for old (generation 0) agents, but it eliminates the incentives of young (generation 1) agents since the bad composition of name buyers eliminates the wage differential. However, if one resorts to an economy in which being inherently good is scarce \( (\gamma < \frac{1}{2}) \) then this cannot happen.

5.2 Equilibrium Selection

The analysis above shows that it is somewhat cumbersome to perform a welfare comparison of the two models, with and without a market for names. Proposition 4 looks at the worst equilibrium for incentives, which may be one way to compare models when there are multiple equilibria. Another way to proceed is to identify a reasonable equilibrium selection and then compare the two models where each has a unique equilibrium. It turns out that adding a second dimension of agent heterogeneity would naturally break the indifference demonstrated in Lemma 5 and pin down a unique equilibrium. One realistic alternative is to make the model richer by having agents vary with respect to their cost of purchasing a firm versus building a new one. This would capture the idea that when an agent creates his own firm then it is tailored to his or her specifications, whereas buying an existing enterprise may require some adaptations or modifications.\(^{16}\)

Formally, let \( \pi \in [0, \pi] \) be the extra cost associated with purchasing an existing firm (name) that is identically and independently distributed across all agents with the cumulative distribution function \( G(\cdot) \), and with positive density \( g(\cdot) \) over the domain \( [0, \pi] \). An agent with cost \( \tilde{\pi} \) will buy a \( S \) name only if it is worthwhile given

\(^{16}\)The reverse argument could also be made: when you buy an existing firm you get it lock, stock and barrel, instead of having to set one up from scratch. This will have the exact same effect of pinning down a unique equilibrium.
his costs, that is, only if

\[ v(S) < w_1(S) - w_1(N) - \hat{\pi}, \]

implying that there will exist some \( \pi^* \in [0, \bar{\pi}] \) such that all agents with cost \( \pi < \pi^* \) will buy \( S \) names, and other agents will not, independent of their type. The equilibrium price of a \( S \) name will then be

\[ v(S) = w_1(S) - w_1(N) - \pi^*, \]

and the \( i.i.d. \) assumption guarantees that a proportion \( G(\pi^*) \) of all types will buy \( S \) names (that is, \( \mu = \rho = G(\pi^*) \) in the two period model). Combining this with the market clearing condition yields

\[ \gamma P_G + (1 - \gamma - \beta)e P_G = G(\pi^*)\gamma + G(\pi^*)(1 - \gamma), \]

or,

\[ \mu^* = \rho^* = G(\pi^*) = \gamma P_G + (1 - \gamma - \beta)e P_G. \]

Note that now there is an asymmetry between the young and old \( O \)-types in the first period that is created by the “friction” \( \pi \): the young will get \( w_1(S) - w_1(N) \) if they succeed, while the old will get \( v(S) < w_1(S) - w_1(N) \) because of the purchasing cost \( \pi \). It is therefore convenient to consider the unique equilibrium derived from the limit \( \pi^* \rightarrow 0.17 \) This would yield identical incentives for old and young agents, as shown in Proposition 2 above, and would yield

\[ v(S) = w(S) - w(N). \]

To complete the characterization of equilibrium, the value of \( \mu^* \) will be simultaneously determined with the level of effort \( e \) so that \( e \) is a best response of the \( O \)-types. Precisely, if \( c'(1) \leq \Delta w_G(\mu^*) P_G \) then in equilibrium we must have \( e = 1 \). Similarly, if \( c'(0) \geq \Delta w_B(\mu^*) P_G \) then in equilibrium we must have \( e = 0 \). If both weak inequalities are violated then the unique equilibrium has \( e \in (0, 1) \), which has the \( O \)-types setting \( c'(e) = \Delta w(\mu^*) P_G \). This, together with market clearing, and the determination of wages from the Bayes Rule equations (4) and (5) above, will

\[ ^{17} \text{This can, for example, follow from } G(\cdot) \text{ being Uniform on } [0, \bar{\pi}], \text{ and letting } \pi \rightarrow 0. \]
determine the equilibrium.\textsuperscript{18}

Once attention is focused on this particular equilibrium, it is straightforward to compute the bad and good wage differentials for this equilibrium which are,

\[
\Delta w_B(\mu^*) = \frac{P_G^2(1 - \gamma)}{2(1 - \gamma P_G)},
\]

and,

\[
\Delta w_G(\mu^*) = \frac{\gamma \beta P_G^2}{2(1 - \beta)(1 - P_G + \beta P_G)}.
\]

5.3 Welfare Comparisons

As mentioned earlier, both with and without an active market for names there is under-provision of incentives for the opportunistic types, and social surplus is therefore higher in the setup that generates more total effort. As far as the effects of the market for names go with respect to incentives of the "young" agents of generation 1 it is ambiguous – it may be that the introduction of a market for names (following a policy that hides the identity of name owners) will cause the wage differential to decrease, thus lowering incentives. However, incentives may be created for the "old" agents of generation 0. Thus, for those parameter values in which incentives for generation 1 agents were not reduced there is a clear welfare improvement from the active market for names. Interestingly, even when incentives are reduced for the agents of generation 1, new incentives are provided for the agents of generation 0.

To see how parameters affect the welfare conclusions, it is again illustrative to consider the extreme equilibria associated with the good and bad wage differentials. As before, let \(\Delta w_G\) and \(\Delta w_B\) denote the good and bad wage differentials for the benchmark model without a market for names. If \(\Delta w_B(\mu^*) < \Delta w_B\), then there exists cost functions \(c(\cdot)\) for which \(\Delta w_B(\mu^*) < c(0) < \Delta w_B\). Thus, for this case young agents exerted some effort in the benchmark model, but with trade of names neither young nor old agents will exert effort in the first period. This is clearly a decrease in social surplus. Clearly, if \(\Delta w_B(\mu^*) > \Delta w_B\) then the reverse is true: there exist cost

\textsuperscript{18}It is straightforward to verify that this equilibrium is indeed a selection of the equilibria set determined in Proposition 3. It is interesting to note that this equilibrium is appealing for another reason. In the original model with multiple equilibria, the indifference of all types regarding whether to buy a success name or not, suggests that there is no reason one type should be more likely to buy a name than another. If this is taken to mean that sellers of success names are randomly matched with buyers of both types in each period, it is exactly these proportions that will arise as the random-matching equilibrium.
functions for which no agents exert effort in the benchmark model, but when trade of names is considered then both young and old agents will exert some effort in the first period.

