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Income Distribution and the Allocation
of Public Education Expenditure

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Income Distribution and Political Economy of Allocation of Public Education Spending *

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Abstract

Over the last 30 years, countries with more unequal income distributions tended to spend more on tertiary education and less on secondary education. During the same period of time, countries spending more on tertiary (secondary) education in one decade tended to experience more (less) unequal income distribution in the next decade. Thus, the relationships between the allocation of education spending and income distribution provide a potential explanation for persistent inequality. This paper develops a political economy model capable of accounting for these stylized facts. The allocation of public education expenditure is the outcome of the interaction between an incumbent government and lobbies representing different socio-economic classes. Unlike previous studies of political influences, the formation of lobbies is endogenous in my model, and is determined by the parameters of the economy. The model generates an abundant assortment of lobby formation equilibria and income distribution dynamics. In particular, it shows that in a more unequal economy, the rich tend to capture the political power, which in turn leads to more unequal income distribution for the future generations, and vice versa.

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1 Introduction

Government spending on education is a pervasive feature of modern economies; it covers all education levels in virtually every country. It accounts, on average, for more than 4.5% of a country’s GNP and more than 14% of a country’s total government expenditure.\(^1\) Public education spending is deemed one of the most important redistributive policies.

Traditional economic theories (for example, Meltzer and Richard [1981], Alesina and Rodrik [1994], and Persson and Tabellini [1994]) predict that more unequal societies experience more redistribution. Empirical studies, however, show that the effect of income distribution on aggregate public education expenditure is rarely significant, and its sign varies from one study or even one specification to another (Perotti [1996], Lindert [1996], Rodriguez [1998]). In addition, there is no clear evidence that more public education expenditure leads to a larger reduction in income inequality over time.

This paper goes a step further and examines the relationship between income distribution and the allocation of public education expenditure among primary, secondary, and tertiary education levels. It reveals that once one disaggregates spending over educational levels, more unequal societies indeed appear to have a less redistributive manner of spending on education. Regression results show that countries with a more unequal income distribution tend to spend proportionately more on tertiary education and less on secondary education; countries spending more on tertiary (secondary) education today tend to experience more (less) unequal income distribution in the future. Thus, the relationships between the allocation of education spending and income distribution provide a potential explanation for persistent inequality.

This paper focuses on a political economy explanation for these stylized facts. In a simple general equilibrium political economy model with overlapping generations, the allocation of public education expenditure is determined through the interaction between the government and various lobbies representing different socio-economic classes. Lobbies bid for more education spending on their own children, which leads to higher future income. Lobbies exert their influence on public education policy by proposing a contribution function contingent on the policy outcome. The government maximizes its own utility, which depends on total contributions and the aggregate social welfare.

Thus, the model has the feature of a “common agency” model developed by Bernheim and Whinston (1986), and first applied to studying political influence by Grossman and Helpman (1994). The novelty of my model is that the formation

\(^1\)Data source: various issues of the UNESCO Statistical Yearbook.
of lobbies is endogenous. With three income groups simultaneously making the lobbying decisions, the model can generate an abundant assortment of lobby-entry equilibria and income distribution dynamics.

For certain configurations of the parameters, the model generates a family of equilibria in which the rich have more political influence. In these equilibria, the richer a group is, the more likely it is to enter the lobbying; the more unequal the income distribution, the more likely the rich are the only ones to enter the lobbying. Depending on the initial income distribution, different sets of lobbies may form, leading to different policy outcomes and different income distributions of the next generation. More specifically, a more unequal initial income distribution may lead to a less redistributive allocation that brings about a more unequal income distribution for the next generation, and vice versa. The economy may reach different steady state income distributions in the long run, exhibiting persistent differences in income distribution and redistributive education policy.

This paper fits into the general topic on income distribution and redistributive policies. It contributes to the literature on two fronts. First, it is the first attempt to systematically describe the empirical relationships between income distribution and the allocation of public education expenditure among different school levels. The existing empirical studies examining the links between income inequality and redistributive policies produce, on the whole, ambiguous results. Examples are Persson and Tabellini (1994), Lindert (1996), Perotti (1996), and Rodriguez (1998).2 The redistribution measures in these studies include shares of transfers in GDP (either as a whole or decomposed into different categories such as welfare, unemployment, health, and social security), average and marginal tax rates, and aggregate public education expenditure. The measure of inequality is generally taken to be the income share of the third quintile or the third plus the fourth quintiles, reflecting the underlying median voter model.3 Another drawback of the existing studies is that most of them use a very small sample with limited coverage of mostly OECD countries. In my study, in contrast, the sample includes most of the democratic countries (defined in the next section). By decomposing aggregate public education expenditure into expenditure shares on different school levels, I uncover significant and unambiguous relationships between inequality and public education expenditure.

Second, this paper develops a special interest political economy model capable of accounting for the stylized facts, and it contributes to bridging the gap between the theoretical predictions and empirical findings regarding the relationship

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2Bénabou (1996) summarizes the results of other empirical studies.
3For an exception, Rodriguez (1998) uses the Gini coefficient to measure inequality. This distinction, however, is of more theoretical relevance, since empirically, all of the inequality measures are closely correlated with each other.
between income distribution and redistributive policies. In a positive analysis of the size of government, measured by the share of income redistributed, Meltzer and Richard (1981) apply a majority voting model and conclude that a decrease in the income of the median voter increases the size of government. In other words, greater inequality translates into more redistribution. This argument is also at the heart of Alesina and Rodrik (1994) and Persson and Tabellini (1994) in explaining the negative relationship between initial inequality and subsequent aggregate growth. Glomm and Ravikumar (1992) show that, in a standard majority voting model, a more unequal society chooses public education, which is more redistributive, over private education, which is less. As is summarized in the previous paragraph, however, the cross-country data does not seem very supportive of these theoretical predictions. To reconcile this inconsistency, Bénabou (2000) argues that asymmetries in political influence can break the positive link between inequality and redistributive policy and create persistent inequality. The policy considered is an income tax rate, and it is determined by majority voting. Thus, the fundamental difference between Bénabou (2000) and the previous studies is that he introduces a decisive voter who is different from the median voter. The same can be said about Ferreira (2001), who focuses on the political economy of total public education expenditure and distribution dynamics. My model also intends to overturn the positive relationship between inequality and redistribution, but it differs from Bénabou (2000) in two major aspects. First, the policy in question is public expenditure on different groups. Thus the policy space is multi-dimensional, and the model departs further from the traditional majority voting model. Second, I establish the asymmetry of political influence from a political economy model of campaign contributions with an endogenous formation of lobbies.

Rodriguez (1998), using a political economy model with campaign contributions, concludes that greater inequality can lower the effective redistributive tax rates. There are three distinctions between his model and mine. First, his model is static with an endowment economy, hence not suitable for studying distribution dynamics. Second, although the policy space in his model is multi-dimensional, he circumvents this complexity by assuming that each lobby negotiates with the government to affect only his own effective tax rate. From this perspective, it is a special and degenerate case of the more general common agency model. Third, and most important, he rules out, ad hoc, the possibility of lobbying by the poor.

This paper is organized as follows. Section 2 shows that relationships between income distribution and redistributive policy through education expenditure are pronounced once one disaggregates across educational levels. Section 3 sets up the political economy model and describes the Subgame Perfect Nash Equilibrium.
concept. Section 4 characterizes the static equilibria of the model. Section 5 demonstrates the income distribution dynamics generated by the model. Section 6 concludes.

2 Stylized Facts

This section presents the cross-country empirical evidence regarding the relationship between income distribution and the allocation of public education expenditure among primary, secondary, and tertiary levels.

2.1 Data

Countries in my sample are those with democratic political institutions, reflecting the emphasis on the political economy of governmental policy making. Specifically, I include only countries scoring between 1 and 5, on average, over the period from the early 1970s to the late 1990s, according to Gastil’s index of political rights. Similar sample selection criterion has also been adopted by Persson and Tabellini (1999, 2001). This selection rule produces a sample of 78 countries.

The variables of primary interest are the percentages spent on various education levels out of total public education expenditure; therefore, “tershare” is the percentage spent on the tertiary level, and “secshare” that on the secondary level. All education spending refers to the current education spending. The data is obtained from various issues of the UNESCO Statistical Yearbook. Another important variable is the Gini coefficient (gini); I use the “accepted” subset of the database compiled by Deininger and Squire (1996). The accepted data has national coverage. Finally, I use a number of socio-economic variables to control for the demand for education: (i) the log of per capita income, denoted as gdp; (ii) the shares of the three age groups that officially correspond to the three education levels in the total population, denoted as ter_age/pop, sec_age/pop, and pri_age/pop. The data sources are the UNESCO Statistical Yearbook and the World Bank World Development Indicator CD-ROM (2001).

I use decade averages of all the variables for the following reasons. First, one does not expect income distribution or public education policy to change dramatically from year to year. Second, missing observations for different variables in different years is the norm rather than the exception; averaging alleviates this problem. Third, averages help to mitigate short-term fluctuations due to business cycles. The decade average data set for the 90s includes 66 countries, the 80s set

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4On a scale from 1 to 7, Gastil (1986-1998) classifies countries scoring 1-2 as “free”, and those scoring 3-5 as “semi-free”.

5
includes 49, and the 70s set includes 38.\textsuperscript{5} Missing data for the Gini coefficient is the major cause of the reduction in sample size. Since the sample size of the 70s data set is far smaller, it has only limited use.

2.2 Stylized Facts

First, consider the relationship between the current income distribution and the current allocation of education expenditure. Countries with more unequal income distributions tend to spend proportionately more on tertiary education and less on secondary education. Table 1 reports the results from cross-country ordinary least-squares regressions of spending shares at the tertiary and secondary levels on income, three demographic variables, and the Gini coefficient for both the 90s and 80s. As can be seen from the table, the Gini coefficient has a positive relationship with the spending share at the tertiary level and a negative relationship with the spending share at the secondary level; these relationships are significant for both the 90s and 80s.\textsuperscript{6} Since the spending shares on primary, secondary, and tertiary levels sum to 1, a regression of spending share on primary education is redundant. One can indeed infer from the regressions for \textit{tershare} and \textit{secshare} that the Gini coefficient has a significantly positive relationship the spending share at the primary level in the 80s.

A potential concern with these estimates is that the Gini coefficient in the Deininger and Squire (1996) database may not be directly comparable. Some numbers come from consumption expenditure surveys, others from income surveys; some pertain to household income, others to personal income; finally, some concern gross income, others net (after tax) income. I deal with this issue by including in the regressions dummy variables for Gini coefficients generated from expenditure data, personal income data, and net income data. The coefficient on the Gini coefficient stays the same qualitatively in all four regressions. The regression results are reported in Appendix-1. Thus, the patterns summarized in Table 1 are robust to the incomplete comparability of the estimates for the Gini coefficient.\textsuperscript{7}

Next, consider the relationship between the education expenditure allocation of decade \(t - 1\) and the income distribution of decade \(t\). Table 2 reports the results from ordinary least-squares regressions of the Gini coefficient on spending shares at the tertiary and secondary levels during the previous period for both

\textsuperscript{5}Seven eastern European countries are included in the 90s data set; their inclusion has virtually no effect on the regression results below.
\textsuperscript{6}The Gini coefficient takes values between 0 and 1. The higher the Gini coefficient, the more unequal the income distribution.
\textsuperscript{7}Atkinson and Brandolini (2001) critically review the problems of and corrections for the “secondary” data sets, in particular, data sets for income inequality.
### Table 1: Cross Country Regressions of Tertiary and Secondary Spending on the Gini Coefficient in the 90s and 80s

<table>
<thead>
<tr>
<th></th>
<th>1990s</th>
<th>1980s</th>
<th>1990s</th>
<th>1980s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>tershare</td>
<td>secshare</td>
<td>tershare</td>
<td>secshare</td>
</tr>
<tr>
<td>gdp</td>
<td>0.005</td>
<td>0.040</td>
<td>-0.004</td>
<td>0.038</td>
</tr>
<tr>
<td></td>
<td>(0.38)</td>
<td>(2.56)</td>
<td>(-0.225)</td>
<td>(1.51)</td>
</tr>
<tr>
<td>ter_age/pop</td>
<td>-0.297</td>
<td>-2.04</td>
<td>1.65</td>
<td>-0.27</td>
</tr>
<tr>
<td></td>
<td>(-0.275)</td>
<td>(-1.87)</td>
<td>(1.76)</td>
<td>(-0.31)</td>
</tr>
<tr>
<td>sec_age/pop</td>
<td>0.063</td>
<td>1.41</td>
<td>-0.64</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>(0.204)</td>
<td>(3.06)</td>
<td>(-1.52)</td>
<td>(3.50)</td>
</tr>
<tr>
<td>pri_age/pop</td>
<td>-0.444</td>
<td>-0.444</td>
<td>-0.29</td>
<td>-0.155</td>
</tr>
<tr>
<td></td>
<td>(-1.37)</td>
<td>(-0.796)</td>
<td>(-0.73)</td>
<td>(-0.286)</td>
</tr>
<tr>
<td>gini</td>
<td>0.311</td>
<td>-0.262</td>
<td>0.205</td>
<td>-0.553</td>
</tr>
<tr>
<td></td>
<td>(2.532)</td>
<td>(-1.8)</td>
<td>(2.1)</td>
<td>(-4.04)</td>
</tr>
<tr>
<td># of Observations</td>
<td>60</td>
<td>60</td>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.117</td>
<td>0.647</td>
<td>0.191</td>
<td>0.579</td>
</tr>
</tbody>
</table>

Numbers in parentheses are t-statistics.

