

# **Zvi Griliches on Diffusion, Lags and Productivity Growth ...Connecting the Dots**

By

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## **ABSTRACT**

The three most extensively cited papers by Zvi Griliches deal with the diffusion of innovations, distributed lags and the sources of the growth of measured total factor productivity, respectively. The close economic connections between these dynamic phenomena remained largely unexplored and were at best only implicit in his published writings until late in his career. Yet, from his later reflective writings, it is clear that Griliches not only recognized the existence of those connections, but regarded them to be critically important in understanding the determinants of the pace of economic growth. The present paper proceeds in that spirit. It examines the relationship between Gliches' pioneering study of the diffusion of hybrid corn and the subsequent development of economic theories explaining diffusion phenomenon. Rather than offering a comprehensive survey of the literature, its aim is to expose the connections with lagged investment in capital-embodied innovations, and formalize of the micro-to macro links between technological diffusion dynamics and the pace of measured productivity growth. The heterodox, "evolutionary economics" aspects of this approach to explaining technological 'transitions' may be thought to be a significant yet under-appreciated part of Griliches' intellectual legacy.

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## PART 1

### **Prologue: Long Ago, When Two ‘City-Boys’ Looked at Agricultural Innovations**

As we are refreshing our memories of Zvi Griliches’ contributions to modern economics, it seems to me quite appropriate to preface the substance of this paper by recalling that I spent the 1972-73, academic year visiting Harvard, where, in addition to teaching American economic history, I was encouraged by Zvi to offer a mini-course of lectures on the diffusion of innovation for economics graduate students. To my great satisfaction, the small turnout for those lectures included Zvi - on at least two occasions that I can recall.

Yet, it soon became apparent to me that the interests of the most distinguished and formidable auditor in the class had moved on from the topics with which I was wrestling; not only from the questions raised by his early work on the introduction and diffusion of hybrid corn, but also from the controversies about the measurement and interpretation of total factor productivity change that had occupied Griliches and his colleague Dale Jorgenson in the late 1960s; and from the possible bearing of the former phenomenon upon the latter.

Those less than fully informed impressions on my part subsequently have been confirmed by people who at the time were working closely with Zvi. Indeed, in 1972-73 he had turned his attention to human capital and was engrossed in estimating the returns to education using the new National Longitudinal Survey (NLS) data, which represented the first large-scale micro dataset used by economists. Zvi’s attention at that point was focused on trying to use econometrics to control for left-out characteristics (such as ability) in the relationship between wages and schooling. He was also involved in developing the use of census micro data for measuring the R&D -productivity relationship, a project that came to fruition during the following decade.<sup>1</sup> Research on diffusion phenomena had most certainly dropped far out of sight.

In addition, were any additional reasons needed, there were a number of methodological aspects in what I was saying about diffusion with which Zvi was not especially sympathetic – a matter to which I will return. Nonetheless, as he was personally sympathetic with me, he listened closely and asked questions from which it was apparent that he had thoroughly grasped what I was trying to do. Far from

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<sup>1</sup> Thanks to Bronwyn Hall for these crisp recollections. See also the account of the work on human capital by Reuben Gronau (2003)

being discouraging, he was unpatronizingly tolerant toward the approach I was developing. I recall that this left me feeling relieved, but also rather disappointed.

I suppose the reckless part of me had wanted a more intense engagement with the Master. And, in truth, it would have been entirely reasonable for Zvi to have responded in a much tougher way to the approach to the microeconomics of technology adoption on which my of lectures were based, seeing in it a not-so-implicit critique of his own pioneering studies of hybrid corn and the trajectory of empirical studies that had sprung from it—notably those by Edwin Mansfield, about which I was making some youthfully ungenerous comments. Consequently, whatever potential I believed my work on diffusion held, my distinct sense of relief in the presence of Zvi's tolerant attitude surely reflected my awareness at the time that some parts of it could be characterized as incomplete, if not "half-baked."

Beyond that level of vulnerability, there was a discernable strand of deviance from the "mainstream" canon in those lectures. It surfaced most clearly in my view that the growth of aggregate productivity measures might reflect the dynamics of diffusion phenomena of the sort that would be generated by the micro-level processes which I was trying to model. Inasmuch as diffusion can be represented as the resultant of a process involving sequential technology *selection* behaviors, my approach was rather vaguely Darwinian, featuring a competition between evolving alternative techniques. It was thus amenable to the spirit of what would emerge a decade later — from Dick Nelson and Sid Winter — as an "evolutionary" alternative to the aggregate production function paradigm within which Zvi and Dale Jorgenson had been working.

More provocative still, I was rejecting the prevailing emphasis in economic studies of diffusion — attributable largely to the work of Griliches and Mansfield — which conceptualized the phenomenon as one of disequilibrium transition, and cited information imperfections as the cause of protracted lags in the adoption of innovations. Instead, I had clung determinedly to microeconomic models of technology selection that posited full information and profit maximization on the part of the agents. (Not that I asserted the realism of that assumption, but it served to bring out more clearly the alternative line of explanation for adoption lags that I was developing.) I readily concede that this aspect of the course must have been a source of no little perplexity for some Harvard students at that time; certainly it has remained a puzzling inconsistency for others who subsequently welcomed the non-

neoclassical, evolutionary overtones of other things that I was saying about the sources of macro-level productivity growth.<sup>2</sup>

In any event, such was the vaguely heterodox approach that I tried to elaborate in the closing lectures of my mini-course. Alas, by that stage, Zvi no longer was in the audience. Most probably he had already surmised what was coming. But maybe not. In any event, it is to the same line of thought that I want to return here, by explicitly presenting the some approaches to understanding the microeconomic determinants of adoption behavior, and showing the resultant paths of innovation diffusion to be potentially powerful drivers of macro-level productivity growth.

### *1.1 Diffusion, lags and the productivity residual – Zvi’s 3 “biggest hits”*

To do that seems to me to be especially fitting for this occasion. It happens that the three most widely cited papers by Zvi Griliches deal with the diffusion of innovations, distributed lags and the explanation of changes in measured total factor productivity (a work co-authored with Dale Jorgenson).