More precisely, the economy can move from an equilibrium with effort to an equilibrium without effort if and only if \( \Delta w_B(\mu^*) < \Delta w_B \), which reduces to,

\[
P_G < \frac{2}{3\gamma}.
\]

(6)

Similarly, if \( \Delta w_G(\mu^*) < c'(1) < \Delta w_G \) then without the market for names we have all young agents choosing \( e = 1 \), whereas with the market for names both young and old will choose \( e < 1 \). This can happen if and only if \( \Delta w_B(\mu^*) < \Delta w_B \), which reduces to,

\[
P_G < \frac{2 - 2\beta}{3 - 6\beta + 3\beta^2}.
\]

(7)

Note that here there is no clear welfare change: it may be that the added effort of old agents more than compensates for the loss in effort of the young agents, and this depends on the shape of the cost function \( c(\cdot) \) near \( e = 1 \).

If, in equilibrium, the choice of effort by \( O \)-types is in the interior of \([0,1]\) then the first order condition for the benchmark model without a market for names is,

\[
c'(e) = \frac{2\gamma P_G^2(1 - \gamma - (1 - \gamma - \beta)e)}{(\gamma + (1 - \gamma - \beta)e)(2 - \gamma P_G - (1 - \gamma - \beta)eP_G)}
\]

(8)

while the FOC for the model with an active market for names is,

\[
c'(e) = \frac{\gamma P_G^2(1 - \gamma - (1 - \gamma - \beta)e)}{2(\gamma + (1 - \gamma - \beta)e)(1 - \gamma P_G - (1 - \gamma - \beta)eP_G)}.
\]

(9)

Now consider the equilibrium effort level \( e^B \) that solves (8) above. If the right hand side of (8) is greater than the right hand side of (9) when both are evaluated at \( e^B \), then by the convexity of \( c(\cdot) \) it follows that in the model with trade both young and old will choose and effort level \( e^* > e^B \). This is clearly an increase in social surplus.
Several steps of algebra conclude that this is the case if and only if,\(^{19}\)

\[
(\gamma + (1 - \gamma - \beta)e^B)3P_G - 2 > 0.
\]

Note that for \(e^B = 0\) or \(e^B = 0\) this inequality reduces to either (6) or (7) above for the extreme wage differentials.

What can be concluded from (10) above are some comparative statics. The social surplus when the market for names is active (relative to the benchmark model) increases (that is, \(e^*\) increases) when \(P_G\) or \(\gamma\) increase, and when \(\beta\) decreases. The intuition follows quite simply: As either \(P_G\) or \(\gamma\) increase, or as \(\beta\) decreases, the wage differential and the proportion of good types buying names increase, which has two positive effects in the model with trade of names. the latter does not appear in the benchmark model.

Without a specific cost function, however, it is impossible to make general claims on the social surplus which would be lost or gained if the market for names is shut down. It is possible to construct cases where an interior solution always exists, and the total effort level is higher when the market for names are active. Simulations show that for simple quadratic cost functions such results are true for a wide range of parameter values. One should remember that this is a stylized model of incentives and reputation that captures the possible economic forces at work. I do not claim that there are "reasonable" welfare comparisons between the models but rather wish to illustrate that more information, that is, making name transfers observable, will shut down the market for names, and this in turn will eliminate incentives for older agents. In many cases this can be detrimental for social surplus.

Note that the analogy of generation 1 agents with "young" agents and generation 0 agents with "old" agents is clear. The next section considers the infinite horizon economy in which there is no terminal period and shows that the results obtained above are robust.

\(^{19}\)Namely, this happens if the right hand side of (9) minus the right hand side of (8) is positive, which reduces to,

\[
\frac{\gamma(1 - \gamma - (1 - \gamma - \beta)e)((\gamma + (1 - \gamma - \beta)e)3P_G - 2)}{2(\gamma + (1 - \gamma - \beta)e)(2 - (\gamma + (1 - \gamma - \beta)e)P_G)(1 - (\gamma + (1 - \gamma - \beta)e)P_G)} > 0
\]

Each of the three bracketed terms in the denominator are positive, as is the first bracketed term in the numerator. Thus, the second bracketed term in the numerator will determine the sign of this fraction, which is the inequality demonstrated above.
6 The Infinite Horizon Model

To analyze the incentive effects of a continuing market for names, it is natural to consider the infinite horizon version of the model described above. That is, every period a new cohort of agents – good, bad and opportunistic – enter the economy and can buy names from the cohort who is retiring, so that there is no termination period. Then, after one period of performance they can either continue with their name or change it, and finally, after two periods they can sell their name. The following Proposition is the parallel of proposition 1 for the infinite horizon model:

**Proposition 6:** Names with a history of one success must be traded in all equilibria.

The proof of this proposition is omitted since it is almost identical to the proof of Proposition 1. The intuition is the same: these names have a positive supply, and under the assumption of no trade it must be that they convey positive information and therefore have value.

Since names are associated with histories, this introduces a rather demanding and complex problem for the analysis of the infinite horizon model. Formally, the set of histories \( H \) is the set of all finite and infinite length histories consisting of successes and failures (including \( N \), no history). Then, even when concentrating only on Steady State Equilibria (SSE), the analysis becomes intractable. For example, Proposition 5 states that \( NS \) names will be traded in every period (where \( NS \) means that the name was created last period and had a success then). This in turn implies that \( NSS \) names must also be traded, and this iterative argument implies that any SSE must have all names with any consecutive number of successes (with no failures) traded. But then any SSE must satisfy a countable number of market clearing conditions, and must satisfy a countable number of inequalities to ensure trade (non-negative values of such names).