The results suggest that the relationships between income distribution and the allocation of public education expenditure tend to perpetuate an initial income distribution. Those with high initial Gis have proportionately lower secondary and higher tertiary spending, and experience higher Gis in the future. Previous empirical studies (see, for example, Cameron and Heckman [1998], Filmer and Pritchett [1998], and Shavit and Blossfeld [1993]) demonstrate that a child’s education attainment is positively correlated with his parents’ income. Children from richer families are more likely to attend college, whereas children from less wealthy families are more likely to drop out early. This regularity holds for both developed and developing countries. Therefore, public spending on primary edu-

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8The regression analysis has not tried to control every socio-economic factor that might affect the evolution of both income distribution and education spending, for example, corruption, religion, racial and gender discrimination, and technology progress. There may exist different stories to explain income distribution persistence through these other channels.
Table 2: Cross Country Regressions of the Gini Coefficient in the 90s and 80s on Tertiary and Secondary Spending in the 80s and 70s

<table>
<thead>
<tr>
<th></th>
<th>gini$_{90}$</th>
<th>gini$_{80}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gini$_{t-1}$</td>
<td>0.89</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>(17.6)</td>
<td>(11.9)</td>
</tr>
<tr>
<td>tershare$_{t-1}$</td>
<td>0.17</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>(3.47)</td>
<td>(1.79)</td>
</tr>
<tr>
<td>secshare$_{t-1}$</td>
<td>-0.1</td>
<td>-0.18</td>
</tr>
<tr>
<td></td>
<td>(-2.52)</td>
<td>(-2.43)</td>
</tr>
<tr>
<td># of Observations</td>
<td>49</td>
<td>38</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.92</td>
<td>0.83</td>
</tr>
</tbody>
</table>

Numbers in parentheses are t-statistics.

cation benefits the population more universally, that on secondary education less so, and the rich benefit disproportionately more from public spending on tertiary education. Thus, a division of public spending over different education levels is, to a significant extent, a division of spending over different socio-economic classes. This is key to understanding the mechanism of income distribution perpetuation through the allocation of education spending.

To determine quantitatively the importance of the relationships, I consider the following counterfactual question: what would the Ginis have been in decade $t$ if the educational expenditure in the previous decades had been allocated the same way irrespective of the initial level of inequality? Two counterfactual exercises are conducted. In the first one, starting from the 80s, countries “adjust” the education spending allocation to the median country in the 80s only; in the second, starting from the 70s, countries “adjust” their education spending allocation to the median country in both the 70s and the 80s. The “adjusted Ginis” are calculated based on the regression coefficients in Table 2. Details of the counterfactual exercises are in Appendix-2. The results are illustrated in Figure 1. Figure 1 plots the actual and “adjusted” Gini coefficients of the 90s against the Ginis of the 70s; the respective regression lines are also delineated. The regression lines for the “adjusted Ginis” are flatter than that for the actual Ginis, and it is even flatter if the adjustment is implemented for two periods rather than one. This flattening indicates that inequality would have been less persistent for countries with high Ginis if they had followed a more equal allocation of education spending. Indeed, with the one-period adjustment, the persistence in the Gini coefficient is reduced by 8.64%; with the two-period adjustment, it is reduced by 14.21%. In sum, public education spending contributes significantly to the persistence of income distribution.
3 Model

The above analysis shows that there exist significant relationships between income distribution and the division of public education spending among different socio-economic classes. My approach here is to offer a theoretical model of education expenditure allocation capable of accounting for these observed relationships. The model highlights the political economy of governmental expenditure decisions.

3.1 The Economy

Consider a two-period overlapping generations model with three individuals in each generation. At the beginning of the second period of life, each individual has one child. Let $I = \{1, 2, 3\}$ be the set of households in each period. Each household belongs to a different socio-economic class: the rich, the middle class, and the poor. The economic structure of the model is deliberately kept simple to highlight the impact of political interaction.

Individuals go to school when young. Each child $i$ goes to a different school with spending level $e_i$; schools are completely publicly financed. Thus, there is, in effect, a separate public school system for each household, and public spending on each school is targeted to the corresponding household. The division of public
spending over different schools is a simplified way to modelling the division of public spending on different socio-economic classes.

Children are of the same ability and a child’s accumulated human capital is simply a concave function of $e_i$, $f(e_i) = 2e_i^{1/2}$. As an adult, $i$ earns a wage income $w_i = f(e_i)$. The aggregate income of the economy is $y = w_1 + w_2 + w_3$. Let $s_i$ be the income share of household $i$, then $s_i = w_i/y$ and $\sum s_i = 1$. Let $s = (s_1, s_2, s_3)$.

The utility function of adult $i$ is:

$$u_i = c_i + \delta_i f(e_i)$$

where $c_i$ is consumption; $\delta_i$ measures adult $i$’s preference for the education of his child. This specification assumes paternalistic altruism of the parents. This is more realistic than assuming infinite-horizon altruism, see, for example, Pollak (1988) and Altonji et al (1992).

I assume that, for the first generation, $\delta_i \geq \delta_j$ if $w_i \geq w_j$. One argument for this assumption is that own consumption and child’s education are complements in a parent’s utility function. A wealthier parent values higher his child’s education. Modelling directly this complementarity in a common agency framework is quite complicated; therefore, I use this quasi-linear specification of the utility function to simplify the analysis.

The difference in valuation of education leads to the difference in the demand for education expenditure, which, mediated through the political process, eventually leads to the difference in actual education expenditure on each child.

### 3.2 Public decision making

The only role the government plays in this economy is raising revenue and providing public education to every child. In each generation, an incumbent government determines the proportional income tax rate $\tau$ and how to allocate the tax revenue among the three schools by choosing $\{e_1, e_2, e_3\}$. Since an adult does not have a labor-leisure choice, the tax is nondistortionary and $\tau$ is residually determined as $\tau = (e_1 + e_2 + e_3)/y$. Denote the policy scheme as $p = \{e_1, e_2, e_3\}$ and $\tau = \tau(p)$. Assume that the government of every generation has to maintain a balanced budget, then the feasible policy space is defined as $\mathcal{P} = \{p = (e_1, e_2, e_3) | \tau(p) \leq 1\}$.

A central assumption of this model is that lobbies representing different socio-economic classes influence the policy making of the incumbent government. Lobbying by socio-economic classes is not uncommon in the real world. For example, it is believed that lobbying by the middle- and upper-class families is the driving force for the change in the federal college student aid program during the 1980s and 1990s in the United States(Spencer [1999]); in Britain, middle-class lobbying
may have helped shape the recent education policy under the New Labour (Thrupp [2001]). Moreover, lobbying by educational organizations is extensive, and these lobbies tend to articulate the demands of their clients, i.e., the parents from different socio-economic classes.

Lobbies exert their influence by making political contributions to the incumbent government contingent on the policy outcome. The political game proceeds in three stages. In the first stage, each adult decides whether or not to enter the lobbying competition; he incurs a fixed sunk cost $F > 0$ if he does. I dub this stage of the game the “entry phase”. Let $\mathcal{L} \subseteq \mathcal{I}$ denote the set of lobbies. In the second stage, lobbies simultaneously and noncooperatively choose their contribution schedules, looking ahead to the response of the government in the next stage. In the third stage, the government chooses a policy scheme $p$ optimally (to be specified below), given the contribution functions of all the lobbies, and collects from each lobby the contribution associated with its policy choice. I dub these two stages of the game the “post-entry phase”. The post-entry phase of the game has the feature of the common agency model developed by Bernheim and Whinston (1986), and first applied to studying political influence by Grossman and Helpman (1994).

An equilibrium is a set of lobbies, a set of contribution functions by the lobbies, and a policy vector that satisfy subgame perfection. More specifically, given the contribution functions of all the entrants, the government selects a policy scheme to maximize its own utility; taking the government’s optimization behavior and other entrants’ contribution functions as given, each entrant proposes a contingent contribution function to maximize his utility; each individual decides whether or not to enter the political game, taking as given other individuals’ entry decision and looking ahead to the optimal contribution functions of the entrants as well as the government’s optimal response.\(^9\)

Note that given the fixed cost incurred when entering the lobbying game, it never pays to buy just a little political influence. Therefore, one is willing to buy into the political market only when the utility gain from lobbying is large enough. This feature is indispensable for producing multiple equilibria at the entry phase. The simplest justification for the fixed cost is the transaction cost in building the necessary connection with government officials.

Denote $v_i = (1 - \tau(p))w_i + \delta_i f(e_i)$, then $v_i$ is interpreted as $i$’s net utility if $i \notin \mathcal{L}$ or $i$’s gross utility if $i \in \mathcal{L}$. Let $C_i(p)$ denote the contribution function of $i \in \mathcal{L}$; it is feasible if $C_i(p) \geq 0$. $i$’s net utility is,

$$u_i = \begin{cases} 
  v_i & \text{if } i \notin \mathcal{L}; \\
  v_i - C_i(p) - F & \text{if } i \in \mathcal{L}.
\end{cases}$$  \hspace{1cm} (2)

\(^9\)Due to the paternalistic altruism assumed, no intertemporal strategic consideration is involved. Thus, the game can be solved sequentially.
The government maximizes a weighted sum of aggregate social welfare $u_1 + u_2 + u_3$ and political contributions. Let $V = v_1 + v_2 + v_3$, the government utility function boils down to:

$$W(p, C(p)) = kV + \sum_{j \in \mathcal{L}} C_j(p) - k \sum_{j \in \mathcal{L}} F$$  \hspace{1cm} (3)$$

where $k > 0$ is the weight the government places on the aggregate social welfare.\(^{10}\)

Since each generation plays the same political game, in the next section, I shall concentrate on the stage game, with a subsequent section devoted to the distribution dynamics the model can generate.

4 Static Outcome

The political equilibrium can be characterized by backward induction. Making use of the existing results, I first characterize the equilibrium spending allocation at the post-entry phase for any given set of lobbies $\mathcal{L}$. With these results, I then proceed to unravel the equilibrium formation of lobbies at the entry phase.

4.1 Useful Tools

As noted before, treating $\mathcal{L}$ as given, the interaction between the various lobbies and the government in this economy has the feature of a common agency problem. Bernheim and Whinston (1986) and Dixit et al (1997) have characterized the equilibrium for a class of such problems. Their results are useful in characterizing the political equilibrium in this economy.

**Proposition 1** $\{(C^o_j)_{j \in \mathcal{L}}, p^o\}$ is a Subgame Perfect Nash Equilibrium (SPNE) of the post entry game if and only if:

(i) $C^o_j(p) \geq 0$ for all $j \in \mathcal{L}$ and $p \in \mathcal{P}$.

(ii) $p^o$ maximizes $W(p, C^o(p))$ on $\mathcal{P}$;

(iii) for all $m, j \in \mathcal{L}$, $(C^o_j(p^o), p^o)$ solves the following problem

$$\max_{\{p, C_j\}} \quad (1 - \tau(p))w_j - C_j(p) - F + \delta_j f(e_j)$$

s.t. $kV + \left( \sum_{m \neq j} C^o_m(p) + C_j(p) \right) \geq \max_{\{p \in \mathcal{P}\}} kV + \sum_{m \neq j} C^o_m(p)$ \hspace{1cm} (5)

\(^{10}\)The government maximizes $W = k \sum_{i=1}^{3} u_i + (k + 1) \sum_{j \in \mathcal{L}} C_j(p)$, i.e., the government values more highly a dollar in its own pocket than a dollar in the hands of the public. Equation (3) follows immediately.
The first condition simply states that lobby $j$'s proposed contribution function has to be feasible. The second condition describes the best response of the government, given the optimal contribution functions. The third condition is the essential part of the proposition; it prescribes the optimal behavior of every lobby. Consider lobby $j$. He takes as given contribution schedules of other lobbies when deciding his own. The constraint, Equation (5), says that $j$ has to provide the government at least the same payoff it can receive were $j$ not to make a contribution. Subject to this constraint, $j$ proposes a policy scheme and contribution schedule to maximize his utility, as in Equation (4). In equilibrium, this has to be true for every lobby. Therefore, in equilibrium, each lobby contributes exactly the right amount so that the government is indifferent between whether he contributes or not.

**Corollary 1** Let $\{C^o, p^o\}$ be a SPNE. Then for each $j \in \mathcal{L}$,

$$W(p^o, C^o(p^o)) = \max_{p \in P} W(p, \{C^o_m(p)\}_{m \neq j}, 0)).$$

The corollary follows immediately from Proposition 1. It says that the utility level of the government in the equilibrium is the same as what it would get if any one of the lobbies were to contribute zero whereas all others maintained their equilibrium contribution functions, and the government chose the optimal policy in response to this deviation. In other words, each lobby must ensure that the government gets a utility equal to its outside opportunity.

Let $p^{-j} = \arg\max_{p \in P} W(p, \{C^o_m(p)\}_{m \neq j}, 0))$. By Corollary 1,

$$C^o_j(p^o) = W(p^{-j}; \{C^o_m(p^{-j})\}_{m \neq j}, 0)) - W(p^o, \{C^o_m(p^o)\}_{m \neq j}, 0)).$$

(6)

It follows that $C^o_j(p^o) > 0$ for all $j \in \mathcal{L}$.

The above model can have multiple subgame perfect Nash equilibria. In a refinement of the Nash equilibrium developed by Bernheim and Whinston(1986), each lobby offers the government a contribution function that is truthful, which rewards the government for every change in the policy choice exactly the amount of change in the lobby’s gross-of-contribution utility, provided that the contribution both before and after the change is strictly positive. In other words, the shape of the contribution schedule mirrors the shape of the lobby’s indifference surface. Therefore, a lobby gets the same net-of-contribution utility for all policies that induce a positive contribution from him.

