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Invention under Uncertainty and the Threat of Ex Post Entry

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Invention under uncertainty and the threat of ex post entry*

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Abstract

This paper proposes a theoretical framework for studying radical innovation—the invention of new products—when demand is uncertain. In this framework, under general conditions, the threat of ex post entry by a competitor can deter invention ex ante. Asymmetric market power in the ex post market exacerbates the problem. The implications of these general results are examined in a series of examples that represent important markets in the computer industry. The first is a model that shows how an operating system monopolist, by its mere presence, can deter the invention of complements, to its own detriment as well as that of society. The implications of policies such as patent protection, price regulation, and mandatory divestiture are considered. Three additional examples consider the ability of a monopolist in one market to commit to bundling an unrelated product, a pair of horizontally differentiated firms that can add a new feature to their products, and a platform leader that can be challenged in its base market by the supplier of a complementary product.

JEL Classifications: L12 (Monopoly; Monopolization Strategies), L13 (Oligopoly and Other Imperfect Markets), O31 (Innovation and Invention: Processes and Incentives)

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1 Introduction

This paper addresses the question of whether the threat of ex post entry by a strong competitor deters invention when demand is unknown ex ante. The invention of new products implies uncertainty, whether over technological feasibility or over the magnitude and character of market demand. I model the uncertainty on the demand side: opportunities to invent new products exist, and the first firm to invest in developing a new product reveals demand once it begins selling.[1] I develop a basic two-period framework for studying the problem (Section 2), and show that invention is foregone when a high–entry cost firm free rides on the demand information revealed by a low–entry cost first mover. Because the first mover cannot internalize this informational externality, some invention that may be both privately and socially beneficial is foregone due to the threat of ex post entry by the high entry cost firm. Though symmetric firms may also deter each other from investing, the problem is exacerbated when a high–entry cost firm has an ex post selling advantage.

Though the framework and its results are general, the clearest applications lie in the computer software industry. The invention of software products is fraught with uncertainty, and complementarities among markets create asymmetries among firms. I apply the framework first to a model (Section 3) motivated by Microsoft's dominance over markets for both software operating systems and a range of complementary software applications. The model considers a base system monopolist that also competes in a complementary application market. The monopolist owns a selling advantage since consumers must purchase an operating system in order to use the application. In this setting, the base system monopolist, by its mere presence, depresses the invention of complements to its own detriment. Since this model relates to the fundamental asymmetry presented by Microsoft, I give it the most attention. In Section 4, I adapt the basic framework to three other examples of competition common in software markets: the bundling of unrelated applications, as Microsoft bundles its Office applications; the addition of a new feature to horizontally differentiated products; and an example inspired by the "divided technical leadership" hypothesis of Bresnahan and Greenstein [7], in which the salient threats to a platform leader are complementors rather than new entrants. Section 5 discusses the results and review the related literature.

The main result—that invention is foregone due to the threat of ex post entry—depends on two key forces. The first is the uncertainty inherent in radical innovation, which puts a wedge between ex ante and ex post incentives. The second is entry cost heterogeneity across firms, which implies that some firms may have an incentive to enter an empty product

[1] In this paper I use "development", "investment", and "entry" as synonyms; "invention" and "innovation" refer to the first act of investment in a particular product category.

market while other firms will not enter until the level of demand has been revealed. A third force, asymmetry in the ex post selling stage, amplifies the problem because a high entry cost firm with a selling advantage has a greater incentive to free ride on the information externality that is generated when a low entry cost firm reveals demand.

In software markets, uncertainty takes two primary forms: technological uncertainty and demand uncertainty. Technological uncertainty stems directly from complexity. Although it may be simple to determine in theory whether a program can be written to accomplish a task, the amount of effort and coordination among programmers required to write the program is unknown. Demand uncertainty also has its source in the inherent complexity of computer software—potential consumers for a product that does not exist may have difficulty perceiving or articulating their needs for the product. And even after purchasing a software product, they may engage in what Bresnahan and Greenstein [6] term "co-invention": the process of discovering how the software's capabilities interact with existing organizations and habits and of applying the software to uses not foreseen by its inventors. In this paper all the uncertainty is on the demand side, but a similar result would obtain if demand were known but the technological uncertainty faced by an early entrant were more severe than the uncertainty faced by subsequent entrants.

Cost heterogeneity across firms is a fundamental consideration in a dynamic theory of industrial organization, and assuming homogeneous entry costs is often highly restrictive. In the model of Section 3, for example, without entry cost heterogeneity there would be no reason for a base system monopolist to ever decline to enter a promising complementary application market, and as a consequence there would be no reason for an independent firm to ever contemplate entry. Firms might face substantially different entry costs for any of a host of reasons. There might be legal constraints, such as patents or antitrust concerns; there might be history-dependent effects, such as learning-by-doing or unique human capital retained by a particular firm; there might be consequences of past decisions regarding internal organization, financing, or employee compensation that are costly to change. Perhaps most importantly, firms of different sizes face different problems of providing their employees with incentives.

Asymmetry in the ex post selling stage can arise due to complementarity: a firm with a monopoly over an existing product can enter a complementary market with a low price, yet still profit by the accompanying increasing in demand for its existing product. In such cases, the monopolist's ex post advantage may be self-defeating ex ante, because it deters the invention of complements. Asymmetry can also arise as a consequence of substitutability: if a newly introduced product is complementary to a substitute for a firm's existing product, the firm may be forced to defend itself by developing a competing product.

The problem of foregone invention is fundamentally one of appropriability: when a firm cannot guarantee itself a monopoly over its invention, its incentives to invent are reduced. To solve the appropriability problem, governments commonly employ patent systems that guarantee a monopoly to each inventor. However, patents are not a panacea because they are desirable only when inventions would not otherwise be undertaken, and undesirable otherwise. In the models I present, under the same parameterizations for which there exist entry cost profiles that leave privately profitable opportunities unexploited, there are also cost profiles for which one or more firms undertake invention even in the absence of patents. More desirable would be a finely tuned patent system that awards a patent only when necessary to induce invention, but the information requirements of such a system are heavy: it would need to know the prior distribution of demand as well as the cost structures of all potential entrants. I discuss the applicability of patents briefly, but in the computer software industry patents are often ruled out as a matter of law (entire applications are usually not patentable) or because they conflict with patents for pre-existing technologies.

2 The basic framework

The basic framework illustrates the problem of foregone invention in a simple game that can be mapped onto a variety of interesting situations. The goal is to characterize, in terms of intuitive assumptions, situations under which privately profitable invention is not undertaken in equilibrium. The first step, after introducing the model, is to solve for the subgame perfect equilibria. This is accomplished in Lemma 1, which gives the conditions for foregone invention in terms of the payoff function. Next I restrict the payoff function according to some appealing assumptions, from which we learn for which entry cost profiles the threat of ex post entry may lead an individual firm to refrain from entering a market that it would otherwise find profitable. Applying this lesson to both firms yields the main result, Proposition 1, which states gives the conditions under which neither firm enters a potentially profitable market. Proposition 1, although it does not require asymmetry, indicates that asymmetry in the ex post market exacerbates the problem.

The framework consists of a two-period game, $t \in \{1, 2\}$, between two firms. There is an empty potential market with unknown demand, and either firm $j \in \{M, X\}$ may enter the market after expending an idiosyncratic entry cost $c_j \geq 0$. Firm j's entry decision in period t is written as in_j^t or out_j^t. Demand in the market is revealed to both firms once any firm enters. Entry decisions are made simultaneously at the start of each period; exit is not considered and entry costs cannot be recovered. In the basic framework, instead of modeling price and quantity decisions explicitly, I take per-period profits as the primitive element.[2] (The examples in later sections model price and quantity decisions in detail.) Let Ω be the universe of possible levels of demand, and let $F(\omega)$ be the probability distribution over demand. The payoff for firm j in period t given demand $\omega \in \Omega$ is indicated by $\pi_j^t(s,\omega)$, where s is the *market structure*, or the set of firms that have already entered. Rather than write s in set notation, I adopt the shorthand $s \in \{\emptyset, M, X, MX\}$ for brevity. In the absence of entry ($s = \emptyset$), ω is irrelevant, so the payoff for firm j is written simply as $\pi_j^t(\emptyset)$. All these details—including c_M and c_X—are common knowledge.

The extensive form of this game is shown in Figure 1. The probabilistic nature of ω is indicated by the choice of "Nature," which chooses at the nodes marked "N". Note that if neither firm enters in the first period, then neither of them is allowed to enter in the second period. This restriction is not crucial, but it aids in applying the results of this finite horizon game to models with infinite horizons, as in the later sections.

The subgame perfect equilibria are found by backward induction, and the outcomes are parameterized by c_M and c_X. Let $s_j^*(\omega, c_j)$ indicate the market structure that results from Firm j's sequentially optimal choice after Firm $-j$ enters and reveals ω:

$$s_j^*(\omega, c_j) = \begin{cases} MX & \text{if } \pi_j^2(MX,\omega) - c_j > \pi_j^2(-j,\omega) \\ -j & \text{otherwise} \end{cases} \tag{1}$$

The following lemma gives the conditions under which neither firm enters the empty market.

Lemma 1 *There exists a subgame perfect equilibrium in which neither firm enters the empty market if and only if*

$$E\left[\pi_j^1(j,\omega) + \pi_j^2\left(s_{-j}^*(\omega, c_{-j}), \omega\right)\right] - c_j \leq \pi_j^1(\emptyset) + \pi_j^2(\emptyset) \tag{2}$$

for both j. Furthermore, this is the unique subgame perfect outcome if and only if the inequalities in (2) are strict and also, for some j,

$$E\left[\pi_j^1(MX,\omega) + \pi_j^2(MX,\omega)\right] - c_j < E\left[\pi_j^1(-j,\omega) + \pi_j^2(-j,\omega)\right] \tag{3}$$

Proof. By backward induction. ■

Lemma 1 requires no structure on the profit functions, but to model realistic situations it helps to impose some assumptions on the structure of profits. The following assumptions are maintained throughout this section and are satisfied by all the examples in later sections.

Assumption 1 (Relevance) $E[\pi_j^1(j,\omega) + \pi_j^2(j,\omega)] > \pi_j^1(\emptyset) + \pi_j^2(\emptyset)$ *for all j.*

[2] Differences in profits across time do not play a crucial role in the basic framework. However, expressing the profits as a function of time aids in mapping the framework onto applications.

Assumption 2 (Common demand) $\Omega \subset [0,1]$ *and* $\pi_j^t(s,\omega)$ *is increasing in* ω *for all* $j \in s$.

Assumption 3 (Competitiveness) $\pi_j^t(j,\omega) > \pi_j^t(MX,\omega)$ *for all* j*, all* t*, and all* $\omega > 0$.

