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Performance Evaluations over Time

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Abstract

This paper examines the benefits of evaluating employees halfway on long projects. Both the firm and the worker prefer halfway evaluations because they provide information on intermediate progress and the option to terminate employment. Both parties gain with more information revelation during the evaluation. The paper compares efficient effort levels across time under the various evaluation schemes, and solves for the optimal contracts. The model shows that turnover rates are higher among less able workers and younger workers. Also, more precise evaluation procedures, such as the 360 degree performance appraisal, will generate higher turnover.

1 Introduction

"The three main jobs of a manager are goal setting, coaching, and performance evaluation," said Hank McKinnell, CEO of Pfizer, Inc., at a recent speech at Stanford business school. Performance evaluations are critical and costly procedures in any organization. They not only assess current progress, but also define incentives for future work. How often should bosses evaluate their workers? How much information about that evaluation should the boss reveal to the worker? What are the effects of the evaluation on the worker’s effort levels, both before and after? These are the questions that this paper tries to answer.

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To answer these questions, I construct three models that build upon each other but have the same basic foundation. Workers exert (continuous) effort to produce output that accumulates across two stages. The boss will pay the worker at the end of the project if his output is sufficiently high. As is standard in moral hazard models, the worker (the agent) observes his effort but the boss (the principal) does not. A new feature here is that the agent does not observe his output unless the principal chooses to reveal it to him. This aims to describe white-collar work in large organizations, where the individual worker cannot precisely determine the relation between his effort (preparing casework for a trial, writing business plans for VC funding, assembling data for a potential client) and the final output (probability of winning the trial, obtaining the funding, convincing the client). It may be helpful to interpret $q$ as quality of output, which a manager can (choose to) observe but the employee himself cannot. Think of a rookie employee who prepares a sales presentation for his boss before the firm pitches the presentation to a prospective client. By virtue of his seniority and greater experience, the boss can more accurately assess quality than the employee. In fact, take this to be the definition of senior management: better observation of true output.

The benchmark model considers no halfway evaluation. Neither party observes output in the first stage, so the principal has no information to share after the first stage, and more importantly, no grounds for firing the agent. So the agent does not observe output at all, and the principal only observes whether final output is high enough to warrant payment. The second model considers halfway evaluation with coarse feedback. The principal conducts the evaluation by setting an intermediate output target after the first stage. If the worker’s output falls below the target, the principal fires the worker; otherwise, the worker advances to the second stage. The feedback is coarse in the sense that a worker who advances to the second stage knows only that his output cleared the target, but not by how much. The third model considers halfway evaluation with detailed feedback. Again, the principal sets an intermediate output target. This time, if the agent clears the target, the principal reveals to him his first stage output precisely. Now the agent who advances to the second stage conditions his effort on the realization

Instead of the principal observing output perfectly and the agent not observing output at all, a slightly more realistic model would involve both the principal and the agent receiving noisy signals of output, where the principal’s signal is less noisy (lower variance) than the agent’s. In this case, revealing information means that the principal would tell the agent her more informative signal. This would complicate the model without qualitatively changing the results.
of first stage output.

There are three main conclusions from the paper. First, it is good (for everyone!) to evaluate halfway. Total surplus rises with a halfway evaluation. The main benefit of evaluation is the option to terminate: the boss can fire the worker if his progress is unsatisfactory. Workers with very low early stage output have low chances of final success, and are costly to employ. Hence society will choose to employ only the valuable (high output) workers, and sack the rest. Note that this is an efficiency result. Neither the boss nor the worker desire continued employment if the worker is no good at the job. His labor is expensive, and he has better opportunities elsewhere.

Second, more information is better. Revealing information to the worker also increases total surplus. The worker can now condition his second stage effort on first stage output. It is efficient for workers with very successful early stages to work less in the later stage. Revealing output to the worker allows him to fine-tune his later effort in order to capture this efficiency. For example, students with the highest scores on a midterm exam should study less for the final than those who barely passed. This benefits the principal as well as the agent since the principal pays for the agent’s effort through the optimal contract.

Thirdly: under course feedback, work hard late; under detailed feedback, work hard early. These results concern the optimal allocation of effort across stages under the various evaluation schemes. Without evaluation, he splits effort evenly across both stages. This makes sense since effort in both stages are perfect substitutes. Under coarse feedback, he works harder in the second stage than in the first stage. This itself is surprising because one might expect an employee to work harder in the early stages of a project in order to secure a spot in the later stages, and then relax once he has "made it". Instead, the reverse effect dominates: the uncertainty of not advancing to the second stage causes the agent to shade his effort downward in the first stage. Once he has passed the test, he no longer faces the possibility of termination, and thus works harder. Finally, under detailed feedback, the agent works harder in the first stage than he does on average in the second stage. The agent optimally decreases his effort after a very successful early stage. He can do this because he sees his output.

In each of the three scenarios above, I solve for the efficient effort levels and the efficient termination rule (when applicable). I show how the principal can implement the efficient solution with a relatively simple contract. This contract is similar to a tournament which offers discrete payments (win and loss prizes) after each stage. Since
both parties are risk neutral, the contract makes the agent the full residual claimant. This paper examines only the efficiency gains of halfway evaluations. It does not include the costs of evaluations, which are easy to model but whose inclusion would detract from the paper’s main argument.

Welfare comparisons of all three schemes makes clear that no evaluation is a special case of evaluation with coarse feedback, which itself is a special case of evaluation with detailed feedback. With coarse feedback, the boss can always set the target as low as possible, letting everybody through. With such a low target, this is equivalent to no evaluation at all. Evaluating halfway gives the boss an extra instrument that makes everybody better off. Similarly, under detailed feedback, the principal permits the agent’s second stage effort to be an arbitrary function of first stage output. Under coarse feedback, this function is restricted to be constant. Since the latter is a subset of the former, detailed feedback dominates coarse feedback.

Section X discusses some applications of the model. First, this is an efficiency explanation of up-or-out contracts common in law firms and universities. The halfway evaluation is the first review (associate level in law firm, or associate professor in a university), which gives the institution the opportunity to cut deadweight, i.e. low performers. The second application is to entrepreneurial seed funding. A venture capitalists disburses start-up funding in stages to screen out successful projects from doomed projects. Again, doing so is more efficient than funding all projects at once.

The last section extends the model to add an ability parameter that persists through time. Production now has three elements: skill, luck, and effort. For example, suppose a student fails a midterm exam, and needs to decide whether to drop the class. His decision will depend on whether he failed because he is dumb, unlucky that day, or lazy. More generally, this describes when to stop any long-term production process: when to switch jobs after a poor performance review, when to fold the start-up after a low early round VC valuation, when to stop a long project after weak early results. The agent will use his intermediate performance measure to update his information on his ability. There are three implications for turnover rates within the firm.

First, turnover rates are higher for less able workers. Said differently, high ability workers set low termination thresholds for themselves since they are more willing to tolerate early failures. High types who fail in the first stage will blame bad luck and continue onto the second stage, but low types will not. This makes clear that the halfway evaluation has a sorting effect, keeping the more able and terminating the less
able workers.

Second, turnover rates are higher for workers who are less informed about their own ability. The more precise information an agent has on his ability, the lower he will set his termination threshold. The better he knows how good he is, the more likely he is to tolerate early failures, as he will blame bad luck instead of low ability. Since workers learn their own ability through age and experience, this explains why turnover rates fall at the more senior levels of an organization.

Third, turnover rates are higher on more precise evaluation procedures. A highly able worker will set a lower termination threshold under a noisy evaluation. In this case, a high type will discount an early failure to bad luck, and continue on. For example, a top student will not drop a class after he fails the midterm if he knows the test is noisy and hence uninformative. In the firm, this means that more exhaustive performance appraisal systems, such as the 360 degree appraisal, will generate more turnover. In the 360 degree appraisal, performance information comes from many different sources and hence is more precise.

2 The Model

Production takes place across two stages, and there is no discounting. The agent exerts effort $e_t$ in stage $t = 1, 2$ at cost $C(e_t)$, where $C', C'', C'''$ are strictly positive. So costs are increasing and convex, and marginal costs are convex. He produces output $q_1 = e_1 + \epsilon_1$ and $q_2 = q_1 + e_2 + \epsilon_2$. The noise terms $\epsilon_t$ are i.i.d., and distributed symmetrically around a mean of zero, with cdf $G(\cdot)$ and density function $g(\cdot)$. A project is an exogenous pair $(V, \bar{q})$, where $V > 0$ is the value of the project and $\bar{q} > 0$ is the final threshold: the principal collects $V$ if $q_2 > \bar{q}$, and zero otherwise.

Let $\bar{u}_t$ be the agent’s reservation utility in each stage $t$. The worker has the option to quit and take his reservation utility either before he begins work on the project, or halfway through after the first stage.

The assumptions on the observability of output are critical in this paper. The principal can always verify if final output clears the final threshold, but she observes first stage output only if she conducts an evaluation. This fits the realistic view that bosses are not continuously monitoring workers. Instead, they may choose to monitor workers, often

\footnote{Comparing output against a fixed standard is equivalent to comparing output against another worker’s output, as in a tournament. Standards are not picked out of thin air, but are usually some weighted average of past output.}
at high costs, in discrete intervals. I will present and discuss the specific assumptions on observability of output as they arise in each of the three scenarios.

3 No Evaluation

Suppose that the principal observes whether \( q_2 > \bar{q} \), but does not observe \( q_1 \). Because the agent does not observe his output, neither party is any better informed about output after the first stage than before. Importantly, there are no grounds for terminating the worker after the first stage. This describes most work in organizations, where bosses cannot observe quality of employee’s work without costly and extensive evaluation. The probability of success is

\[
P = Pr(q_2 > \bar{q}) = Pr(\epsilon_1 + \epsilon_2 > \bar{q} - e_1 - e_2) = \int \int_{\bar{q}-e_1-e_2} g(x)g(y)dydx
\]

by the independence of the errors. After integrating and using the symmetry of the errors,

\[
P = \int g(x)[1 - G(\bar{q} - e_1 - e_2 - x)]dx = \int g(x)G(x + e_1 + e_2 - \bar{q})dx.
\]

\[
\frac{\partial P}{\partial e_t} = \int g(x)g(\bar{q} - e_1 - e_2 - x)dx > 0,
\]

so the probability of success increases by the same amount with effort in either stage. Since total effort is additive, stage one effort and stage two effort are perfect substitutes, so they will be the same. This is shown formally next.