**Definition 1** A feasible contribution function $C_j$ is truthful relative to $p^o$ if and only if for all $p \in P$, either

$$(i) \ u_j(p) = u_j(p^o) = u^o_j,$$
or

\[(ii) \ u_j(p) < u_j(p^\circ) \quad \text{and} \quad C_j(p) = 0.\]

\(\{C_j\}_{j \in \mathcal{L}}, p^\circ\) is a Truthful Nash Equilibrium (TNE) if and only if it is a Nash Equilibrium and \(\{C_j\}_{j \in \mathcal{L}}\) are truthful relative to \(p^\circ\).

Thus, a competition in truthful strategies boils down to noncooperative choices of the constant \(\{u_j\}_{j \in \mathcal{L}}\), which determines the equilibrium net utilities of the lobbies. For the rest of the paper, I focus on the Truthful Nash Equilibrium, since it greatly simplifies the analysis.

Bernheim and Whinston (1986) prove that truthful strategies and Truthful Nash Equilibria have the following properties, which justify our focusing on the truthful equilibria. First, the best response set of lobby \(j\) to any set of contribution functions by his opponents contains a truthful contribution function. Thus, a lobby bears essentially no cost from playing truthful strategies. Second, the Truthful Nash Equilibrium always results in an efficient policy choice. The equilibrium policy choice maximizes the joint payoff to the government and the lobbies. Third, the set of Truthful Nash Equilibria coincides with the set of coalition-proof Nash Equilibria which are stable when non-binding communication among lobbies is possible. Therefore, nothing is lost by restricting attention to this class of equilibria.

Focusing on the Truthful Nash Equilibrium provides a straightforward way of calculating the equilibrium policy scheme. By Corollary 1, Equation (5) is always binding. Substitute Equation (5) in Equation (4) for \(C_j\) and replace \(C_m\) with its truthful form, the constrained maximization problem for every lobby is equivalent to

\[
\max_{\{p\}} kV + \sum_{j \in \mathcal{L}} v_j. \tag{7}
\]

It is straightforward to verify that the solution to (7) also solves the government’s optimization problem. Thus, in a Truthful Nash Equilibrium, the government reaches the optimal policy scheme by maximizing a weighted sum of the aggregate social welfare and the aggregate welfare of all the lobbies. This is equivalent to maximizing a weighted sum of the welfare of all the individuals, with a higher weight placed on the welfare of the lobbies. The equilibrium policy in a Truthful Nash Equilibrium is efficient in that it maximizes the joint welfare of the government and the lobbies. Moreover, since the government also cares about the welfare of the non-lobbies, their welfare will not be driven down to a minimum.

### 4.2 Allocation of Education Spending

I solve for the optimal policy scheme prescribed by (7) for four generic cases: no-lobby, one-lobby, two-lobby, and three-lobby, denoted as \(\mathcal{L} = \emptyset, \mathcal{L} = \{1\}, \mathcal{L} = \{1, 2\}, \mathcal{L} = \{1, 2, 3\}\).
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Case & Government Utility & Equilibrium Policy, p & Next Generation Income Distribution \\
\hline
\(\mathcal{L} = \emptyset\) & \(kV\) & \(\delta_1^2\) & \(\frac{\delta_1}{\delta_1 + \delta_2 + \delta_3}\) \\
& & \(\delta_2^2\) & \(\frac{\delta_2}{\delta_1 + \delta_2 + \delta_3}\) \\
& & \(\delta_3^2\) & \(\frac{\delta_3}{\delta_1 + \delta_2 + \delta_3}\) \\
\hline
\(\mathcal{L} = \{1\}\) & \(kV + v_1\) & \(\frac{\delta_1^2(k+1)^2}{(k+s_1)^2}\) & \(\frac{\delta_1(k+1)}{\delta_1(k+1)+\delta_2k+\delta_3k}\) \\
& & \(\frac{\delta_2^2}{k^2}\) & \(\frac{\delta_1(k+1)}{\delta_1(k+1)+\delta_2k+\delta_3k}\) \\
& & \(\frac{\delta_3^2}{(k+s_1)^2}\) & \(\frac{\delta_1(k+1)}{\delta_1(k+1)+\delta_2k+\delta_3k}\) \\
\hline
\(\mathcal{L} = \{1, 2\}\) & \(kV + v_1 + v_2\) & \(\frac{\delta_1^2(k+1)^2}{(k+s_1+s_2)^2}\) & \(\frac{\delta_1(k+1)}{\delta_1(k+1)+\delta_2(k+1)+\delta_3k}\) \\
& & \(\frac{\delta_2^2}{(k+s_1+s_2)^2}\) & \(\frac{\delta_1(k+1)+\delta_2(k+1)+\delta_3k}{\delta_1(k+1)+\delta_2(k+1)+\delta_3k}\) \\
& & \(\frac{\delta_3^2}{(k+s_1+s_2)^2}\) & \(\frac{\delta_1(k+1)}{\delta_1(k+1)+\delta_2(k+1)+\delta_3k}\) \\
\hline
\(\mathcal{L} = \{1, 2, 3\}\) & \((k+1)V\) & Same as \(\mathcal{L} = \emptyset\) & Same as \(\mathcal{L} = \emptyset\) \\
\hline
\end{tabular}
\caption{Equilibrium Policy and Next Generation Distribution}
\end{table}

\(\mathcal{L} = \{1, 2\}\), and \(\mathcal{L} = \{1, 2, 3\}\). The results are summarized in Table 3. Note that it is possible that \(\mathcal{L} = \{2\}\) or \(\{3\}\) in the one-lobby case, and \(\mathcal{L} = \{1, 3\}\) or \(\{2, 3\}\) in the two-lobby case, but the equilibrium results are similar to those for \(\mathcal{L} = \{1\}\) and \(\{1, 2\}\). I leave for the next section the discussion of the equilibrium net utilities.

\(\mathcal{L} = \emptyset\) is the benchmark case, in which the government maximizes the aggregate social welfare. In this case, a child receives a share of the total education spending in accordance with his parent’s preference for his education, and the education policy favors no one in particular in excess of his own preference. The next generation income distribution is determined accordingly. In the one-lobby and two-lobby cases, the government attaches a higher weight to the utility of the lobby. Compared to the outcome in the no-lobby case, the child of the lobby receives a larger share, as well as a larger amount, of the public education spending, while the child of the non-lobby receives smaller shares; consequently, the former gets a considerably larger share of the total income than he could have in the no-lobby case.

When all three individuals are lobbying, the government maximizes \(kV + v_1 + v_2 + v_3 = (k+1)V\). The education policy is identical to that in the no-lobby case; so is the income distribution of the next generation. Therefore, political competition has an impact on the equilibrium education policy and the future income distribution only when a subset of the individuals enter the lobbying.

Since the equilibrium policy is the same, the government receives the same level of aggregate social welfare as in the no-lobby case, in addition to the positive.
contributions from all the lobbies. The net utility of every individual, in contrast, is diminished due to the fixed cost and the positive contribution payment. The government thus captures all the surplus from the political relationships, while the entire society incurs an efficiency loss from paying the fixed cost.

To summarize, lobbying increases education spending on a lobby’s child and decreases it on a non-lobby’s child. Moreover, when more people enter lobbying, hence more fierce political competition, the increase in spending on a lobby diminishes and the decrease in spending on a non-lobby enlarges. Lobbying therefore has an unambiguous implication for the future income of every individual. The tax rates are nevertheless not comparable across these cases; thus, the implication of lobbying for one’s current consumption is not clear-cut. This insight turns out to be critical for the intuitive analysis of individuals’ entry decisions in the next section.

When a subset of the individuals organize into lobbies, the government’s preference over the aggregate social welfare, $k$, plays an important role in determining the relative amount spent on each child and the future income distribution. The higher the $k$, the more concerned the government is about social welfare, and the less biased the policy is toward the lobbies. At the limit, when $k$ approaches infinity, the allocation of education spending, as well as the income distribution of the next generation, is identical to that which emerges in the no-lobby case, regardless of which lobby forms.

4.3 Formation of Lobbies in the Entry Phase

Based on the equilibrium analysis of the post-entry phase, I now consider the “entry phase” of the game and endogenize the formation of lobbies. More specifically, I show that in most circumstances, not every individual enters lobbying, and the entry decision is closely related to an individual’s income.

An individual enters the lobbying competition if that leaves him a higher payoff. When $F = 0$, the unique equilibrium at the entry phase is that $\mathcal{L} = \{1, 2, 3\}$.

Let $G_j(\mathcal{L}) \equiv u_j(p^e) - v_j(p^{-j})$ denote the difference in $j$’s net utilities when $j \in \mathcal{L}$ and when $j \notin \mathcal{L}$, taking as given the entry decision of all $i \in \mathcal{I}$ and $i \neq j$. Clearly, $j$ enters the lobbying game if and only if $G_j \geq 0$. Therefore, $\mathcal{L}$ is an equilibrium set of lobbies when $G_j(\mathcal{L}) \geq 0$ for all $j \in \mathcal{L}$ and $G_j(\mathcal{L} \cup j) < 0$ for all $j \notin \mathcal{L}$.

Appendix-3 gives the expressions for $G_j(\mathcal{L})$ for all $j \in \mathcal{I}$ and all possible $\mathcal{L}$. These expressions are functions of model parameters ($k$, $F$, and $\delta$’s) and the initial income distribution; income levels are irrelevant.
Several features of the utility gain functions stand out. First, individual $j$’s utility gain from lobbying increases with $\delta_j$. Intuitively, the higher $\delta_j$, the more individual $j$ values his child’s future income. Given other individuals’ strategies, since $j$’s child receives more education spending hence higher future income when $j$ lobbies than when he does not, a higher $\delta_j$ translates into a larger utility gain from lobbying. As a result, $j$ has more incentive to lobby.

Second, individual $j$’s utility gain from lobbying is unmistakably positively correlated with $\delta_i$, $\forall i \neq j$. As discussed in the previous section, as $j$ comes to lobby, whether $i$ is a fellow-lobby or a non-lobby, the education spending on $i$’s child drops. This drop contributes to a reduction in the tax rate; the higher $\delta_i$, the larger the reduction in the tax rate. Therefore, other people’s greater taste for education provides an indirect incentive for one to enter lobbying.

Third, the effect of income share on one’s utility gain from lobbying is not clear. This is essentially due to the fact that the equilibrium tax rates are not comparable. For instance, given other individuals’ entry decisions, if the equilibrium tax rate is lower when one lobbies as opposed to when he does not, then his current net-of-tax income rises as a result of his lobbying. The higher the income, the larger the rise. Conversely, if the equilibrium tax rate is higher, then his current net-of-tax income in fact drops as a result of his lobbying. The higher the income, the larger the drop. When this happens, unless higher income is associated with a larger utility gain derived from future income, higher income actually hurts one’s incentive to enter lobbying.

For the rest of the paper, I assume $s_1 \geq s_2 \geq s_3$ for the first generation, and define the feasible space of income distribution of the first generation as $S^3 = \{s = (s_1, s_2, s_3) | s_1 \geq s_2 \geq s_3; s_1 + s_2 + s_3 = 1\}$. Thus, individual 1 represents the rich, 2 the middle class, and 3 the poor. In addition, let $D_{12} = \frac{\delta_1}{s_2}$ and $D_{23} = \frac{\delta_2}{s_3}$; $D_{12}$ and $D_{23}$ measure the differences in individuals’ taste for education.

The following proposition establishes the existence, and in certain circumstances the uniqueness, of a pure strategy Nash equilibrium at the entry phase of the game. To establish these results, some restrictions on $D_{12}$ and $D_{23}$ are necessary.

**Proposition 2** For $k$ sufficiently large, $\exists D_{12} > D_{12} > 1$ and $D_{23} > D_{23} > 1$, such that when $\overline{D_{12}} < D_{12} < \underline{D_{12}}$ and $\overline{D_{23}} < D_{23} < \underline{D_{23}}$:

1. If $F \leq \frac{\delta_2(k+1)(\delta_1^2 + \delta_2^2)}{(k+1)^2(\delta_1^2 + \delta_2^2) + k^2\delta_3^2} = F_0$, then $\forall s \in S^3$, $\mathcal{L} = \{1, 2, 3\}$ is a unique equilibrium.

---

\[11\] Thus $\frac{1}{3} \leq s_1 \leq 1$, $0 \leq s_2 \leq \frac{1}{2}$, and $0 \leq s_3 \leq \frac{1}{3}$, but the income order of the future generations may change.
2. If $F_0 \leq F \leq \frac{\delta_2^2}{2k+1}$, \(\forall s \in S^3\), a unique equilibrium exists; the equilibrium is either \(L = \{1, 2, 3\}\) or \(L = \{1, 2\}\).

3. If \(\frac{\delta_2^2}{2k+1} < F \leq \frac{\delta_2^2}{k+1}\), \(\forall s \in S^3\), an equilibrium exists; the equilibrium is either \(L = \{1, 2, 3\}\), \(L = \{1, 2\}\), \(L = \{1, 3\}\), or \(L = \{1\}\); whenever \(L = \{1, 3\}\) is an equilibrium, it overlaps with the equilibrium \(L = \{1, 2\}\).

4. If \(F \geq \frac{\delta_2^2}{k}\), then \(\forall s \in S^3\), a unique equilibrium exists; the equilibrium is either \(L = \{1, 2\}\), \(L = \{1\}\), or \(L = \emptyset\).

In other words, when the fixed cost is low enough, everyone enters the lobbying game. The intuition is clear: the utility gain from lobbying does not have to be substantial to justify incurring the fixed cost. When \(F\) increases, a substantial utility gain from lobbying is necessary to justify incurring the fixed cost to begin with. Thus, the number of lobbies on the whole decreases, but the rich are more likely to remain.