Assumption 4 (Positive profits) $\pi_j^t(MX,\omega) > \pi_j^t(-j,\omega)$ *for all* j*, all* t*, and all* $\omega > 0$.

These assumptions define a class of interesting situations. If *relevance* holds, then the empty market offers enough profit potential that each firm would enter if it had sufficiently low entry costs and knew that it would gain a monopoly. If *common demand* holds, then demand is characterized by a single parameter and an increase in that parameter leads to higher profits for every firm in the market. If *competitiveness* holds, then each firm prefers to monopolize the market rather than share it with a competitor. *Positive profits* means that revenues for any firm in the market will always cover its operating costs, so that the act of entry risks at most the entry costs.

An additional definition formalizes the idea that a firm faces a disadvantage in the ex post market.

Definition 1 ***J-submodularity*** *is satisfied if, for all* t*,*

$$E\left[\pi_j^t\left(MX,\omega\right) - \pi_j^t(-j,\omega)\right] < E[\pi_j^t(j,\omega)] - \pi_j^t\left(\emptyset\right) \tag{4}$$

X-submodularity (for example) means that Firm X perceives Firm M's investment as a strategic substitute ex ante. That is, Firm X gains less from investing when Firm M also invests than it gains from investing when Firm M does not invest. If Firm X's profits when it does not enter the market do not depend on Firm M's action (i.e., $\pi_X^t\left(M,\omega\right) = \pi_X^t\left(\emptyset\right)$ for all ω and all t), then X-submodularity is implied by competitiveness. Otherwise, X-submodularity implies that Firm M's entry does not depress Firm X's profits (e.g., in some related market) too much when Firm X does not invest. J-submodularity can be satisfied for both firms, but in the results that follow it need only be satisfied for at least one firm—which I name Firm X.

Under X-submodularity, there are situations in which Firm X does not enter a promising market. Define $\hat{c}_j$ as the entry cost at which firm j is exactly indifferent between investing and not investing in the absence of firm $-j$:

$$\hat{c}_j \equiv E[\pi_j^1(j,\omega) + \pi_j^2(j,\omega)] - \pi_j^1\left(\emptyset\right) - \pi_j^2\left(\emptyset\right) \tag{5}$$

If Firm M were for some reason unable to enter the market, then at entry cost $\hat{c}_X$ Firm X would be indifferent between entering and not entering. Since Firm M is able to enter ex post, there exists a threshold entry cost $\tilde{c}_M > 0$ below which it will indeed enter with positive probability once demand is revealed. Then Firm X's strict best response given entry cost $\hat{c}_X$ is not to enter. X-submodularity means that Firm X has less incentive to enter when Firm M enters, so if Firm M were to enter ex ante then Firm X's strict best response given $\hat{c}_X$ also would be not to enter. Since these strict best responses also hold in a neighborhood around $\hat{c}_X$, there are costs $c_X < \hat{c}_X$ at which, given $c_M < \tilde{c}_M$, Firm X does not enter in any subgame perfect equilibrium even though it would strictly prefer to enter given $c_M > \tilde{c}_M$. Lemma 2, in the Appendix, makes this argument formally.

Combining the results for the individual firms reveals the conditions under which neither firm enters a promising market. It is helpful to visualize each firm's optimal first period entry decision as a function of its entry cost profile, as is shown in Figure 2A for Firm X. The region in which Firm X does not enter the market solely due to the possibility of entry by Firm M covers intermediate values of c_X and low values of c_M. Overlaying the regions in which Firm M and Firm X do not enter in the first period, as is shown in Figure 2B, reveals a region of concern in which neither firm enters the market even though at least one firm would enter if it could be assured of a monopoly. I call this the region of *foregone invention*, and in the results that follow I will identify the conditions under which this region has positive measure.[3]

The region of foregone intervention covers an area of intermediate entry costs, because very high entry costs or very low entry costs make the deterrence of invention either difficult or moot. More specifically, if Firm X has very low entry costs it is not likely to be deterred by the prospect of Firm M's entry, while if Firm M has very high entry costs it is less likely to enter and thus poses less of a threat to Firm X. On the other hand, if Firm X has very high entry costs then it is unlikely to enter even without the threat of Firm M's subsequent entry, while if Firm M has low entry costs then it is likely to enter ex ante regardless of whether it deters Firm X. It is quite possible that these effects that operate against foregone invention may overlap, in which case the region of foregone invention does not exist. Figure 2B suggests that the region of foregone invention will have positive measure if the curves that divide entry from non-entry for each firm do not intersect at $(\hat{c}_M, \hat{c}_X)$. In fact this is the case, as is implied by the following proposition.

Proposition 1 *In the basic framework, if X-submodularity holds and*

$$E[\pi_X^2(X,\omega)] > E\left[\pi_X^2\left(s_M^*\left(\omega,\hat{c}_M\right),\omega\right)\right] \tag{6}$$

then the region of foregone invention has positive measure.

[3] Lebesgue measure in $\mathcal{C}_A \times \mathcal{C}_B$, where $c_j \in \mathcal{C}_j = \mathbb{R}_+$.

Condition (6) means that Firm M has an ex post incentive to enter when its entry cost is high enough that it does not enter ex ante. The proof (in the Appendix) uses Assumptions 1–4 and condition (6) to establish that $(\text{out}_M^1, \text{out}_X^1)$ is an equilibrium at entry cost profiles near $(\hat{c}_M, \hat{c}_X)$, while X-submodularity rules out the possibility that $(\text{in}_M^1, \text{in}_X^1)$ is also an equilibrium.[4]

Under what conditions is (6) likely to be satisfied? The left hand side is high when Firm X's monopoly profits in the new product market are likely to be high; the right hand side is low when Firm X's duopoly profits are likely to be low and when Firm M is likely to enter ex post. Thus the condition is more likely to be satisfied if Firm M has an advantage over Firm X in the ex post market, since Firm M is more likely to enter ex post if its duopoly profits are likely to be high. Although there are symmetric conditions that can lead to foregone invention, symmetry increases or decreases both firms' duopoly profits in tandem, leading to counteracting effects on the right hand side of (6). But as Firm M grows stronger ex post at the expense of Firm X, ex post entry becomes more likely and Firm X's expected profits fall unambiguously. Hence foregone invention is a more serious problem when ex post market power is asymmetric. In subsequent sections I explore several examples of asymmetric market power, in which the asymmetry derives from one firm's market power in a related market.[5]

In sum, the basic framework provides a convenient characterization of situations in which invention may be foregone. The key conditions that lead to foregone invention are two: one firm must perceive its opponent's ex ante investment as a strategic substitute (j-submodularity), and the other firm must have an ex post incentive enter when it would not enter ex ante (condition (6)). The region (in entry cost space) of foregone invention, if it exists, occupies entry cost profiles with intermediate entry cost levels for both firms. There is more likely to be a significant region of foregone invention when ex post market power is asymmetric, because one firm is more likely to be deterred from investing if the other firm has an ex post advantage. Because they are based on reduced form payoff functions, these insights can be applied to a wide range of market situations, including infinite horizon problems, as shown in the sections that follow.

[4] These are sufficient, but not necessary, conditions for foregone invention. It is possible for invention to be foregone when (6) does not hold, such as when the curves that divide entry from non-entry for each firm intersect at $(\hat{c}_M, \hat{c}_X)$ but also intersect at other points.

[5] There are other possible sources of asymmetry, such as asymmetric operating costs. For example, in a two period model with Cournot competition the following parameterization yields a region of foregone invention with positive measure: demand $Q = \omega(10 - p)$, uncertainty $\omega = 1$ with probability γ and $\omega = 0$ otherwise, and marginal costs of 0 for Firm M and 3 for Firm X.

3 Invention to complement a base system

In this section, I show that a computing platform with a base system monopolist is likely to experience foregone invention. The problem is that the base system monopolist possesses an ex post advantage in markets for complementary products since it can enter a complementary market with a low price yet still benefit from the increased demand for its base system. In this "Base System" model, Firm M has a pre-existing and exogenously protected monopoly in the market for software operating systems, while the empty market is for a software application that requires Firm M's base system in order to run. Firm X is a startup firm that can enter the empty market. The asymmetric complementarity between the base system and the application amplifies the problem of foregone invention, to the detriment of both firms as well as to social welfare.

Following the initial discussion, Section 3.1 introduces the model. Section 3.2 derives the static market outcomes as a function of the market structure. Section 3.3 analyzes the dynamic equilibrium, and Section 3.4 discusses potential private and public remedies.

Although designed to represent the computer software industry, the model applies to any set of markets in which one product (a "base system") is required for the operation of others ("applications") and in which there is potential for the invention of new applications. Gawer and Henderson [17] and Gawer and Cusumano [16] pose an example in the computer hardware industry: Intel, given its dominant position in the market for the microprocessors, has an incentive to enter complementary markets because it can internalize the complementarity. But Intel also benefits when other firms innovate in these areas, because such innovations increase the market for Intel's core products. Intel has developed organizational structures that may help it commit not to "trample all over everybody's business," and it has succeeded in keeping most of its efforts focused on its core business. However, Intel has entered a number of already-occupied complementary markets, including videoconferencing, digital cameras, chipsets, motherboards, home networking, and network adapters. The model in this section gives a theoretical rationale for the contrast between Intel's stated concern for complementors and its inability to fully commit not to behave aggressively toward them.

Of course, another obvious application of the model is to Microsoft's domination of the market for personal computer operating systems and the many applications that can run on top of Microsoft's Windows operating systems. Microsoft's ex post success in the markets for Windows applications—internet browsers, e-mail clients, media players, spreadsheets, word processors, etc.—indicates the strength of its ex post advantage. Since the foregone invention result predicts the non-existence of certain products, this variety of observed products raises the question of whether the model's predictions are falsifiable. However, the steadily falling development costs that characterize computer technology imply that the development costs for any given application will eventually leave of the region of foregone invention. Accordingly, the results in this section present a broad warning that, whatever may be the benefits of standardization on Microsoft's operating systems, a market structure with a base system monopolist poses hazards that can retard radical innovation.

3.1 The model

This model examines a base operating system market and a complementary application market, and two firms involved in these markets. The application market is initially empty, with uncertain demand.

Technology and firms Let $K = \{b, a\}$ be the set of possible products—b for "base system" and a for "application." The base system has already been developed and is produced by Firm M, while Firm X is exogenously barred from the base system market. Either firm may enter the application market by incurring a fixed cost $c_j > 0$. The application requires the base system in order to provide value, but the base system can provide value without the application; i.e., the base system is "essential." One way to interpret this assumption is that it represents the potential for users to program their own software to run on the base system. More broadly, it reflects the ability of the base system to serve as a platform upon which any number of other software products can run.[6]

Markets and consumers Once firms have had the opportunity to develop the application, they simultaneously set prices, produce at zero cost any products that they have developed, and sell to consumers. The set of branded products available for sale to consumers is $s \subset K \times \{M, X\}$, where, for shorthand, $k_j \in s$ is product $k \in K$ sold by firm j at price p_j^k. The set s also represents the "industry structure"—the combination of firms present in each market. For brevity, s will often be written in shorthand as $\emptyset$, M, X, or MX to indicate which which firms have entered the application market.