[Figure 1 here]

I reproduce in Figure 1 a set of graphs from Arthur Diamond’s forthcoming article on “Zvi Griliches’ Contributions to the Economics of Technology and Growth.” They trace the historical evolution of annual journal citations to these three all-time “top cited” papers.<sup>3</sup> Obviously, these citation statistics testify immediately to the sustained influence of each of the individual studies. The fact that the 1957 *Econometrica* paper on hybrid corn heads the list of his articles in terms of the cumulative number of citations is a pleasingly quantitative confirmation of my decision that I should start by focusing upon the significance of this path-

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<sup>2</sup> “Which side is he on?,” must have been the question perplexing the students (then alive to the lingering controversies between ‘neoclassicals’ and their radical political economy critics). “Both sides are right” I would have said, thereby following the example of the famous Rabbi who agreed with both of the disputing parties who had sought his judgment, and also agreed with the student who criticized his inconsistency.

<sup>3</sup> The figure has omitted the graph for the fourth paper, although it is closely to the others in substance, being Zvi’s pioneering effort to quantify the social rate of return on public investment in hybrid corn.

breaking empirical study of the diffusion of innovations.<sup>4</sup> But there is more information to be extracted from the graphs in Figure 1.

### *1.2 Remarkable coincidences, significant connections*

Indeed, there is a remarkable, truly wonderful aspect of these time series that made the thought of exhibiting them here simply irresistible. You may already have noticed, as I did, that the time-series of citations in each of the three publications is self-exemplifying of the subject matter treated by the article in question. The hybrid corn article is unique in that they have continued to grow over time, but, more than that, they are following a *sinusoidal* path – the canonical profile of a diffusion curve. Look next at the graph of citations to Zvi’s survey article on “Distributed Lags” published by *Econometrica* in 1967: does it not display a typical distributed lag profile, acquiring its peak shortly after the event (publication in this case) and thereafter dying away geometrically? <sup>5</sup> And lastly, the annual citation record for the 1967 Griliches-Jorgenson contribution in the *Review of Economic Studies* goes along from year to year throughout the ensuing period at the same level, emulating the profile of the proportionate annual movements around a constant trend rate of growth in total factor productivity.

Although these pictorial coincidences certainly are striking and amusing, there is really nothing further that I can make of them. Yet, there is another coincidence in the citation record that does appear to have some further import. While the three top-cited publications have been regarded as largely unrelated in the literature, so that their joint salience owes virtually nothing to cross-citations, there are significant substantive connections among them. At least that is my contention.

Until late in his career, the economic relationships between these three dynamic phenomena remained largely unexplored by Griliches himself and at best were only implicit in his published writings. From his later reflective writings,

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<sup>4</sup> Corroborative reassurance is available, from the *New York Times* (5 November 1999: p. 11) obituary by Michael Weinstein, which reports Dale Jorgenson’s reference to this study, along with the 1958 *JPE* article (measuring the social rate of return on R&D in hybrid corn) as the best known and most-mentioned of Griliches’ contributions. Diamond (forthcoming in *Economics of Innovation and New Technology*, 2004) also notes Ariel Pakes’ (2000) description of the 1957 *Econometrica* article as “seminal.”

<sup>5</sup> It would be entertaining to see how well one could fit a Koyck lag specification to this series. The model would be based on the notion that the advent of publication within a year had created a disequilibrium state in which there was an excess demand (for something in the neighborhood of 400 citations to such a survey!

however, it is clear that Zvi recognized the existence of important connections between two of the topics – diffusion and productivity; and that he regarded these to be critically important in understanding the determinants of the pace of economic growth. In *R&D and Productivity: the Econometric Evidence*, the collection of his papers that appeared in 1998, one may read Griliches' considered judgment:

“Real explanations [of productivity growth] will come from understanding the sources of scientific and technological advances and from identifying the incentives and circumstances that brought them about and that facilitated *their implementation and diffusion*.”<sup>6</sup>

The present paper proceeds with that endorsement. It examines the relationship between Griliches' pioneering study of the diffusion of hybrid corn and the subsequent formal modeling of diffusion phenomenon. The latter expose the links with lagged investment in capital-embodied innovations, and provide a basis for formalizing of the micro-to macro links between technological diffusion dynamics and the pace of productivity growth. The heterodox, “evolutionary economics” aspects of this approach to explaining ‘transitions’ may be thought to form a significant yet under-appreciated part of Griliches' intellectual legacy.

## PART 2

### The Nature of Zvi's Legacy – Economics and Technology Diffusion

The first of my essay's two main substantive strands will occupy us in this Part. Its' focus on Zvi's seminal contribution to the literature on the topic of diffusion will be appreciative, but (in the best Griliches tradition) a little bit critical of the famous hybrid corn papers for lacking any real micro-level technology choice model; also for ignoring at the technical level the elements of fixed costs which implied that this innovation would not necessarily be profitable to adopt at all scales of adoption. I shall point up that criticism rather more sharply in regard to Ed Mansfield's studies during the 1960's of the diffusion of certain industrial innovations, an important program of the same representative empirical research that followed the same conceptual and econometric approach that had been pioneered in the study of hybrid corn.

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<sup>6</sup> In this passage, quoted in full by Diamond (forthcoming, 2004) Griliches (1998: pp.89-90) went on to point out that this would lead economists “back to the study of the history of science and technology and the diffusion of their products, a topic that we have left largely to others.”