Details of the proof can be found in Appendix-3. The restrictions on \(D_{12}\) and \(D_{23}\) require that the \(\delta\)’s are sufficiently different. The intuition for maintaining these restrictions is as follows. As discussed above, one’s utility gain from lobbying is ambiguously correlated with his income, whereas it is unequivocally positively correlated with the taste for education of everyone. Therefore, by widening the gap in the taste for education between each pair of individuals, I essentially give the rich more incentive to enter lobbying: the gain in future income is sufficiently large; by not widening the gap too much, I essentially restrict the incentive of the poor to enter lobbying: the tax reduction due to the drop in education spending on others as the poor come to lobby is moderate, which contributes to a moderate utility gain for the poor.

The above arguments imply that, when the entry cost becomes prohibitively high, it will outweigh any utility gain from lobbying. Thus, no one enters the lobbying game; the unique equilibrium at the entry phase is \(L = \emptyset\).

The cutoff values of \(F\) are all negatively affected by the value of \(k\), the government’s valuation of aggregate social welfare. Intuitively, as \(k\) increases, the public education policy exhibits less bias toward the lobby, thus reducing the utility gain from lobbying. The opposite happens when \(k\) goes down.

Several remarks are in order. First, Proposition 2 demonstrates that with some restrictions on the differences in individuals’ taste for education, for all initial distribution \(s \in S^3\), an entry phase equilibrium exists for a wide range of values of the entry cost \(F\); moreover, numerical exercises show that the equilibrium is unique in most circumstances.
Second, an equilibrium at the entry phase might not exist if the parameters do not satisfy the restrictions above. For example, when \( \frac{\delta^2}{k+1} < F < \frac{\delta^2}{k} \), or when the restrictions on \( D_{12} \) and \( D_{23} \) are absent. The fundamental reason for this non-existence lies in the discreteness of the choice set: one can only choose between entering and not entering. This discontinuity naturally gives rise to the problem of existence. One could potentially deal with this problem by making the choice set continuous and allowing individuals to choose at what capacity to enter the lobbying competition. Thus at the entry phase, an individual decides on an optimal lobbying intensity, which is associated with an optimal level of entry cost he will incur. In the subsequent stage, the optimal contribution function he can propose will be subject to this predetermined lobbying intensity. An equilibrium now consists of a lobbying intensity, or equivalently, an entry cost, a contribution function for every individual, and a policy outcome determined by the government. With certain additional assumptions, this modification might get around the existence problem.

Third, a concave production function generates utility gains that are not monotonic in income; i.e., in subsets of the feasible income distribution space \( S^3 \), one’s utility gain from entry does not monotonically increase with one’s income. The rich are not always the most politically powerful. This non-monotonicity may bring about income mobility in the long run. Nonetheless, for certain configurations of the model parameters, the utility gain from entry exhibits a monotonic increase with income; hence the richer an individual, the more likely he is to enter lobbying.

5 Income Distribution Dynamics

So far, I have concentrated on the static equilibrium of the political game. Joining the equilibrium entry decision with the ensuing equilibrium public expenditure allocation, I now proceed to explore how the political interaction acts upon the evolution of the income distribution.

The specific question in this section is, for any initial income distribution \( s \in S^3 \), what is the equilibrium education spending allocation and what is the resulting next period income distribution? Moreover, is there a steady state income distribution in the long run? I answer these questions chiefly by exploring numerical examples of the model. To do so meaningfully, I concentrate on configurations that generate a unique entry phase equilibrium for all \( s \in S^3 \).

As discussed above, the stage game can produce an abundant assortment of entry phase equilibria, hence equilibrium allocations of public expenditure. The resulting income distribution dynamics, however, falls into three broad categories: income distribution converges to a unique steady state in the long run; income
distribution converges to different steady states in the long run; and income distribution does not converge in the long run and oscillates between different steady states.

It is convenient to introduce the following notations. Let $S_n$ represent a subset of $S^3$ such that $\forall s \in S_n$, a type-$n$ equilibrium forms at the entry phase, and $n \in N = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$, and $S^3 = \bigcup_n S_n$. Each $S_n$ is associated with a next-generation income distribution $\sigma(n)$ irrespective of the specific initial income distribution $s \in S_n$. This is because, recalling Section 4.2, for any given set of lobbies, even though the education spending policy $p$ and hence the next generation income levels are dependent upon the current income distribution, the next generation income distribution is completely characterized by the model parameters.

5.1 Income Distribution Convergence

From Proposition 2, when the fixed entry cost $F$ is sufficiently low or high, $\forall s \in S^3$, the only entry phase equilibrium sustained is either the three-lobby equilibrium or the no-lobby equilibrium, with the same resulting policy, $p = (\delta_1^3, \delta_2^3, \delta_3^3)$, which only depends on individuals’ tastes for education. The second generation income distribution is also the same, $\sigma(\emptyset) = \sigma(\{1, 2, 3\}) = \left(\frac{\delta_1}{\delta_1 + \delta_2 + \delta_3}, \frac{\delta_2}{\delta_1 + \delta_2 + \delta_3}, \frac{\delta_3}{\delta_1 + \delta_2 + \delta_3}\right)$. Since the determination of the entry phase equilibrium does not depend on individuals’ income levels, for the second generation, the only entry phase equilibrium sustained is still either the three-lobby equilibrium or the no-lobby equilibrium, with $\sigma(\emptyset) = \sigma(\{1, 2, 3\})$ the third generation income distribution. By the same argument, therefore, in the long run the economy converges to this single steady state distribution.

Convergence to a single steady state distribution is also possible in other cases. For example, if for the first generation, depending on the income distribution the entry phase equilibrium is either $\mathcal{L} = \{1, 2, 3\}$ or $\{1, 2\}$, i.e., $S^3 = S_{\{1, 2, 3\}} \cup S_{\{1, 2\}}$, then the associated second generation distribution is either $\sigma(\{1, 2, 3\})$ or $\sigma(\{1, 2\})$. By the same argument as in the previous paragraph, the possible entry phase equilibrium for the second generation is either $\mathcal{L} = \{1, 2, 3\}$ or $\{1, 2\}$ depending on its income distribution. If, however, $\sigma(\{1, 2, 3\}), \sigma(\{1, 2, \}) \in S_{\{1, 2\}}$, then the only entry equilibrium that occurred in the second generation is $\mathcal{L} = \{1, 2\}$, with $\sigma(\{1, 2\})$ as the single equilibrium income distribution for the third generation. The economy converges to $\sigma(\{1, 2\})$ in the long run.

In general, if $\forall n \in N, \sigma(n) \in S_m$, where $m \in N$, then the income distribution of the third generation is $\sigma(m)$; the economy in the long run converges to $\sigma(m)$, regardless of the initial income distribution. In principal, $\sigma(m)$ can be any one of the possible equilibrium distributions.
5.2 Income Distribution Persistence

Consider an example in which the vector of individuals’ tastes for education is given by $\delta^2 = \{.45, .15, .1\}$; the government’s preference for social welfare is $k = 2$; the fixed entry cost is $F = .051$. Figure 2 depicts the equilibrium outcome of the stage game and the steady state distributions.

In the figure, the horizontal axis represents the initial income share of the middle class, $s_2 \in [0, 1/2]$, and the vertical axis that of the rich, $s_1 \in [1/3, 1]$. Along line $OE$, $s_1 = s_2$; along line $DE$, $s_2 = s_3$. The income share of the poor is zero along line $OD$ and reaches its highest possible level, $1/3$, at point $E$. The triangle $OED$ thus represents the feasible space of income distribution $S_3$. For all initial income distribution in the dark-shaded area, $\mathcal{L} = \{1\}$ is the unique entry phase equilibrium; this area is thus $S_{\{1\}}$. For all initial income distribution in the light-shaded area, $\mathcal{L} = \{1, 2\}$ is the unique entry phase equilibrium; this area is thus $S_{\{1,2\}}$. These two entry phase equilibria are associated with equilibrium income distributions of the second generation denoted by points $\sigma(\{1\})$ and $\sigma(\{1, 2\})$ respectively.

This numerical example is consistent with the observed relationships between income distribution and the allocation of public education spending described in Section 2. It also illustrates the persistence of the differences in income distribu-
First, a distribution in $S_{1}$ is more unequal than one in $S_{1,2}$. A simple way to see this is that, given the income share of the poor, the middle class has a higher income in $S_{1,2}$ than in $S_{1}$. The resulting policies for $S_{1}$ and $S_{1,2}$ are, with a little abuse of notation,

$$p(\{1\}) = \left( \frac{\delta_1^2 (k+1)^2}{(k + s_1)^2}, \frac{\delta_2^2 k^2}{(k + s_1)^2}, \frac{\delta_3^2 k^2}{(k + s_1)^2} \right),$$

and

$$p(\{1, 2\}) = \left( \frac{\delta_1^2 (k+1)^2}{(k + s_1 + s_2)^2}, \frac{\delta_2^2 (k+1)^2}{(k + s_1 + s_2)^2}, \frac{\delta_3^2 k^2}{(k + s_1 + s_2)^2} \right).$$

Comparing $p(\{1\})$ and $p(\{1, 2\})$ reveals that, the spending share on the rich is higher in $S_{1}$ than in $S_{1,2}$, and the spending share on the middle class is lower in $S_{1}$ than in $S_{1,2}$, but the spending share on the poor is higher in $S_{1}$ than in $S_{1,2}$. Moreover, if we take the spending differences between the rich and the middle class, and between the middle class and the poor as approximate measures for per student spending on tertiary and secondary levels, then

$$\begin{align*}
(tershare, secshare)_{\{1\}} &= \left( \frac{\delta_1^2 (k+1)^2 - \delta_2^2 k^2}{\delta_1^2 (k+1)^2 + \delta_2^2 k^2 + \delta_3^2 k^2}, \frac{2(\delta_2^2 k^2 - \delta_3^2 k^2)}{\delta_1^2 (k+1)^2 + \delta_2^2 k^2 + \delta_3^2 k^2} \right), \\
(tershare, secshare)_{\{1, 2\}} &= \left( \frac{\delta_1^2 (k+1)^2 - \delta_2^2 (k+1)^2}{\delta_1^2 (k+1)^2 + \delta_2^2 (k+1)^2 + \delta_3^2 k^2}, \frac{2(\delta_2^2 (k+1)^2 - \delta_3^2 k^2)}{\delta_1^2 (k+1)^2 + \delta_2^2 (k+1)^2 + \delta_3^2 k^2} \right).
\end{align*}$$

The number “2” appears in the $secshare$ expression because both the rich and the middle class choose a secondary education. It is obvious that $tershare_{\{1\}} > tershare_{\{1, 2\}}$, $secshare_{\{1\}} < secshare_{\{1, 2\}}$, and moreover $prishare_{\{1\}} > prishare_{\{1, 2\}}$.

Second, the resulting second generation distribution is given by $\sigma(\{1\})$ and $\sigma(\{1, 2\})$ respectively. Recalling the results in Section 4.3, $\sigma(\{1, 2\})$ represents a more equal distribution than $\sigma(\{1\})$, since the Gini coefficient for $\sigma(\{1, 2\})$ is lower than that for $\sigma(\{1\})$, which, in this example, are 0.2417 and 0.2690 respectively.

Since $\sigma(\{1\}) \in S_{1}$ and $\sigma(\{1, 2\}) \in S_{1,2}$, the entry equilibrium for the second generation is $L = \{1\}$ for $\sigma(\{1\})$, and $L = \{1, 2\}$ for $\sigma(\{1, 2\})$. The associated third generation income distributions are in turn $\sigma(\{1\})$ and $\sigma(\{1, 2\})$ respectively.

To summarize, in the long run, for any economy starting in $S_{1}$, its income distribution converges to $\sigma(\{1\})$; for any economy starting in $S_{1,2}$, its income distribution converges to $\sigma(\{1, 2\})$. Since income distributions in $S_{1}$ are in general
more unequal than those in $S_{\{1,2\}}$, and $\sigma(\{1\})$ represents a more unequal distribution than $\sigma(\{1,2\})$, the model generates persistence of differences in income distribution.

5.3 Income Distribution Oscillation

Finally, suppose there exist $l, m$ pairs, where $l, m \in N$, such that $\sigma(m) \in S_l$ and $\sigma(l) \in S_m$, then for all initial income distribution in $S_l \cup S_m$, in the long run, the income distribution oscillates between $\sigma(l)$ and $\sigma(m)$.

I demonstrate this case in a numerical example with $\delta^2 = \{.45,.15,.1\}$, $k = 2$, and $F = .068$. The only difference between this example and the previous one is that the entry cost is higher. Figure 3 depicts the equilibrium outcome. For any initial income distribution $s$, the entry phase equilibrium is either $L = \emptyset$, $L = \{1\}$, or $L = \{1,2\}$, thus $S^3 = S_0 \cup S_{\{1\}} \cup S_{\{1,2\}}$. The associated second generation distributions are $\sigma(\emptyset)$, $\sigma(\{1\})$, and $\sigma(\{1,2\})$ respectively. Two features stand out. First, $L = \{1,2\}$ is merely a short-run entry phase equilibrium, since no equilibrium distribution falls into $S_{\{1,2\}}$. Second, in the long run, income distribution of the economy oscillates between $\sigma(\{1\})$ and $\sigma(\emptyset)$, since $\sigma(\{1\}) \in S_{\emptyset}$ and $\sigma(\emptyset) \in S_{\{1\}}$. The income rank of the three individuals does not alter, but the income differences among them constantly change.