There is a continuum of consumers with unit total mass, and each consumer i values product k by the amount $\theta_i^k \in [0, 1]$.[7] Each consumer demands at most one of each product

[6] This assumption contrasts with several earlier models in the literature which assume that both products are used in fixed proportions, as in Economides and Salop [14], Matutes and Regibeau [19], Farrell and Katz [15], and Choi and Stefanadis [11] However, when attempting to model Microsoft's base system monopoly, it is important to caputre the asymmetry that is inherent when applications require a base system in order to run. Heeb [18] is a recent theoretical work that shares this assumption.

[7] This implies that applications a_M^1 and a_X^1 are perfect substitutes. Adding a measure of product differ-

in any period. Consumer i's valuations for the two products are written as a vector $\theta_i \equiv (\theta_i^b, \theta_i^a) \in \Theta \equiv [0,1]^2$. In each period, consumer i purchases a basket of branded products $s_i \subset s$ to maximize quasilinear utility: $\sum_{k_j \in s_i} (\theta_i^k - p_j^k)$. Since a consumer must own the base system in order to make use of an application, utility maximization is subject to the constraint that if s_i is non-empty then it must contain b_M. θ^b is distributed uniformly on $[0,1]$, while, prior to the first period, demand in the application market is unknown. With probability γ, θ^a is distributed uniformly on $[0,1]$; otherwise $\theta^b = 0$ for all consumers.

Dynamics The profits for each firm in each type of market constitute a payoff function that can be plugged into the basic framework. This is accomplished by looking at the problem through the lens of an infinite period model[8] in which the firms share a discount factor $\delta \in (0,1)$, and letting the second period of the basic framework summarize the discounted profits in periods $2, \ldots, \infty$. Since $s = MX$ is an absorbing state, the equilibrium can still be found by backward induction. As in the basic framework, each period t consists of two stages, an investment stage and a selling stage. Once any firm enters the application market, demand is revealed to all. Though the context is dynamic, I assume that market pricing reflects the static Nash equilibrium of a simultaneous pricing game. Collusive pricing is considered in Section 3.4.1.

The state vector at the start of each period consists of the sets of products that have been developed by the firms together with the common prior distribution over demand for the application. The state can be summarized in shorthand (and with some abuse of notation) by the pair $\langle s, \omega \rangle$, where $s \in \{\emptyset, M, X, MX\}$, $\omega = 1$ if demand for the application is known to be high, $\omega = 0$ if demand for the application is known to be low, and $\omega = \gamma$ if demand for the application is yet unknown.

3.2 Static prices and profits

This section characterizes the static market equilibrium of the Base System model under the various industry structures that can arise. These static outcomes enable us to analyze the dynamic equilibrium in Section 3.3. The paragraphs that follow display the static equilibrium prices and profits for each firm under each industry structure in turn. The derivations of these static equilibria are straightforward and thus omitted. Consumer choices are shown in Figures 3A–D, where $\Theta = [0,1]^2$ is divided into regions according to which consumers purchase which baskets, with θ^b on the horizontal axis and θ^a on the vertical

entiation could quantitatively change the problem of foregone invention, but would not alter the qualitative conclusions.

[8] The infinite horizon provides stationary payoffs, whereas a two period model would force payoffs in the second period to be perceived differently from the perspectives of the first period and the second period.

axis. Observe that whenever Firm M offers the application, assuming that it prices its two products separately is without loss of generality, since offering a bundle would be equivalent to setting a price of zero for the application. I call this "trivial bundling" when it arises in equilibrium.

Base system monopoly When there is no application (i.e., ω is either γ or 0), Firm M faces a simple monopoly problem. It sets its price for the base system at $p_M^b = 0.5$, yielding per-period profits of $\pi_M(\emptyset) = 0.25$. Consumer choices are displayed in Figure 3A.

Integrated monopoly When Firm M monopolizes both markets (with $\omega = 1$), it sets prices $p_M^b \approx 0.667$ and $p_M^a = 0.167$, resulting in per-period profits of $\pi_M(M, 1) = 0.546$. Consumer choices are displayed in Figure 3B.

The price for the application is relatively low because the the application makes the base system more valuable, allowing Firm M to price the base system well above the price it would set as merely a base system monopolist. This illustrates how Firm M internalizes the complementarity, yielding benefits for consumers as well as itself.[9]

Bilateral monopoly When Firm M monopolizes the base system market and Firm X monopolizes the application market (with $\omega = 1$), they set equilibrium prices of $p_M^b = 0.586$ and $p_X^a = 1 - p_M^b$. The per-period profits are $\pi_M(X, 1) = 0.343$ and $\pi_X(X, 1) = 0.172$. Consumer choices are displayed in Figure 3C.

The price for the application is higher than under an integrated monopoly because Firm X cannot reap the positive externality that its application provides to Firm M. Thanks to this externality, Firm M can raise the price of its base system relative to the price it would set in the absence of the application, but not to the level it would set as an integrated monopolist. Both total industry profits and consumer surplus are lower than under integrated monopoly, illustrating the social consequences of Firm X's inability to internalize the complementarity. Despite the essentiality assumption maintained here, the contrast between bilateral monopoly and integrated monopoly in this model follows the same logic as Cournot's 1838 result [12] and other static models such as Economides and Salop [14] and Chen and Ross [10].

Application duopoly When Firm M monopolizes the base system market but competes with Firm X in the application market (with $\omega = 1$), both firms set a price of zero for the

[9]Note that consumer surplus also increases over the base system monopoly, although it is not a Pareto improvement because some consumers who would buy the base system under a base system monopoly are priced out of the market under integrated monopoly.

application, while Firm M sets a price of $p_M^b = 0.816$ for the base system. The resulting per-period profits are $\pi_M(MX, 1) = 0.544$ and $\pi_X(MX, 1) = 0$. Consumer choices are displayed in Figure 3D.

Firm M trivially bundles its application with the base system, forcing Firm X to sell its application at a zero price. By this strategy, Firm M achieves profits nearly as high as if it enjoyed an integrated monopoly. It is akin to the strategy of bundling demonstrated by Nalebuff [21], except that here the bundling arises as a result of competitive pricing in the application market and monopoly pricing in the base system market. This illustrates the power of a base system monopoly. Ordinarily, no firm would enter a pre-existing market with undifferentiated Bertrand competition if in order to do so it would have to pay a positive entry cost. But Firm M benefits from selling the application even at a zero price because by doing so it increases the price of its base system. As with integrated monopoly, both total industry profits and consumer surplus are higher than under bilateral monopoly, so from an ex post perspective Firm M's entry into the application market is both privately and socially desirable so long as its investment costs are not too high.

3.3 Dynamic equilibrium

Although the game is constructed with an infinite horizon, it can be solved by backward induction from the absorbing states. The analysis is equivalent to that of the basic framework. Recall that Proposition 1 gave the conditions for foregone invention in the two period basic framework. Proposition 2 applies that result to this infinite horizon model.

Proposition 2 *In the Base System model, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.679$) then the region of forgone invention has positive measure.*

In order to gain a more economic interpretation of this result, look a little more closely at the conditions that define the region of foregone invention. I first identify three effects that drive Firm M's investment incentives, and then analyze the equilibrium outcome as a function of the entry cost profile. This helps visualize the region of foregone invention and identify two effects that summarize how Firm X responds to Firm M's incentives.

Begin with Firm M's entry rules. When $\omega = 1$, Firm M chooses to invest ex post if the gain from investing more than offsets the fixed investment cost; i.e., it enters if

$$c_M < \frac{1}{1-\delta}\left(\pi_M(MX, 1) - \pi_M(X, 1)\right) \tag{7}$$

Compare this to Firm M's ex ante entry rule when Firm X is unable to invest: if $c_X = \infty$ then Firm M enters if

$$c_M < \frac{\gamma}{1-\delta} \left(\pi_M \left(M, 1 \right) - \pi_M \left(\emptyset \right) \right) = \hat{c}_M \tag{8}$$

Most of the difference between (7) and (8) is due to two effects: First, if Firm X has already invested and revealed high demand, Firm M receives a higher payoff even if it does not invest because the availability of Firm X's application makes Firm M's base system more valuable; i.e., $\pi_M (X, 1) > \pi_M (\emptyset)$. This *complementarity effect* reduces the incentive for Firm M to invest. Second, if Firm X has already revealed that there is high demand for the application, then Firm M's expected payoff from investing is higher, increasing its incentive to invest. This *demand revelation effect* is reflected in the absence of the uncertainty term γ from from Firm M's ex post entry rule (7). A potential third effect, the *competitive effect*, stems from the prospect of competition in the application market. It exerts almost no influence on Firm M's decision because Firm M can achieve almost as much profit by trivial bundling against Firm X as it can by monopolizing both markets; i.e., $\pi_M (MX, 1) \approx \pi_M (M, 1)$. The demand revelation effect swamps the competitive effect except when γ is extremely close to 1. The demand revelation effect outweighs both the complementarity effect and the competitive effect when $\gamma < 0.679$.

To graph the equilibrium outcome, suppose for a moment that $\delta = 0.9$, and consider the two cases $\gamma = 0.8$ and $\gamma = 0.4$. When $\gamma = 0.8$, equilibria in which Firm X and Firm M invest ex ante share a region of overlap. This is illustrated in Figure 4A, where the region of overlap is labeled as the region of first mover advantage.[10] Since $\gamma = 0.8 > 0.679$,there is no region of foregone invention. Note that even if the firm with lower costs is the first mover, there is still productive inefficiency (conditional on some firm investing, it is the high–entry cost firm that invests) over a significant range of entry costs. This is a natural consequence of entry cost heterogeneity combined with Firm M's internalization of the complementarity.

When $\gamma = 0.4$, the regions in which Firm X and Firm M may invest ex ante do not overlap, and a region of foregone invention fills the gap: in equilibrium, neither firm invests even in situations in which investment could be privately profitable under non-equilibrium strategies. This is illustrated in Figure 4B. There are three factors that contribute to the region of foregone invention, all of which are necessary in the Base System model. First,

[10]There are some regions of the parameters in which either Firm M or Firm X, but not both, could invest in pure strategy equilibrium. This implies the existence of an unstable mixed strategy equilibrium. However, this mixed strategy equilibrium is not robust to a small first-mover advantage for either firm. Accordingly, I rule out mixed strategy equilibria and merely indicate that only the first mover will invest, while remaining agnostic regarding the identity of the first mover.

the threat of encroachment: Firm X's entry costs are high enough that it doesn't want to invest if Firm M will subsequently invest (when $\omega = 1$). Second, demand uncertainty: Firm M's entry costs are high enough that it invests only if it knows that $\omega = 1$. Third, for $\gamma = 0.4$ the demand revelation effect outweighs the complementarity and competitive effects, leading Firm M to invest if Firm X has revealed that $\omega = 1$.