The reason there is a countable number of market clearing conditions is because the supply of each name is endogenous to the proportions of good, opportunistic, and bad types who buy other names. That is, even though it is common knowledge that current agents cannot be responsible for performance generated more than one period ago (they live for only 2 periods), the proportions of different types who buy names can (artificially) be made history dependent.\(^{20}\)

\(^{20}\)For the pure adverse selection case this issue is addressed in Tadelis (2000).

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A way to simplify the analysis and make the model tractable is to distinguish between histories and reputations in a way that clients do not observe complete histories, but rather some summary of histories which represents reputations. Formally, let $r : H \to R$ be a reputation mapping from the set of possible histories to a reputation range which is arbitrarily defined. In particular, let $R = \{S, F, M, N\}$ so that the countable number of histories $h \in H$ are mapped into four possible reputations that are observed by the clients: $S$ (successful), $F$ (failed) $M$ (mediocre), and $N$ (no reputation). This can be justified by imposing some bounded recall on the clients. To illustrate this, imagine that clients can only remember the outcomes of the last two periods, and the reputational mapping will map all histories ending with $SS$ and $NS$ to reputation $S$, all histories ending with $SF$ to reputation $M$, and all other histories to $F$.\footnote{Note that $FF$ names should not exist because no one would buy either a $F$ name or a $FF$ name. This is easily supported off the equilibrium path by assigning average beliefs to clients.}

**Definition 1:** Let $r : H \to \{S, F, M, N\}$ be a reputation mapping satisfying:

\[
\begin{align*}
r(N) &= N; \\
r(h) &= S \text{ for all } h \in H_S \equiv \{NS, NSS, NSSS, \ldots\}; \\
r(h) &= M \text{ for all } h \in H_M \equiv \{NSF, NSF, NSF, \ldots\}; \\
r(h) &= F \text{ for all } h \in H \setminus (N \cup H_S \cup H_M).
\end{align*}
\]

Given this restriction of histories, the analysis that follows will focus on the unique stationary SSE consistent with the heterogeneity described by $\pi$. Namely, a proportion $G(\pi^*)$ of each of the three types will buy $S$ names.\footnote{Recall that we are considering the limit with $\pi^* = 0$. The difference here compared to the two period model is that the buyers of names cannot be classified as good and bad since there are opportunistic types who may choose to exert effort in equilibrium (there is no terminal period). Also, the analysis will restrict attention to trade in $S$ names, and it will be shown that other names will not be traded given the correct equilibrium beliefs.\footnote{Without the added heterogeneity given by $\pi$ then there would be a continuum of equilibria as in the two period case. Furthermore, there would be a possibility to have trade in $M$ names. See Tadelis (2000) for a detailed analysis of the pure adverse selection framework.}}

Since only $F$, $M$ and $S$ are observed by clients, the equilibrium constructed will have all successful names (with reputation $S$) traded at the same price $v$, and all other names will not be traded. (It will be confirmed that $F$ or $M$ names are not worth
buying given clients’ beliefs, and agents with \( M \) names would prefer to change them to \( N \) names.) Denote the SSE wages as \( w(N) \), \( w(S) \), \( w(M) \) and \( w(F) \) respectively. To write down the market clearing condition, the effort choice of \( O \)-types must be determined. In each period there are “young” and “old” agents as in any standard OLG model. The following result is parallel to Proposition 2:

**Proposition 7:** In any SSE, all young and old \( O \)-types have identical incentives and thus choose the same effort level.

The intuition is simple: since the steady-state price of a name will be \( v(S) = w(S) - w(N) \) (similar to the two-period model) then the premium wage that young \( O \)-types get from working in the first period is equal to the sales premium that old \( O \)-types get from having a successful name to sell. Thus, in any SSE all \( O \)-types will choose the same effort level \( e \) in every period.

The argument above implies that the supply of \( S \) names has the same structure as for the two-period model; this follows from the fact that old agents who sell a \( S \) name are those who succeeded in their last period, regardless of what happened in their first period.\(^\text{24}\) The market clearing condition can now be established and is,

\[
\gamma P_G + (1 - \gamma - \beta) e P_G = G(\pi^*),
\]

where \( G(\pi^*) \) is the proportion of good, bad and opportunistic types that buy \( S \) names. We can therefore characterize the SSE by the tuple \( \{\pi^*, v, e, w(S), w(N), w(F), w(M)\} \).

To compute the SSE wages, note that every period has young and old agents with \( N \) and \( S \) names (since other names will be changed to \( N \) names). It is useful to illustrate the proportion of each type, in each generation, as follows:

<table>
<thead>
<tr>
<th>New Agents</th>
<th>Old Agents</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )-types</td>
<td>( O )-types</td>
</tr>
<tr>
<td>( N )</td>
<td>( S )</td>
</tr>
<tr>
<td>( 1 - G(\pi^*) )</td>
<td>( G(\pi^*) )</td>
</tr>
<tr>
<td>( 1 - G(\pi^*) )</td>
<td>( G(\pi^*) )</td>
</tr>
<tr>
<td>( 1 - G(\pi^*) )</td>
<td>( G(\pi^*) )</td>
</tr>
</tbody>
</table>

**Table 1: Steady State Name Proportions**

\(^\text{24}\)If an agent failed in his first period he will change his name, potentially selling a \( S \) name after he retires. Thus, the potential of selling a name depends only on the probability of success in the agent’s second period.
Using Table 1 above it is easy to write the Bayes formula to obtain,

\[
\Pr\{G|S\} = \frac{\gamma(P_G + G(\pi^*)) + e(1 - \gamma - \beta) (eP_G + G(\pi^*))}{\gamma P_G + e(1 - \gamma - \beta) P_G + G(\pi^*)}
+ \frac{\gamma(P_G + \gamma P_G + e(1 - \gamma - \beta) P_G) + e(1 - \gamma - \beta) (eP_G + \gamma P_G + e(1 - \gamma - \beta) P_G)}{2(\gamma P_G + e(1 - \gamma - \beta) P_G)}.
\]