Figure 3: Income Distribution Oscillation: Long Run Mobility
6 Conclusion

This paper has carried out the first systematic study of the relationships between income distribution and the allocation of public education expenditure. It demonstrates that a more unequal society tends to allocate public education expenditure in a way that is more biased toward the rich, resulting in persistent inequality. It then develops a political economy model that is capable of explaining these stylized facts. In particular, it shows that for certain parameter configurations, in a more unequal economy, the rich tend to capture the political power, leading to a less redistributive spending allocation that brings about a more unequal income distribution for the next generation, and vice versa. Thus, the economy may reach different steady states in the long run, exhibiting persistent differences in income distribution and redistributive education policy. When multiple steady states exist, history matters. Temporary shocks to the economic or political systems could permanently move a society from one equilibrium to the other, or more generally, have long-lasting effects on the income distribution.
## Appendix-1 Regression Results with Dummy Variables for Gini Types

Table A-1 reports the results from ordinary least-squares regressions of spending shares at the tertiary and secondary levels on income, three demographic variables, the Gini coefficient, and three dummy variables for Gini types. $exp_{-}dum = 1$ if an estimate of the Gini coefficient is from consumption expenditure surveys, and 0 if it is from income surveys. $per_{-}dum = 1$ if an estimate of the Gini coefficient is based on personal income, and 0 if it is based on household income. $net_{-}dum = 1$ if an estimate of the Gini coefficient is based on net(after tax) income, and 0 if it is based on gross-of-tax income.

<table>
<thead>
<tr>
<th></th>
<th>1990s tershare</th>
<th>1990s secshare</th>
<th>1980s tershare</th>
<th>1980s secshare</th>
</tr>
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<tbody>
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<td>$gdp$</td>
<td>0.012</td>
<td>0.043</td>
<td>0.0005</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(0.943)</td>
<td>(2.91)</td>
<td>(0.032)</td>
<td>(1.36)</td>
</tr>
<tr>
<td>$ter_{-}age/pop$</td>
<td>-0.262</td>
<td>-2.242</td>
<td>2.05</td>
<td>-0.227</td>
</tr>
<tr>
<td></td>
<td>(-0.235)</td>
<td>(-2.12)</td>
<td>(2.35)</td>
<td>(-0.221)</td>
</tr>
<tr>
<td>$sec_{-}age/pop$</td>
<td>0.204</td>
<td>0.90</td>
<td>-0.81</td>
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<td></td>
<td>(0.604)</td>
<td>(1.92)</td>
<td>(-1.96)</td>
<td>(3.24)</td>
</tr>
<tr>
<td>$pri_{-}age/pop$</td>
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<td>-0.697</td>
<td>-0.67</td>
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</tr>
<tr>
<td></td>
<td>(-1.147)</td>
<td>(-1.32)</td>
<td>(-1.63)</td>
<td>(-0.285)</td>
</tr>
<tr>
<td>$gini$</td>
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<td>-0.238</td>
<td>0.154</td>
<td>-0.538</td>
</tr>
<tr>
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<td>(2.645)</td>
<td>(-1.57)</td>
<td>(1.56)</td>
<td>(2.524)</td>
</tr>
<tr>
<td>$exp_{-}dum$</td>
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<td>0.09</td>
<td>0.053</td>
<td>0.011</td>
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<tr>
<td></td>
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<td>(3.41)</td>
<td>(2.08)</td>
<td>(0.21)</td>
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<tr>
<td>$per_{-}dum$</td>
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<td>-0.002</td>
<td>0.037</td>
<td>0.0001</td>
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<td>(0.25)</td>
<td>(-0.12)</td>
<td>(2.11)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>$net_{-}dum$</td>
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<td>-0.044</td>
<td>-0.033</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>(1.16)</td>
<td>(-1.99)</td>
<td>(-1.34)</td>
<td>(-0.06)</td>
</tr>
<tr>
<td># of Observations</td>
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<td>59</td>
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<td>43</td>
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<tr>
<td>$R^2$</td>
<td>0.138</td>
<td>0.703</td>
<td>0.302</td>
<td>0.58</td>
</tr>
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</table>

Numbers in parentheses are t-statistics.

Table A-1: Cross Country Regressions of Tertiary and Secondary Spending on the Gini Coefficient in the 90s and 80s, with dummy variables for the Gini types.
Appendix-2 The Counterfactual Exercise

Adjusting Education Spending in the 80s. In this exercise, countries start from the 80s and “adjust” their education spending allocations to the median country in the 80s.

Run the regression:

\[ Gini_{90} = C_1 + \alpha_1 Gini_{80} + \beta_1 tershare_{80} + \beta_2 secshare_{80} + \epsilon_1 \]  \hspace{1cm} (A-1)

Calculate the adjusted values of \( Gini_{90} \).

\[ \hat{Gini}_{90} = \hat{C}_1 + \hat{\alpha}_1 Gini_{80} + \hat{\beta}_1 tershare_{80} + \hat{\beta}_2 secshare_{80} + \hat{\epsilon}_1 \]

\[ = Gini_{90} + \hat{\beta}_1 (tershare_{80} - tershare_{80}) + \hat{\beta}_2 (secshare_{80} - secshare_{80}) \]

where \( tershare_{80} \) and \( secshare_{80} \) are the percentages spent on tertiary and secondary levels in the 80s by a median country.

Adjusting Education Spending in both the 70s and the 80s. In this exercise, countries start from the 70s and “adjust” their education spending allocations to the median country in both the 70s and 80s.

Stage 1: adjusting in the 70s. Run the regression:

\[ Gini_{80} = C_2 + \alpha_2 Gini_{70} + \gamma_1 tershare_{70} + \gamma_2 secshare_{70} + \epsilon_2 \] \hspace{1cm} (A-2)

Calculate the adjusted values of \( Gini_{80} \).

\[ \hat{Gini}_{80} = \hat{C}_2 + \hat{\alpha}_2 Gini_{70} + \hat{\gamma}_1 tershare_{70} + \hat{\gamma}_2 secshare_{70} + \hat{\epsilon}_2 \]

\[ = Gini_{80} + \hat{\gamma}_1 (tershare_{70} - tershare_{70}) + \hat{\gamma}_2 (secshare_{70} - secshare_{70}) \]

where \( tershare_{70} \) and \( secshare_{70} \) are the percentages spent on tertiary and secondary levels in the 70s by a median country.

Stage 2: adjusting in the 80s. Run regression (A-1) and calculate the adjusted values of \( Gini_{90} \),

\[ \hat{Gini}_{90} = \hat{C}_1 + \hat{\alpha}_1 \hat{Gini}_{80} + \hat{\beta}_1 tershare_{80} + \hat{\beta}_2 secshare_{80} + \hat{\epsilon}_1 \]

\[ = Gini_{90} + \hat{\beta}_1 (tershare_{80} - tershare_{80}) + \hat{\beta}_2 (secshare_{80} - secshare_{80}) \]
Including the residual term in calculating the adjusted Gini coefficients reflects
the fact that in the counterfactual exercise, the education spending allocation is
the only factor altered, and all the other socio-economic factors are kept the same.

Appendix-3 Proofs of the Existence of Equilibrium for the Entry Phase Game

Appendix-3.1 Net-of-Contribition Utility Characterization

In this section, I directly derive \( j \)'s payoff net of contribution but gross of the fixed
cost, \( \omega_j = v_j - C_j \), and \( j \)'s net payoff \( u_j \) is simply \( \omega_j - F \). In the derivation below,
a superscript number refers to the number of lobbies.

**Case 1: \( j \) is the single lobby.** By Corollary 1, \( W(p^1, C_j^1(p^1)) = W(p^0, 0) \).
Since \( W(p^1, C_j^1(p^1)) = kV(p^1) + C_j^1(p^1) - kF \) and \( W(p^0, 0) = kV(p^0) - kF \), we have \( C_j^1(p^1) = kV(p^0) - kV(p^1) \). Thus,

\[
\omega_j^1 = [kV(p^1) + v_j(p^1)] - kV(p^0).
\]

I use the results of Theorem 2 in Bernheim and Whinston (1986) to characterize
the lobbies' net utilities for the two-lobby and three-lobby cases. Theorem 2 states
that in all truthful Nash equilibria, the government selects \( p \in P^*_L \), and the lobbies
receive payoffs in the set \( E(p) \). \( P^*_L = \arg\max \sum_{i \in L} v_i \) is the set of efficient
policies that maximize the joint payoff of the lobbies and the government, and

\[
E(p) \equiv \{ \omega \in R^l| \omega \in \Pi(p) \text{ and there does not exist } \omega' \in \Pi(p), \text{ with } \omega' \geq \omega \},
\]

\[
\Pi(p) \equiv \{ \omega \in R^l| \text{for all } L \subseteq L, W_L \leq [kV(p) + V_L(p)] - [kV(p_T) + V_T(p_T)] \},
\]

where \( l \) is the number of lobbies in \( L \), \( \omega \) is an \( l \)-dimensional vector of gross-of-
fixed-cost payoffs to the lobbies in \( E(p) \), \( W_L = \sum_{i \in L} \omega_i \), \( T \) is the complement of \( L \)
in \( L \), \( p \in P^* \), \( p_T \in P_T \) with \( P_T = \arg\max \ kV + \sum_{i \in L} v_i \), \( V_L(p) = \sum_{i \in L} v_i \), and
\( V_T = \sum_{i \in T} v_i \).

**Case 2: \( j \) is one of the two lobbies.** Let \( L = \{i, j\} \), using the definition of
\( \Pi(p) \), we have

\[
\omega_i^2 \leq [kV(p^2) + v_i(p^2) + v_j(p^2)] - [kV(p_T^2) + v_i(p_T^2)]
\]

\[
\omega_j^2 \leq [kV(p^2) + v_i(p^2) + v_j(p^2)] - [kV(p_T^2) + v_i(p_T^2)]
\]

\[
\omega_i^2 + \omega_j^2 \leq [kV(p^2) + v_i(p^2) + v_j(p^2)] - kV(p^0)
\]
The first two inequalities thus give the upper bounds on the values of $\omega_i^2$ and $\omega_j^2$. They give the exact values of $\omega_i^2$ and $\omega_j^2$ if the third inequality is redundant. The third inequality can be shown to be redundant when $\mathcal{L} = \{1, 2\}$; or when $\mathcal{L} = \{1, 3\}$, and the difference between $\delta_2$ and $\delta_3$ is not excessively large; or when $\mathcal{L} = \{2, 3\}$, and the difference between $\delta_1$ and $\delta_2$ is not excessively large.

**Case 3: $j$ is one of the three lobbies.** Let $\mathcal{L} = \{1, 2, 3\}$ and $s_1 \geq s_2 \geq s_3$, we have

\[
\begin{align*}
\omega_1^3 & \leq (k + 1)V(p^3) - [KV(p_{1,3}^2) + v_2(p_{1,3}^2) + v_3(p_{1,3}^2)] \\
\omega_2^3 & \leq (k + 1)V(p^3) - [KV(p_{1,3}^2) + v_1(p_{1,3}^2) + v_3(p_{1,3}^2)] \\
\omega_3^3 & \leq (k + 1)V(p^3) - [KV(p_{1,2}^2) + v_1(p_{1,2}^2) + v_2(p_{1,2}^2)] \\
\omega_1^3 + \omega_2^3 & \leq (k + 1)V(p^3) - [KV(p_{3}^3) + v_3(p_{3}^3)] \\
\omega_1^3 + \omega_3^3 & \leq (k + 1)V(p^3) - [KV(p_{1}^3) + v_2(p_{1}^3)] \\
\omega_2^3 + \omega_3^3 & \leq (k + 1)V(p^3) - [KV(p_{1}^3) + v_1(p_{1}^3)] \\
\omega_1^3 + \omega_2^3 + \omega_3^3 & \leq (k + 1)V(p^3) - kV(p^0)
\end{align*}
\]

The first three inequalities thus give the upper bounds on the values of $\omega_1^3$, $\omega_2^3$, and $\omega_3^3$. Since $p^3 = p^0$, the last inequality is redundant. The fourth inequality is redundant if $k \geq 1$ (sufficient condition); otherwise, it is redundant if the $\delta$’s are not too close to each other. The fifth inequality is redundant if the difference between $\delta_2$ and $\delta_3$ is not excessively large. The sixth inequality is redundant if the $\delta$’s are sufficiently close to each other; otherwise, there is no general condition for its redundancy.

In what follows, I use these upper bounds to approximate the values of $\omega$’s in the two-lobby and three-lobby cases. This compromise may, for some parameter values, alter the cutoff values of $F$ in the existence proof, but it does not change qualitatively the properties of the entry phase equilibrium.

**Appendix-3.2 Conditions for Possible Entry Equilibria**

There are eight possible pure strategy Nash equilibria at the entry phase. The conditions for each of the equilibria, after some tedious manipulation, are as follows.