In general, the region of foregone invention reaches its maximum area at an intermediate level of uncertainty, $\gamma \approx 0.340$. This occurs for two reasons: First, if demand is less likely to be high, then entry would be relatively unlikely even if foregone invention were not a problem, so foregone invention occurs over only a small range of entry costs. Second, if demand is more likely to be high (but still $\gamma < 0.679$), then ex post incentives are more in line with ex ante incentives. For any fixed $\gamma \in (0, 0.679)$, the region of foregone invention grows with increasing δ. Although greater patience gives firms a greater incentive to invent new products, it does not help align ex ante incentives with ex post incentives, so the range of entry costs over which invention is foregone increases.

3.4 Strategies to encourage invention

The region of foregone invention is troubling for social welfare: opportunities to expand social welfare by inventing new products go unexploited even though they offer potential for private profit. It is also troubling for Firm M, because it means that opportunities to expand its core business go unexploited. In this section I first consider how Firm M might try to encourage the invention of complements. I then consider the effects of several public policy interventions that a social regulator might consider. I interpret these policies both within the model and in the context of some considerations outside the model.

3.4.1 Private strategies for encouraging invention

There are several strategies that Firm M might pursue in order to attempt to overcome the problem of foregone invention. For example, it can merge with outside inventors like Firm X, it can sponsor a research tournament, it can attempt to collude in the ex post market, or it can attempt to coordinate on a long run equilibrium in which it commits not to enter certain types of markets.

Mergers From both a social standpoint and from the firms' perspective, a situation in which the application is first invented by the firm with lower costs and then marketed by Firm M is usually preferable to a situation in which Firm M invests to enter an application market already occupied by Firm X. When Firm X has lower entry costs, there are two classes of simple contractual remedies: one, Firm M buys Firm X (or the rights to sell its application) ex post; two, Firm M buys Firm X ex ante. I consider each in turn.

Suppose that Firm M can offer to buy Firm X ex post. For the moment, restrict attention to situations in which c_M is low enough that invention may be foregone. Once Firm X has revealed the state of the world as $\omega = 1$, Firm M prefers to buy Firm X rather than pay the investment cost to compete in the application market. However, since Firm M's entry costs are relatively low, it can credibly threaten to enter the application market on its own, giving Firm X an outside option of $\pi_X(MX, 1) = 0$. Firm M's influence over Firm X's outside option leads to a low buyout price, making it difficult for ex post merger to solve the ex ante incentive problem.

When instead Firm M's entry costs are high enough that it will not enter ex post, it offers Firm X a buyout price at least as high as Firm X's outside option, $\frac{1}{1-\delta}\pi_X(X)$. Though this has the helpful effect of increasing the range of c_X over which Firm X will invest when Firm M poses no threat to enter the application market, the prospect of ex post merger actually increases the region of foregone invention, because it increases the maximum entry cost at which Firm X could profit from a monopoly more than it increases the maximum entry cost at which Firm X would invest when Firm M can threaten to enter.

If the firms could merge ex ante, then they could jointly internalize the complementarity and eradicate the problem of foregone invention. There are two important reasons to think that merging ex ante would be problematic, however. First, if entry costs were private information then Firm M would face an adverse selection problem in identifying which firms to choose as merger partners. In return for merging with Firm M, many firms might offer their development services even if their development costs were not low, since after the merger they would be able to "hold up" Firm M and compel it to pay higher development costs than it had anticipated. Second, even if the correct Firm X could be identified, the transaction costs of the merger might outweigh the difference in entry costs between the two independent firms. Such transaction costs could include not only the accounting costs of the merger, but also the effect on the incentives of the software engineers of moving from a small firm to a larger firm.

Research tournaments Firm M could attempt to solve the adverse selection problem posed by ex ante mergers by offering a tournament prize that independent firms could win by developing the application. But, aside from the difficulty of identifying the correct application to invent, writing an appropriate contract for such a tournament would be difficult. Since software applications are complex objects, any contract that does not completely specify the desired product will give the contestants distorted incentives that may lower the quality of the winning application (which can be interpreted as raising the costs that Firm M must incur to perfect the application after paying out the prize). For the specific setting of computer software, the mere act of writing a substantive tournament contract could require accomplishing a significant portion of the development work.

Collusion Firm M and Firm X, having both introduced competing applications, could collude to keep the prices of their applications above zero. Joint profit maximization implies that $p^b_M \approx 0.667$ and $p^a_M = p^a_X \approx 0.167$, yielding per period profits of $\pi_M \approx 0.495$ and $\pi_X \approx 0.051$ if the two firms split the application market equally. However, Firm M can actually earn higher per period profits ($\pi_M \approx 0.544$) by driving the application price to zero. Collusion on positive application prices is possible only if Firm X restricts its output to much less than half of the total number of applications. This, of course, leaves Firm X with little in the way of profits, and offers little inducement to innovate in situations under which Firm M will subsequently enter and collude.

Building a reputation for enabling complements The Base System model considers a single application, but a fundamental characteristic of software operating systems is that their functionality is open-ended. If a new potential application market arises in each period, we might expect there to be an equilibrium in which Firm M does not enter certain complementary markets already occupied by independent firms, because the independent firms will cease innovating if they ever observe Firm M enter a forbidden complementary market. Gawer and Cusumano [16] theorize that Intel has pursued the strategy of attempting to coordinate on such an equilibrium, characterizing its strategy as "communicating commitment to third parties." However, extending the model to allow for new applications each period is problematic because it generates a dynamic game with an infinite state space, for which there is no folk theorem to invoke and for which static prices and profits are difficult to compute.[11]

[11]It can be shown that if total expenditure in the markets for applications and base systems is bounded, then for Firm M cannot commit to enabling complements even if the number of applications is unbounded.

If instead the useful lifetimes of applications are uniformly bounded, then Dutta's folk theorem [13] applies. However, the idea that applications have uniformly bounded lifetimes should be applied with caution, since in the model an "application," which may have a short useful lifetime, should be interpreted as a proxy for an application category, which is likely to have a long lifetime. Once the first application in any category has been invented and revealed demand, subsequent applications in that category are not subject to the same level demand uncertainty.

3.4.2 Public policies for inducing invention

This section considers several potential policy interventions: a property rights approach that gives Firm X a patent if it innovates first, a structural regulation approach that forces Firm M to divest from any applications it invents, and a price regulation approach that prevents Firm M from driving application prices down. Throughout, I assume that the policy planner cannot observe firms' investment costs.

Patent protection The appropriability problem facing Firm X can be completely eliminated by a simple patent policy. Unfortunately, though patent protection eliminates the problem of foregone invention, it poses other problems. In general, since patent protection prevents competition, it can reduce social surplus whenever it is not necessary to induce investment. This phenomenon manifests in at least three specific ways: First, the costs of reduced competition are evident in the Base System model whenever Firm X's entry costs can be so low that Firm M's subsequent entry does not dissuade Firm X from innovating ex ante. In these situations, it is socially desirable ex ante for Firm M to enter a market occupied by Firm X, but a patent policy prevents it from doing so. Second, though the Base System model does not allow for vertical differentiation, a lack of competition can be particularly problematic in markets in which incremental vertical (quality) improvement is important because competition can induce firms to invest more in improving quality. Heeb [18] shows that Firm M is actually the stronger incremental innovator in such a situation, so long as Firm X is not induced to exit. Third, the Base System model does not allow for horizontal differentiation, and in markets in which consumers have varying horizontal tastes it may be beneficial to have multiple vendors to satisfy different types of consumers.

In the case of computer software, the issue of patent protection is often moot, because an attempt to patent an entire software application could be blocked by any firm that holds a patent on some aspect of the underlying technologies included in the application. For example, many contemporary applications make use of patented compression, encryption, and networking technologies. In particular, base system vendors like Microsoft and Intel tend to hold many of these sorts of patents—enough to potentially head off attempts by complementors to patent application categories.[12]

Mandatory divestiture Because the model features undifferentiated Bertrand competition, requiring Firm M to divest itself of any applications that it develops makes little sense: the spun-off firm (call it Firm Y) would drive the application price to zero if $s = XY$, while if $s = X$ it would lead to a bilateral monopoly, which is inferior to an integrated monopoly.

[12] See Bessen [2] for a closer examination of this idea.

However, a mandatory divestiture rule could potentially be useful in a model with horizontal or vertical product differentiation. Horizontal product differentiation would allow both Firm X and Firm Y to earn modest profits when they compete, partially alleviating the problem of foregone innovation. Not only would mandatory divestiture offer greater incentives for Firm X to innovate ex ante, it would also give Firm X a greater opportunity to enter a market already occupied by Firm Y to provide a horizontally differentiated alternative.[13] Vertical product differentiation combined with horizontal differentiation would give Firms X and Y incentives for incremental quality improvement. On one hand, Heeb [18] shows that Firm Y would be a stronger incremental innovator if it were integrated with Firm M, as long as Firm X is not induced to exit. On the other hand, Firm X is more likely to be induced to exit when facing an integrated competitor than when facing a symmetric competitor.

Unlike patent protection, mandatory divestiture cannot eliminate the problem of foregone invention: Firm M still has some ex post incentive to develop the application in order to induce Firm X and Firm Y to drive down application prices and increase demand for the base system. Another downside of mandatory divestiture is that Firm M's static pricing policy no longer internalizes the complementarity between the base system and complementary applications, weakening Firm M's ex ante incentives to invest. Whether a mandatory divestiture policy would be socially beneficial depends on the balance among the social benefits of radical innovation, incremental improvement, product variety, and static efficiency, as well as the probability distribution over entry costs.

Price regulation There are also more and less attractive policies of price regulation that can be used to reduce Firm M's threat to innovation without reducing its incentives to innovate. A first policy would be to prevent Firm M from excluding rivals by means other than pricing, as the Justice Department's recent settlement with Microsoft putatively attempts.[14] A more severe second policy would be to regulate Firm M's pricing so that, when it enters an already-occupied market ex post, it cannot undercut its rival or bundle its application with any other products. In the Base System model presented here, pegging Firm M's price to Firm X's price would restore some of the Firm X's ex post profits, alleviating but not completely solving the foregone invention problem. Because it prevents trivial bundling, it also reduces the incentive for Firm M to enter ex post. Outside the model, such a policy would not harm incremental quality improvement—in fact, it gives Firm M an incentive to create excellent products because it cannot compete on price. On the

[13] Such entry is not necessarily socially beneficial; whether it increases or decreases social surplus depends on the specification of demand and entry costs.