Although Zvi Griliches is quite properly cited as the path-breaking economist in this field, subsequent development of work on the economics of diffusion has discarded many specific details of this contribution. This is not unusual in the history of science; the formative legacies often are quickly outgrown. But, in this case, progress towards more sophisticated theoretical understanding has not been costless; the data requirements for consistent econometric studies became more exacting, but the profession has responded weakly to that challenge. The result has been that systematic econometric research on diffusion continues to be rather neglected, and the gap between theoretical modeling and empirical studies has tended to widen.<sup>7</sup>

### *2.1 The hybrid corn study – an econometric paradigm is born*

Zvi Griliches' early econometric studies of the commercial introduction and diffusion of hybrid corn in the U.S. were influential for two reasons that were somewhat in tension with each other. First, he construed the phenomenon of diffusion in economic rather than sociological terms, and so opened a new avenue to examining the economics of technological change. Second, the quantitative approach he adopted was primarily inductive, rather than dependent upon the postulating of a particular theoretical model from which deductive propositions could be formulated and subjected to statistical testing. Instead, it demonstrated a way to quantitatively characterize differential features of the generic phenomenon, providing a methodology that could be widely applied while leaving room for many alternative explanatory hypotheses.

Both aspects of Griliches' approach are reflected in his implicit characterization of the diffusion process as one that occurred successively within distinct geographical regions, and in his selection of a statistically convenient descriptive specification for the diffusion path – namely, the cumulative logistic distribution. He could then proceed immediately to hypothesize that the speed of diffusion (and hence the overall shape of the S-shaped path determined by the slope parameter in the diffusion function) would reflect economic conditions having to do with the innovation's profitability for a representative adopter. Similarly, economic factors could be supposed to affect the location of the onset date for the diffusion process under examination, and therefore to reflect themselves statistically in the value of that second (scaling) parameter of the logistic.<sup>8</sup>

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<sup>7</sup> Recent exceptions prove the rule: see Forman, Goldfarb and Greenstein (2003); Bresnahan and Yin (2003).

<sup>8</sup> When the upper asymptote of the diffusion path is taken to be unity (universal adoption), signifying the assumption made by Griliches' (1957), there are only two free parameters to fit econometrically

Yet, Griliches' classic (1957) paper on hybrid corn offered no theoretical justification for its reliance on the logistic specification in the econometric analysis; nor did it actually provide an economic (or sociological) explanation for the failure of the innovation to be taken up instantaneously and universally as soon as it was introduced in any particular region. It is reasonable to suppose that this was not regarded as necessary, in view of the compelling empirical patterns exhibited in that famous chart of the changing proportions of corn acreage planted with hybrid seed in the Midwestern states. Figure 2 reproduces the picture from the 1957 *Econometrica* article – repeated in his compact account published by *Science* in 1960, and so widely reproduced thereafter as to have become emblematic of the phenomenon of diffusion.

[Figure 2 here]

Griliches thereby was able to characterize the diffusion path in terms of two readily obtained parameters – the slope coefficient of the logistic function, and the 'intercept' coefficient (which sets the initial or conventionally perceptible) proportion of adopters from whence the observed diffusion process proceeds. The elegance of this simplification had a major initial influence in stimulating econometric research on diffusion: characteristics of various technological innovations, or of the industries and markets into which these had been introduced, could simply be entered as regressors that might account for inter-innovation variations of the logistic slope coefficients.

As will be seen, this soon was seized upon by economists as an "obvious" way proceed in quantitative studies of the role of demand-side conditions in determining the adoption of new production techniques and new goods. But, an equally novel aspect of Griliches' paper in *Econometrica* (1957) was concerned with factors operating on the supply side of the market for hybrid corn seed in the U.S. Like many other innovations, hybrid plants are most efficient as elements of a production system when they have been designed for a specific environment. In some cases the relevant "environment" is economic, in the sense of being defined by the structure of relative prices of the array of inputs used by the production system; in others, it is the physical environment to which the process required being adapted. In the case of hybrid corn, Griliches noted, local variations in soil types, climate, and pests called for the suppliers of seeds to develop particular varieties that would be best suited to the requirements of farmer in the various sub-regions of

for the logistic distribution. But the conventional approach is to estimate a slope parameter and an intercept constant from a (linear) model of the log-odds ratio. See discussion below, in this section.



the U.S., ranging southwards from Wisconsin and Iowa, to Texas and Alabama (see Fig. 2).

Consequently, considerations of the profitability of the incremental “development” investments this entailed – such as the extent of the existing corn-acreage in the sub-region, and the typical size of the farm that the company’s seed-salesmen would have to visit – would be expected to affect the spatio-temporal sequencing of the innovation’s introduction. In this part of his analysis, Zvi clearly had anticipated the eventual efforts of the builders of microeconomic models to incorporate the effects of incremental supply-side adaptations upon the diffusion process. Yet, this feature of his path-breaking study, about which I will want to say something more in a few moment, attracted comparatively little notice at the time.

## 2.2 *Explanations and interpretations: the contagion model emerges*

Not long thereafter, Edwin Mansfield’s (1961, 1963a, 1963b, 1966, 1968b) inquiries into the diffusion of industrial innovations began to erect an impressive empirical edifice by applying the econometric approach that Griliches (1957) had pioneered. It was Mansfield (1961) who felt obliged to propose a formal economic rationale for the dependence of his econometric studies upon the logistic specification. This he did by hypothesizing that information imperfections effectively constrained adoption, but gradually were eliminated as knowledge about of the innovation became more and more widely disseminated. The supposed mechanism of dissemination was a social contact, ‘word-of-mouth’ transmission of the relevant knowledge, rather than one based upon economic consideration of the benefits and costs of searches for information by members of the potential population of adopters; or of investment in the broadcast of information by seed-companies and agricultural extension agents.

Instead of trying to explain why such information seemed to flow along particular channels, the so-called ‘contagion’ model of diffusion made use of a very simple conceptualization of the random propagation of information concerning an innovation. By positing that information was transferred (as an infection) through random social contacts between those who had already adopted the innovation and those who had not, one arrives almost immediately at the differential equation:  $dP = \phi \{ P(1-P) \}$ , where  $P$  is the mean probability that a randomly drawn member of the population of potential adopters will already have adopted the innovation and hence possess information as to its benefits. Under conditions of complete and random social inter-mixing, the product term  $P(1-P)$  is probability that a random dyadic contact will thus inform a non-adopter. If the constant  $\phi$  is taken to be mean

probability that a newly informed non-user will join the ranks of adopters, the left hand-side of the equation represents the resulting increase in the expected increment in the share of adopters among the population. Now  $P(t)$ , thus defined, is a measure of the extent of the innovation's diffusion  $D(t)$  at time  $t$ , and so, by integration of the differential equation for  $dP/dt$ , one may arrive immediately at the result that  $D(t)$  is a logistic function of  $t$ , with the slope coefficient  $\varphi$ .