**Conditions for $\mathcal{L} = \emptyset$**

For every individual $i \in \mathcal{I}$, the net utility when entering is smaller than that when
not, given that the other two individuals do not enter.

\[
\begin{align*}
\{s_1^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_1(2\delta_1^2 + F) + (\delta_1^2 - k \cdot F)\} \frac{1}{k + s_1} &< 0 \quad (1a) \\
\{s_2^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_2(2\delta_2^2 + F) + (\delta_2^2 - k \cdot F)\} \frac{1}{k + s_2} &< 0 \quad (1b) \\
\{s_3^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_3(2\delta_3^2 + F) + (\delta_3^2 - k \cdot F)\} \frac{1}{k + s_3} &< 0 \quad (1c)
\end{align*}
\]

**Conditions for** \( L = \{1\} \)

1. Individual 1 receives a higher net utility when entering than not, given that 2 and 3 do not enter.
2. Given that 1 enters and 3 does not, 2 receives a lower utility when entering than not.
3. Given that 1 enters and 2 does not, 3 receives a lower utility when entering than not.

\[
\begin{align*}
\{s_1^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_1(2\delta_1^2 + F) + (\delta_1^2 - k \cdot F)\} \frac{1}{k + s_1} &> 0 \quad (2a) \\
\{s_2^2(\delta_1^2(k+1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_2(k + s_1)(2k\delta_2^2 + F(k+s_1)) + (k + s_1)^2(\delta_2^2 - F(k + s_1))\} \frac{1}{(k + s_1 + s_2)(k + s_1)^2} &< 0 \quad (2b) \\
\{s_3^2(\delta_1^2(k+1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_3(k + s_1)(2k\delta_3^2 + F(k+s_1)) + (k + s_1)^2(\delta_3^2 - F(k + s_1))\} \frac{1}{(k + s_1 + s_3)(k + s_1)^2} &< 0 \quad (2c)
\end{align*}
\]

**Conditions for** \( L = \{2\} \)

1. Individual 2 receives a higher net utility when entering than not, given that 1 and 3 do not enter.
2. Given that 2 enters and 3 does not, 1 receives a lower utility when entering than not.
3. Given that 2 enters and 1 does not, 3 receives a lower utility when entering than not.

\[
\begin{align*}
\{s_2^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_2(2\delta_2^2 + F) + (\delta_2^2 - k \cdot F)\} \frac{1}{k + s_2} &> 0 \quad (3a) \\
\{s_1^2(\delta_2^2(k+1)^2 + (\delta_1^2 + \delta_3^2)k^2) - s_1(k + s_2)(2k\delta_1^2 + F(k+s_2)) + (k + s_2)^2(\delta_1^2 - F(k + s_2))\} \frac{1}{(k + s_1 + s_2)(k + s_2)^2} &< 0 \quad (3b) \\
\{s_3^2(\delta_2^2(k+1)^2 + (\delta_1^2 + \delta_3^2)k^2) - s_3(k + s_2)(2k\delta_3^2 + F(k+s_2)) + (k + s_2)^2(\delta_3^2 - F(k + s_2))\} \frac{1}{(k + s_2 + s_3)(k + s_2)^2} &< 0 \quad (3c)
\end{align*}
\]
Conditions for $\mathcal{L} = \{3\}$

(1) Individual 3 receives a higher net utility when entering than not, given that 1 and 2 do not enter.

(2) Given that 3 enters and 2 does not, 1 receives a lower utility when entering than not.

(3) Given that 3 enters and 1 does not, 2 receives a lower utility when entering than not.

\[
\begin{align*}
\{ s_3^2(\delta_1^2 + \delta_2^2 + \delta_3^2) - s_3(2\delta_3^2 + F) + (\delta_3^2 - k \cdot F) \} \frac{1}{(k + s_3)} &> 0 \quad (4a) \\
\{ s_1^2(\delta_3^2(k + 1)^2 + (\delta_1^2 + \delta_2^2)k^2) - s_1(k + s_3)(2k\delta_1^2 + F(k + s_3)) \\+(k + s_3)^2(\delta_1^2 - F(k + s_3)) \} \frac{1}{(k + s_1 + s_3)(k + s_3)^2} &< 0 \quad (4b) \\
\{ s_2^2(\delta_3^2(k + 1)^2 + (\delta_1^2 + \delta_2^2)k^2) - s_2(k + s_3)(2k\delta_2^2 + F(k + s_3)) \\+(k + s_3)^2(\delta_2^2 - F(k + s_3)) \} \frac{1}{(k + s_2 + s_3)(k + s_3)^2} &< 0 \quad (4c)
\end{align*}
\]

Conditions for $\mathcal{L} = \{1, 2\}$

(1) Individual 1 receives a higher net utility when entering than not, given that 2 enters and 3 does not.

(2) Individual 2 receives a higher net utility when entering than not, given that 1 enters and 3 does not.

(3) Individual 3 receives a lower net utility when entering than not, given that both 1 and 2 enter.

\[
\begin{align*}
\{ s_1^2(\delta_2^2(k + 1)^2 + (\delta_1^2 + \delta_2^2)k^2) - s_1(k + s_2)(2k\delta_1^2 + F(k + s_2)) \\+(k + s_2)^2(\delta_1^2 - F(k + s_2)) \} \frac{1}{(k + s_1 + s_2)(k + s_2)^2} &> 0 \quad (5a) \\
\{ s_2^2(\delta_1^2(k + 1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_2(k + s_1)(2k\delta_2^2 + F(k + s_1)) \\+(k + s_1)^2(\delta_2^2 - F(k + s_1)) \} \frac{1}{(k + s_1 + s_2)(k + s_1)^2} &> 0 \quad (5b) \\
\{ s_3^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_3(k + 1)(\delta_3^2 - F) \\+(k + 1)(\delta_3^2 - F(k + 1)) \} \frac{1}{(k + s_1 + s_2)^2} &< 0 \quad (5c)
\end{align*}
\]

Conditions for $\mathcal{L} = \{1, 3\}$

(1) Individual 1 receives a higher net utility when entering than not, given that 3 enters and 2 does not.

(2) Individual 3 receives a higher net utility when entering than not, given that 1 enters and 2 does not.
(3) Individual 2 receives a lower net utility when entering than not, given that both 1 and 3 enter.

\[
\begin{align*}
\{ s_2^2(\delta_3^2(k + 1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_1(k + s_3)(2k\delta_1^2 + F(k + s_3)) \\
+ (k + s_3)^2(\delta_2^2 - F(k + s_3)) \} \frac{1}{(k + s_1 + s_3)(k + s_3)^2} &> 0 \quad (6a) \\
\{ s_3^2(\delta_1^2(k + 1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_3(k + s_1)(2k\delta_3^2 + F(k + s_1)) \\
+ (k + s_1)^2(\delta_3^2 - F(k + s_1)) \} \frac{1}{(k + s_1 + s_3)(k + s_3)^2} &> 0 \quad (6b) \\
\{ s_2^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_2(k + 1)(\delta_2^2 - F) \\
+ (k + 1)(\delta_2^2 - F(k + 1)) \} \frac{1}{(k + s_1 + s_3)^2} &< 0 \quad (6c)
\end{align*}
\]

**Conditions for \( L = \{2, 3\} \)**

(1) Individual 2 receives a higher net utility when entering than not, given that 3 enters and 1 does not.

(2) Individual 3 receives a higher net utility when entering than not, given that 2 enters and 1 does not.

(3) Individual 1 receives a lower net utility when entering than not, given that both 2 and 3 enter.

\[
\begin{align*}
\{ s_1^2(\delta_3^2(k + 1)^2 + (\delta_1^2 + \delta_2^1)k^2) - s_1(k + s_3)(2k\delta_1^2 + F(k + s_3)) \\
+ (k + s_3)^2(\delta_1^2 - F(k + s_3)) \} \frac{1}{(k + s_2 + s_3)(k + s_3)^2} &> 0 \quad (7a) \\
\{ s_3^2(\delta_1^2(k + 1)^2 + (\delta_2^2 + \delta_3^2)k^2) - s_3(k + s_2)(2k\delta_3^2 + F(k + s_2)) \\
+ (k + s_2)^2(\delta_3^2 - F(k + s_2)) \} \frac{1}{(k + s_2 + s_3)(k + s_2)^2} &> 0 \quad (7b) \\
\{ s_1^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_1(k + 1)(\delta_1^2 - F) \\
+ (k + 1)(\delta_1^2 - F(k + 1)) \} \frac{1}{(k + s_2 + s_3)^2} &< 0 \quad (7c)
\end{align*}
\]

**Conditions for \( L = \{1, 2, 3\} \)**

For every individual \( i \in I \), the net utility when entering is higher than that when
not, given that the other two individuals enter.

\[
\begin{align*}
\{ s_1^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_1(k + 1)(\delta_1^2 - F) \\
+ (k + 1)(\delta_1^2 - F(k + 1)) \} \frac{1}{(k + s_2 + s_3)^2} & > 0 \quad (8a) \\
\{ s_2^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_2(k + 1)(\delta_2^2 - F) \\
+ (k + 1)(\delta_2^2 - F(k + 1)) \} \frac{1}{(k + s_1 + s_3)^2} & > 0 \quad (8b) \\
\{ s_3^2((k + 1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F) - 2s_3(k + 1)(\delta_3^2 - F) \\
+ (k + 1)(\delta_3^2 - F(k + 1)) \} \frac{1}{(k + s_1 + s_2)^2} & > 0 \quad (8c)
\end{align*}
\]

The denominator of the left-hand side of each of the inequalities (1a)-(8c) is positive. In the proofs below, I consider only the numerator, which is a quadratic function of the corresponding individual income share.

**Appendix-3.3 Proofs**

**Definition A-1** The discriminant of a quadratic equation \( f(x) = ax^2 + bx + c = 0 \) is defined as \( \Delta = b^2 - 4ac \).

A quadratic equation has no real root if \( \Delta < 0 \); it has two identical real roots if \( \Delta = 0 \); it has two different real roots if \( \Delta > 0 \). When \( a > 0 \), \( \Delta \leq 0 \) is equivalent to \( f(x) > 0 \) on its domain. When \( a > 0 \) and \( f(x) = 0 \) has two different real roots \( x^{\text{low}} \) and \( x^{\text{high}} \), \( f(x) > 0 \) for \( x < x^{\text{low}} \) and \( x > x^{\text{high}} \).

I prove Proposition 2 by proving the following lemmas. In the proofs, when an equation number is referred to, it means the numerator of the left-hand side of the associated inequality.

**Lemma A-1** Let \( F_0 = \frac{\delta_1^2(k+1)(\delta_1^2+\delta_2^2)}{(k+1)^2(\delta_1^2+\delta_2^2)+k^2\delta_3^2} \). If \( F < F_0 \), then \( \forall s \in S^3, \mathcal{L} = \{1,2,3\} \) is an equilibrium.

**Proof.** \( F_0 = \{ F|\Delta(8c) = 0 \} \). When \( F < F_0 \), \( \Delta(8a) < \Delta(8b) < \Delta(8c) < 0 \), hence \( (8a) > 0 \), \( (8b) > 0 \), and \( (8c) > 0 \). ■

**Lemma A-2** If \( D_{23} \geq \frac{2k+1}{k+1} \) and \( D_{12} \geq \max \left\{ 1 + \frac{1}{4k(2k+1)^2}, \frac{8(k+1)^2+D_{23}(8k^2+1)}{16(k+1)^2+D_{23}(8k^2-8k)} \right\} \), then for \( F_0 \leq F \leq \frac{\delta_2^2}{2k+1}, \forall s \in S^3 \),
(i) A unique equilibrium exists;

(ii) the equilibrium is either $L = \{1, 2, 3\}$ or $L = \{1, 2\}$.

Moreover, for $F < F_0$, $L = \{1, 2, 3\}$ is the unique equilibrium for all $s \in S^3$.

Proof. I prove the proposition by showing that, regardless of his income, individual 1’s dominant strategy is to enter; given that individual 1 enters irrespective of others’ actions, individual 2’s iterated dominant strategy is also to enter regardless of his income; individual 3’s strategy, in contrast, depends upon his income, as well as specific values of the model parameters.

(1) Individual 1’s dominant strategy is to enter, regardless of his income.

First, $D_{23} \geq \frac{2k+1}{k+1} \frac{\delta_2^2(k+1)\delta_2^2 + \delta_2^2}{(k+1)^2(k_2^2 + \delta_2^2) + k^2 \delta_2^2}$ is sufficient for $F_0 < \frac{\delta_3^2}{k+1} < \frac{\delta_2^2}{2k+1} < F_1$, where $F_1 = \frac{\delta_3^2(k+1)(\delta_2^2 + \delta_2^2)}{(k+1)^2(k_2^2 + \delta_2^2) + k^2 \delta_2^2}$. Since $F_1 = \{F|\Delta_{(8b)} = 0\}$, for $F \leq \frac{\delta_3^2}{2k+1}$, $\Delta_{(8a)} < \Delta_{(8b)} < 0$, hence $(8a) > 0$. Given that 2 and 3 enter, 1 will enter.

Second, $\Delta_{(5a)} < 0$ for $F \leq \frac{\delta_3^2}{2k+1}$, hence $(5a) > 0$. To see this,

$$\Delta_{(5a)} = \{F^2(k + s_2)^2 + 4F(k + s_2)(\delta_1^2 k (k + 1) + \delta_2^2(k + 1) + \delta_3^2 k^2)$$
$$- 4\delta_1^2 (\delta_2^2 (k + 1)^2 + \delta_3^2 k^2)\}(k + s_2)^2$$
$$\leq \{F^2(k + \frac{1}{2})^2 + 4F(k + \frac{1}{2})(\delta_1^2 k (k + 1) + \delta_2^2(k + 1) + \delta_3^2 k^2)$$
$$- 4\delta_1^2 (\delta_2^2 (k + 1)^2 + \delta_3^2 k^2)\}(k + s_2)^2$$
$$\leq \{\delta_1^2 + 8\delta_2^2(\delta_2^2 (k + 1)^2 + \delta_3^2(k + 1)^2 + \delta_3^2 k^2)$$
$$- 16\delta_1^2 (\delta_2^2 (k + 1)^2 + \delta_3^2 k^2)\}(k + s_2)^2$$
$$< 0$$

is equivalent to

$$D_{12} > \frac{D_{23}(1 + 8(k + 1)^2) + 8k^2}{D_{23}(8(k + 1)^2 + 8(k + 1)) + 16k^2} = RHS,$$

which holds since $D_{12} \geq 1 > RHS$. Given that 2 enters and 3 does not, 1 will enter.