[14] Bresnahan [5] argues that this attempt will not succeed.

down side, such a policy invites possibly inefficient ex post entry by low-cost third parties. It also begs the question of how to define the boundaries between application markets.

4 Additional examples

In this section I examine three additional examples that complement the Base System model of Section 3. The purpose is to demonstrate how the ideas of the basic framework and the Base System model can be applied to additional contexts. Each example is motivated by an interesting phenomenon in the computer software market, each leading to foregone invention in a qualitatively different way. What all the examples share is a vertical relationship between a potential new product an established product over which some firm has market power. The nature of the vertical relationship determines the conditions under which invention is foregone.

All the examples here rely on a demand system similar to that in the Base System model. The derivation of period profits under various market structures is accordingly analogous, so I do not present the details in the text.

4.1 Bundling unrelated products

The first example is bundling with commitment, by which a firm can leverage market power even from an unrelated market. The ability to commit to bundling creates an artificial vertical relationship where one does not arise naturally. In the Base System model, Firm M was able to leverage its market power through trivial bundling because it controlled the base system upon which the application depended. This "Bundling" example instead assumes that products b and a are completely unrelated. This implies that the monopolist in the market for product b can extend its market power to the market for product a only if it can find a way to create a relationship between the two products. The device I consider is an ability to commit to offer unrelated products only as a bundle.

This example is motivated in part by Microsoft's dominance in the market for business productivity software, and its practice of bundling a word processor, spreadsheet, personal information manager, and presentation editor together under the brand "Microsoft Office." Nalebuff [20], Carlton and Waldman [8], and Bakos and Brynjolfsson [1] show that bundling can serve as an entry deterrent, and Bakos and Brynjolfsson speculate informally that innovation incentives in an empty market will be distorted by the presence of a potential bundler. Here, I show that Microsoft's ex post option to bundle future products into Office can deter the ex ante invention of these products. In the works cited above, the monopolist initially monopolizes multiple markets and tries to prevent entry into one of them. Here, the incumbent monopolizes only one market, while another market lies unoccupied, with unknown demand.

As in Section 3.1, θ^b is distributed uniformly on $[0,1]$, while θ^a is either distributed uniformly on $[0,1]$, with probability γ, or otherwise $\theta^a = 0$ for all consumers. There are two firms, M and X, and four relevant market structures, $s \in \{\emptyset, M, X, MX\}$. In each static market, Firm M first chooses which bundles to offer from among the products it has developed, and then both firms set prices simultaneously. Since the two products are unrelated, independent pricing is no longer without loss of generality. Accordingly, I allow non-trivial bundling, including "mixed" bundling (in which a firm offers a bundle alongside independent products).The static prices and profits are as follows:[15]

- When $s = \emptyset$, Firm M monopolizes its existing market, setting price $p_M^b(\emptyset) = 0.5$ and earning profits $\pi_M(\emptyset) = 0.25$, as in the Base System model.
- When $s = M$, Firm M monopolizes both markets, and it offers a bundle ba at price $p_M^{ba}(M) \approx 0.862$ as well as each product independently at prices $p_M^b(M) = p_M^a(M) \approx 0.667$, giving it profits of $\pi_M(M) \approx 0.549$.
- When $s = X$, the two markets are independent and each firm sets $p_M^b(X) = p_X^a(X) = 0.5$ and earns monopoly profits $\pi_M(X) = \pi_X(X) = 0.25$.
- When $s = MX$, Firm M offers a bundle at price $p_M^{ba}(MX) \approx 0.607$, while Firm X offers its independent product at price $p_X^a(MX) = 0.245$, a combination that yields profits of $\pi_M(MX) = 0.369$ for Firm M and $\pi_X(MX) = 0.067$ for Firm X. Firm M does not offer either of its products independently: if it offered product a independently then Bertrand competition would drive its price to zero, while it cannot gain from offering product b independently because the bundle price is less than 0.667.

Proposition 3 demonstrates that the period profits in this model can lead to foregone invention.

Proposition 3 *In the Bundling example, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.398$) then the region of foregone invention has positive measure.*

The condition on γ is more restrictive than in the Base System model, because Firm X can earn some profits competing against Firm M's bundle. In this sense, a new product

[15] Though this example shares the demand setup of Nalebuff's model, these prices and profits differ slightly due to differences in timing and because Nalebuff does not consider mixed bundling.

that is not dependent on a base system is better insulated from ex post entry and therefore is more likely to be invented.

Commitment is necessary for invention to be foregone. When $s = MX$, Firm M would want to undercut Firm X's price in the application market if it could renege on its bundling commitment, so the price of product a would fall to zero under Bertrand competition. Thus Firm M would not enter the market for a ex post if it could not commit to bundling. This implies a simple public policy remedy: do not allow Firm M to bundle any unrelated products together with a product over which it has market power.

In this example, the two products are completely unrelated. But many the results carry over to the case of partial complementarity. If the new product a were complementary to Firm M's existing product b, then Firm M might enter in order to be able to increase the price on product b. The intuition is similar to that of the Base System model: Firm M prefers that complements to its own monopoly product should have low prices, and entering the complementary markets in order to increase price competition is one way to accomplish that goal. However, in the Base System model bundling was trivially implied when Firm M set a zero price for its application because consumers needed to purchase Firm M's base system in order to use the application. Here, although complementarity makes price competition in the new market valuable to Firm M in its monopoly market, Firm M would prefer to commit to a bundle whenever the complementarity between the two products is less than perfect. Firm X is able to earn positive profits when Firm M enters with bundle pricing, but Firm X earns zero profits if Firm M enters without bundling. Thus when complementarity is strong enough to induce Firm M to enter even if it could not commit to bundling, an ability to commit to bundling has a beneficial effect on Firm X's ex ante invention incentives. In this way, bundling as an entry strategy is less destructive to social surplus when complementarity is high.

4.2 Product features

The second example considers the addition of the same new feature to two horizontally differentiated products: the "Product Features" example. The necessity of bundling the new feature with an existing product is assumed as a technological constraint. In this example, neither firm is privileged in the ex post market; instead, it is the fact that the new feature brings them into closer competition that leads to an excessive incentive to invest ex post.This model illustrates that foregone invention may occur even in a symmetric situation. It also shows that market power does not need to take the form of monopoly in order to pose a threat to related market.

In the example, there are initially two existing products, which are sufficiently differentiated horizontally that their vendors' desired market shares do not come into conflict. In the world of software, one example of such a pair might be a visual web page editor and an HTML code editor. Though the products differ, there is an opportunity to add a particular feature (such as the ability to integrate with a new type of back-end data source) to either or both of them. The demand for this feature, though unknown, is orthogonal to the horizontal differentiation between the existing products. If only one of the two products offers the new feature, some customers may switch to the one with the new feature. Thus the invention of the new feature brings the firms into closer competition.

The example is constructed as follows. Consumer i's valuation of Firm M's product is θ_i^0, while her valuation of Firm X's product is $1 - \theta_i^0$. θ^0 is distributed uniformly on $[0, 1]$. Consumer i's valuation of the new feature is θ_i^1, which with probability γ is distributed uniformly on $[0, 1]$, and is 0 for all consumers otherwise. There are four possible market structures, $s \in \{\emptyset, M, X, MX\}$, and their static prices and profits are as follows:

- When $s = \emptyset$, each firm sets $p_j(\emptyset) = 0.5$ and earns profits $\pi_j(\emptyset) = 0.25$. In this example, the horizontal differentiation is calibrated so that all consumers are served in equilibrium and each firm is able to earn monopoly profits.

- When $s = M$, both firms set higher prices, $p_M(M) = 0.832$ and $p_X(M) = 0.534$, than when $s = \emptyset$, even though only Firm M has the new feature. This is because for any fixed θ^1 it is the consumers with intermediate values of θ^0 who are most willing to switch from Firm X to Firm M. As a consequence, Firm X now serves a population more heavily weighted toward having high values for its basic product (i.e., low θ^0), so it raises its price to extract more surplus from them. Firm M, of course, now sees higher demand for its product, and raises its price accordingly. Since both firms raise their prices, some customers with low values for the new feature and intermediate values for each basic product now go unserved. The resulting profits are $\pi_M(M) = 0.472$, and $\pi_X(M) = 0.195$.

- When $s = X$, $p_X(X) = 0.832$, $p_M(X) = 0.534$, $\pi_X(X) = 0.472$, and $\pi_M(X) = 0.195$.

- When $s = MX$, $p_M(MX) = p_X(MX) = 0.75$ and $\pi_M(MX) = \pi_X(MX) = 0.352$.

Proposition 4 shows that the profit functions in this model lead to foregone invention.

Proposition 4 *In the Product Features example, if the prior distribution over demand is sufficiently pessimistic ($\gamma < 0.707$) then the region of foregone invention has positive measure.*

This conclusion indicates that firms may be deterred from developing new features for their products by the possibility that adding those features will bring them into closer competition with currently distant competitors. In this specification of demand, the new feature is quite valuable relative to the original products, so a firm will lose badly if only its competitor offers the new feature. Thus the ex post incentive to develop the new feature is strong, and invention is forego over a large range of γ. Thus, in contrast to the Base System model and the Bundling example, it is the prospect of losing market power, rather than gaining market power, that drives ex post entry and thus reduces ex ante investment.

4.3 Divided Technical Leadership

The third example presents a situation in which Firm M's existing power in a related market is so threatened by Firm X's introduction of a new product that Firm M is induced to develop a new competing product ex post in order to protect its existing product. This leads to a qualitatively different set of conditions for foregone invention. The example exemplifies some aspects of Bresnahan and Greenstein's [6] "divided technical leadership" (DTL) hypothesis as applied to the recent history of the market for internet browsers. It is also closely related to Carlton and Waldman [8], which examines the ability of a monopolist to extend its monopoly to the market for a newly emerging substitute. My purpose is not to construct the definitive model of the relationship among Microsoft, Netscape, and Sun, but rather to propose that DTL suggests that platform leaders may seek to discourage the invention of complementary platform layers. The example also illustrates how the conditions for foregone invention are qualitatively different when a dominant firm perceives its competitors' investments as strategic complements.