3.1 Efficiency

The social planner maximizes total surplus, so his problem and FOC is

\[
\max_{e_t} PV - C(e_1) - C(e_2)
\]

\[
V(\frac{\partial P}{\partial e_t}) = C'(\hat{e}_t)
\]

The marginal cost of effort is equal to its marginal return, which is the marginal probability of success times the value the project. Since the left-hand side is independent of
t, the right hand side must be as well. Hence $e_1 = e_2 \equiv \hat{e}$; the agent splits effort evenly across periods since the cost of effort per stage is the same. Note that convexity of the cost function is not what guarantees that effort in both stages is the same; effort is the same because the cost functions are the same. Convexity does, however, guarantee that efficient effort increases with $V$. Collecting terms, the efficient per stage effort level $\hat{e}$ solves
\[ C'(\hat{e}) = V \int g(x)g(\bar{q} - 2\hat{e} - x)dx. \]

The remaining constraint is a bound on the reservation utilities. The total surplus from having the worker undertake the project must be at least as large as the social return from the agent capturing his outside option. Call this the social rationality (SR) constraint: $PV - C(\hat{e}_1) - C(\hat{e}_2) \geq \bar{u}_1 + \bar{u}_2$. It makes no sense on a social scale to employ workers on projects when they can generate higher returns for themselves (and for society) elsewhere.

### 3.2 Optimal Contract

If the principal can contract on $q_2$, then he will offer a payment $t(q_2)$ for each $q_2$, and the contract will be a function $t(\cdot)$. In the appendix, I derive this general contract that induces efficient effort. In particular, under this contract the agent will work equally hard in both stages. Fortunately, a much more simple contract implements efficient effort levels. I will restrict attention to step contracts which take the form of $t(q_2) = W$ for $q_2 > \bar{q}$ and $t(q_2) = L$ otherwise, which can be represented by the win/loss prizes $(W, L)$. I can restrict attention to step contracts without loss of generality because they are optimal, both here and throughout the paper.

By construction of the contract, the agent earns $W$ with probability $P$ and $L$ with probability $1 - P$. His optimization and first-order conditions are
\[ \max_{\hat{e}_t} PW + (1 - P)L - C(e_1) - C(e_2) \]
\[ (\partial P/\partial e_t) \Delta = C'(\hat{e}_t). \]

where $\Delta = W - L$ is the prize spread. Comparing this first order condition with the social planner’s first order condition, it is clear that the principal can implement the first best by setting $\Delta = V$. So the principle induces efficient effort by promising a bonus equal to the full value of the project. This confirms the standard intuition: in order
to provide first-best incentives, the agent must become the full residual claimant. This fixes the prize spread, and the individual rationality condition will fix the absolute levels of the prizes. The principal extracts all the rents from the agent, so that in equilibrium the agent receives his reservation utility. The principal can do this by lowering the win and loss prizes by the same amount, keeping the prize spread fixed. Formally, this means that $\hat{PW} + (1 - \hat{P})L - C(\hat{e}_1) - C(\hat{e}_2) = \bar{u}_1 + \bar{u}_2$, where $\hat{P}$ is $P$ evaluated at the efficient effort level $\hat{e}$. Solving for the prizes, we have

$$W = \bar{u}_1 + \bar{u}_2 + C(\hat{e}_1) + C(\hat{e}_2) + V(1 - \hat{P})$$
$$L = \bar{u}_1 + \bar{u}_2 + C(\hat{e}_1) + C(\hat{e}_2) - \hat{P}V$$

These prizes are analogous to a single two stage tournament in the spirit of Lazear and Rosen (1981). The main difference here is that the equilibrium probability of success $\hat{P}$ may not be equal to $1/2$ since it depends on the exogenous standard $\bar{q}$. One can interpret the prizes as the price for a gamble. The agents pays $\hat{P}V$ in order to enter a lottery which pays out a base payment which covers the agent’s cost of effort and time for both stages, and a bonus of $V$ with probability $\hat{P}$ and nothing otherwise. In particular, if the loss payment is negative (as shown below), this means the agent pays a price if he loses the gamble. The prizes are chosen such that in equilibrium, the risk neutral agent breaks even and is indifferent between accepting the lottery and rejecting it to receive his reservation utility.

Now consider the division of surplus between the two parties. Since the principal is holding the agent to his (IR) constraint, the principal takes all the rents. In particular, the principal’s utility is

$$PV - [PW + (1 - P)L] = -L + P[V - \Delta] = -L.$$  

The principal pays out a loss payment for sure, and earns back the surplus $V - \Delta$ with probability $P$. But since the agent is the full residual claimant, this surplus is zero. Note that the social rationality (SR) condition holds if and only if $L \leq 0$, which means that the principal’s (IR) constraint will also hold: $-L \geq 0$. As long as society deems the project worthwhile to undertake, so will the principal. In particular, if (SR) is slack, so there is positive net social value to employing the agent on the project, the principal captures this entire social value, and the agent nothing. If the agent loses, he pays a penalty equal to the net social value of the project.
All that remains to show is that neither the principal nor the agent will choose to deviate from the contract \((W, L)\) after the first stage. The (IR) constraints of both parties are sufficient to show this. If the agent quits after the first stage, he captures his reservation utility \(\bar{u}_2\) whereas if he stays at the contract he earns the expected return \(\hat{P}W + (1 - \hat{P})L - C(e_2)\). But the agent’s IR constraint shows that \(\hat{P}W + (1 - \hat{P})L - C(e_1) - C(e_2) = \bar{u}_1 + \bar{u}_2 > \bar{u}_2\), so that the agent will not quit. Since the principal does not see any new information after the first stage, if she chose to employ the agent prior to the first stage, she will choose to employ him after the first stage as well. If the principal fires the agent halfway, she earns nothing but pays nothing either. If she sticks to the contract and advances the agent, she earns \(PV - [PW + (1 - P)L] = -L \geq 0\) from the principal’s ex-ante (IR) constraint. In conclusion, the principal and the agent will not deviate from the contract.

3.3 Motivation for Evaluation

The previous sections derived the efficient effort levels given the information constraints of both parties. There were no grounds for separation after the first stage, since no information was revealed. A natural question to ask is whether revealing some information will increase total surplus. Consider the following thought experiment. In the game from the previous section, suppose that the agent has chosen his effort levels for both periods. Suppose that the social planner observes \(q_1\) after the first stage. The next proposition shows that the social planner will terminate workers with sufficiently low \(q_1\).

**Proposition 1** There exists a cut off point \(q^*\) such that it is efficient only for workers with \(q_1 > q^*\) to advance to the second stage.

**Proof:** The planner sees \(q_1\). The ex-post probability of passing the final threshold is \(P(q_1) \equiv P\{q_2 > \bar{q}\} = G(q_1 + e_2 - \bar{q})\). Let \(h(q_1) \equiv P(q_1)V\) be the ex-post expected benefit. Clearly there exists an \(x, y\) such that \(h(x) = 0 < C(e_2) + \bar{u}_2 < V = h(y)\) where the last inequality must hold for the project to be sufficiently valuable to society. Since \(G(\cdot)\) is monotonically increasing and continuous in \(q_1\), so is \(h(\cdot)\), and thus by the intermediate value theorem there exists a \(q^*\) such that \(h(q^*) = P(q^*)V = C(e_2) + \bar{u}_2\).

Now put this to use. If the worker continues to the second stage, society earns ex-post expected benefit and bears the costs from both periods, yielding net utility \(P(q_1)V - C(e_1) - C(e_2)\). If the worker does not continue, society nets \(\bar{u}_2 - C(e_1)\). The
such that these two terms are equal will define \( q^* \). So 
\[
P(q^*)V - C(e_2) = \bar{u}_2.
\]
By the previous paragraph, this \( q^* \) in fact exists. In words, the optimal target is chosen such that society is indifferent between advancing and terminating the worker.

If 
\[
P(q_1)V - C(e_2) < \bar{u}_2,
\]
society prefers to terminate such a worker and let him capture his outside option. Since \( P(q_1) \) is monotonically increasing in \( q_1 \), this holds for \( q_1 < q^* \). Hence the social planner’s termination rule will take the form of a cut-off. ■

This shows that after the principal and the agent have agreed to a contract and chosen their prizes and effort levels, it is efficient to terminate workers with a low first stage output. Because this is an efficiency result, both parties prefer separation: the principal wants to fire low output workers, and such low-output workers want to quit. They realize that they are no good at the job, and have better opportunities elsewhere. Of course, without any revelation of \( q_1 \), separation is impossible. The next section examines the minimum possible revelation of information that provides the option for halfway termination.

### 4 Evaluation with Coarse Feedback

Now suppose that the principal conducts an halfway performance evaluation with coarse feedback. Precisely, the principal sets some target (or milestone) \( q^* \) after the first stage, and observes whether first stage output clears the target. Note that the principal does not observe \( q_1 \) precisely but only whether \( q_1 > q^* \). This assumption makes sense for three reasons. First, any evaluation is at best an approximation of true quality, and the coarseness of the feedback reflects the level of noise in the evaluation technology. For example, in the annual review of their graduate students, faculty need only see letter grades from classes (coarse feedback) and not the precise scores on all exams and assignments within those classes (detailed feedback). Second, it may be too costly or even impossible to provide full, detailed feedback. A college admissions committee does not care whether a student cannot integrate by parts or if he can’t remember the chain rule; it only cares that he received a B in calculus. Third, the evaluation may contain subjective elements that only makes sense in broad categories (rankings such as outstanding, good, satisfactory, etc.). The most common form of this is the letter of recommendation.