It takes nothing away from Ed Mansfield's achievements to say that the empirical research program that began to spew forth papers on the adoption of industrial process innovations in the early 1960's had been catalyzed, if not inspired by Zvi's study of hybrid corn. Lacking comparable inductive reasons for adopting the logistic specification, he had come up with a plausible theoretical rationale. Moreover, whereas Griliches had been able to construct diffusion time-series from the published USDA data, Mansfield had to build his own datasets to trace the penetration of the range of new techniques (many of them, like continuous rolling mills, continuous iron casting, and continuous annealing machines) based upon new fixed capital equipment. Mansfield had mounted a serious and sustained data collection effort focused on the adoption of new technology both within firms and across firms in the manufacturing and transport sectors. Most of what we came to know about the central tendencies and variations in the patterns of diffusion of industrial processes stemmed from this very data-oriented research program, in which several generations of Mansfield's graduate students at the University of Pennsylvania were enlisted. But his style was to develop his own industrial data sources, rather than set about persuading government statistical agencies to institutionalize the collection of information to support work in this field, an approach quite different from that which Zvi was eventually to develop to a fine art.

Having justified the econometric specification, Mansfield's empirical studies thus followed Griliches' by hypothesizing that the slope coefficient of the logistic would be a positive function of the expected profitability of the innovation for a representative agent in the population of potential adopters. As he was concerned for the most part with the adoption decisions of industrial firms, and writing for an audience interested in the economics of technological change, Mansfield escaped the skeptical reception that Griliches' (1957 and 1960) articles drew from the members of another social discipline that had developed strong pre-existing views about the ways farmers behaved in response to the stimulus of novelty.

### 2.3 *The unanticipated benefits of upsetting rural sociologists*

The interest of economists in the topic of diffusion of technology, and their appreciation of young Griliches,<sup>9</sup> undoubtedly, was raised further by the criticisms it soon drew from rural sociologists. Perhaps Zvi had found his conclusions so compelling as to be beyond cavil, or he may have underestimated the extent to which his plain expression of them would be read in some quarters as fighting words. Anyway, here is what he said in the original 1957 paper:

“It is my belief that in the long run, and cross-sectionally, [sociological] variables tend to cancel themselves out, leaving the economic variables as the major determinants of the pattern of technological change.”<sup>10</sup>

Perhaps not altogether surprisingly, rural sociologists – even before they reached this passage – were finding it difficult to square the complex and nuanced picture that had been projected by Ryan and Gross’s (1943) influential study of the response of Iowa’s farmers to the introduction of hybrid corn,<sup>11</sup> with Griliches’ stark emphasis upon the differential profitability of hybrid corn as the principal systemic factor affecting the speed of its diffusion. Ryan and Gross (1943) had reported a complicated array of objective and subjective considerations as having shaped the

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<sup>9</sup> Many readers now may not be aware that at the end of 1950’s the Stanford Economics faculty, impelled by the enthusiasm of Kenneth Arrow, was trying to recruit two young “agricultural economists”. One was Marc Nerlove, and the other was Zvi Griliches. Unfortunately, for Stanford, in the then very small and theoretically oriented department, the idea of two “empirical” assistant professors both doing agriculture was felt to be more than reasonable, and most likely more than the Dean would support (the two conditions not always being congruent). Unfortunately, Arrow’s effort to rescue the plan by persuading the Food Research Institute to share the cost of an assistant professorship for Zvi, was unavailing. How differently would things have turned out if an extra seven thousand dollars had been added to the Stanford Department’s budget?

<sup>10</sup> These fighting words still rankled in some quarters many years afterwards. They are quoted in the 3rd Edition of Everett M. Rogers (1983) survey of research on the diffusion of innovations; only to be immediately dismissed as exemplifying a “ridiculous” subscription to the naive *homo economicus* conceptualization popular among members of the “Chicago School.” In truth, Rogers completely missed the point of the argument advanced by Griliches, which had to do not with whether human action was influenced by non-economic considerations, but with the issue of whether those factors were sufficiently correlated over time, or in the cross-section, to exert a significant influence on the observed aggregate outcomes.

<sup>11</sup> This had been a landmark contribution to the methodology and substance of interview-based research on the adoption of innovations. Indeed, the tradition among quantitative sociologists springing from the work of Ryan and Gross (1943) continues to shape empirical studies in the field of marketing, as well as academic sociological inquiries into the adoption of technological and other innovations among rural communities in the developing economies.

reception of hybrid corn cultivation as a substitute for the traditional farming regime based on open-pollinated corn-seed. Thus, in criticism of Griliches' paper, in their home journal, *Rural Sociology*, they argued for greater attention to the roles of the "congruence" or "compatibility" of the innovation with pre-existing practices and beliefs about their efficacy; and also for recognition of the structure of social interactions that affected access to persuasive information about hybrid corn's advantages. These underlying structures were held to be the real determinants of the alacrity with which individuals embraced.<sup>12</sup>

Zvi's defense was spirited, but, his rebuttals adroitly sought to blunt the force of these criticisms, rather than counter-attacking. Essentially, he pointed out that such considerations affected both the reality and the perception of the innovation's profitability; to portray their influence as distinct from that of profitability was therefore a "false dichotomy." On the one hand, this was something of a retreat from the unqualified dismissal of the relevance of "sociological factors" in his 1957 text, while on the other hand, it made the tent of "profitability" bigger – so that everything could be taken in beneath it, and no real concession need be made to his critics. Both sides withdrew, and, among themselves the sociologists and the economists alike each declared victory.