(12)

The numerator accounts for the agents with a past success who will be “good” again. This includes all the old G-types who succeeded, a proportion \(e\) of the old O-types who succeeded, a proportion \(G(\pi^*)\) of the new G-types, and a proportion \(G(\pi^*)\) of the new O-types who will work. The second equality follows from substituting for \(G(\pi^*)\) from the market clearing condition (11). Similarly, the conditional probability of a good type running a firm with no history is,

\[
\Pr\{G|N\} = \frac{\gamma (1 - G(\pi^*) + 1 - P_G) + e(1 - \gamma - \beta)(1 - G(\pi^*) + 1 - eP_G)}{\gamma (1 - P_G) + \beta + (1 - \gamma - \beta)(1 - eP_G) + 1 - G(\pi^*)}
+ \frac{\gamma (2 - \gamma P_G - e(1 - \gamma - \beta) P_G - eP_G) + e(1 - \gamma - \beta)(2 - \gamma P_G - e(1 - \gamma - \beta) P_G - eP_G)}{2(1 - \gamma P_G - e(1 - \gamma - \beta) P_G)},
\]

(13)

where the second equality follows from substituting for \(G(\pi^*)\) from the market clearing condition (11).

In Appendix B it is verified that all types will discard their name after any failure (that is, both M and F names). Using the conditional probabilities in (12) and (13) above we can calculate the good and bad differentials in the same way we did in the previous section. That is,

\[
\Delta w^*_B = P_G^2(\Pr\{G|S, e = 0\} - \Pr\{G|N, e = 0\})
= P_G^2 \left(1 - \frac{\gamma}{P_G}\right)
= \Delta w^*_B(\mu^*),
\]

and,

\[
\Delta w^*_G = P_G^2(\Pr\{G|S, e = 1\} - \Pr\{G|N, e = 1\})
= \frac{\beta P_G^2}{2(1 - P_G + \beta P_G)}
= \Delta w^*_G(\mu^*) \left(\frac{1 - \beta}{\gamma}\right) > \Delta w^*_G(\mu^*).
\]

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The values of the wage differentials are not surprising. In the two period model, all
O-types who bought S names did not work (the economy ended), and this is basically
the same thing as having \( e = 0 \) in the infinite horizon model. This implies that the
bad wage differential is the same for the two period model and the infinite horizon
model, and is verified by the equality \( \Delta w^*_B = \Delta w^*_B(\mu^*) \). However, when \( e = 1 \) in
the infinite horizon model then O-types who buy a S name will act as good types,
which increases the value of having a S name, and causes the good wage differential
to increase. Therefore, the incentives in the infinite horizon model are improved
compared to the two period model due to O-types having a continuing value for
working in their second period.

Note that the equilibrium identified in the finite horizon benchmark model is the
unique stationary SSE for the infinite horizon model without trade of names.\(^{25}\) Thus,
when the infinite horizon model is considered, the incentive properties of the market
for names are enhanced, and social welfare is more likely to be higher when the market
for names is active.

7 Sorting by Reputations

One might conjecture that the value of a good reputation would be higher for a
good type who is more likely to maintain it, than for a bad type who cannot, or
an opportunist type who has to exert effort to do so. This reasoning would be
consistent with the theories of Klein and Leffler (1981) and of Kreps (1990), if one were
to incorporate their ideas in an adverse selection framework. That is, if good types
find it easier to maintain a reputation, then they should be able to outbid bad types
who are more likely to ruin a reputation. Notice, however, that in the equilibrium
analyzed in the previous section (and those analyzed in the two period model) this
did not occur – all types had the same value from buying a good reputation. It turns
out that this is no coincidence: situations in which the good reputations are bought
only by good types, and possibly by opportunistic types who plan to be good, cannot
be supported in equilibrium. Formally:

Proposition 8: For the infinite horizon model there is no equilibrium in which S
 names are traded, and these are bought only by good types and by opportunistic

\(^{25}\) It is easy to construct cyclical equilibria for the model without trade of names. Intuitively, if in
every odd period O-types work harder than in every even period, then the wage differential in odd
periods is lower than that in even periods, which supports this type of cyclical behavior.
types who choose to be good.

To see this, consider a proposed dynamic equilibrium in which only good types buy $S$ names. This means that clients must (correctly) predict that any good name is bought by a good type, so that a name with a history that starts with a success will be attributed to a good type no matter what the continuation history will be. This may at first seem somewhat strange, but the intuition is quite simple: If a name had a success in period $t$, then the person who continues with this name at period $t + 1$ must be good (or an opportunist who plans to be good). Thus, failures will not be “punished” with low wages due to the inability of clients to update their beliefs, nor will they reduce the value of the name.

This, in turn, will cause bad types to value a good name more than good types. The reason this is true follows from the fact that the value of buying a success name depends on the alternative option of not buying such a name. Bad types face a very bleak future if they start with a new name because they cannot build their own good reputation, while good types can. Therefore, if the stream of payments generated by having a good name is the same (or close) for both bad and good types, then the fact that bad types have a poor alternative will make them value a good name even more than good types. This clearly points to the nature of any equilibrium: It must be that enough bad types buy success names so that clients will sufficiently update their beliefs after a name with a good reputation starts performing poorly.\textsuperscript{26}

This proposition generalizes a result in Tadelis (1999) that identifies two reputational effects. The reputation maintenance effect comes from the fact that good types are more likely to maintain a good name than bad types are. This allows good types to reap benefits over a longer period of time (on average), which in turn gives them a higher willingness to pay for a good name than bad types would have. The reputation start-up effect is that good types can build a good name of their own while bad types cannot. Therefore, if a firm’s reputation is hard to depreciate (i.e., failures do not cause a strong enough depreciation of the reputation) then good types will have a lower willingness to pay than bad types –bad types gain more until the name is depreciated.