Coarse feedback is especially prevalent when the principal involves a third party
(or even more) to conduct all or part of the evaluation. This is the case for most of
the big decisions in economic and social life, because such decisions must aggregate
large amounts of information from many sources. Think of outside referees for tenure
decisions; multiple letters of review for an employee’s promotion; headhunting for a CEO;
grades, standardized test scores, and letters a recommendation for college admissions,
etc. One example of a very coarse evaluation is a consulting firm recruiting from a top
graduate school of business (call it GSB). Here, \( q \) is the quality of the potential job
candidate; \( q_1 \) is quality prior to entering GSB, and \( q_2 \) is quality (grades, interviews)
during GSB. Hence the GSB is conducting the halfway performance evaluation, which
is the decision to grant or deny admission (pass or fail a target) to a large pool of
applicants. By recruiting at the GSB, as opposed to hiring people off the street, the firm
makes use of a third party to screen out the low types.

The probability of reaching the target is

\[
P_l = Pr(q_1 > q^*) = 1 - G(q^* - e_1)
\]

as expected, this probability increases in first-stage effort since \( \partial P_l / \partial e_1 = g(q^* - e_1) > 0 \).

The probability of success is now

\[
P = Pr(q_2 > \bar{q}, q_1 > q^*) = Pr(\epsilon_1 + \epsilon_2 > \bar{q} - e_1 - e_2, \epsilon_1 > q^* - e_1) = \int_{q^* - e_1}^{\bar{q}} \int_{q^* - e_1 - e_2 - x}^{\bar{q} - e_1 - e_2} g(x)g(y)dydx.
\]

which we can rewrite as

\[
P = \int_{q^* - e_1}^{\bar{q}} g(x)G(x + e_1 + e_2 - \bar{q})dx.
\]

Notice that

\[
\partial P / \partial e_2 = \int_{q^* - e_1}^{\bar{q}} g(x)g(x + e_1 + e_2 - \bar{q})dx > 0,
\]

\[
\partial P / \partial e_1 = \partial P / \partial e_2 + \partial P_1 / \partial e_1 G(q^* + e_2 - \bar{q}) > \partial P / \partial e_2.
\]

The returns to effort are positive for both stages, but are higher for the first stage.
Additional first stage effort increases the probability of success in two ways. First, it
increases total output \( (q_2 = e_1 + e_2 + \epsilon_1 + \epsilon_2) \), and thus directly increases the probability
of final success. Second, it increases first stage output \( (q_1 = e_1 + \epsilon_1) \) and so improves the
chance of advancing to the second stage. It is incorrect to conclude from this, however,
that the agent works harder in the first stage, since this ignores the cost of effort.
Increasing first stage effort increases the chances of advancing to the second stage, and
thus increases the cost that society must bear for second stage effort. At the optimum,
this cost is enough to push \( e_1 \) below \( e_2 \).
4.1 Efficiency

Now consider the social planner’s problem. Let \( Q = Pr(q_2 > \bar{q} | q_1 > q^*) \) be the conditional probability of clearing the final threshold, given that the agent has reached the target. Of course, \( P = P_1 Q \). Total surplus is \( P_1 [QV - C(e_1) - C(e_2)] + (1 - P_1) [-C(e_1) + \bar{u}_2] \), or \( PV - C(e_1) + (1 - P_1) \bar{u}_2 - P_1 C(e_2) \). The social planner now has two social rationality constraints. Ex-ante (SR) guarantees that society will employ the worker in a project and not send them off to capture his outside options: \( PV - C(e_1) + (1 - P_1) \bar{u}_2 - P_1 C(e_2) \geq \bar{u}_1 + \bar{u}_2 \). Ex-post (SR) guarantees that society will continue to employ the worker after the first stage, and not have him capture his second stage upset option: \( QV - C(e_2) \geq \bar{u}_2 \). If the outside options satisfy these two constraints, the social planner’s problem is now

\[
\max_{e_1, e_2, q^*} PV - C(e_1) + (1 - P_1) \bar{u}_2 - P_1 C(e_2)
\]

The last term is the cost of advancing to the second stage. This cost is increasing in first stage effort and decreasing in the level of the target. As the principal raises the target, the agent is less likely to advance to the second stage, so the social planner is less likely to capture \( V \) but also less likely to bear the cost \( C(e_2) \). The first order conditions with respect to \( e_1, e_2, q^* \) are, respectively,

\[
\begin{align*}
C'(e_2) &= \frac{V}{1 - G(q^* - e_1)} \int_{q^* - e_1}^{q_2} g(x) g(\bar{q} - e_1 - e_2 - x) dx \\
C'(e_1) &= V \int_{q^* - e_1}^{q_2} g(x) g(\bar{q} - e_1 - e_2 - x) dx \\
V G(q^* + e_2 - \bar{q}) &= C'(e_2)
\end{align*}
\]

Hence marginal costs of effort are set to their marginal return, which is the marginal probability success times \( V \). Note that since \( P = P_1 Q \),

\[
C'(e_1) = V \frac{\partial P}{\partial e_2} < V \frac{\partial P/\partial e_2}{P_1} = V \frac{\partial Q}{\partial e_2} = C'(e_2)
\]

so the marginal return of an agent who cleared the target exceeds the marginal return of an agent in the first stage. Those who make it to the second stage are more valuable precisely because their first stage output was sufficiently high and second state output is cumulative. Another consequence is that \( C'(e_2) = C'(e_1)/(1 - G(q^* - e_1)) \). First, this shows that the agent will set the ratio of the marginal costs equal to the probability of reaching the target. As the principal lowers the target, the agent will allocate more
effort into the first stage. Second, since the marginal costs are strictly increasing, this shows that $\hat{e}_1 < \hat{e}_2$. Hence it is efficient for the agent to overwork in the second stage relative to the first.

Working too hard in the first stage is inefficient because the agent may not advance to the second stage. In this case, society bears the cost of effort $C(e_1)$ but does not acquire the benefit $V$. Instead the worker will allocate more of his effort in the later stages of the project, when he can focus exclusively on clearing the final threshold and need not worry about being sacked halfway. Since the agent does not advance to the second stage for sure, first stage effort carries more uncertainty than second stage effort. This uncertainty makes it efficient to underwork in the early stages, as society captures more value from his second stage effort. Once the young scholar is promoted from assistant to associate professor, she knows that she is a serious candidate for tenure, and hence works even harder than before.

It will be useful to write the solution in terms of the marginal continuation utility. Let $U(q_1) \equiv P(q_1)V - C(e_2)$ be the continuation utility, where $P(q_1) = Pr(q_2 > \bar{q}|q_1) = G(q_1 + e_2 - \bar{q})$ as before. Then

\[
C'(e_2) = \frac{1}{1 - G(q^* - e_1)} \int_{q^*} U'(q_1)g(q_1 - e_1)dq_1 = E[U'(q_1)|q_1 > q^*]
\]

\[
C'(e_1) = \int_{q^*} U'(q_1)g(q_1 - e_1)dq_1 = E[U'(q_1)]
\]

\[
U(q^*) = \bar{u}_2
\]

### 4.2 Discussion

Lazear’s paper on The Peter Principle (2001) addresses similar issues but arrives at different conclusions. His model allows firms to promote workers from an easy into a hard job. When workers are paid piece rates, high ability employees work harder before promotion, and low ability employees work harder after promotion. The reason is that the high types want to be promoted since their output is greater in the hard job than in easy job. Conversely, the low types do not want to be promoted since their output is greater in the easy job than in the hard job; so they work less in the first stage.

Lazear’s argument is based on sorting. Workers know their types prior to exerting effort, and the firm uses promotion to match the right worker with the right job. Given that the firm is doing this, the workers adjust their effort to assist in proper placement.
Moreover, given that the workers are strategically choosing their effort levels, the firm sets the promotion standard accordingly. The main point is that the employee makes the decision to underwork or overwork in the first stage (relative to the second) based on getting the right job in the second stage.

My argument is based on a combination of sorting and incentives. The optimal prizes provide the incentives, and the halfway evaluation does the sorting. The principal uses the prize spread in order to induce the agent to work at the socially efficient level. The halfway evaluation sets the level of the target, which sorts out the low-performers (low $q_1$) from the high.

4.3 Optimal Contract

This section shows that the principal can implement the efficient solution with a contract $(W_t, L_t, q^*)$: Win and Loss prizes after stage $t = 1, 2$ and in intermediate output target $q^*$. Since the principal only knows whether $q_1 > q^*$ and not $q_1$ precisely, the only information she can reveal to the agent is whether he cleared the target or not. If the agent fails to clear the target, the principal pays the agent a loser’s prize $L_1$ and the game ends. If he clears the target, the principal rewards the agent with $W_1$ and allows him to continue to the second stage. As before, if the agent’s second stage output $q_2$ clears the threshold $\overline{q}$, the principal collects $V$ and pays the agent $W_2$. If $q_2 < \overline{q}$, the principal pays the agent $L_2$, collects nothing, and the game ends. To reinforce, *target* refers to $q^*$ and *threshold* refers to $\overline{q}$. It is important to bear in mind that the worker is a high type if and only if he clears the target; it is the target that determines whether the worker is good enough to advance.