The scenario is based on Microsoft's entry into the internet browser market once Netscape's browser became popular. Microsoft feared that the combination of Netscape's browser and Sun's Java Virtual Machine (JVM), as a middleware layer on top of Windows, could potentially "commoditize" the underlying operating system. Microsoft would be left to compete on price against other operating system vendors while Netscape and Sun assumed the mantle of platform leadership. The DTL hypothesis claims that such transitions are the usual course of change in the computer industry: a new entrant develops a complementary computing layer to serve a market segment distinct from that of the current platform leader, but once it achieves success in its own market it exploits changing trends in the industry to wrest platform leadership from the incumbent. Before Netscape popularized the browser, Microsoft faced only moderate incentives to enter the browser market because it could gain at most monopoly profits in that market. Microsoft judged, perhaps rightly, that it could better allocate its development resources to other projects ex ante. But once the browser-JVM combination threatened to become successful, Microsoft entered the browser market in order to snuff out any challenge to its leadership role.[16] From Microsoft's perspective, invesments were strategic complements: its incentives to invest were greater once Netscape and Sun also invested, because if the Netscape-Sun middleware product had succeeded then Microsoft could have faced severe consequences in its existing markets.

In this example, Firm X can invent a complementary application, and if it turns out to be popular then Firm X can invent a rival base system (product d) two periods later by investing c_d in the second period. Firm X's application works with either base system. Firm M can also develop a complementary application, but Firm M's application works only with Firm M's base system; Firm M cannot invent a new base system. The only uncertainty is over the demand for the application, which, as in previous models, is distributed uniformly on $[0, 1]$ with probability γ and is zero for all consumers otherwise. I assume that each consumer buys only one application when Firm M is the only base system vendor (in which case the cross-platform compatibility of Firm X's application's offers no advantage), so if Firm M bundles its application then no consumers buy Firm X's application, regardless of its price. Now there are six relevant market structures, $s \in \{\emptyset, M, X, MX, XX, MXX\}$, where XX indicates that Firm X has developed both an application and a base system. The value of Firm M's base system is $\theta^b \sim U[1, 2]$ and the value of Firm X's base system is $3 - \theta^b \sim U[1, 2]$.[17] The static prices and profits in each market structure are as follows:

- When $s = \emptyset$, Firm M sets $p_M^b(\emptyset) = 1$ and earns $\pi_M(\emptyset) = 1$.
- When $s = M$ and $\omega = 1$, $p_M^b(M) = 1.272$, $p_M^a(M) = 0.340$, and $\pi_M(M) = 1.319$.
- When $s = X$ and $\omega = 1$, $p_M^b(X) = 1.222$, $p_X^a(X) = 0.488$, $\pi_M(X) = 1.060$, and $\pi_X(X) = 0.238$. Firm X's application is popular.
- When $s = MX$ and $\omega = 1$, $p_M^a(MX) = p_X^a(MX) = 0$, $p_M^b(MX) = 1.549$ and $\pi_M(MX) = 1.316$. Competition drives the application price to zero. Since Firm M trivially bundles its application with its base system, no consumers purchase Firm X's application.
- When $s = XX$ and $\omega = 1$, $p_M^b(XX) = 0.928$, $p_X^d(XX) = p_X^{da}(XX) = 1.297$, and $p_X^a(XX) = 0.655$. Firm X trivially bundles the application with its base system,

[16] See Bresnahan [4] for a discussion of Microsoft's motivations based on internal strategy documents.

[17] Consumers' base system valuations are higher by 1 than in the product features model. This is so that under symmetric conditions Firm X's new base system captures some of Firm M's potential base system customers.

but those who buy Firm M's base system must pay a high price for the application. Profits are $\pi_M(XX) = 0.431$ and $\pi_X(XX) = 0.776$.[18]

- When $s = MXX$ and $\omega = 1$, $p_M^a(MXX) = p_X^a(MXX) = 0$, $p_M^b(MXX) = p_X^d(MXX) = 1$, and $\pi_M(MXX) = \pi_X(MXX) = 0.5$.

Because there may be up to three investments in this model, it does not fit directly into the basic framework. Instead, it is necessary to solve backward from the absorbing state, MXX. This is accomplished for $c_d < \frac{1}{1-\delta}0.5$ in Lemma 3 (in the Appendix) and the following Proposition.

Proposition 5 *In the DTL example, if $c_d < \frac{1}{1-\delta}0.5$ and $\delta > 0.113$ then the region of foregone invention has positive measure, regardless of the prior disribution over demand for the application.*

In contrast to the previous examples, the conditions for foregone invention in the DTL example do not involve the level of uncertainty, γ. This is because Firm M's ex post incentive to use trivial bundling as an entry barrier to its base system market is higher than its ex ante incentive to invent a complementary application of its own, even if $\gamma = 1$. Social welfare, of course, is best served if Firm X is able to enter both markets, a second best result is if both firms enter the application market, and the worst result is for neither firm to invent the complementary application.[19]

5 Discussion

This paper deals with the dynamic consequences of ex post competition when heterogeneous inventors face uncertain demand. Demand uncertainty leads only low–entry cost firms to invent ex ante, but high–entry cost firms may wish to enter ex post if demand is revealed to be high. The prospect of ex post competition from high–entry cost firms dampens the ex ante incentives of low–entry cost firms, so that invention is foregone over an intermediate region of entry cost profiles. Proposition 1 gave sufficient conditions in a simple two-stage

[18]There is actually a superior, but quantitatively insignificant solution in which Firm X also offers its base system sans application at a slightly lower price. The change in profits for each firm is less than 10^{-4}.

[19]In reality, Netscape did develop a browser and Sun did develop its JVM. In the context of the model, this could mean that their development costs were sufficiently low that Microsoft's ability to force the browser price to zero and to exclude them from the base system market was not sufficient to deter their initial investment. Outside the model, it is also possible that Netscape and Sun misjudged Microsoft's ability to overcome the network externalities that they had built up through wide distribution of the browser and JVM. On the other hand, Netscape and Sun may have judged the situation properly and merely experienced a bad realization from among a range of possible outcomes.

game for the region of foregone invention to have positive measure: j-submodularity, which means that at least one firm j perceives its opponent's investment as a strategic substitute; and expected ex post competition (condition (6)), such that with positive probability demand is high enough that firm $-j$ enters ex post when its entry costs are high enough that it would not enter ex ante. Although foregone invention can occur in symmetric situations, the problem is more likely to occur when one firm earns high monopoly profits and low duopoly profits, while the other firm earns high duopoly profits.

Of the three main factors that contribute to foregone invention—demand uncertainty, entry cost heterogeneity, and asymmetric market power—asymmetric market power has received the most attention in the literature on innovation. When it is not the simple consequence of an inherent entry cost advantage or demand advantage, asymmetric market power typically stems from some sort of complementarity. A number of static results, including Chen and Ross [10], Economides and Salop [14], and stemming back to Cournot in 1838 [12], demonstrate that complementarity between markets makes a vertically integrated monopoly the socially preferred market structure. A number of authors have investigated the dynamic consequences of these static results. Whinston [23], Bakos and Brynjolfsson [1], and Nalebuff [20,21] indicate that a firm with products in multiple markets can tie its products together to deter entry or induce exit, to the detriment of society. Stefanadis [22] and Choi and Stefanadis [11] consider investment in cost-reducing improvements with uncertain technological success, and show that the threat of bundling or downstream foreclosure causes invention to be foregone. Unlike my basic framework, their models do not allow firms to take advantage of information revealed by their competitors. Carlton and Waldman [8] demonstrate how a monopolist in one market can extend its monopoly to another market. Farrell and Katz [15] show that a monopolist in one market may invest too much in uncertain quality improvement for its complement while alternative suppliers invest too little; their analysis assumes ex ante symmetric investment costs and ascribes the uncertainty to technology rather than demand. Heeb [18] investigates incremental quality improvement in a series of models related to my Base System model, and finds that the base system monopolist faces greater incentives to invest in incrementally improving the application than an independent supplier. Adding heterogeneous entry costs and demand uncertainty to these types of models can lead to foregone invention.

Patents are the usual answer to problems of insufficient ex ante invention incentives. But for some entry cost profiles invention occurs without patents, and in such cases patents are harmful because they reduce ex post competition. Such competition is particularly important when there is scope for incremental quality improvement, because a firm that faces no competition ex post has less incentive to improve on its invention. There is a significant literature on the dynamic incentives posed by various patent regimes, of which I will mention just two works that are closely related. Chang [9] investigates the incentives of firms that undertake sequential innovation under a patent system, and finds that the firm that can invent the primary innovation may fail to do so if some other firm will gain the benefits of a secondary innovation. But in his model, the two firms have exogenous monopolies over the ability to invest in their respective innovations; the question arises why the second firm cannot also invest in the primary innovation. My basic framework can address such questions by varying the ability of each firm to invest in the primary innovation, i.e., by taking investment costs as a parameter. Bessen and Maskin [3] examine a sequential innovation model with hidden entry costs, uncertain technology, and free imitation. They find that ex post competition (when patents are not imposed) increases the incentive to innovate, because inventors look forward to being able to imitate subsequent improvements invented by others. In their model, each firm that invests in developing a sequential improvement draws its probability of success independently, so it always has some incentive to invest even when its competitor also invests. I conjecture that putting the uncertainty on the demand side and allowing asymmetric ex post market power would lead to the foregone invention problem instead. One final problem with patents is that, even if they could be helpful in concept, it is difficult to apply the legal framework of the patent system to categories of software applications, both because software applications typically make use of previously patented technologies (see Bessen [2]) and because reverse engineering is legal and relatively easy compared to development without imitation.

As for Microsoft, the results in this paper suggest that its very existence is a serious threat to innovation, but also that many policies that could eliminate or reduce this threat could have other negative effects on innovation. Of course, one policy to reduce Microsoft's threat with almost no negative effects is to reduce Microsoft's ability to exclude its rivals other than by pricing. But the models show that such a policy falls far short of solving the problem. Perhaps it is helpful to think of Microsoft as merely a dominant firm, rather than a true monopolist in the operating system market. It is my view that healthy competition in operating system markets can be a driving force for improvements in computing, because competition spurs incremental quality improvement in operating systems and also prevents any one base system vendor from seriously threatening a complementor's ex post profits.[20] In a static framework, encouraging operating system competition would require forgoing some benefits of standardization, but the dynamic losses from allowing Microsoft a true monopoly could be much greater than the static gains.[21] Accordingly, preventing Microsoft

[20]The complementor can market its product for several platforms, while an operating system vendor can trivially bundle only with its own operating system.

[21]For instance, the Microsoft Windows user interface itself is based on innovations developed by Apple,

from using its market power to eliminate or marginalize its operating system competitors should be a high policy priority, and an effective policy will necessarily involve restrictions on Microsoft's behavior in both operating system markets and complementary markets as long as Windows enjoys market dominance. Finally, such restrictions should be made explicit ex ante—rather than relying on ex post litigation—so as to encourage complementors and competitors to innovate.