\textsuperscript{26}This idea is also robust to $1 > P_G > P_B > 0$ (so that bad types can succeed). If $P_B > 0$ then the algebraic expression of Bayes Rule will be more cumbersome but the driving forces would remain in place – good names would depreciate too slowly if only good types bought them. The relevant way to treat opportunistic agents in this case is by choosing $e \in [0, 1]$ and yielding a probability of success equal to $P(e) = eP_G + (1 - e)P_B$. 

29
The additional moral hazard in the model of this paper does not alter these effects but instead shows their generality. The market-based approach suggested here naturally lends to the alternative of buying a good name, which is building one.\textsuperscript{27} The value of these two alternatives depends on the correct beliefs that clients have with respect to which are (in proportions) choosing which alternatives, and in equilibrium the dynamics of Bayes’ rule have to be consistent with the market values of these alternatives. That is, good names must depreciate enough after they fail, and in turn new names will gain in value after a success is realized. The dynamics, which arise uniquely in the model suggested here, are missing as a prediction of the standard repeated game approach to modelling reputation.

8 Discussion

This papers suggests that the anonymous separation of entity from identity, and the market for names it entails, help provide incentives to mitigate moral hazard. When names can change hands without clients’ awareness, the latter must constantly update their beliefs about the type of agent who is running the firm. Furthermore, their updating must be confined to a sensible rule: good performance causes higher expectations, while bad performance causes lower expectations. These dynamics cause good reputations to have value, which in turn give agents incentives to maintain a good reputation throughout their career and realize this value.

This suggests that a regulatory action that increases the amount of information by making the event of a name transfer public information can be harmful. If trade in names is public information then the market for names can collapse, thereby destroying the endogenous incentives created by this market.\textsuperscript{28} Furthermore, any friction in the market for firm names will create a gap between career concerns of young agents

\textsuperscript{27}Indeed, a similar set of forces is identified by Mailath and Samuelson (2000) who, as mentioned earlier, investigate a repeated game model with incomplete information and imperfect monitoring. They show that when a reputation is “too good” it is more likely to be bought by a bad type, for otherwise beliefs would be inconsistent with equilibrium. Given the game-theoretic, partial equilibrium approach they use, they need to exogenously assume that good types have a better outside option than bad types. This is a natural assumption, and in the model presented here this is derived endogenously from the general equilibrium analysis.

\textsuperscript{28}As for the conclusion that revealing information might destroy social value, recall Hershleifer’s (1971) seminal paper, which shows that private information acquisition may destroy markets for socially efficient risk sharing. In this paper, information disclosure will not have any effect on risk sharing which is irrelevant here, but may destroy markets that provide incentives.
and name-reputation concerns of old ones. Thus, policies that impose asymmetry between the market for firms' services and the market for firms' names, will create a distortion in the dynamics of career concerns. This insight may be relevant for firm taxation: if taxes on profits (income/corporation taxes) are different from taxes on the sale of a name (capital gains taxes), then this will affect the dynamic allocation of effort.

As mentioned in the introduction, an important difference between the model in this paper and standard models of reputation is in the general competitive equilibrium approach employed here. This is key in deriving two intriguing results. First, young and old agents face the same incentives created by the market. The two markets in this model – one for services and one for names – are linked: good names are scarce, and thus their price captures their full value, which is the wage differential they generate in the market for services. Second, reputations cannot fully sort good agents from bad ones. Models of reputation that use a partial equilibrium repeated games approach, such as Kreps (1990) or Klein and Leffler (1981), show that good reputations support good behavior. If one would try to take this argument a step further, it is possible to conclude that good reputations will be valued more by agents who intend to be good, either by characteristic (type) or by choice (action). Using a general equilibrium analysis, however, this paper shows that such separation is impossible. The value of a reputation depends on the updating that clients perform, which depends on the types of agents that are buying these reputations. If "too many" of the agents buying good names are expected to perform well, then failure causes weak updating of client beliefs. This in turn causes good reputations to be more valued by agents who cannot perform well, because the alternative of starting with a clean record is rather bleak for them.

The empirical relevance of this paper is immediate to small owner-operated firms. It seems, however, that the forces identified here may apply to more complex organizations. For example, the results may imply that key figures in an organization should have a stake in the organization's future reputation. Gibbons and Murphy (1992) show that the loss of career concerns of managers close to retirement can be supplemented by explicit compensation contracts. They support their theoretical results with empirical analysis, yet the strength of explicit incentives (a share of current profit) that is observed is remarkably low. Since future, and not current profits are at the heart of reputational incentives, it may be wise to compensate managers with stock options that have future expiration dates. In a recent survey article, Murphy (1999) indicates that top executive compensation has a large component of long-term
stock options. This idea complements Fama’s argument: competitive forces in the market for managerial labor alleviate the moral hazard problem, but for this to hold there must be a stake in the future of the firm, beyond a manager’s finite career. This can be done by allocating some of the value of a firm’s name to managerial labor.

In organizations with many agents, there may be a free rider problem associated with maintaining a good name. In their seminal paper, Alchian and Demsetz (1972) raise an important question related to free riding in an organization: “One method of reducing shrinking is for someone to specialize as a monitor to check the input performance of team members. But who will monitor the monitor?” (pp. 781-2). They suggested that the monitor is provided with correct incentives when he is the residual claimant to the team’s profits. The argument is that market forces will cause the monitor to internalize the social costs and benefits of monitoring. But monitors do not have finite life-spans, implying that monitors should lose incentives as their productive horizon come to its end. This paper suggests that the residual claimant to a firm’s profits should also be the residual claimant to the value of its name, thus internalizing the full current and future value of his monitoring efforts.