#### *2.4 In the aftermath of the debate some questions remained*

Two points concerning this long-past controversy seem to justify my revisiting. First, their specific criticisms aside, the defenders of the rural sociological tradition at that point were thoroughly aligned with the same implicitly "pro-innovation" disposition that can be found in the studies of Griliches, and later Mansfield. They, too, viewed innovations as being universally superior in some important objective sense vis-à-vis the "traditional" techniques that were in use by the population under examination. Further, that superiority was taken to be established from the first moment of the innovation's introduction, and to persist thereafter. The issue in contention with the economists, therefore, was simply whether or not the index of that superiority was comparative profitability.

My second comment concerns the awkward implications that followed from acknowledging, as Zvi's rejoinder had done, that an innovation's profitability might be affected by its "congruence" and "compatibility" with other elements in the established farming regime. In other words, the comparison in the case under debate might not be properly characterized merely by looking at the costs and yields of

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<sup>12</sup> See, for example, Babcock (1962), Rogers and Havens (1962), and the review of this controversy – very much from the side of the sociologists – in Rogers (3rd Ed., 1983: pp. 32-34, 56, 214-215).

hybrid corn seeds and the open-pollinated alternatives. Hybridization in effect had created a more efficient pump for nutrients, and, as the agricultural extension officers of the day were explaining to farmers – albeit in different terms – this pump would not deliver what it could unless it first was properly set up. Chemical fertilizers would have to be supplied, and that meant fertilizer tanks would need to be purchased and installed; more water would be required to go along with the fertilizer, and that might mean digging new wells, or otherwise improving irrigation capacity. Providing more nutrients would heighten the problem of weeds, and so chemical or mechanical means would need to be introduced to suppress these competitors for the expensive nourishment that the hybrid plants were supposed to pump up. Even that was not the end of the matter. In addition to the direct financial costs of those fixed inputs, access to working capital would be critical when a wholesale switch was made to hybrid seed, because if bad weather or pests spoiled the harvest, the wherewithal to purchase new seed for following year would become a critical condition for the farm family's survival on the land.

Looked at from this angle, the “representative agent” version of the “profitability counts” story about hybrid corn appears rather too facile. Objective economic differences existed among the farms of the Midwest in this era, quite noticeably in regard to their current and their expected future corn acreage, the terms of their access to bank finance, their family labor supply situation, and also in the educational attainments of the farms' operators – which might well affect their capabilities to grasp and manage critical aspects of the new, more intricate system of cultivation.<sup>13</sup> Surely the heterogeneity of the population in these respects might be expected to show up in cost, realized yield, and farm revenue differences. Hence, by the very same argument that Zvi had used to deflect the criticisms from his sociological antagonists, the determinants of perceived profitability might well be said to govern the extent to which the innovation would be adopted within a farming community. But profitability was not simply a function of seed yields and prices that were essentially the same for everyone.

Supposing, then, that awareness of the requirements for commercially successful deployment of hybrid corn had become thoroughly disseminated as a result of the efforts of those agricultural extension officers, there could nonetheless be “rational non-adopters.” If that was the case, something else in the objective situation would have to change in order for there to be the further expansion in the

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<sup>13</sup> The significance of human capital intensity among the sources of the growth of U.S. farm productivity would emerge as a notable finding in Griliches' (1964), an aggregate production function study.

proportion of acreage under hybrid corn. That, at least, was the way it appeared to this city-boy when, without ever having set foot on an Iowa corn-farm, he starting to think about the determinants of the diffusion of grain harvesting in the antebellum Midwest, and came upon the debate that had gone on between Zvi and his critics.<sup>14</sup>

There was another bothersome matter — not unrelated to the one just noticed, but having to do with the neat empirical strategy that Zvi had devised. The family of diffusion paths exhibited in Figure 2 clearly is an artifact of the U.S. Department of Agriculture's statistical reporting practices: total corn acreage under cultivation, and acreage under hybrid corn were collected at the county level, aggregated and published for the states. Surely these political units had to be viewed as rather arbitrary aggregations from an economic standpoint; there were not even any apparent state-level farm policy issues that would render it of economic interest to concern ourselves with the dynamics of the diffusion of hybrid corn on a state-wide basis. But, what was the theoretically appropriate level of aggregation? Supposing that the data could be obtained on a county-by-county basis, would that be a more 'natural' population unit within which to examine the course of diffusion? Or should the county-level data be re-aggregated to form some economically distinct larger regions within which there could be said to be substantial homogeneity? Would such a thing be feasible, let alone appropriate for analytical purposes?

The answers were not obvious. It was not even clear that the idea of regional differences could be specified clearly, and if so, whether or not it should be independent of fixed features — such as climate, or soil types — that might have a bearing on micro-level adoption decisions. What did seem clear is that if all the state data for corn farmers was aggregated, it would exhibit an adoption path which was far more protracted than those shown in Figure 2. Further, because that diffusion curve would have a unique inception date, the slope parameter estimated from the logistic regression would need to describe the more protracted time path, and consequently would be smaller than that for many of the sub-regions. On the Griliches-Mansfield interpretation, however, differential profitability of hybrid corn would affect the slope coefficient of the logistic and consequently govern the speed

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<sup>14</sup> The influence of those doubts about the sufficiency of the 'contagion' model found its way into the approach taken in David (1966). But, as that publication was meant to be a contribution to economic history — in a festschrift for Alexander Gerschenkron, the advisor of my yet unfinished doctoral dissertation — and not about diffusion theory, its pages contained no explicit references to Griliches' study of hybrid corn and the controversy it had ignited. Such matters would wait until my incipient heterodoxy could be formalized for a different audience, in David (1969).

of the contagion process; hence, for the larger aggregate it would have to be said that the differential profitability of the innovation for farmers was weaker in the aggregate (and *a fortiori* weaker in the late adoption regions) than was the case in the early adopting sub-regions.