Third, $D_{12} > \frac{8(k+1)^2 + D_{23}(8k+1)}{16(k+1)^2 + D_{23}(8k+1)}$ is sufficient for $\Delta_{(6a)} < 0$, hence $(6a) > 0$. 

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To see this, we have
\[
\Delta_{(6a)} = \left\{ F^2(k+s_3)^2 + 4F(k+s_3)(\delta^2_1 k(k+1) + \delta^2_2 k^2 + \delta^2_3 (k+1)^2) \\
-4\delta^2_1 (\delta^2_2 k^2 + \delta^2_3 (k+1)^2) \right\}(k+s_3)^2
\]
\[
\leq \left\{ F^2(k + \frac{1}{2})^2 + 4F(k + \frac{1}{2})(\delta^2_1 k(k+1) + \delta^2_2 k^2 + \delta^2_3 (k+1)^2) \\
-4\delta^2_1 (\delta^2_2 k^2 + \delta^2_3 (k+1)^2) \right\}(k+s_3)^2
\]
\[
\leq \left\{ \delta^4_2 + 8\delta^2_2 (\delta^2_1 k(k+1) + \delta^2_2 k^2 + \delta^2_3 (k+1)^2) \\
-16\delta^2_1 (\delta^2_2 k^2 + \delta^2_3 (k+1)^2) \right\}(k+s_3)^2
\]
\[
< 0
\]
is equivalent to
\[
D_{12} > \frac{8(k+1)^2 + D_{23}(8k^2 + 1)}{16(k+1)^2 + D_{23}(8k^2 - 8k)}.
\]

Given that 3 enters and 2 does not, 1 will enter.

**Last.** $D_{12} \geq 1 + \frac{1}{4k(2k+1)}$ is sufficient for $\Delta_{(2a)} < 0$, hence $(2a) > 0$. To see this, we have
\[
\Delta_{(2a)} = F^2 + 4F((k+1)\delta^2_1 + k\delta^2_2 + k\delta^2_3) - 4\delta^2_1 (\delta^2_2 + \delta^2_3)
\]
\[
\leq \frac{\delta^2_2}{(2k+1)^2} + \frac{4\delta^2_2}{2k+1}((k+1)\delta^2_1 + k\delta^2_2 + k\delta^2_3) - 4\delta^2_1 (\delta^2_2 + \delta^2_3)
\]
\[
< 0
\]
is equivalent to
\[
D_{12} > \frac{4k(2k+1) + D_{23}(4k(2k+1) + 1)}{4(2k+1)^2 + D_{23}(4k(2k+1))} = \text{RHS},
\]
which holds if $D_{12} > 1 + \frac{1}{4k(2k+1)}$. Given that neither 2 nor 3 enters, 1 will enter.

(2) Given that individual 1 enters irrespective of others’ actions, individual 2’s iterated dominant strategy is to enter regardless of his income.

**First.** For $F \leq \frac{\delta^2_2}{2k+1}$, $\Delta_{(8b)} < 0$, hence $(8b) > 0$. Given that 1 and 3 enter, 2 will enter.

**Second.** $D_{12} \geq 1 + \frac{1}{4k(2k+1)}$ is sufficient for $\Delta_{(5b)} < 0$, hence $(5b) > 0$. To see
this,
\[
\Delta_{(5b)} = \{ F^2(k + s_1)^2 + 4F(k + s_1)(c_2^2(k + 1)^2 + c_2^2k(k + 1) + c_3^2k^2) \\
- 4c_2^2(c_2^2(k + 1)^2 + c_3^2k^2)\}(k + s_1)^2
\]
\[
\leq \{ F^2(k + 1)^2 + 4F(k + 1)(c_2^2(k + 1)^2 + c_2^2k(k + 1) + c_3^2k^2) \\
- 4c_2^2(c_2^2(k + 1)^2 + c_3^2k^2)\}(k + s_1)^2
\]
\[
\leq \{ \frac{\delta 2^4(k + 1)^2}{(2k + 1)^2} + \frac{4\delta 2^4(k + 1)}{2k + 1}(c_2^2(k + 1)^2 + c_2^2k(k + 1) + c_3^2k^2) \\
- 4c_2^2(c_2^2(k + 1)^2 + c_3^2k^2)\}(k + s_1)^2
\]
\[
< 0
\]
is equivalent to
\[
D_{12} > 1 + \frac{1}{4k(2k + 1)} - \frac{k^2}{D_{23}(k + 1)^2},
\]
which holds if \( D_{12} > 1 + \frac{1}{4k(2k + 1)} \). Given that 1 enters and 3 does not, 2 will enter.

(3) Individual 3’s action depends on his income, as well as specific values of the model parameters.

(8c) = 0 has two real roots: \( s_3^{low} \) and \( s_3^{high} \).

- If \( 0 < s_3^{low} < s_3^{high} < \frac{1}{3} \), then (8c) > 0 for \( s_3 \in (0, s_3^{low}) \cup (s_3^{high}, \frac{1}{3}) \); given that 1 and 2 enter, 3 will enter. Otherwise, 3 will not enter.
- If \( s_3^{low} < 0 \) and \( s_3^{high} > \frac{1}{3} \), then (8c) < 0 for all feasible values of \( s_3 \); given that 1 and 2 enter, 3 will not enter.
- If \( s_3^{low} < 0 \) and \( s_3^{high} < \frac{1}{3} \), then (8c) > 0 for all \( s_3 \in (s_3^{high}, \frac{1}{3}) \); given that 1 and 2 enter, 3 will enter. Otherwise, 3 will not enter.
- If \( s_3^{low} > 0 \) and \( s_3^{high} > \frac{1}{3} \), then (8c) > 0 for all \( s_3 \in (0, s_3^{low}) \); given that 1 and 2 enter, 3 will enter. Otherwise, 3 will not enter.

(4) For \( F < F_0 \), steps (1) and (2) hold. Moreover, individual 3 also has a dominant strategy—to enter—regardless of his income. Therefore, the unique equilibrium is \( \mathcal{L} = \{1, 2, 3\} \).

Lemma A-3 If \( \max\{1.5, \frac{2k + 1}{k + 1}\} \leq D_{23} \leq \frac{\delta}{k + 1} \), where \( \frac{\delta}{k + 1} \) is a decreasing function of \( k \) and \( \lim_{k \to \infty} \frac{\delta}{k + 1} = 3.732 \), and \( D_{12} \geq \max\left\{ \frac{(k+1)(D_{23}+1)}{D_{23}+k+1}, \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} \right\} \),

then for \( \frac{\delta}{2k+1} < F \leq \frac{\delta}{k+1}, \forall s \in S^3 \),
(i) An equilibrium exists;

(ii) the equilibrium is either \( L = \{1, 2, 3\}, L = \{1, 2\}, L = \{1, 3\}, \text{ or } L = \{1\}; \)

(iii) whenever \( L = \{1, 3\} \) is an equilibrium, it overlaps with the equilibrium \( L = \{1, 2\}. \)

**Proof.** I prove the proposition by showing that individual 1 has a dominant strategy–to enter–regardless of his income, while the actions of individuals 2 and 3 depend upon their income, as well as specific values of the model parameters.

(1) Individual 1’s dominant strategy is to enter, regardless of his income.

First, \( D_{12} \geq \frac{(k+1)(D_{23}+1)}{k+D_{23}+1} \) is sufficient for \( F_1 < \frac{\delta_2^2}{k+1} < F_2, \) where \( F_2 = \frac{\delta_2^2(k+1)(\delta_2^2+\delta_3^2)}{(k+1)^2(\delta_2^2+\delta_3^2)+k^2\delta_1^2}. \) Since \( F_2 = \{F|\Delta_{(8a)} = 0\}, \) for \( \frac{\delta_2^2}{2k+1} < F \leq \frac{\delta_2^2}{k+1}, \Delta_{(8a)} < 0, \) hence \((8a) > 0.\) Given that 2 and 3 enter, 1 will enter.

Second, \( D_{12} > \frac{(k+1)(D_{23}+1)}{k+D_{23}+1} \) is sufficient for \( \Delta_{(5a)} < 0, \) hence \((5a) > 0.\) To see this,

\[
\Delta_{(5a)} \leq \frac{\delta_2^4(2k+1)^2}{4(k+1)^2} + \frac{2\delta_2^2(2k+1)}{k+1}(\delta_1^2k(k+1) + \delta_2^2(k+1)^2 + \delta_3^2k^2) - 4\delta_1^2(\delta_2^2(k+1)^2 + \delta_3^2k^2) \leq 0 \]

is equivalent to

\[
D_{12} > \frac{8D_{23}(k+1)^3(2k+1) + D_{23}(2k+1)^2 + 8(2k+1)(k+1)k^2}{8D_{23}(k+1)^2(3k+2) + 16k^2(k+1)^2} = RHS. \]

It holds since \( D_{12} > \frac{(k+1)(D_{23}+1)}{k+D_{23}+1} > RHS. \) Given that 2 enters and 3 does not, 1 will enter.

Third, \( D_{12} > \max \left\{ \frac{k(D_{23}+1)}{k+1}, \frac{D_{23}}{4(k+1)^2}\right\} \) is sufficient for \( \Delta_{(2a)} < 0, \) hence \((2a) > 0.\) To see this,

\[
\Delta_{(2a)} \leq \frac{\delta_2^4}{(k+1)^2} + \frac{4\delta_2^2}{k+1} (k+1)\delta_1^2 + k\delta_2^2 + k\delta_3^2 - 4\delta_1^2(\delta_2^2 + \delta_3^2) < 0 \]

is equivalent to

\[
D_{12} > \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} = RHS. \]

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The RHS may or may not be greater than 1. Thus, \( D_{12} > \max \left\{ 1, \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} \right\} \).

Given that neither 2 nor 3 enters, 1 will enter.

**Last,** as long as \( D_{23} \) is not too large, \( D_{12} > \max \left\{ 1, \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} \right\} \) is sufficient for \( \Delta_{(6a)} < 0 \), hence \((6a) > 0\). To see this,

\[
\Delta_{(6a)} \leq \frac{\delta_2^4(3k+1)^2}{9(k+1)^2} + \frac{4\delta_2^2(3k+1)}{3(k+1)}(\delta_1^2k(k+1) + \delta_2^2k^2 + \delta_3^2(k+1)^2) \\
-16\delta_1^2(\delta_2^2k^2 + \delta_3^2(k+1)^2)(k+s_3)^2
\]

< 0

is equivalent to

\[
D_{12} > \frac{12(k+1)^3(3k+1) + D_{23}(12k^2(3k+1)(k+1) + (3k+1)^2)}{12(k+1)^2(3(k+1)^2 - kD_{23})} = \text{RHS}.
\]

It can be shown that \( \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} > \text{RHS} \) as long as \( D_{23} < \bar{D}_{23}(k) \), where \( \bar{D}_{23}(k) \) is a decreasing function of \( k \), and \( \lim_{k \to \infty} \bar{D}_{23}(k) = 3.732 \).

Given that 3 enters and 2 does not, 1 will enter.

(2) The actions of individual 2 and 3 depend on their income, as well as specific values of the model parameters.

**First,** \((8c) = 0\) has two roots of opposite signs: \( s_3^+ \) and \( s_3^- \).

- If \( s_3^+ \geq \frac{1}{3} \), then \((8c) < 0\) for all \( s_3 \in (0, \frac{1}{3}) \). \( \mathcal{L} = \{1, 2, 3\} \) is not an equilibrium.

- If \( s_3^+ < \frac{1}{3} \), then \((8c) > 0\) for \( s_3 \in (s_3^+, \frac{1}{3}) \); 3 will enter given that 1 and 2 do.

- For \( F < F_1 \), \( \Delta_{(8b)} < 0 \) and \((8b) > 0\).

- For \( F > F_1 \), \( \Delta_{(8b)} > 0 \) and \((8b) = 0\) has two positive roots: \( s_2^{\text{high}} \) and \( s_2^{\text{low}} \). Since \((8b) \) and \((8c) \) belongs to the same equation family

\[
G = Ax^2 - 2(k+1)(\delta^2 - F)x + (k+1)(\delta^2 - F(k+1)) = 0,
\]

where \( A = (k+1)(\delta_1^2 + \delta_2^2 + \delta_3^2) - F \); and

\[
\frac{\partial x^{\text{high}}}{\partial \delta^2} = -\frac{\partial G/\partial \delta^2}{\partial G/\partial x^{\text{high}}} = -\frac{(k+1)(1 - 2x^{\text{high}})}{\sqrt{\Delta G}} < 0
\]

if \( x^{\text{high}} < \frac{1}{2} \).
Since $s_3^+ < \frac{1}{3}$, for $\delta_2^2$ sufficiently close to $\delta_3^2$, $s_2^{\text{high}} < s_3^+$. Moreover, 
\[
\frac{\partial s_2^{\text{high}}(8b)}{\partial \delta_2^2} < 0.
\]
Therefore, for all $\delta_2^2 > \delta_3^2$, we have $s_2^{\text{high}} < s_3^+ < \frac{1}{3}$.

Since $s_2 \geq s_3 > s_3^+ > s_2^{\text{high}}$, $(8b) > 0$.

Thus for $s_3 \in (s_3^+, \frac{1}{3})$, $L = \{1, 2, 3\}$ is an equilibrium.

**Second**, consider the subspace of $S^3$ with $s_3 < s_3^+$, such that $(8c) < 0$, or equivalently, $(5c) < 0$. Moreover, we know $(5a) > 0$ and $(2a) > 0$.