It is interesting to note that casual empiricism suggests that modern capitalist economies have legal systems that support the separation of entity from identity, and facilitate well functioning markets for names. These legal systems identify names as proprietary assets so that property rights are well defined and owners of names can capture their value via a market. The analysis of this paper suggests that without such property rights and markets for names, incentives are eroded as agents approach retirement. Therefore, such legal systems are indeed beneficial from an efficiency perspective since the market for firms’ names is complementary to both product and labor markets in a well functioning capitalist economy.
Appendix A: Proofs

proof of Proposition 1: First, let \( c'(0) \geq \Delta w_B \) and assume that all \( O \)-types choose \( c = 0 \) in the first period. In this case clients beliefs are \( \Delta w = \Delta w_B \), and no \( O \)-type has an incentive to increase \( c \) since \( c'(0) \geq \Delta w_B \) and \( c''(e) > 0 \), implying that this is an equilibrium. If \( c'(0) \geq \Delta w_B \) then no other equilibrium can be sustained since any effort level \( \hat{e} > 0 \) implies a lower value \( \Delta \hat{w} < \Delta w_B \), in turn implying that \( c'(\hat{e}) > \Delta \hat{w} \), thus causing all \( O \)-types to lower effort. Similarly, if \( c'(1) \leq \Delta w_G \) then the unique equilibrium has all \( O \)-types of generation 1 choosing \( c = 1 \) in the first period. Now assume that neither of the weak inequalities above are satisfied, that is, both \( c'(0) < \Delta w_B \) and \( c'(1) > \Delta w_G \) hold. Neither \( e = 1 \) nor \( e = 0 \) can be supported in equilibrium due to the \( O \)-type’s FOC. From (1) and (2) it is easy to see that \( \Delta w \) is continuous and decreasing in \( e \), implying that there exists some \( e \in (0, 1) \) such that \( c'(e) = \Delta w \), which is an equilibrium. Since \( c'(e) \) is increasing and \( \Delta w \) is decreasing in \( e \), this is the unique equilibrium. \( Q.E.D. \)

proof of Proposition 2: First observe that if there is an equilibrium with no trade in names then all \( O \)-types must choose \( e = 0 \) in the first period: Assume that there exists an equilibrium in which no names are traded at \( t = 1 \). This implies that the value of a name with a past success must be zero since the supply of \( S \) names is positive and is equal to the measure \( \gamma P_C + (1 - \gamma - \beta) e P_C \) (the agents of generation 0 who have succeeded and who exit the economy for retirement). This in turn implies that \( w_1(S) \leq w_1(N) \), since otherwise \( S \) names would be valuable to agents at \( t = 1 \) (since owning such a name would generate a higher wage from clients compared to having a name with no past). Consider the \( O \)-types of generation 0 in the first period. Their wage at \( t = 0 \) does not depend on their action, and since names are not traded it is a dominant strategy to choose \( e = 0 \). Now consider the \( O \)-types of generation 1 in the first period. Their wage at \( t = 0 \) also does not depend on their action, but their performance can affect their wage at \( t = 1 \). However, since \( w_1(S) \leq w_1(N) \), such an agent cannot lose from changing his name in the second period, which implies that choosing \( e = 0 \) in the first period is a dominant strategy. This establishes that all \( O \)-types must choose \( e = 0 \) in the first period. Thus, under the no trade presumption the model is reduced to a two-type model (all \( O \)-types are bad). Furthermore, if names are not traded then Assumption A4 implies that \( \Pr[S|S] = P_C \) and since \( 1 - \gamma > 0 \) it is always true that some new agents are either bad or opportunistic so that \( \Pr[S|N] < P_C \). This in turn implies that \( w_1(S) > w_1(N) \). But then, any agent who has no past will be willing to pay a positive price for a \( S \) name since it has value, contradicting no-trade as an equilibrium. \( Q.E.D. \)

proof of Lemma 1: Assume first that some agents strictly prefer buying a \( S \) name to not buying one. Observe, however, that the only effect a name has for agents at \( t = 1 \) is to increase their wages relative to the wage of an agent without a name, and this effect is identical for all types. So, if some agents prefer buying a \( S \) name then all agents entering the economy at \( t = 1 \) would share these preferences. But the measure of new agents is 1 while the measure of supplied \( S \) names is \( |\gamma + e(1 - \gamma - \beta)| P_C < 1 \) which creates excess demand, in turn causing

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20Assumption A4 guarantees that \( \Pr[G|S] = 1 \) when no names are traded. This assumption helps rule out “bad” equilbria of the following form: All agents attempt to abandon their name after the first period, and in the second period clients believe that a firm with any history is worse than the average firm with no track record. Since a proportion \( \varepsilon \) of the agents will not be able to abandon their name, these beliefs cannot be sustained in equilibrium for the following reason: No new agent would choose to buy any name, and all agents would attempt to lose their name. But, a proportion \( \varepsilon \) of the population will have their name stick to them, which in turn implies by Bayes rule that \( \Pr[G|S] = 1 \).
the price of a $S$ name to rise. Therefore, in equilibrium no type of agent can strictly prefer to buy an $S$ name. From Proposition 1 we know that trade of $S$ names must occur, and the only price that guarantees indifference between buying and selling is $v(S) = w_1(S) - w_1(N)$. This concludes the proof of this lemma. Q.E.D.

Proof of Proposition 3: In equilibrium O-types of generation 1 set $e$ to satisfy $c'(e) = (w_1(S) - w_1(N))P_G$, and O-types of generation 0 set $e$ to satisfy $c'(e) = v(S)P_G$. But from Lemma 5, $v(S) = w_1(S) - w_1(N)$ which concludes the proof of this lemma. Q.E.D.