Now I will solve for the prizes $(W_1, L_1, W_2, L_2)$ that induce the socially efficient effort levels. Consider the agent’s problem. If the agent fails the target, he earns $L_1$ and bears costs $C(e_1)$. If the agent clears the target, he earns $W_1$ and an expected payment of $Q W_2 + (1 - Q)L_2$ and bears costs $C(e_1) + C(e_2)$. Then his maximization problem and FOCs are

$$\max_{e_t} P_1[W_1 - C(e_1) - C(e_2) + Q W_2 + (1 - Q)L_2] + (1 - P_1)[L_1 - C(e_1) + \overline{u}_2]$$

$$C'(e_2) = \frac{\Delta_2 \frac{\partial P}{\partial e_2}}{P_1}$$

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\[ C'(e_1) = \Delta_2(\partial P/\partial e_1) + g(q^* - e_1)[\Delta_1 + L_2 - C(e_2) - \bar{u}_2] \]

where \( \Delta_t = W_t - L_t \) is the stage \( t \) prize spread. Comparing this with the social planner’s first order conditions, the principal can implement efficient effort by setting \( \Delta_2 = V \) and \( \Delta_1 = -L_2 \). As before, the principal passes the full value of the project to the agent if he is successful at the end. Since each spread must be nonnegative, this means that \( L_2 \leq 0 \), so the second stage loss payment is in fact a penalty. If the agent passes in the first stage, he wins a bonus which he must forfeit if he fails in the second stage. So the agent receives the same payments whether he fails early, or whether he passes early but fails later. The real bonus comes when he passes twice, in which case he earns the value of the project. Thus the four prizes \( (W_t, L_t) \) generate only two levels of wealth, \( W_1 + W_2 \) and \( L_1 = W_2 - L_2 = \Delta_2 = V \). On purely monetary grounds, this two stage contest (against the standard \( \bar{q} \)) is equivalent to a single contest that pays \( W_1 + W_2 \) if successful and \( L_1 \) otherwise, making the agent the full residual claimant. This shows that even though effort takes place in two stages, the prizes are structured to reflect a single contest which only rewards total success, nothing halfway.

In order to solve for the optimal prizes, we need two individual rationality constraints. The first guarantees that the agent will choose to advance to the second stage, given that he has reached the target. The principal will reduce the second stage prizes by the same amount (keeping the spread fixed) until the agent’s expected payment, conditional on passing in the first stage, less his cost of effort is set to his outside option. So, \( \hat{Q}W_2 + (1 - \hat{Q})L_2 - C(\hat{e}_2) = \bar{u}_2 \). The second constraint is the pooled constraint over both stages. The principal will set the prizes such that the agent’s equilibrium utility is set equal to his outside option, which is the sum of his reservation utilities. The agent must receive at least this amount if he is to begin work on the project at all. Let \( U(e_1, e_2) \) be the agent’s utility over both stages; it is the objective function in the maximization above. The (IR) constraint becomes \( U(\hat{e}_1, \hat{e}_2) \equiv \hat{P}_1[W_1 - C(\hat{e}_1) - C(\hat{e}_2) + \hat{Q}W_2 + (1 - \hat{Q})L_2] + (1 - \hat{P}_1)[L_1 - C(\hat{e}_1) + \bar{u}_2] = \bar{u}_1 + \bar{u}_2 \). This simplifies to \( L_1 + \hat{P}_1\Delta_1 + \hat{P}_1[\hat{Q}W_2 + (1 - \hat{Q})L_2 - C(\hat{e}_2)] - C(\hat{e}_1) = \bar{u}_1 + P_1\bar{u}_2 \), where the term in brackets is just \( \bar{u}_2 \) from the first (IR) constraint. Solving for the prizes, we have

\[
W_1 = \bar{u}_1 - (1 - P_1)\bar{u}_2 + C(\hat{e}_1) - (1 - \hat{P}_1)C(\hat{e}_2) + V(1 - \hat{P}_1)\hat{Q} \\
L_1 = \bar{u}_1 + P_1\bar{u}_2 + C(\hat{e}_1) + \bar{u}_2 + \hat{P}_1C(\hat{e}_2) - V\hat{P}
\]
\[
W_2 = \bar{u}_2 + C(\hat{e}_2) + V(1 - \hat{Q})
\]
\[
L_2 = \bar{u}_2 + C(\hat{e}_2) - \hat{Q}V
\]

The second round prizes are analogous to a single tournament in the second stage. As before, this is analogous to a lottery in the second stage where the agent pays an entrance fee of \(\hat{Q}V\) and receives a base payment that reimburses his cost of effort and time, along with a bonus of \(V\) with probability \(\hat{Q}\), and nothing otherwise. The first stage prizes by themselves are difficult to interpret. They must be seen in the context of all four prizes. Since \(L_1 = W_1 + L_2\), this means that the four prizes only give the agent two levels of wealth whose difference is \(V\), as argued earlier. Finally, the expected prize in the first stage is
\[
EP_1 \equiv P_1 W_1 + (1 - P_1) L_1 = L_1 - P_1 L_2 = \bar{u}_1 + C(\hat{e}_1) + \bar{u}_2(1 - P_1)
\]
since \(\Delta_1 = -L_2\) and \(P_1 \hat{Q} = P\). The expected prize in the second stage is
\[
EP_2 \equiv Q W_2 + (1 - Q) L_2 = Q V + L_2 = \bar{u}_2 + C(\hat{e}_2)\]
since \(\Delta_2 = V\). In general both terms will depend on the reservation utilities, but in the case that \(\bar{u}_1 = \bar{u}_2 = 0\), \(EP_1 < EP_2\). If the outside options are both zero, then not only does the agent work harder in the second stage, but he is paid more as well. Finally, note that in order to satisfy the ex-ante and ex-post (SR) constraints, we must have \(L_t \leq 0\). The loss payments in both stages are penalties.

It is necessary to check that both parties will not deviate from the contract after the first stage. First suppose that the agent hears that he has cleared the target (i.e. \(q_1 > q^*\)). If he chooses to stay, he earns \(Q W_2 + (1 - Q) L_2 - C(e_1)\), whereas if he leaves here earns \(\bar{u}_2\). His second stage (IR) constraint guarantees that he will stay. Now suppose that he hears that he has missed the target. If he chooses to stay, he earns \(S W_2 + (1 - S) L_2 - C(e_1)\) where \(S \equiv P(q_2 < \tilde{q} | q_1 < q^*)\) is the conditional probability of success given a first stage failure. If he chooses to leave, he earns \(\bar{u}_2 = Q W_2 + (1 - Q) L_2 - C(e_1)\), which is strictly greater since \(S < Q\). So the agent will not deviate. Now consider the principal’s decision. If the principal fires the agent after he clears the target, her utility is \(-L_1\). If she instead advances the agent, her utility is
\[
Q[V - W_2] + (1 - Q)[-L_2] - W_1 = -W_1 - L_2 = -L_1
\]
where the first equality follows from \(\Delta_2 = V\) and the second from \(\Delta_1 = -L_2\). In other words, if the principal advances the agent, she gets \(-W_1 - L_2 + Q[V - \Delta_2]\), which means that the principal pays the win payment from the first stage, the loss payment
from the second stage, and she earns back the surplus $V - \Delta_2$ with probability $Q$ if the agent is successful. But in order to provide efficient incentives to the agent, the principal’s surplus is zero; the agent is the full residual claimant. So regardless of the probability of success from the first stage, zero surplus for the principal always ensures that she will be indifferent between firing and keeping the worker. Since the principal’s payoff is independent of $q_1$, the same inequality above holds with $S$ replacing $Q$. This is consistent with the social planner’s decision rule of firing if $q_1 < q^*$, so the principal will not deviate.

5 Evaluation with Detailed Feedback

Now suppose that the principal conducts a halfway performance evaluation and learns and reveals $q_1$ to the agent after the first stage. Now the agent will condition his second stage effort on first stage output. Efficient effort levels will take the form of $e_1$ from the first stage and the function $e_2(q_1)$ for each $q_1$. In particular, second stage effort functions $e_2(\cdot)$ that are constant for $q_1 > q^*$ will no longer be efficient. Such constant effort functions precisely characterize effort under coarse feedback. So if the principal learns first stage output, everyone is better off if she reveals it to the agent.

5.1 Efficiency

Who will the social planner terminate after the first stage? The agent produced output $q_1$, and the ex-post probability of success is $P(q_1) \equiv G(q_1 + e_2(q_1) - \bar{q})$. If the agent does not continue to the second stage, society gets $\bar{u}_2 - C(e_1)$. If the agent advances to the second stage, society earns $P(q_1)V - C(e_2(q_1)) - C(e_1)$. Hence it is efficient for the social planner to terminate the worker if $U(q_1) \equiv P(q_1)V - C(e_2(q_1)) \leq \bar{u}_2$, that is, if the continuation utility $U(q_1)$ is sufficiently low. The first result here, proved in the appendix, is that the social planner’s termination rule will take the form of a cut-off rule.

**Proposition 2** If first stage output is public knowledge after its realization, it is inefficient for workers with sufficiently low output to continue to the second stage. There exists cut-off level $q^*$ such that the social planner will terminate workers with $q_1 < q^*$. Moreover, as functions of first stage output, second stage effort increases up to a point but then decreases afterwards.
As before, let $P_1 = Pr(q_1 > q^*)$ and $Q = Pr(q_2 > \bar{q}|q_1 > q^*)$. Let $P = P_1Q$ be the probability of final success. Total surplus is $P_1 [QV - C(e_1) - E[C(e_2(q_1))|q_1 > q^*] + (1 - P_1) (\bar{u}_2 - C(e_1))]$. Rewriting this as the social planner’s problem and solving it gives the main result on the optimal division of effort between stages.

$$\max_{e_1, e_2, q^*} PV - P_1 E_{q_1} [C(e_2(q_1))|q_1 > q^*] - C(e_1) + (1 - P_1) \bar{u}_2.$$  

**Proposition 3** If the agent observes $q_1$, he will work harder in the first stage than he will on average in the second.