But Zvi had provided a different explanation for the separation between early- and late-adopters: it was supposed to have stemmed from the differences in the *innovation-suppliers'* expected profitability, which gave rise to the sequence of introduction dates. Looking at the process from the aggregate level, it now seemed that the heterogeneity of corn-growing conditions across the U.S. played a role in the diffusion process that was not acknowledged when Griliches focused his empirical analysis at the state level. Yet, if this was a valid conclusion, might it not also be one that would hold in regard to the diffusion process at the state level? And then, why not also at the county-level, and below that?

All this was still rather inchoate in my thinking in the mid-1960's, when I began trying to understand the determinants of the timing and extent of the mechanical reaper's adoption by farmers in the Midwest during the era before the Civil War. What resulted, in the paper (David 1966) contributed to the festschrift for Alexander Gerschenkron, was a story about a 'moving equilibrium' at the micro-level, in which farmers with more acreage under small-grain (i.e., wheat, oats, rye and barely—-but mainly wheat in the relevant parts of the Midwest) were first to adopt. But, for others (who I supposed were no less rational and intent on cost-cutting) to follow the leaders it was not sufficient that news about the wonderful McCormick reaping machine was percolating throughout the region. The relative wage-rental rate had to fall, so that it would become profitable for farmers with smaller acreages under wheat to invest in the purchase of that labor-saving and fixed-capital using innovation; and the McCormick original 1834 design of the reaper itself had to undergo a significant improvement, as was recorded by McCormick's 1845 Patent.

At the time, this was a distinctly heterodox way to be thinking about diffusion, fitting neither the rural sociologist's nor the economists' paradigm. Summing up the latter, the phenomenon of lagged adoption response to the stimulus of a novel product or process was presented (and almost universally accepted) as a transitional *disequilibrium* process. The objective circumstances of all potential adopters were more or less the same, as were the net benefits offered to them by the innovation. Although the extent of adoption was constrained at every moment by ignorance on the part of some portion of the potential adopter

population, over time the process was supposed to be driven forward by the inevitable contagious spread of information about its comparative profitability.

### *2.5 The multiplying models of logistic diffusion*

Nevertheless, Mansfield's (1961) invocation of a random contact process of contagion was only one among the multiplicity of interpretations that might equally have been attached to the phenomenon of logistic diffusion. At the reduced form level at which econometric work in this area was conducted, most of the alternative formulations were observationally equivalent. The following paragraphs demonstrate this explicitly without attempting to be exhaustive.

#### *2.5a: An 'evolutionary' economic interpretation?*

Consider, for example, a variant that has a distinctly evolutionary flavor; not surprisingly, perhaps, as it has its roots in the mathematics of population genetics. Lucca Cavalli-Sforza and Marc Feldman (1981) provide a genetic model based upon random population inter-mixing, in which the proportion of the population (again, we may label it  $P$ ) carrying the mutant trait evolves according to a logistic function of time. They show that the slope coefficient of the logistic (again, call it  $\varphi$ ) in this case is simply  $\varphi = \ln(k_m / k_o)$ , where the ratio  $k_m / k_o$  measures the Darwinian fitness of the mutant gene relative to the old gene. Translating this into more familiar terms, we could say that the slope coefficient is a log-transform of the 'fitness' measure of the innovation's advantage vis-à-vis the established (cultural) trait.

To go from this metaphor to a formal "evolutionary economics"-style model of diffusion some further bits must be added, making the replicator dynamics explicit. This is not so hard: suppose that the innovation (mutant cultural trait) is a production method that reduces unit production costs for the adopting firms, and that the latter are operating in a competitive market. Next, suppose the rate of capacity growth via investment in the facilities required by the innovation is equal to the profit rate.<sup>15</sup> Since the firms enjoying the lower unit costs will get more profit per unit of capacity, and hence will do proportionately more investment, the capacity of the firms adopting the innovation must grow relative to that of the non-innovating remnant by  $k_m / k_o$ . The result will be a logistic path for the proportion of (full capacity utilization) output that is produced with the new technique. From this angle, Zvi Griliches' intuitive identification of the slope parameter of the logistic

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<sup>15</sup>If we are entertaining evolution, why not also have a Cambridge-Pasinetti style theory of savings, in which the capitalists save everything and the workers nothing?



with some measure of the relative profitability of the innovation in question might entitle him to further esteem – surprisingly in this case, as a pioneering evolutionary economist in spite of himself!<sup>16</sup>

The ‘evolutionary economics’ overtones of the foregoing sketch- model notwithstanding, it follows the work of Griliches and Mansfield faithfully in its assumption that the coefficient of relative fitness for the innovation (i.e., the new ‘cultural trait’ in question) is inherent in the innovation itself. An obvious justification for this supposition is to dismiss as inconsequential the possible variations of the environments encountered by those who acquire the trait. More complex models of population genetics subsequently have abandoned that simplification; they allow for both the heterogeneity of environments and the dynamic transformation of the latter as a consequence of the diffusion of the new (behavioral) trait within the population.

In a sense, research on the microeconomics of the diffusion of innovations began to move in the same direction as that taken by the population genetics modelers (although the evolutionary parallels were not consciously noted at the time). The impetus for that development derived from acknowledgment of the implications of heterogeneities in the adopter population, and biases in the properties of innovations. Together, these empirical realities posed a challenge to the casual assumption that innovations were universally dominant vis-à-vis pre-existing technologies. They therefore pointed to the possibility that diffusion lags were not necessarily explained by incomplete information. This generated a new family of formal economic models of diffusion; and they, in turn, raised questions about the rationale of policy programs designed to promote technology adoption by providing demonstration programs and identifying efficient channels for the propagation of information about the innovation in question.<sup>17</sup>

The approach finding its way into corners of the economics literature focused less upon information contagions, and more upon the implications of population heterogeneities, combined with fixed costs of adoption and variable input-saving biases in process innovations. It allowed for the possibility that expected scale of

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<sup>16</sup> This Molière-like denouement should not obscure the credit for the replicator dynamic in this formulation, which was introduced much later by Nelson and Winter (1982), who created a stochastic version of a system with this structure and examined its path of adjustment (through selection) in response to the recurrent emergence of mutations characterized by varying degrees of “relative fitness.”