- Since $\Delta_{(5b)}$ increases with $s_1$, $\exists s_1^*$, such that when $s_1 < s_1^*$, $\Delta_{(5b)} > 0$; and when $s_1 \geq s_1^*$, $\Delta_{(5b)} < 0$. If $s_1^* > 1$, then $\Delta_{(5b)} < 0$ for all $s$ in this subspace of $S^3$, hence $(5b) > 0$, and $\{1, 2\}$ is an equilibrium. If $s_1^* < 1$, then for $\frac{1}{3} < s_1 < s_1^*$, $\Delta_{(5b)} < 0$, hence $(5b) > 0$, and $\{1, 2\}$ is an equilibrium; for $s_1^* < s_1 < 1$, $(5b) = 0$ partitions the relevant space into two parts: $(5b) < 0$ for $s_2 \in (s_2^{\text{low}}, s_2^{\text{high}})$, and $(5b) > 0$ hence $\{1, 2\}$ is an equilibrium, for otherwise.

- Consider the subspace $s_2 \in (s_2^{\text{low}}, s_2^{\text{high}})$, such that $(5b) < 0$, or equivalently, $(2b) < 0$. I show that $(2c) < 0$ for this region.
  - Since $\Delta_{(2c)} > \Delta_{(2b)}$ when $\delta_2^2(k + 1)^2 > Fk(k + s_1)$, which holds trivially for $F < \frac{\delta_2^2}{k+1}$, $\Delta_{(2c)} > 0$. Thus, $(2c) = 0$ has two real roots: $s_3^{\text{low}}$ and $s_3^{\text{high}}$. $(2c) < 0$ is equivalent to $s_3^{\text{low}} < s_3 < s_3^{\text{high}}$.
  - $s_3 < s_3^{\text{high}}$ if $s_2 < s_2^{\text{high}}$. To see this, note that $(2b)$ and $(2c)$ belong to the same equation family

$$G = Ax^2 - (k + s_1)(2k\delta^2 + F(k + s_1))x + (k + s_1)^2(\delta^2 - F(k + s_1)) = 0$$

where $A = \delta_1^2(k + a)^2 + \delta_2^2k^2 + \delta_3^2k^2$; and

$$\frac{\partial x^{\text{high}}}{\partial \delta^2} = \frac{\partial G/\partial \delta^2}{\partial G/\partial x^{\text{high}}} = -\frac{(k + s_1)(k + s_1 - 2kx^{\text{high}})}{\sqrt{\Delta G}} < 0$$

if $x^{\text{high}} < \frac{1}{2}$. Moreover, $\frac{\partial x^{\text{high}}(2b)}{\partial \delta_2^2} < 0$.

Therefore, if $s_3^{\text{high}} < \frac{1}{2}$, then $s_2^{\text{high}} < s_3^{\text{high}} < \frac{1}{2}$ for all $\delta_2 > \delta_3^2$. Since $s_2 < s_2^{\text{high}}$ and $s_3 < s_3^{\text{high}}$. If, on the contrary, $s_3^{\text{high}} > \frac{1}{2}$, then $s_3 < s_3^{\text{high}}$ by definition.

- To show $s_3 > s_3^{\text{low}}$, it is sufficient if $s_3^{\text{low}} < 0$. Since $s_3^{\text{low}} + s_3^{\text{high}} > 0$, we need $s_3^{\text{low}}, s_3^{\text{high}} < 0$; this in turn is sufficient if $\delta_3^2 < F(k + s_1)$, or equivalently, $s_1 > \frac{\delta_3^2}{F} - k = s_1^*$. Since the relevant subspace exhibits
In sum, in equilibrium, the equilibrium is either D or A unique equilibrium exists; let Individual 3’s dominant strategy is to not enter regardless of his income, i.e.,

\[
s_3^* < s_1 < 1,
\]

we want to show \( s_1^* < 1 \) and \( s_3^* < s_3^* \). First,

\[
s_1^* = \frac{\delta_3^2}{F} - k < \frac{\delta_3^2}{\delta_2^2(2k+1)} - k < (2k+1)\frac{k+1}{2k+1} - k = 1.
\]

The first inequality holds because \( F > \frac{\delta_3^2}{2k+1} \), and the second because \( \frac{\delta_3^2}{\delta_2^2} > \frac{2k+1}{k+1} \). Second, \( s_3^* < s_1^* \) is equivalent to \( \Delta_{(2b)}(s_1^*) < 0 \), which, after tedious manipulation, is equivalent to \( D_{12} > \frac{1+4k^2+4kD_{23}}{4(k+1)^2(D_{23}-D_{23})} \).

When \( D_{12} > \max \{ 1, \frac{k(D_{23}+1)}{k+1} + \frac{D_{23}}{4(k+1)^2} \} \), it holds as long as \( D_{23} \) is not too small. A sufficient condition is \( D_{23} \geq \max \{ 1, 2 + \frac{4k+1}{k+1} \} \).

Thus \( \mathcal{L} = \{ 1 \} \) is an equilibrium for this region.

(3) In sum, in equilibrium, \( \forall s \in S^3 \), an equilibrium exist, and it can be either \( \mathcal{L} = \{ 1, 2, 3 \} \), \( \mathcal{L} = \{ 1, 2 \} \), or \( \mathcal{L} = \{ 1 \} \). We cannot however rule out \( \mathcal{L} = \{ 1, 3 \} \) as an equilibrium, but we know that whenever it arises, it has to overlap with \( \mathcal{L} = \{ 1, 2 \} \), since it is mutually exclusive with \( \mathcal{L} = \{ 1, 2, 3 \} \) and \( \mathcal{L} = \{ 1 \} \).

Lemma A-4 Let \( \widehat{D}_{12} = \min \left\{ 3 + \frac{2}{k} - \frac{1}{2}, \frac{8 + \frac{1}{k+1}}{k}, -\frac{1}{2} \right\} \). For \( F \geq \frac{\delta^2_3}{k} \), if \( k \) is sufficiently large, and \( D_{23} > 1 \), \( D_{12} \leq \widehat{D}_{12} \), then \( \forall s \in S^3 \),

(i) A unique equilibrium exists;

(ii) the equilibrium is either \( \mathcal{L} = \{ 1, 2 \} \), \( \mathcal{L} = \{ 1 \} \), or \( \mathcal{L} = \emptyset \).

Proof. I prove the proposition by showing that individual 3’s dominant strategy is to not enter regardless of his income, while the actions of individual 1 and 2 depend on their income, as well as specific values of the model parameters.

(1) Individual 3’s dominant strategy is to not enter regardless of his income, i.e., \( \forall s_3 \in [0, \frac{1}{3}] \), \( 1c < 0 \), \( 2c < 0 \), \( 3c < 0 \), and \( 8c < 0 \) when \( F \geq \frac{\delta^2_3}{k} \). It is obvious that \( 1c = 0 \), \( 2c = 0 \), \( 3c = 0 \), and \( 8c = 0 \) each have two real roots of opposite signs when \( F \geq \frac{\delta^2_3}{k} \), and the positive roots increase with \( F \); therefore, it is sufficient to show the positive roots are greater than \( \frac{1}{3} \).

First, \( D_{12} \leq 8 + \frac{3}{k} - \frac{4}{D_{23}} = \overline{D}_a \) is sufficient for \( s_3^+ (1c) \geq \frac{1}{3} \).

Second, a sufficient condition for \( s_3^+ (2c) \geq \frac{1}{3} \) is that \( s_3^+ (s_1 = \frac{1}{3}) (2c) \geq \frac{1}{3} \). This is because

\[
\frac{\partial s_3^+ (2c)}{\partial s_1} = -\frac{\partial (2c)/\partial s_1}{\partial (2c)/\partial s_3^+ (2c)} = \frac{3(F(k^2 + s_1^2) + 2Fk + 2s_1(3kF + F - \delta^2_3))}{\sqrt{\Delta(2c)}} > 0
\]
when $F \geq \frac{\delta^2}{k}$. Therefore, $D_{12} \leq \frac{(3k+1)^2(3k+2)+3k^3}{3(k+1)^2} - \frac{(2k+1)^2}{k+1^2} = \overline{D}_b$ is sufficient for $(2c) < 0$.

**Third**, similarly, a sufficient condition for $s_3^+(3c) \geq \frac{1}{3}$ is that $s_3^+(s_2 = 0)(3c) \geq \frac{1}{3}$, which is equivalent to $D_{12} \leq 8 + \frac{k-1}{k^2} - \frac{4}{D_{23}} = \overline{D}_c$.

**Last**, $D_{12} \leq \frac{(3k+2)^2}{k(k+1)} - 1 - \frac{4}{D_{23}} = \overline{D}_a$ is sufficient for $s_3^+(8c) \geq \frac{1}{3}$.

(2) It is obvious that $(1b) = 0$ has two real roots of opposite signs when $F \geq \frac{\delta^2}{k}$, and its positive root increases with $F$. Thus we can show that $(1b) < 0$ by showing $s_3^+(F = \frac{\delta^2}{k})(1b) \geq \frac{1}{2}$. A sufficient condition is $D_{12} \leq 3 + \frac{\delta^2}{k^2} - \frac{1}{D_{23}} = \overline{D}_c$. In other words, with more a stringent constraint on $D_{12}$, we have that $2$ will not enter given that $1$ and $3$ do not.

(3) There are only three possible equilibria: $\emptyset$, $\{1\}$, and $\{1,2\}$.

**First**, $(1a) = 0$ may or may not have real roots. If it does not have real roots, then $\forall s \in S^3$, $(1a) > 0$. If it has real roots $s_1^{low}$ and $s_1^{high}$, and $\frac{1}{3} < s_1^{low} < s_1^{high} < 1$, then for the region $s_1 \in (s_1^{low}, s_1^{high})$, $(1a) < 0$ and $\mathcal{L} = \emptyset$ is an equilibrium; $(1a) > 0$ for otherwise.

**Second** Consider the relevant subspace of $S^3$ in which $(1a) > 0$. It is obvious that $(2b) = 0$ has two real roots of opposite signs when $F \geq \frac{\delta^2}{k}$. Thus, the relevant subspace can be partitioned into two parts by the curve $(2b) = 0$. For the part $(2b) < 0$, $\mathcal{L} = \{1\}$ is an equilibrium. For the other part $(2b) > 0$, or equivalently, $(5b) > 0$, it can be shown that as long as $D_{12}$ is not too large, $(5a) > 0$; thus $\mathcal{L} = \{1,2\}$ is an equilibrium. This is proved as follows.

- Since $(5b) = 0$ has two real roots of opposite signs: $s_2^+$ and $s_2^-$, $(5b) > 0$ only for $s_2 \in (s_2^+, min\{s_1, 1-s_1\})$.
- When $F \geq \frac{\delta^2}{k}$,
  \[\frac{\partial(5a)}{\partial s_2} = -3F(k + s_2)^2 + 2(k + s_2)(\delta_1^2 - Fs_1) - 2k s_1 \delta_1^2 < 0,\]
  as long as $D_{12} < \frac{3(k+s_2)^2+2(k+s_2)s_1}{2k(k(1-s_1)+s_2)} = \overline{D}_f$. I only need this to hold for $s_2 \in (s_2^+, min\{s_1, 1-s_1\})$.
- It is straightforward that $(5a) > (5b) > 0$ when $s_2 = s_1 < \frac{1}{2}$. Given the monotonicity of $(5a)$ with respect to $s_2$ shown above, $(5a) > 0$ for $s_2 \in (s_2^+, s_1)$.

\[\text{Indeed, } s_3^+(s_2 = 0)(3c) \geq \frac{1}{3} \text{ is probably too strong. What we need precisely is } s_3^+(s_2)(3c) \geq min\{s_2, \frac{1}{3}\}.\]
− When $s_1 > \frac{1}{2}$, it can be shown that $(5a) > (5b) > 0$ for $s_2 = 1 - s_1$. To see this,

$\begin{align*}
(5a) - (5b) &= \delta_1^2 (k + 1) \{(k + 1)[4s_1^3 - 6s_1^2 + 4s_1 - 1 + (2s_1 - 1)k] \\
&\quad + (1 - 2s_1)(k + s_1)^2 \} \\
&\quad + \delta_2^2 (k + 1) \{(k + 1)[4s_1^3 - 6s_1^2 + 4s_1 - 1 + (2s_1 - 1)k] \\
&\quad + (1 - 2s_1)(k + 1 - s_1)^2 \} \\
&\quad + \delta_3^2 (k + 1) \{(k + 1)[4s_1^3 - 6s_1^2 + 4s_1 - 1 + (2s_1 - 1)k] \},
\end{align*}$

and the coefficients on $\delta^2$’s are all positive. By monotonicity of $(5a)$, $(5a) > 0$ for $s_2 \in (s_2^+, 1 - s_1)$.

It can be verified that, for $k$ sufficiently large, $\overline{D}_d > \overline{D}_a > \min \{\overline{D}_b, \overline{D}_f\} > \min \{\overline{D}_c, \overline{D}_e\}$. Let $\overline{D}_{12} = \min \{\overline{D}_c, \overline{D}_e\}$, then $D_{12} \leq \overline{D}_{12}$ is sufficient for arguments (1)-(3) to hold.

(4) It is possible mathematically that $\mathcal{L} = \{1, 2\}$ overlaps with $\mathcal{L} = \emptyset$. One of them, however, can be ruled out by comparing the net utilities of 1 and 2 in both cases. When the net utilities of both 1 and 2 are greater in $\mathcal{L} = \{1, 2\}$, it emerges as a unique equilibrium. Otherwise, $\mathcal{L} = \emptyset$ is the unique equilibrium. ■
References