**Proof:** Rewrite the problem as

$$\max_{e_1, e_2, q^*} \int_{q^*} [VG(q_1 + e_2(q_1) - \bar{q}) - \bar{u}_2 - C(e_2(q_1))]g(q_1 - e_1) dq_1 + \bar{u}_2 - C(e_1).$$

Differentiate with respect to $e_2(q_1)$ and $q^*$ to get

$$C'(e_2(q_1)) = VG(q_1 + e_2(q_1) - \bar{q}) \text{ for a.e. } q_1 > q^*$$

$$VG(q^* + e_2(q^*) - \bar{q}) = C(e_2(q^*)) + \bar{u}_2$$

Now change variables to write the objective function as

$$\int_{q^* - e_1} [VG(e_1 + x + e_2(e_1 + x) - \bar{q}) - \bar{u}_2 - C(e_2(e_1 + x))]g(x) dx + \bar{u}_2 - C(e_1)$$

which gives $C'(e_1) = X + Y + Z$ where

$$X = VG(q^* + e_2(q^*) - \bar{q}) - C(e_2(q^*))$$

$$Y = \int_{q^* - e_1} VG(e_1 + x + e_2(e_1 + x) - \bar{q}) g(x) dx$$

$$Z = \int_{q^* - e_1} e_2'(e_1 + x)[VG(e_1 + x + e_2(e_1 + x) - \bar{q}) - C'(e_2(e_1 + x))] g(x) dx$$

The derivatives with respect to $e_2(q_1)$ and $q^*$ show that $Z = 0$ and $X = 0$, respectively. Changing variables of $Y$ back yields

$$C'(e_1) = \int_{q^*} [VG(q_1 + e_2(q_1) - \bar{q}) g(q_1 - e_1)] dq_1$$

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Observe that \( C'(e_1) = E_{q_1}[C'(e_2(q_1))]. \) The error distribution \( g(q_1 - e_1) \) induces a distribution on \( e_2 \) since \( q_1 \sim g(q_1 - e_1) \) and \( e_2 \) is a function of \( q_1 \). Since marginal costs are convex, Jensen’s inequality on the random variable \( e_2 \) gives

\[
C'(e_1) = E_{q_1}[C'(e_2(q_1))] = E_{e_2}[C'(e_2)] > C'(E_{e_2}[C'(e_2)]) = C'(E_{q_1}[e_2(q_1)]).
\]

Since costs are increasing, this means that \( e_1 > E_{q_1}[e_2(q_1)] \).\[\blacksquare\]

Observe that this result seems to contradict the earlier results under coarse feedback, where the agent work harder in the second stage rather than the first. The difference is that because the agent can now fine-tune his second stage effort on first stage output, he does not work as hard for very high realizations of \( q_1 \). More precisely, in the proof of proposition 2 I show that \( e_2(q_1) \) increases up to a point, but then decreases afterwards. Observing \( q_1 \) allows the agent to (rationally) decrease effort for a very successful early stage. Coarse feedback, on the other hand, forces the agent to work at a uniform level for all \( q_1 \). At high \( q_1 \), this turns out to be wasteful effort, as the section on welfare comparisons will show that conditioning on \( q_1 \) increases his overall utility.

### 5.2 Optimal Contract

As with coarse feedback, the principal can implement the efficient effort levels and termination rule with a contract \((W_t, L_t, q^*)\): win and loss prizes after each stage and an intermediate output target. Importantly, even though \( q_1 \) becomes public after the second stage, and even though the agent conditions \( e_2 \) on \( q_1 \), it is not necessary for the principle to condition her contract on \( q_1 \) in order to implement efficiency. The discontinuity in the principal’s payoff \((V \text{ if } q_2 > \bar{q}, 0 \text{ otherwise})\) ensures that a simple step contract will suffice.

**Proposition 4** If \( q_1 \) becomes public knowledge after the first stage, the principal can implement the efficient solution with a contract \((W_t, L_t, q^*)\), where \( W_1 = L_1 \leq 0, (W_2, L_2) = (V, 0) \), and \( q^* \) is the efficient termination rule.

**Proof:** The agent’s objective function is

\[
P_1[W_1 - E_{q_1}[C(e_2(q_1))|q_1 > q^*] + Q W_2 + (1 - Q) L_2] + (1 - P_1)[L_1 - \bar{w}_2] - C(e_1).
\]
Expanding this and taking derivatives gives

\[
\max_{e_1,e_2(\cdot),q^*} \int_{q^*} \left[ \Delta_2 G(q_1 + e_2(q_1) - \bar{q}) - \bar{u}_2 - C(e_2(q_1)) \right] g(q_1) dq_1 + \bar{u}_2 - C(e_1) + P_1[\Delta_1 + L_2] + L_1.
\]

\[
C'(e_2(q_1)) = \Delta_2 g(q_1 + e_2(q_1) - \bar{q}) \text{ for a.e. } q_1 > q^*
\]

\[
C'(e_1) = \int_{q^*} \left[ \Delta_2 g(q_1 + e_2(q_1) - \bar{q}) g(q_1) dq_1 + g(q^* - e_1) \right] \Delta_1 + L_2
\]

\[
\Delta_2 G(q^* + e_2(q^*) - \bar{q}) = C(e_2(q^*)) + \bar{u}_2 - g(q^* - e_1) \left[ \Delta_1 + L_2 \right]
\]

comparing these equations to the social planner’s first order conditions, the principal can implement efficient solution by setting \( \Delta_1 = -L_2 \) and \( \Delta_2 = V \).

As before, the ex-post probability of passing the final threshold is \( P(q_1) \equiv Pr(q_2 > \bar{q}) = G(q_1 + e_2 - \bar{q}) \). A worker with first stage output \( q_1 \) will choose to work in the second stage if his expected return exceeds his outside option, or \( P(q_1) W_2 + (1 - P(q_1))L_2 - C(e_2(q_1)) \geq \bar{u}_2 \). We can write this as \( L_2 + f(q_1) \geq \bar{u}_2 \), where \( f(q_1) = P(q_1)V - C(e_2(q_1)) \) is the ex-post social value of the worker. This inequality must hold for every \( q_1 > q^* \), and because \( f(\cdot) \) is strictly increasing, it is sufficient to require \( L_2 + f(q^*) \geq \bar{u}_2 \). Recall that on the margin society is indifferent between advancing and terminating the worker, so \( f(q^*) = \bar{u}_2 \), and the agent’s ex-post (IR) constraint reduces to \( L_2 \geq 0 \). The principal’s (IR) constraint is \( P(q_1)V - [P(q_1)W_2 + (1 - P(q_1))L_2] = -L_2 \geq 0 \). Combining the two (IR) constraints show that \( L_2 = 0 \). The principal makes zero profits, and she holds the lowest-type worker (for whom \( q_1 = q^* \)) to his (IR) constraint. Moreover, workers with \( q_1 > q^* \) capture strictly positive rents \( R(q_1) = P(q_1)V - C(e_2(q_1)) - \bar{u}_2 \). These rents are increasing in \( q_1 \).

The principal will choose \( L_1 \) such that the agent’s ex-ante (IR) constraint binds, so

\[
L_1 = \bar{u}_1 + P_1 \bar{u}_2 + C(e_1) + P_1 E_{q_1} \left[ C(e_2(q_1)) | q_1 > q^* \right] - PV.
\]

The principal’s (IR) constraint gives

\[
P_1[QV - [L_2 + Q\Delta_2] - W_1] + (1 - P_1)[-L_1] = -L_1 \geq 0.
\]

Observe that this is equivalent to the ex-ante social rationality (SR) constraint. Finally, since \( \Delta_1 = -L_2 = 0 \), we have \( W_1 = L_1 \leq 0 \), and \( (W_2,L_2) = (V,0) \).
To check that neither party will deviate from the contract halfway, suppose the agent produces \( q_1 \) after the first stage. If the agent quits, he earns \( L_1 - C(e_1) + \bar{u}_2 \). If instead the agent stays on the job, he earns a win payment from the first stage, an expected payment from the second stage, and he bears the cost of effort for both stages. His utility is

\[
W_1 + P(q_1)[W_2] + (1 - P(q_1))[L_2] - C(e_1) - C(e_2(q_1)) = W_1 + L_2 + P(q_1)\Delta_2 - C(e_2(q_1)) - C(e_1).
\]

Since \( \Delta_2 = V \) and \( \Delta_1 = -L_2 \), this reduces to \( P(q_1)V - C(e_2(q_1)) + L_1 - C(e_1) \). The agent is indifferent between quitting and staying if \( P(q_1)V - C(e_2(q_1)) = \bar{u}_2 \). Comparing this to the final first order condition from the social planner’s optimization (with respect to the target), this show is that the agent is indifferent if and only if \( q_1 = q^* \), he prefers to stay if \( q_1 > q^* \), and he prefers to quit if \( q_1 < q^* \). In other words, the agent’s decision rule is exactly the same as the social planner’s. How about the principal? If the principal fires the agent, her utility is \( -L_1 \). If she instead advances the agent, her utility is

\[
P(q_1)[V - W_2] + (1 - P(q_1))[L_2] - W_1 - L_2 = -L_1
\]

where the first equality follows from \( \Delta_2 = V \) and the second from \( \Delta_1 = -L_2 \). As before, the principal is indifferent between firing and keeping the worker for any realization of first stage output. This is consistent with the social planner’s decision rule of firing if \( q_1 < q^* \). This shows that the optimal contract implements the efficient termination rule.

As before, the principal makes the agent full residual claimant in the second stage. The fact that \( W_1 = L_1 \leq 0 \) has two implications. First, the agent pays a penalty after the first stage, and he is willing to do this because he is risk neutral and the second stage prize is sufficiently large. Second, the first stage prize spread is zero. So the principal pays the agent with option value: the agent works in the first stage in order to continue to the second.

Now consider the division of surplus. By holding the agent to his ex-ante (IR) constraint, the principal captures all the rents. Interestingly, the principal captures none of the ex-post surplus. In the second stage, the principal must use a single ex-post (IR) constraint to induce workers with \( q_1 \geq q^* \) to stay on the job. The principal accomplishes this by forcing the (IR) constraint for the lowest-type worker (the one who produces \( q_1 = q^* \)) to bind and letting all other workers \( (q_1 > q^*) \) capture positive rent.
In particular, the ex-post rents for the worker are increasing in $q_1$, and the principal earns no rents in the second stage. Recall that under coarse feedback, the principal captured both the ex-ante and ex-post surplus. Hence revealing $q_1$ transfers the ex-post surplus from the principal to the agent.