<sup>17</sup> See e.g., Rogers and Shoemaker (1971); Rogers and Kincaid (1981) for programs of that genre, and the implicit critique by Stoneman and David (1986).

operations might enter into investment decisions involving choices between new and old techniques, such, that given the relative prices of the fixed and variable inputs, there would be a “threshold” output scale below which adoption would not occur, so long as the decision agents were myopic cost-minimizers.

### *2.5b Moving equilibrium: logistic diffusion with the threshold model*

My initial and subsequent contribution to the modeling of diffusion innovations introduced and generalized that approach for the case of process innovations. But, this hardly is the occasion on which to review the details of all those papers.<sup>18</sup> Rather, the point in bringing up the matter here is simply to underscore the previous assertion that there are many different models that will account for the phenomenon characterized of logistic and logistic-like diffusion at the macro-level. The class of so-called ‘threshold models’ – in which a variety of formulations is subsumed – can perform that trick without having any recourse to imperfections in the information states of the agents. Furthermore, their simpler formulations suppose that adopters and non-adopters alike at each moment are in profit-maximizing (or, at least, cost-minimizing) equilibrium. The essence of the approach is to view the diffusion path itself as *a moving equilibrium*, the dynamics of which can have exogenous or endogenous drivers, or both.

Consequently, I found it quite striking many decades later to read that in the interview conducted by Krueger and Taylor (2000: p.181) Zvi had ascribed the economics profession’s failure to further develop his early work of the adoption innovation to economists’ over-riding preoccupation with models of equilibrium:

“We never have had a good theory of transitions. And the field, by and large, moved toward an interpretation where everything was in equilibrium, all the time. So the diffusion story, as such didn’t seem like the model people wanted to develop....[M]ost of the economy is quite far away from the boundaries of the current state of knowledge. Some of it is because it is equilibrium—it’s not profitable at the existing cost structures. But some of it is because it’s new and it hasn’t been fully developed yet. It’s in the process of being adopted.”

One might phrase essentially the same insight somewhat differently: the profession’s predilection for modeling the behavior of agents in equilibrium terms

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<sup>18</sup> See David (1969, 1986, 1991, 1997); David and Olsen (1984, 1986, 1992). I do not review here to a separate line of my publications that are concerned with models of inter-innovation rivalry, particularly those driven by network externalities.

posed an obstacle to explaining macro-transitions simply in terms micro-level disequilibrium. Models of temporally extended diffusion processes that allow feedback from the process itself to provide dynamic drivers for a moving equilibrium therefore could restore the conceptualization of the macro-phenomenon as a transition. But the cost to the neoclassical world view of pursuing the latter interpretation was the acceptance of externalities (such as various forms of “learning”) as a vital source of the system’s dynamics.

In order to fix ideas for subsequent use in the remainder of this Part, and in Part 3, we may start with the basic formalization of the “threshold” model of technology selection, and develop a simple specification that allows this model to mimic the performance of the very different contagion model — by generating a logistic diffusion path. It is best at this stage not to burden the exposition with mathematical formalism, so the notation here is kept to a minimum — even at the cost of leaving some loose ends that can be tidied up afterward.

The basic notion of an adoption “threshold” is that there is a variate  $z$  that enters the discrete choice problem of individual agent  $i$  who is characterized by the value  $z^i$ , such that the agent will select the novel option — the innovation — over others when  $(z^i) \leq z^*$ . Thus,  $z^*$  is implicitly defined as the “threshold adoption level” of the key variate.<sup>19</sup> Let us assume that the technology choice is for all intents and purposes irreversible, perhaps because the decision to adopt the innovation entails acquisition of a highly durable physical asset.

If we then suppose that the critical variate  $z$  has a continuous frequency density function in the population of potential adopters,  $f(z)$ , the proportion of the population among which the condition for adopting the innovation is not fulfilled will be just the value of the corresponding (stationary) cumulative density function,  $F(z^*)$ . Therefore, we have as a measure of the extent of diffusion the proportion of the population that should adopt:

$$D(z^*) = 1 - F(z^*) .$$

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<sup>19</sup> The roots of this particular formulation lay in a simple characterization of the choice between an innovative piece of farm machinery, the horse-drawn grain-reaper, which saved harvest labor but entailed extra fixed capital costs for the farmer (see David 1966). Given the indivisibility of the machine and the difficulties (during the era under study) attending commercial rental of its services, the prevailing wage-rental rate would define a break-even scale of production, above which the purchase of the machine would be profitable. Here  $z^*$  is the break-even output scale implied by the wage-rental ratio.

Under the assumption that  $F(z)$  is stationary,  $D(z^*)$  can increase if and only if  $z^*$  becomes smaller. In other words, the threshold point has to pass downwards through the  $z$ -distribution.

If we know the shape of  $F(z)$ , and can characterize the dynamics of  $z^*(t) = g(t)$ , it is straightforward to deduce how the latter's motion must re-map  $F(z^*) \Rightarrow F(t)$ , thereby generating a 'moving equilibrium' path for the diffusion index in the time domain,  $D(t)$ . It's really no more complicated than that!