Appendix

Lemma 2 *In the basic framework, let* $\tilde{c}_M = \sup\{c_M : \Pr[s_M^*(\omega, \tilde{c}_M) = MX] > 0\}$*. Suppose that X-submodularity holds; then there exists* $\varepsilon > 0$ *such that, given entry cost* $\hat{c}_X - \varepsilon$*,*

1. *If* $c_M < \tilde{c}_M$ *then Firm X does not enter the empty market in the first period in any subgame perfect equilibrium*
2. *If* $c_M > \tilde{c}_M$ *then Firm X enters the empty market in the first period in every subgame perfect equilibrium*

Proof. Positive profits and common demand imply that $\tilde{c}_M > 0$. This fact combined with common demand and competitiveness implies that

$$E[\pi_X^1(X,\omega) + \pi_X^2(X,\omega)] > E\left[\pi_X^1(X,\omega) + \pi_X^2(s_M^*(\omega, c_M), \omega)\right] \tag{9}$$

for all $c_M < \tilde{c}_M$. Given an entry cost profile $(c_M, \hat{c}_X)$ with $c_M < \tilde{c}_M$, Firm X's strict best response (in the first period reduced game) to out_M^1 is out_X^1:

$$E\left[\pi_X^1(X,\omega) + \pi_X^2(s_M^*(\omega, c_M), \omega)\right] - \hat{c}_X < \pi_X^1(\emptyset) + \pi_X^2(\emptyset) \tag{10}$$

X-submodularity further implies that Firm X's strict best response (in the first period reduced game) to in_M^1 is out_X^1:

$$E\left[\pi_X^1(MX,\omega) + \pi_X^2(MX,\omega)\right] - \hat{c}_X < E\left[\pi_X^1(M,\omega) + \pi_X^2(M,\omega)\right] \tag{11}$$

Choose $\varepsilon > 0$ sufficiently small that both of these strict best responses continue to hold at $c_X = \hat{c}_X - \varepsilon$. Then, given an entry cost profile $(c_M, \hat{c}_X - \varepsilon)$ with $c_M > \tilde{c}_M$, Firm X's optimal first period action is in_X^1. The argument for $c_M > \tilde{c}_M$ is similar. ■

without which graphical user interfaces might not have gained popularity until many years later. For a second instance, the open source programmers of Linux, BSD, and other server operating systems have demonstrated the ability to identify and fix security vulnerabilities quickly, putting pressure on Microsoft to improve the security of Windows. Furthermore, Microsoft's dominance gives it an incentive to avoid or subvert industry standards to keep rival operating systems from working well with Windows, so interoperability among operating systems ought to improve if the market share of Windows declines.

Proof of Proposition 1 (page 7). Condition (6) implies that $\hat{c}_M < \tilde{c}_M$, so by Lemma 2, given entry cost profile $(\hat{c}_M, \hat{c}_X - \varepsilon_X)$ with $\varepsilon_X > 0$ sufficiently small, Firm X does not enter in any subgame perfect equilibrium even though it would strictly prefer to enter given an entry cost profile $(c_M, \hat{c}_X - \varepsilon_X)$ with $c_M > \tilde{c}_M$. This is also true at $(\hat{c}_M + \varepsilon_M, \hat{c}_X - \varepsilon_X)$, for $\varepsilon_M > 0$ sufficiently small, where Firm M also does not enter in any subgame perfect equilibrium. ■

Proof of Proposition 2 (page 13). First,

$$\hat{c}_M \approx \gamma \left(\frac{1}{1-\delta} 0.546 \right) + (1-\gamma) \left(\frac{1}{1-\delta} 0.25 \right) - \frac{1}{1-\delta} 0.25 = \frac{\gamma}{1-\delta} 0.296 \tag{12}$$

Second,

$$s_M^\star(1, \hat{c}_M) = \begin{cases} MX & \text{if } c_M < \frac{1}{1-\delta}(0.544 - 0.343) = \frac{1}{1-\delta} 0.201 \\ X & \text{otherwise} \end{cases} \tag{13}$$

while $s_M^\star(0, \hat{c}_M) = X$. Third, by assumption $\hat{c}_M < \frac{1}{1-\delta} 0.201$, so

$$E\left[\pi_X^2(X, \omega)\right] \approx \frac{\gamma\delta}{1-\delta} 0.172 > E\left[\pi_X^2\left(s_M^\star(\omega, \hat{c}_M), \omega\right)\right] = 0 \tag{14}$$

Finally, X-submodularity is satisfied because Firm X always earns zero profits when it does not invest. Thus the conditions of Proposition 1 are satisfied. ■

Proof of Proposition 3 (page 21). First,

$$\hat{c}_M \approx \gamma \left(\frac{1}{1-\delta} 0.549 \right) + (1-\gamma) \left(\frac{1}{1-\delta} 0.25 \right) - \frac{1}{1-\delta} 0.25 = \frac{\gamma}{1-\delta} 0.299 \tag{15}$$

Second,

$$s_M^\star(1, c_M) = \begin{cases} MX & \text{if } c_M < \frac{1}{1-\delta}(0.369 - 0.25) = \frac{1}{1-\delta} 0.119 \\ X & \text{otherwise} \end{cases} \tag{16}$$

while $s_M^\star(0, c_M) = X$. Third, by assumption $\hat{c}_M < \frac{1}{1-\delta} 0.119$, so

$$E\left[\pi_X^2(X, \omega)\right] \approx \frac{\gamma\delta}{1-\delta} 0.25 > E\left[\pi_X^2\left(s_M^\star(\omega, \hat{c}_M), \omega\right)\right] \approx \frac{\gamma\delta}{1-\delta} 0.067 \tag{17}$$

Finally, X-submodularity is satisfied because Firm X earns zero profits whenever it does not invest. Thus the conditions of Proposition 1 are satisfied. ■

Lemma 3 *When $c_d < \frac{1}{1-\delta} 0.5$, the Divided Technical Leadership model is equivalent to the basic framework with*

$$\pi_M^2(X, 1) \approx \delta 1.060 + \frac{\delta^2}{1-\delta} \begin{cases} 0.431 & \text{if } c_M > \frac{\delta}{1-\delta} 0.069 \\ 0.5 & \text{if } c_M < \frac{\delta}{1-\delta} 0.069 \end{cases} \tag{18}$$

$$\pi_X^2(X, 1) \approx \delta 0.238 + \frac{\delta^2}{1-\delta} \begin{cases} 0.776 & \text{if } c_M > \frac{\delta}{1-\delta} 0.069 \\ 0.5 & \text{if } c_M < \frac{\delta}{1-\delta} 0.069 \end{cases} \tag{19}$$

with $\pi_j^1(s,\omega) = \pi_j(s,\omega)$ *and* $\pi_j^2(s,\omega) = \frac{\delta}{1-\delta}\pi_j(s,\omega)$ *for all states* $(s,\omega) \neq (X,1)$ *with* $s \in \{\emptyset, M, X, MX\}$.

Proof. Solve backward from the absorbing state, MXX, to states X, M, and MX. If $s = XX$, then Firm M invests if $c_M < \frac{1}{1-\delta}0.069$. If $s = MX$ then Firm X cannot invest, because its application is not popular. If $s = M$, then Firm X does not invest. If $s = X$ and one period has already elapsed since Firm X invented the application, then Firm X's best response to no investment by Firm M is to invest if $c_d < \frac{1}{1-\delta}0.538$; Firm M's mutual best response is indeed not to invest if $c_M > \frac{1}{1-\delta}0.069$. Firm X's best response to investment by Firm M in this situation is to invest if $c_d < \frac{1}{1-\delta}0.5$; Firm M's mutual best response is indeed to invest if $c_M < \frac{1}{1-\delta}0.069$. If $s = X$ and Firm X has just invented the application, Firm M can switch to state MX by investing, the incentives for which are accounted for in the basic framework. ■

Corollary 1 *Assuming X-submodularity, suppose that second period profits are much more important than first period profits and*

$$E\left[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega)\right] > E\left[\pi_M^2(M,\omega)\right] - \pi_M^2(\emptyset) \tag{20}$$

Then the region of foregone invention has positive measure.

Proof. Since $E\left[\pi_M^2\right]$ is supermodular in investment, the measure of ω for which

$$\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > E[\pi_M^2(M,\omega)] - \pi_M^2(\emptyset) \tag{21}$$

is positive, and since first period profits are small, the measure of ω for which

$$\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > E[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)] - \left(\pi_M^1(\emptyset) + \pi_M^2(\emptyset)\right) \tag{22}$$

is also positive. Thus Firm M enters ex post with positive probability given $\hat{c}_M$:

$$\begin{aligned} 0 &< \Pr\left[\pi_M^2(MX,\omega) - \pi_M^2(X,\omega) > E[\pi_M^1(M,\omega) + \pi_M^2(M,\omega)] - \pi_M^1(\emptyset) + \pi_M^2(\emptyset)\right] \\ &= \Pr\left[\pi_M^2(MX,\omega) - \hat{c}_M > \pi_M^2(X,\omega)\right] = \Pr\left[s_M^*(\omega,\hat{c}_M) = MX\right] \end{aligned} \tag{23}$$

This implies the condition in Proposition 1. ■

Proof of Propositon 5 (page 26). Suppose that $c_d < \frac{1}{1-\delta}0.5$. If $c_M > \frac{1}{1-\delta}0.069$ then satisfying (20) requires

$$\frac{\gamma\delta}{1-\delta}1.316 - \gamma\left(\delta 1.060 + \frac{\delta^2}{1-\delta}0.431\right) > \frac{\gamma\delta}{1-\delta}(1.319 - 1) \tag{24}$$

while if $c_M < \frac{1}{1-\delta}0.069$ then this requires

$$\frac{\gamma\delta}{1-\delta}1.316 - \gamma\left(\delta 1.060 + \frac{\delta^2}{1-\delta}0.5\right) > \frac{\gamma\delta}{1-\delta}(1.319 - 1) \tag{25}$$

Both these conditions are satisfied when $\delta > 0.113$, so by Corollary 1 invention is foregone. ■

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Figure 1:

Extensive form of the basic framework

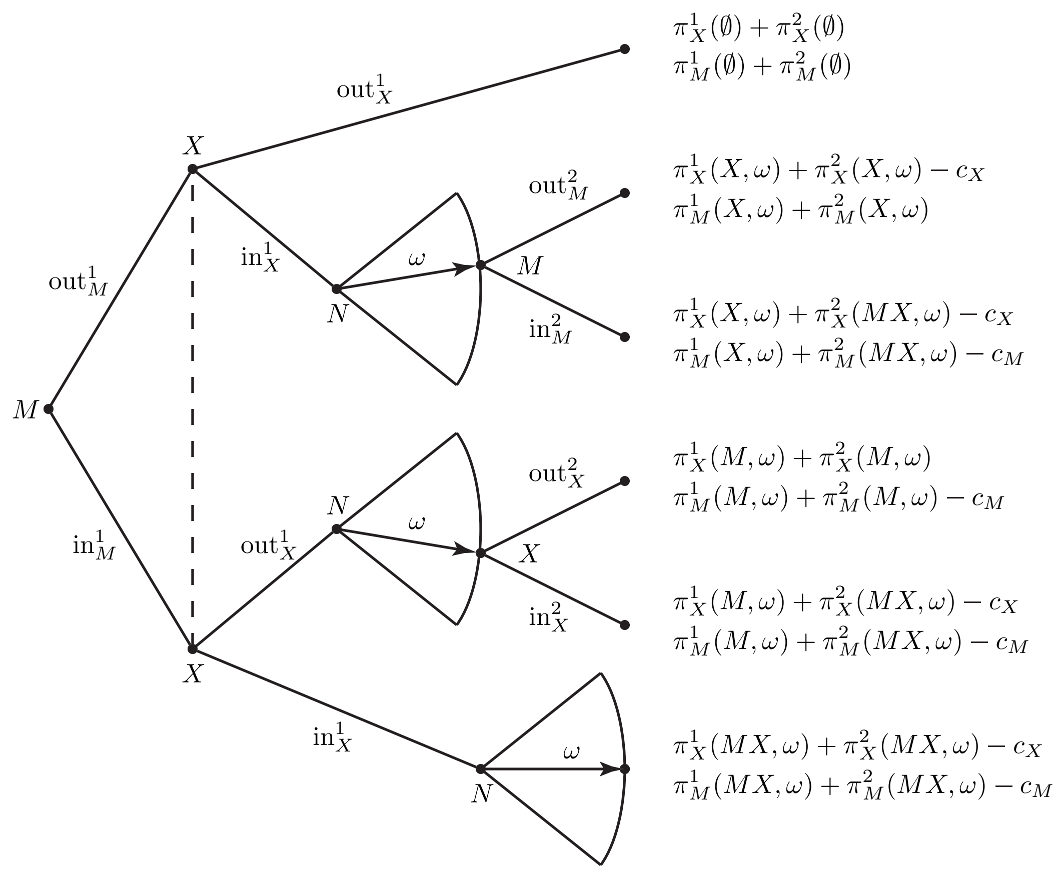
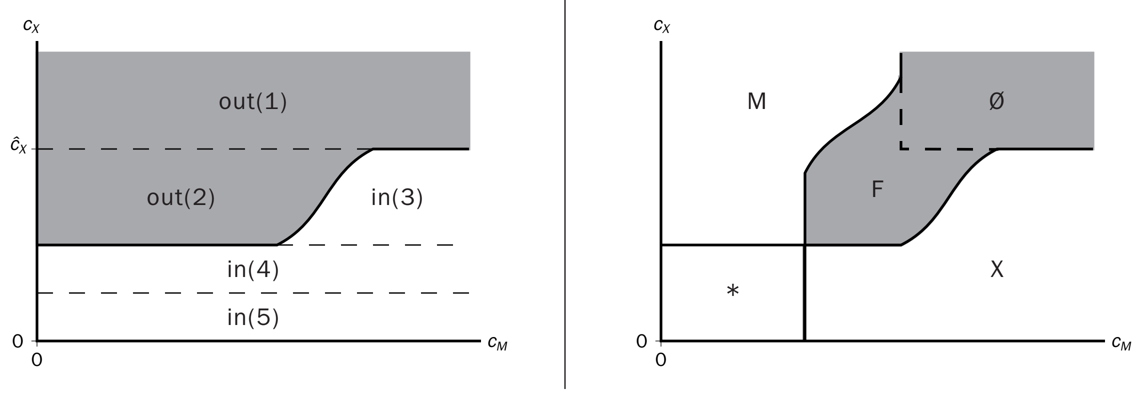


Figure 2:

The region of foregone invention in the basic framework

A. Firm X's optimal action in period 1

B. The region of foregone invention

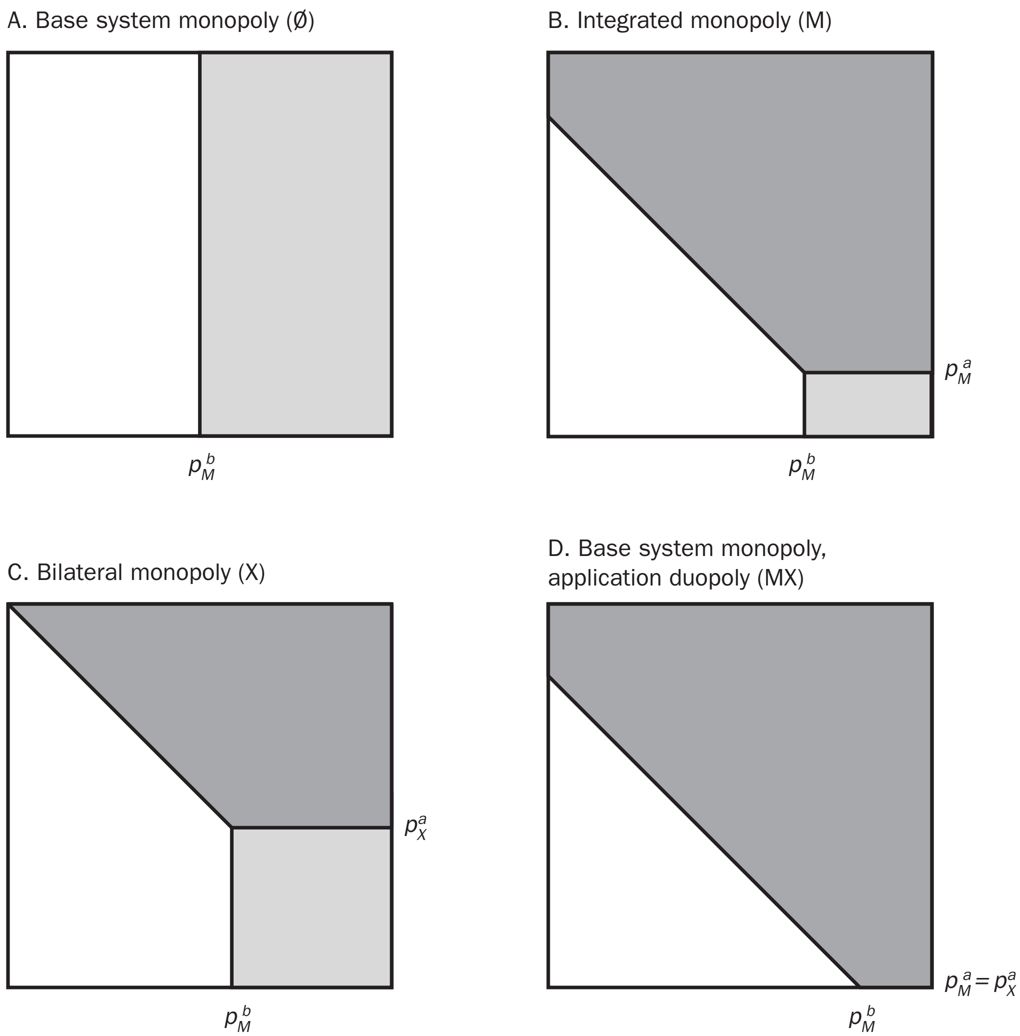


Each graph displays the cost space, with c_M as the horizontal axis and c_X as the vertical axis.

out(1)	Firm X does not enter because c_X is too high	Ø	Neither firm enters because both face high entry costs
out(2)	Firm X does not enter because c_M is low enough that Firm M will subsequently enter for too many ω	F	Foregone invention; neither firm enters, although the market is promising for at least one firm
in(3)	Firm X enters because c_M is high enough that Firm M will not subsequently enter for too many ω	X	Firm X enters in period 1
in(4)	Firm X can enter because c_X is low enough that subsequent entry by Firm A is not a deterrent	M	Firm M enters in period 1
in(5)	Firm X enters because c_X is so low that simultaneous entry by Firm M is not a deterrent	*	At least one firm enters in period 1

Figure 3:

Consumer purchases as a function of tastes
in the Base System model



Each graph displays Θ, the space of consumer tastes, with θ^b as the horizontal axis and θ^a as the vertical axis. Regions of Θ are shaded according to which consumers purchase which products:

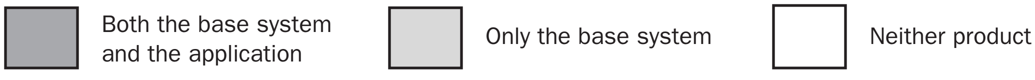
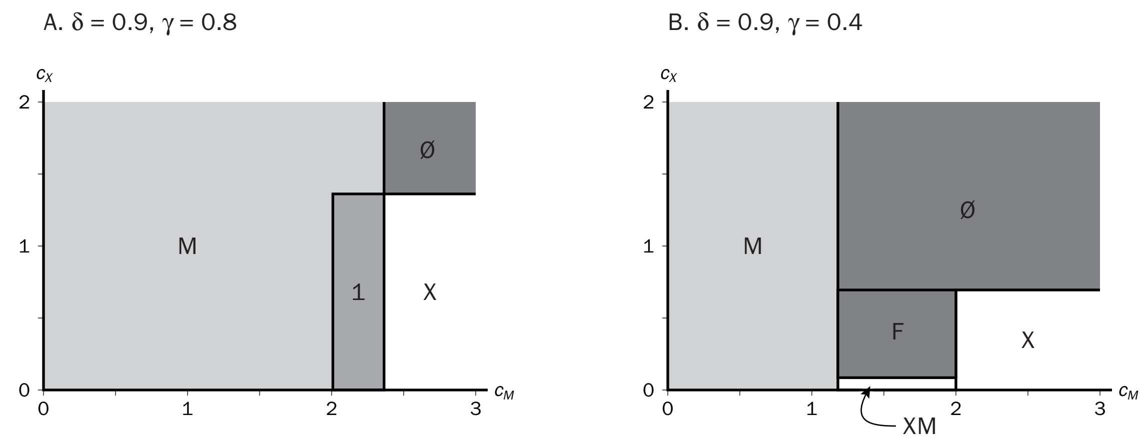


Figure 4:

Dynamic equilibrium as a function of costs in the Base System model



Each graph displays the cost space, with c_M as the horizontal axis and c_X as the vertical axis. Cost regions are shaded according to which firm invests first, and labeled according to the type of dynamic equilibrium that results:

- X: Only Firm X invests
- M: Only Firm M invests
- Ø: Neither firm invests; costs are too high
- XM: First Firm X invests, then Firm M invests if demand is high
- 1: Only the first mover invests
- F: Neither firm invests; foregone innovation