The contract in the proposition implements the efficient solution and gives the principal all the surplus; clearly the principal cannot do any better. Moreover, the contract is quite simple: it does all this with only five instruments ($W_t, L_t, q^*$). In particular, even though $q_1$ is public knowledge after the first stage, the principal does not need to contract on $q_1$. The next proposition shows that this is always the case in every optimal contract.

**Proposition 5** In every optimal contract, the second stage payment will be independent of $q_1$.

**Proof:** Omitted (for now). ■

Said differently, any contract whose second stage payments vary with $q_1$ will be inefficient.

## 6 Welfare Comparisons

This section compares total surplus generated under the three schemes: no evaluation, evaluation with coarse feedback, and evaluation with detailed feedback. The main result is that a halfway evaluation increases surplus over no evaluation, and providing detailed feedback increases surplus over providing coarse feedback.

**Proposition 6** Let the distribution of the errors have finite support $[-m, m]$. Measured by total surplus, detailed feedback weekly dominates coarse feedback, and coarse feedback strictly dominates no evaluation.

**Proof:** Let the superscripts N, C, D stand for no evaluation, coarse feedback, and detailed feedback, respectively. The probability of final success in each of the three cases is

$$P^N = \int_{-m}^{m} \int_{q-e_1-e_2-x}^{m} G(x + e_1 + e_2 - q)g(x)dx$$
\[ P^C = \int_{q^*-1}^{m} \int_{q-e_1-e_2-x}^{m} G(x + e_1 + e_2 - \overline{q})g(x)dx \]

\[ P^D = \int_{q^*-1}^{m} \int_{q-e_1-e_2(e_1+x)-x}^{m} G(x + e_1 + e_2(e_1 + x) - \overline{q})g(x)dx \]

The social planner’s objective function in each of the three cases is

\[ U^N(e_1, e_2) = P^N V - C(e_1) - C(e_2) \]

\[ U^C(e_1, e_2, q^*) = P^C V - C(e_1) - P_1 C(e_2) + (1 - P_1) \overline{u}_2 \]

\[ U^D(e_1, e_2(\cdot), q^*) = P^D V - P_1 E_{q_1}[C(e_2(q_1))|q_1 > q^*] - C(e_1) + (1 - P_1) \overline{u}_2 \]

First I will show that total surplus under coarse feedback dominates total surplus under no evaluation, or \( TS^N < TS^C \). Notice that if \( q^* = \hat{e}_1 - m \), then \( P^N = P^C \) and \( P_1 = 1 \), and thus

\[ U^N(e_1, e_2) = U^C(e_1, e_2, \hat{e}_1 - m) - (1 - P_1) \overline{u}_2 < U^C(e_1, e_2, \hat{e}_1 - m) \forall e_1, e_2 \in \mathbb{R}. \]

So

\[ TS^N \equiv \max_{e_i} U^N(e_1, e_2) < \max_{e_i} U^C(e_1, e_2, \hat{e}_1 - m) \leq \max_{e_i, q^*} U^C(e_1, e_2, q^*) \equiv TS^C. \]

The weak inequality holds because the optimization on the right hand side includes an additional variable. The strict inequality holds because \( \overline{u}_2 > 0 \).

Now compare coarse with detailed feedback. Define the following sets

\[ A = \{ e_1 \in \mathbb{R}, e_2 : \mathbb{R} \to \mathbb{R}, q^* \in \mathbb{R} | e_2(q_1) \text{ is constant for all } q_1 > q^* \} \]

\[ B = \{ e_1 \in \mathbb{R}, e_2 : \mathbb{R} \to \mathbb{R}, q^* \in \mathbb{R} \} \]

Clearly \( A \subset B \). Let \( x = (e_1, e_2(\cdot), q^*) \in A \). Then for all \( q_1 > q^* \), \( e_2(q_1) = e_2 \) for some \( e_2 \). This means

\[ P_1 E_{q_1}[C(e_2(q_1))|q_1 > q^*] = \int_{q^*}^{m} C(e_2)|g(q_1 - e_1) dq_1 = P_1 C(e_2) \]

\[ P^D = \int_{q^*}^{m} [G(q_1 + e_2)g(q_1 - e_1) dq_1 = P^C. \]

So \( U^D \) and \( U^C \) agree on \( A \), or \( U^D(x) = U^C(x) \) for all \( x \in A \). And since \( A \subset B \), we have

\[ TS^C = \max_{x \in A} U^C(x) = \max_{x \in A} U^D(x) \leq \max_{y \in B} U^D(y) = TS^D \]

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The argument in the proof is simple. In progressing from no evaluation to evaluation with coarse feedback, and then from coarse feedback to detailed feedback, the social planner is maximizing surplus over larger and larger sets. Hence no evaluation is a special case of the coarse feedback, and likewise coarse feedback is a special case of detailed feedback. For example, compare no evaluation with coarse feedback. The halfway evaluation gives the social planner an extra variable (the halfway target) in his optimization problem. The social planner can always set the target low enough such that the agent will always pass, regardless of his first stage error. As modelled, this is equivalent to not having a halfway evaluation at all. Now compare coarse feedback with detailed feedback. Under detail feedback, revealing \( q_1 \) halfway allows second stage effort to be an arbitrary function of \( q_1 \). The social planner can always restrict this function to be constant for \( q_1 > q^* \). As modelled, this is equivalent to providing coarse feedback. So surplus under detailed feedback is at least as large as that under coarse feedback.

The three separate schemes allow us to break the efficiency gains into two parts: sorting and incentives. Shifting from no evaluation to coarse feedback is the sorting effect. Think of the first stage shock as the worker’s type: the agent is a star if \( \epsilon_1 > q^* - e_1 \) and a slug otherwise. Prior to starting work on the project, the worker does not know his own type, and this is revealed to him after he tries his hand at the project in the first stage. Because no evaluation lets everyone through (even the slugs), the probability of success is higher than under coarse feedback, but so is the second stage cost. On the margin, the slugs cost more than they are worth. The slugs do not increase the probability of success enough to justify society paying for their second stage effort.

The incentive effect kicks in while shifting from coarse feedback to detailed feedback. Revealing first stage output allows second stage effort to finely adjust to \( q_1 \), whereas not revealing it forces second stage effort to be constant for any \( q_1 > q^* \). The fine adjustment is useful especially at the tails of the error distribution. For example, if the agent receives a big first stage shock, he will want to shade his second stage effort down appropriately (since \( e_2'(q_1) < 0 \)). This fine-tuning avoids wasted effort.

\(^3\)For errors with infinite support, the principal can set the target arbitrarily low, so that agent passes with probability 1. Using the same arguments from the proof, for any effort levels the objective function under final evaluation is arbitrarily close to the objective function under halfway evaluation using this arbitrarily low target. And again, total surplus will in general be higher with halfway evaluations.
One consequence of this is that it does not matter if the principal fires the agent or if the agent voluntarily quits. Because the analysis solves the social planner’s problem, it is in the interest of both parties for the agent to stop working. We can interpret $q^*$ not only as the target that the principal sets, but as the quit threshold that the agent himself sets, abandoning the project halfway if his first stage output falls below this threshold.

**Corollary 1** The principal prefers detailed feedback to coarse feedback to no evaluation. The agent is indifferent to all three schemes, since the principal captures all the surplus and the agent none of it.

**Proof:** As before, let the superscripts N, C, D stand for no evaluation, coarse feedback, and detailed feedback, respectively. Let $V^i_p, V^i_a, L^i_1, TS^i_t$ be the principal’s equilibrium utility, the agent’s equilibrium utility, the loss prize in stage $t$, and total surplus for $i = N, C, D$.

Then inspection of the loss prizes and total surplus levels shows that

$$V^i_p = -L^i_1 = TS^i_t - \bar{u}_1 - \bar{u}_2$$

Since $TS^D \geq TS^C \geq TS^N$, this means $V^D_p \geq V^C_p \geq V^N_p$. Because $V^i_a = \bar{u}_1 + \bar{u}_2$, the agent is indifferent for $i = N, C, D$ and the principal seizes all the rents.

Under limited liability, $V^i_p = -L^i = 0$, so the principal is forced to zero profit, his reservation level. So the agent captures all the rents: $V^i_a = TS^i_t$. Of course, this means $V^D_a \geq V^C_a \geq V^N_a$. ■

7 Ability

This section will extend the model to include an underlying ability parameter that persists through both stages. In particular, suppose that the production function is a general $V(q_1, q_2)$. Suppose now that $q_t = ae_t + \epsilon_t$. Clearly, higher ability makes effort more productive. Ability $a$ is distributed according to a prior distribution $f(\cdot)$ with support $A$. In what follows, we’ll need the condition that the higher the realization of first stage output, the more likely it is that ability is high. This is the monotone likelihood ratio property of the posterior density.

4Let $L^N_1 = L^N$, the loss prize under no evaluation. There is only one stage under no evaluation.
**Definition 1** The posterior $f(a|q_1)$ satisfies monotone likelihood (MLRP) if $L(a) \equiv \frac{f_q(a|q_1)}{f(a|q_1)}$ increases in $a$.

Let $U(q_1)$ be the continuation utility after stage one. This is the expected utility of continuing, given that first stage output is $q_1$. Formally,

$$U(q_1) = \int_A \int V(q_1, ae_2(q_1) + \epsilon_2)g(\epsilon_2)d\epsilon_2f(a|q_1)da - C(e_2(q_1)).$$

As before, a cutoff strategy is a target $q^*$ that the agent sets for himself, such that he continues to work if $q_1 > q^*$ but not otherwise. Let $V_t$ be the partial of $V(\cdot, \cdot)$ with respect to $q_t$. Then we have the following result, proved in the appendix.

**Proposition 7** If $V_t > 0$ and (MLRP) holds, then the agent uses a cutoff strategy.

The agent will continue if his continuation utility exceeds his outside option, or $U(q_1) \geq \bar{u}_2$. The proof amounts to showing that $U(q_1)$ is strictly increasing. The following example will illustrate this.