Proof of Proposition 4: Market clearing must be satisfied in any equilibrium, thus $(\mu, \rho, e)$ must satisfy (3). Also, it must be the case that in equilibrium $v(S) \geq 0$, or equivalently, $w_1(S) - w_1(N) \geq 0$ (otherwise no agent would buy a $S$ name). Since $w_1(h) = \Pr\{G|h\} \cdot P_G$, then using (4) and (5) above this inequality can be written as,

$$\frac{\gamma P_G + \gamma \mu}{2\gamma P_G + 2(1 - \gamma - \beta)eP_G} \geq \frac{2\gamma - \gamma P_G - \mu \gamma}{2 - 2\gamma P_G - 2eP_G(1 - \gamma - \beta)},$$

which after rearranging becomes,

$$\mu \geq (2\gamma - 1)P_G + 2eP_G(1 - \gamma - \beta).$$

Denote the RHS of (14) by $\bar{\mu}$. Observe that,

$$\bar{\mu} = P_G(2\gamma - 1 + 2e(1 - \gamma - \beta))$$

$$< P_G(2\gamma - 1 + 2(1 - \gamma - \beta))$$

$$= P_G(1 - 2\beta)$$

Therefore, the first (weak) inequality follows from $\bar{\mu}$ increasing in $e$, and the second (strong) inequality follows from $\beta > 0$. Define $\hat{\theta} = \max\{0, \bar{\mu}\}$, and note that to satisfy $v(S) \geq 0$ it must be that $\mu \geq \hat{\theta}$. Now, let $\hat{\mu}$ be the proportion of $G$-types that are needed to clear the market with no bad types buying $S$ names at $t = 1$, which is derived from market clearing as follows,

$$\hat{\mu} = P_G \left[1 + e \left(\frac{1 - \beta}{\gamma} - 1\right)\right].$$

Notice that $\hat{\mu} \geq P_G$ since $\frac{1 - \beta}{\gamma} \geq 1$. Also, for certain parameter values (e.g., $\gamma < 1 - \beta$) we can have $\hat{\mu} > 1$, which means that only good types buying $S$ names cannot clear the market. Define $\underline{\hat{\mu}} = \min\{\hat{\theta}, 1\}$. The interval $[\hat{\theta}, \underline{\hat{\mu}}]$ is non-empty which follows from $\bar{\mu} < P_G$ and $\hat{\mu} \geq P_G$. Thus, if $(\mu, \rho, e)$ is an equilibrium then it must be that $\mu \in [\hat{\theta}, \underline{\hat{\mu}}]$, that $(\mu, \rho)$ satisfy market clearing, and that $e$ is a best response of the O-types in the first period which satisfies $c'(e) = \Delta wP_G$. The converse follows immediately: if $\mu \in [\hat{\theta}, \underline{\hat{\mu}}]$, $(\mu, \rho, e)$ satisfy market clearing and $e$ satisfies $c'(e) = \Delta wP_G$ for the O-types in the first period, then the wages and prices generated by correct beliefs will constitute an equilibrium, and $e$ is a best response. Q.E.D.

Proof of Proposition 5: (i) Suppose in negation that O-types choose $e = 0$. The worst equilibrium for incentives in the first period is the one in which $\mu = \hat{\theta}$ since this supports the lowest possible price of a name (as well as the lowest wage premium). If $e = 0$ and $\gamma < \frac{1}{2}$ then the
analysis of proposition 4 implies that $\delta = 0$, and from (4) and (5) above, the corresponding
wages for this equilibrium are:

$$w_1(S) = \frac{P_G}{2}, \text{ and } w_1(N) = \frac{P_G(2\gamma - \gamma P_G)}{2(1 - \gamma P_G)},$$

which together with Lemma 1 imply that the market price of a name is,

$$v(S) = \frac{P_G(1 - 2\gamma)}{2(1 - \gamma P_G)}.$$  

The choice $e = 0$ is a best response if and only if $c'(0) \geq P_Gv(S)$, but this condition is not satisfied by the assumption of the proposition, a contradiction. Thus, $O$-types must choose $e > 0$ under these conditions.

(ii) Let $c'(1) \leq \frac{\gamma P_G^2(2\beta - 1)}{2(1 - \beta)(1 - P_G + \beta P_G)}$ and $\beta > \frac{1}{2}$. To confirm that all $O$-types choosing $e = 1$ in the first period is an equilibrium, consider the worst equilibrium for incentives in which $\mu = \delta$. The analysis of Proposition 4 implies that $\delta = 0$ when $\beta > \frac{1}{2}$ and $e = 1$. From (4) and (5) above, the corresponding wages for this equilibrium are:

$$w_1(S) = \frac{\gamma P_G}{2(1 - \beta)}, \text{ and } w_1(N) = \frac{\gamma P_G(2 - \gamma)}{2(1 - P_G + \beta P_G)},$$

which together with Lemma 1 imply that the market price of a name is,

$$v(S) = \frac{2\gamma \beta P_G - \gamma P_G}{2(1 - \beta)(1 - P_G + \beta P_G)}.$$

The choice $e = 1$ is a best response if and only if $c'(1) \leq P_Gv(S)$, which is the assumption of the proposition. To see that under this condition no positive measure of $O$-types would choose $e < 1$ in equilibrium, observe that if the beliefs of clients are that some positive measure of $O$-types choose $e < 1$ in the first period, then by Bayes rule $w_1(S)$ must be higher and $w_1(N)$ must be lower than their values in the equilibrium above, making it a dominant strategy for all $O$-types to choose $e = 1$. Therefore, if $c \leq \frac{\gamma P_G^2(2\beta - 1)}{2(1 - \beta)(1 - P_G + \beta P_G)}$ and $\beta > \frac{1}{2}$ then all $O$-types must choose $e = 1$ in all equilibria. $Q.E.D.$

proof of proposition 7: If in equilibrium only $S$ names are traded then all other names are never better than a new name (which is confirmed in Appendix B). Thus, the utility an agent who succeeds with probability $P$ gets from buying a $S$ name is

$$u_i(S) = w(S) + P_i w(S) + (1 - P_i) w(N),$$

whereas his utility from not buying a name is,

$$u_i(N) = w(N) + P_i w(S) + (1 - P_i) w(N).$$

Thus, the value from having a $S$ name under the assumption that all other names are not traded is $\Delta u_i = w(S) - w(N)$. Since all agents – regardless of their type – get this benefit, then there is scarcity of names and the price of a $S$ name will be $v(S) = w(S) - w(N)$, which in turn proves the proposition. $Q.E.D.$

proof of proposition 8: Assume in negation to the proposition that only $S$ names are traded and are bought only by $G$-types. If $M$ names are not traded then $\Pr\{G|M\} = 1$, which follows
because such a history can belong only to a good type. This is true because no trade of \( M \) names implies that such a history must belong to an agent who bought a \( S \) name and then failed, and by assumption such an agent must be good. In this case \( w(M) > w(N) \), which in turn implies that \( M \) names must be traded. Thus assume that \( w(M) > w(N) \) and that \( M \) names are traded in equilibrium. The utility that a young good type entering the economy gets out of owning a \( S \) name is,

\[
u_G(S) = w(S) + P_G w(S) + (1 - P_G) w(M) + P_G^2 v(S),
\]

while his utility from not owning a name is,

\[
u_G(N) = w(N) + P_G w(S) + (1 - P_G) w(N) + P_G^2 v(S),
\]

because after failing in his first period such an agent who started with a new name will wish to change his bad (\( F \)) name. The benefit from owning a name is therefore,

\[
u_G(S) - u_G(N) = w(S) - w(N) + (1 - P_G) w(M) - w(N).
\]

Given that the choice of an opportunistic type depends only on \( w(S) - w(N) \) and not on the name he actually has, the preferences (and thus, utility differences) of opportunistic types who choose to be good are identical to good types, and for those who choose to be bad are identical to bad types. Similar to the calculations above, the utility that a young bad type entering the economy gets out of owning a \( S \) name is,

\[
u_B(S) = w(S) + w(M),
\]

while his utility from not owning a name is,

\[
u_B(N) = w(N) + w(N),
\]

and,

\[
u_B(S) - u_B(N) = w(S) - w(N) + w(M) - w(N).
\]

But since \( w(M) > w(N) \) and \( P_G > 0 \) then \( u_B(S) - u_B(N) > u_G(S) - u_G(N) \). That is, \( B \)-types have a larger benefit from owning a \( S \) name at \( t = 1 \), which contradicts the assumption that only \( G \)-types or good \( O \)-types buy \( S \) names at \( t = 1 \) in equilibrium. \( Q.E.D. \)

**Appendix B: Infinite Horizon Analysis**

By Bayes’ rule, similarly to (12) above (at the limit \( \varepsilon = 0 \), a \( M \) name must be generated by a \( S \) name that was followed by a failure and then continues to be active. Such a name can only be generated by young agents who bought a \( S \) name, and then failed and are stuck with their name. Thus,

\[
\Pr\{G|M\} = \frac{G(\pi^0)\gamma(1 - P_G)}{G(\pi^0)(\gamma + \epsilon(1 - \gamma - \beta))(1 - P_G) + G(\pi^0)((1 - \epsilon)(1 - \gamma - \beta) + \beta)}
\]

\[
= \frac{\gamma(1 - P_G)}{1 - \gamma P_G - (1 - \gamma - \beta)\epsilon P_G}.
\]

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Similarly, a $F$ name is generated by a $N$ name that was followed by a failure and then continues to be active. Such a name can only be generated by young agents who did not buy a $S$ name, and then failed and are stuck with their name. Thus,

$$
\Pr\{G|F\} = \frac{(1 - G(\pi^+))\gamma(1 - P_G)}{(1 - G(\pi^+))(\gamma + \epsilon(1 - \gamma - \beta))(1 - P_G) + (1 - G(\pi^+))((1 - \epsilon)(1 - \gamma - \beta) + \beta)}
$$

$$
\Pr\{G|F\} = \frac{\gamma(1 - P_G)}{1 - \gamma P_G - (1 - \gamma - \beta)\epsilon P_G}.
$$

Any other name that is considered to be a $F$ name and is different than the above (by the definition of the reputation mapping) occurs with zero probability and therefore assigning the same beliefs by clients to any such name is valid. (For example, a name that has a history consisting of a failure followed by a success.)

Thus, $w(F) = w(M)$, and to verify that an agent who failed will prefer to change names over sticking to a $F$ or $M$ name it must be shown that $w(N) \geq w(F)$. It suffices therefore to show that $\Pr\{G|N\} \geq \Pr\{G|F\}$. Let $\Delta \equiv \Pr\{G|N\} - \Pr\{G|F\}$. Using (13) and (15) above we have,

$$
\Delta = \frac{\gamma(2 - \gamma P_G - \epsilon(1 - \gamma - \beta)P_G - P_G)}{2(1 - \gamma P_G - \epsilon(1 - \gamma - \beta)P_G) - \gamma(1 - P_G)} - \frac{\gamma(1 - P_G)}{1 - \gamma P_G - (1 - \gamma - \beta)\epsilon P_G}
$$

$$
> \frac{\gamma(2 - \gamma P_G - \epsilon(1 - \gamma - \beta)P_G - P_G)}{2(1 - \gamma P_G - \epsilon(1 - \gamma - \beta)P_G)} - \frac{\gamma(1 - P_G)}{1 - \gamma P_G - (1 - \gamma - \beta)\epsilon P_G}
$$

$$
= \frac{\gamma P_G(1 - \gamma - \epsilon(1 - \gamma - \beta))}{2(1 - P_G(\gamma + \epsilon(1 - \gamma - \beta)))} > 0.
$$
References


Murphy, Kevin J. (1999) “Executive Compensation,” mimeo, University of Southern California.