Suppose the production function is linear, so $V(q_1, q_2) = V(q_1 + q_2)$. In this case, from the first order condition, the optimal effort function will satisfy $C'(e_2(q_1)) = V$. Effort will be a constant function of $q_1$. There is no need to decrease $e_2$ after high $q_1$ because the linearity in the production function allows the agent to capture $V$ for every marginal increase in output. The continuation utility simplifies to

$$U(q_1) = Vq_1 + V(e_2E[a|q_1]) - C(e_2)$$

where $e_2$ solves $C'(e_2) = V$. Now $\frac{\partial}{\partial q_1} E[a|q_1] = \int af_q(a|q_1)da > 0$, where the inequality follows from (MLRP) \(^5\). Hence

$$U'(q_1) = V[1 + e_2\int af_q(a|q_1)da] > 0$$

By the intermediate value theorem, there exists a $q^*$ such that $U(q^*) = \bar{u}_2$. Hence the agent uses a cutoff strategy: he advances if $q_1 > q^*$, or $U(q_1) > \bar{u}_2$. The proof of Proposition 8 is a more general version of this example.

I will extend this example to show how the optimal target $q^*$ varies with the noise in the model. In particular, suppose that errors are distributed $e_t \sim N(0, s^2)$, and the

\(^5\)The full argument is in the proof of Proposition 8. Let $z(a) = a$ in Lemma 1.
ability parameter is distributed $a \sim N(a_0, t^2)$. Calculation shows that the posterior is also normal:

$$a|q_1 \sim N\left(\frac{t^2 e_1^2}{t^2 e_1^2 + s^2} q_1 + \frac{s^2}{t^2 e_1^2 + s^2} a_0, \frac{s^2 t^2 e_1^2}{t^2 e_1^2 + s^2}\right).$$

As usual, the posterior mean weights $q_1$ and $a_0$ according to their relative precision, placing more weight on the random variable with smaller variance. As the prior variance decreases to zero, this distribution collapses to the point $a_0$. Similarly, as the error variance decreases to zero, this distribution collapses to $q_1$.

Now we can write out continuation utility explicitly, and solving for the threshold through the equation $U(q^*) = \bar{u}_2$ gives

$$q^* = \frac{z(t^2 e_1^2 + s^2) - e_2 s^2 a_0}{t^2 e_1^2 (1 + e_2) + s^2}$$

where $z = (\bar{u}_2 + C(e_2))/V$. Observe that $\frac{\partial q^*}{\partial a_0} < 0$. The higher the prior mean, the lower the target. Suppose a student fails a midterm exam, and must decide whether to drop the class or not. Intuitively, keeping all other parameters fixed, if he is dumb (low $a_0$), he will most likely fail the exam as well, so he should drop the class. But if he is smart (high $a_0$), he just got unlucky that day on the midterm, so he should stay in the class and try again the final. Hence there is a range of $q_1$’s such that the dumb student quits and the smart student stays, so the smart threshold must necessarily be lower than the dumb threshold.

Now consider the noise in the model. Some algebra shows that $\frac{\partial q^*}{\partial t^2} > 0$ if $a_0 > z/(1 + e_2)$. In words, if the prior mean is sufficiently high, the agent will set a lower threshold as his prior information improves. Suppose the student is smart, and he fails the midterm. If he knows that he is smart, then he will blame bad luck, and stay in the class. If he doesn’t know his ability well, he thinks that he might be dumb, so he will drop the class (with some probability). Hence there is a range of midterm scores such that the informed student stays and the uninformed student drops. So the informed threshold is less than the uninformed threshold. Smart students (high types) are more willing to tolerate failures as the information on their ability improves.

Among other things, this explains greater turnover and labor mobility among young professionals compared to older workers. Young workers have poor information on their ability. When they observe low output (bad grades, poor performance reviews, failed ventures, etc.), they interpret this as a signal of their low ability and switch jobs, maybe
even careers. Interpreting a more broadly as the quality of the match between worker and job, the young consultant quits his firm after his lousy performance review, figuring that consulting is not his cup of tea. Older workers with more precise information on their ability tolerate more failures, blaming bad luck (poor market conditions, uncooperative team members, etc.) rather than concluding that they are the low-types.

An analogous argument works for the error variance: \( \frac{\partial q^*}{\partial z} < 0 \) if \( a_0 > \frac{z}{(1 + c_2)}. \) If the prior mean is sufficiently high, the agent will set a higher threshold as his error information improves. Suppose a smart student fails the midterm. If he knows his luck precisely, then failing the mid-term means that he is low ability, and he cannot just blame bad luck. So he drops the class. If he does not know his luck very well, then he blames bad luck for his low score, and stays in the class. So the informed threshold is higher than the uninformed threshold. As the noise in the evaluation process increases, the agent is more willing to tolerate failures. A smart student is willing to tolerate low midterm scores under a lousy (noisy) test, as the noise renders the information from the test practically useless.

One application is that firms with very precise evaluation procedures should expect higher rates of turnover and job separation. One such procedure is the 360 degree performance appraisal, in which an employee’s review solicits input from all parties that the employee has contact with (bosses, co-workers, subordinates, customers, etc.). Adding more sources of input to the review makes it more precise, and gives the employee a better signal of his ability. By the theory above, this leads to greater job separation.

### 7.1 Allocation of Effort

Now that the agent is using a cutoff strategy, it is easy to calculate his optimal decision rule, which will be an effort level \( e_1 \), an effort function \( e_2(q_1) \), and an intermediate target \( q^* \). The agent will first condition on \( a \), and then average over all \( a \in A \). So his maximization problem is

\[
\max_{e_1, e_2(\cdot) \mid q^*} E_a \left[ \int_{q^*} U(q_1) g(q_1 - ae_1) dq_1 + G(q^* - ae_1) \bar{w}_2 - C(e_1) \right]
\]

Solving this problem and making an additional assumption gives the following result, proved in the appendix.

**Proposition 8** If \( V_1 = V_2 > 0 \) and (MLRP) holds, then the agent works harder in the first stage than he does on average in the second stage.
The argument here is similar to the analogous proposition in the model without ability. Because marginal costs are convex, Jensen’s inequality guarantees that \( e_1 > E[e_2(q_1)] \). The assumption \( V_1 = V_2 \) is necessary in order to compare \( C'(e_1) \) with \( C'(e_2(q_1)) \). The main difference with adding ability is that now \( U'(q_1) > C'(e_2(q_1)) \); without ability, this would be an equality. Because the ability persists in stage two, the marginal utility of continuing exceeds the marginal cost of second stage effort.

8 Conclusion

This paper examines the timing and information revelation of performance evaluations. It constructs a dynamic moral hazard model to explore whether managers should evaluate halfway through long projects, how much information to reveal, and how to structure the optimal contracts. The most important assumption is that the worker does not observe his own output, and the manager observes output only if he chooses to evaluate. The main conclusions are:

1. Halfway performance evaluations increase total surplus because it gives the principal the option to terminate a low-performer. This is an efficiency result, so the worker who fails to clear the intermediate target would quit because of his outside option.

2. Revealing output to the worker increases total surplus. The worker now conditions second stage effort on first stage output, and rationally decreases effort after a very successful early-stage. Since his labor is expensive, this saves the principal money as well.

3. The allocation of effort between stages depends on the amount of information revealed the worker. If the worker receives coarse feedback, he works harder in the second stage than in the first. If the worker receives detailed feedback, the reverse is true. This happens because the worker who knows his output avoids wasteful effort after a high \( q_1 \).

4. Since both parties are risk neutral and not wealth constrained, the optimal contracts involved the firm selling the project to the worker. More interestingly, even though the principal reveals output to the worker, she does not need to condition the contract on this output in order to implement efficient effort levels. Any contract that does vary with output will be inefficient.

5. All the qualitative results hold if the model includes worker ability. In particular, and they are three implications for turnover in the labor force. Turnover rates are
higher for less able and younger workers, and more precise evaluation procedures (the 360 appraisal) generate higher turnover.

9 Appendix

Proof of Proposition 2

Proof: Recall that $U(q_1) \equiv P(q_1)V - C(e_2(q_1))$. Let $X \equiv \{ q_1 : U(q_1) > \bar{u}_2 \}$ be the social planner’s continuation set: it is efficient to allow a worker with output $q_1$ to advance to the second stage if and only if $q_1 \in X$. Let $P_1 = Pr(X)$ and $Q = Pr(q_2 > \bar{q}|X)$. Let $P = P_1Q$ be the probability of final success. Total surplus is $P_1[QV - C(e_1) - E[C(e_2(q_1))|X] + (1 - P_1)(\bar{u}_2 - C(e_1))]$. Rewrite this as the social planner’s problem

$$\max_{e_1,e_2} PV - P_1E_{q_1}[C(e_2(q_1))|X] - C(e_1) + (1 - P_1)\bar{u}_2$$

which we can expand as

$$\max_{e_1,e_2} \int_X [VG(q_1 + e_2(q_1) - \bar{q}) - C(e_2(q_1))]g(q_1 - e_1)dq_1 - C(e_1) + (1 - P_1)\bar{u}_2.$$ 

Maximizing the integral pointwise yields

$$e_2(q_1) \in \arg \max_{e_1,e_2} VG(q_1 + e_2(q_1) - \bar{q}) - C(e_2)$$

for almost every $q_1 \in X$. For every such $q_1$, $e_2(q_1)$ solves

$$C'(e_2(q_1)) = Vg(q_1 + e_2(q_1) - \bar{q})$$

Using the envelope theorem,

$$e'_2(q_1) = -\frac{Vg'(q_1 + e_2(q_1) - \bar{q})}{Vg'(q_1 + e_2(q_1) - \bar{q}) - C''(e_2)}.$$ 

The denominator is negative by the second order condition, and since costs are convex, we have

$$1 + e'_2(q_1) = -\frac{C''(e_2)}{Vg'(q_1 + e_2(q_1) - \bar{q}) - C''(e_2)} > 0$$

$$P'(q_1) = g(q_1 + e_2(q_1) - \bar{q})(1 + e_2(q_1)) > 0$$

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The ex-post probability of success increases in \( q_1 \). Finally, combining this with \( U'(q_1) = P'(q_1)V - C'(e_2(q_1))e'_2(q_1) \) gives

\[
U'(q_1) = Vg(q_1 + e_2(q_1) - \bar{q}) + e'_2(q_1)[Vg(q_1 + e_2(q_1) - \bar{q}) - C'(e_2(q_1))] > 0
\]

where the term in brackets is zero from the first order condition of \( e_2(q_1) \), and the inequality follows from the positive density. Hence the ex post social value of a worker increases in his first stage output. Since \( U(\cdot) \) is continuous, by the intermediate value theorem, there exists a \( q^* \) such that \( U(q^*) = \bar{u}_2 \). Since \( U(\cdot) \) is strictly increasing, this means that \( X = \{ q_1 : q_1 > q^* \} \).

To see the shape of \( e_2(q_1) \), note that from the envelope condition

\[
e'_2(q_1) > 0 \iff Vg'(q_1 + e_2(q_1)) > 0 \iff q_1 + e_2(q_1) < \bar{q}
\]

since the density \( g \) is symmetric around zero. Since \( 1 + e'_1(q_1) > 0 \), the function \( q_1 + e_2(q_1) \) is strictly increasing, so \( q_1 + e_2(q_1) < \bar{q} \) iff \( q_1 < \hat{q}_1 \), where \( \hat{q}_1 \) solves \( q_1 + e_2(\hat{q}_1) = \bar{q} \). From the first order condition for \( e_2(q_1) \) we have \( C'(e_2(\hat{q}_1)) = Vg(0) \). Combining these last two equations gives \( \hat{q}_1 = \bar{q} - (C')^{-1}Vg(0) \). So \( e_2(\cdot) \) is increasing for \( q_1 < \hat{q}_1 \) and decreasing for \( q_1 > \hat{q}_1 \).

\[\blacksquare\]

**Corollary 2** Under no evaluation, in every optimal contract the agent sets \( e_1 = e_2 \).

**Proof:**

The agent’s problem is

\[
\max_{e_t} E[t(q_2)] - C(e_1) - C(e_2)
\]

where \( E[t(q_2)] = \int \int t(e_1 + e_2 + x + y)g(x)g(y)dxdy \). The first order condition is

\[
\int \int t'(e_1 + e_2 + x + y)g(x)g(y)dxdy = C'(e_t).
\]

Since the left-hand side is independent of \( t \), the right hand side is as well, so \( C'(e_1) = C'(e_2) \). To complete the description of the optimal contract, consider the principal’s problem. The principal will lower the payments until the agent receives his reservation utility. The agent’s ex-ante IR constraint is

\[
E[t(q_2)] = C(e_1) + C(e_2) + \bar{u}_1 + \bar{u}_2
\]
Since the agent does not see $q_1$, there is no ex-post IR constraint. The principal’s problem is

$$\max_{t(\cdot)} PV - E[t(q_2)]$$

Substituting in the agent’s IR constraint, the first order condition yields

$$V \int g(x)g(\bar{q} - 2\hat{e} - x)dx = \int \int t'(e_1 + e_2 + x + y)g(x)g(y)dxdy$$

Any optimal contract satisfies this equality. ■

**Lemma 1** Let (MLRP) hold. If $z(a)$ is strictly increasing, then $\int_A z(a)f_{q_1}(a|q_1)dq_1 > 0$.

**Proof:** As usual, the posterior is given by

$$f(a|q_1) = \frac{g(q_1|a)f(a)}{\int g(q_1|a)f(a)da}.$$  

Now

$$f_{q_1}(a|q_1) = \frac{g'(q_1|a)f(a)\int g(q_1|a)f(a)da - g(q_1|a)f(a)\int g'(q_1|a)f(a)da}{(\int g(q_1|a)f(a)da)^2}.$$ 

So

$$\int_A f_{q_1}(a|q_1)dq_1 = 0.$$ 

Monotone likelihood will show that the function $f_{q_1}$ crosses the x-axis only once. By (MLRP), $L(a) \equiv \frac{f_{q_1}(a|q_1)}{f(a|q_1)}$ is increasing, so there exists an $a^*$ such that $\{a : L(a) < 0\} = (\underline{a}, a^*)$ and $\{a : L(a) > 0\} = (a^*, \overline{a})$, where $\underline{a}, \overline{a}$ are the upper and lower limits of $A$, respectively. By definition of $L(a)$, $f_{q_1} > 0$ if and only if $L(a) > 0$. Hence $\{a : f_{q_1}(a|q_1) > 0\} = (\underline{a}, a^*)$ and $\{a : f_{q_1}(a|q_1) < 0\} = (a^*, \overline{a})$. Then we can rewrite the equation above as

$$\int_{a^*}^{\pi} f_{q_1}(a|q_1)dq_1 = \int_{a^*}^{\pi} |f_{q_1}(a|q_1)|dq_1$$

where $f_{q_1}(a|q_1) < 0$ if $a < a^*$, so $|f_{q_1}(a|q_1)| = -f_{q_1}(a|q_1)$. Then we have

$$\int_a^{a^*} z(a)|f_{q_1}(a|q_1)|dq_1 < \int_a^{a^*} z(a^*)|f_{q_1}(a|q_1)|dq_1 = \int_{a^*}^{\pi} z(a^*)f_{q_1}(a|q_1)dq_1 < \int_{a^*}^{\pi} z(a)f_{q_1}(a|q_1)dq_1.$$
where the inequalities follow from $z(a)$ being strictly increasing. Take the left hand side over to the right gives

$$
\int_A z(a)f_{q_1}(a|q_1)daq_1 = \int_{q_1^*}^\pi z(a)f_{q_1}(a|q_1)dq_1 - \int_a^\pi z(a)|f_{q_1}|(a|q_1)dq_1 > 0.
$$

\[\square\]

\section*{Proof of Proposition 8}

\textbf{Proof:} Let \(X \equiv \{q_1 : U(q_1) > \bar{w}_2\}\) be the continuation set. That is, the agent advances to the second stage if \(q_1 \in X\). Then the agent solves

$$
\max_{e_1,e_2} E_a[\int_X U(q_1)g(q_1 - ae_1)dq_1 + Pr(q_1 \in X) - C(e_1)]
$$

For each \(a \in A, q_1 \in X\),

$$
e_2(q_1) \in \arg\max_{e_2} U(q_1)g(q_1 - ae_1)f(a).
$$

Note that only \(U(q_1)\) contains \(e_2\). So

$$
C'(e_2(q_1)) = \int_A\int V_1(q_1, ae_2(q_1) + \epsilon_2)g(\epsilon_2)d\epsilon_2f(a|q_1)da.
$$

Recall that

$$
U(q_1) = \int\int_A V(q_1, ae_2(q_1) + \epsilon_2)g(\epsilon_2)d\epsilon_2f(a|q_1)da - C(e_2(q_1)).
$$

Taking the derivative with respect to \(q_1\) and substituting in the previous expression for \(C'(e_2)\) gives \(U'(q_1) = W + Y\) where

$$
W = \int\int_A V_1f(a|q_1)dag(\epsilon_2)d\epsilon_2.
$$

$$
Y = \int\int_A V(q_1, ae_2(q_1) + \epsilon_2)f_{q_1}(a|q_1)dag(\epsilon_2)d\epsilon_2.
$$

Since \(V_1 > 0\), \(W\) is positive. Now \(V_2 > 0\), so the function \(z(a) \equiv \int V(q_1, ae_2 + \epsilon_2)g(\epsilon_2)d\epsilon_2\) increases in \(a\). By Lemma 1, this means

$$
Y = \int z(a)f_{q_1}(a|q_1)dag(\epsilon_2)d\epsilon_2 > 0.
$$
Thus $U'(q_1) > 0$. Clearly $U$ is continuous. By the intermediate value theorem, there exists a $q^*$ such that $U(q^*) = \bar{u}_2$. Hence $X = \{q_1 : q_1 > q^*\}$. ■

**Proof of Proposition 9**

**Proof:** First order conditions give

$$U(q^*) = \bar{u}_2$$

$$C'(e_2(q_1)) = \int \int_A V_2 f(a|q_1) dag(\epsilon_2) d\epsilon_2$$

$$C'(e_1) = E_a[-\int_{q^*} U(q_1) g'(q_1 - ae_1) dq_1 - g(q^* - ae_1)\bar{u}_2]$$

Substitute $\bar{u}_2 = U(q^*)$ into the last equation and integrate by parts to yield

$$C'(e_1) = E_a[\int_{q^*} U'(q_1) g(q_1 - ae_1) dq_1.]$$

Recall that

$$U(q_1) = \int \int_A V(q_1, ae_2(q_1) + \epsilon_2) g(\epsilon_2) d\epsilon_2 f(a|q_1) da - C(e_2(q_1)).$$

Using the first order condition for $e_2(q_1)$,

$$U'(q_1) = Z + \int \int_A V_1 f(a|q_1) dag(\epsilon_2) d\epsilon_2$$

where

$$Z = \int \int_A V(q_1, ae_2(q_1) + \epsilon_2) f_{q_1}(a|q_1) dag(\epsilon_2) d\epsilon_2$$

If $V_1 = V_2$ then

$$U'(q_1) = Z + C'(e_2(q_1))$$

Let $z(a) = V(q_1, ae_2(q_1) + \epsilon_2)$. Now $V_2 > 0$ means that $z(a)$ is increasing. By Lemma 2 in the appendix, this means $Z \equiv \int \int_A z(a) f_{q_1}(a|q_1) dag(\epsilon_2) d\epsilon_2 > 0$. Combining, this gives

$$C'(e_1) = E_a E_{q_1}[Z + C'(e_2(q_1))] > E_a E_{q_1} C'(e_2(q_1)) > E_a C'(E_{q_1}[e_2(q_1)]),$$

where the last inequality follows from Jensen’s Inequality, since marginal costs are convex. Since costs increase, $e_1 > E[e_2(q_1)]$. ■
References