All sorts of diffusion paths can be rationalized in terms of this basic framework. Putting it the other way 'round, by specifying  $f(z)$  and  $g(t)$  one may derive the shape of the diffusion path. Thus, let us posit that the  $z$ -distribution is the log-logistic, and that  $g(t)$  declines exponentially with time at the instantaneous rate  $\lambda$ , and see what happens. We now may write

$$D(z) = 1 - F(z^*) = \exp\{-\gamma(\ln z)\} [1 - D(z)] = (z)^{-\gamma} [1 - D(z)],$$

and

$$z^*(t) = g(t) = z^*(0) [\exp\{-\lambda t\}].$$

Upon finding  $\ln(z^*(t))$  and substituting this in the expression for  $D(z=z^*)$ , we immediately obtain a logistic function in  $t$ , the slope parameter of which now is revealed to be  $\phi = \gamma\lambda$ . This is readily confirmed by forming the resulting expression for the familiar log-odds ratio:

$$\ln\left\{\frac{D(t)}{[1-D(t)]}\right\} = (\gamma\lambda)t - \gamma\ln\{z^*(0)\}.$$

This model of logistic diffusion affords an interesting interpretation of the coefficient of  $t$  that may be estimated by linear regression methods; it reflects both the rate at which the threshold point is falling, and the shape of the underlying  $z$ -distribution. Given an extraneous estimate for  $z^*(0)$  – the value observed for the initial adopters – both of the model's structural parameters can be recovered from the intercept and slope coefficients of the regression.<sup>20</sup> It may be noticed that although profitability considerations obviously can enter into the marginal agent's

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<sup>20</sup> A word of caution is in order for those who would follow common econometric practice and estimate the log-odds equation by OLS methods. In this time-series relationship the problem of auto correlated disturbances suggests relying instead on minimum Chi-square estimators for the slope coefficient.

micro-level decision functions, the estimated parameter  $\varphi$  says nothing at all about the relative profitability of the innovation.

### *2.6 Adoption decisions when learning effects are anticipated*

The first serious generalization of the basic threshold modeling approach came in a still unpublished paper by David and Olsen (1984) that depicted technological in the form of “automation” as an extended dynamic process involving the diffusion of an interrelated sequence of minor innovations in production methods.<sup>21</sup> These incremental improvements are embodied in successive vintages of a new class of indivisible capital equipment (“machines”), the first vintage of which is treated as having been introduced by an exogenous discrete innovation. The equilibrium model of diffusion considers a learning process in the industry supplying the automation equipment, and derives explicit micro-level, decision criteria on which the demand for investment in additional machines of this type at each point in time will be based. These criteria are applied by the heterogeneous firms of the machine-using industry to determine their respective optimal dates of adoption. Quite obvious, this will generate the appearance of a distributed lag process of investment in the machine-using industry following the “shock” of the innovation’s introduction.

This model is was a particularization of the general observation that innovations seldom remain in their original form, and that improvements effected by the suppliers play an important part in widening the field of adoption. By the early 1980s abundant empirical evidence was accumulated concerning long-run “learning-by-doing” and “learning-by-using” with new forms of durable investment goods in agriculture, manufacturing, transportation, and communications. The ubiquitous nature of the latter phenomenon supports the plausibility of supposing that a major technological breakthrough would establish a potential for many subsequent incremental improvements whose cumulative effect upon production costs might well overshadow that of the initiating innovation.<sup>22</sup>

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<sup>21</sup> The exposition that follows draws upon David and Olson (1984), and the more compact presentation of essentially the same model published as David and Olson (1986).

<sup>22</sup> See, e.g., Enos (1962, 2001) on petroleum refining; Hollander (1965) on rayon; David (1975, chapters. 2, 3) on cotton textiles, and references to studies of steel, airframe- and ship- building; Leiberman (1984) on chemicals; Sahal (1981), especially on agricultural machinery. On producer-user interactions, see von Hippel (1978), Rosenberg (1982, ch. 6), and Lundvall (1984). Surveys of the empirical literature on learning curves are available in Yelle (1979) and Steinmueller (1985), the latter being the more comprehensive and making special reference to the manufacture of integrated

Endogenous generation of incremental innovations is conventionally represented as a learning process that results in a continuous reduction of the unit reproduction cost of “machines of a constant kind.” This reduction proceeds at a pace governed by the rate of accumulation of collective experience among the ensemble of firms engaged in the business of supplying those machines. Assuming conditions of perfect competition prevail in the latter industry, these real cost reductions equivalent to a relative decline in the *hedonic* price of the indivisible capital goods embodying the new technology.<sup>23</sup> Because the pace at which this kind of learning can proceed remains limited by the rate at which the new technology is being adopted, expectations about the future trajectory of incremental innovations and the continuation of diffusion process itself, in effect, become hostage to one another. Treating this complication due to the feedback from use-experience in the micro level model goes beyond the standard point that anticipations (or expectations) of continued technological innovation may affect adoption decisions.<sup>24</sup>

### *2.6(a) The generalized threshold model*

Consider an industry comprised of firms producing a homogeneous final output, denoted by  $X$ , and marketing it competitively at price  $p$ . Further, there is a capital-goods industry that supplies machinery used in the production of this final good under conditions of perfect competition. To simplify matters, we may assume that the remaining sector of the economy is large in relation to the former two, and its product is the numeraire of the system; machinery from the capital-goods industry is not used by this (residual) sector.

The machines embodying the innovation in this setup are taken to be supplied only as large and indivisible units of capacity, at the unit purchase cost  $k$ . Although only one unit of this automation equipment need be installed by any firm in order for it to acquire access to the latest production technology, in this general formulation one may allow the possibility that the acquiring firms are able to operate it over a wide range of output scales.

circuits. But, see also Cohen and Klepper (2001) on the role that purposive R&D – rather than experience-based learning – may play in the generation of incremental technical progress.

<sup>23</sup> Another connection may thus be noted – between studies of diffusion dynamics and the theory and application of hedonic prices, to which Griliches made pioneering contributions (see the assessment by Pakes (2003)).

<sup>24</sup> See Rosenberg’s (1976) discussion of the latter in a context where “learning effects” are kept in the background.

We may abstract from the possible effects of imperfect information and uncertainty upon the process of diffusion, and assume instead that all firms in the final goods industry have identical (costless) knowledge concerning the benefits and costs associated with use of the new production technology. The population of firms in the industry is taken to be fixed, implying that only firms already established when the new equipment first becomes available will have access to the innovation.<sup>25</sup> Rather than complicate the presentation by introducing realistic considerations such as the depreciability of capital goods, and the possibility of replacement of obsolescent equipment at some future date, one may make the following further assumptions: (a) all investment is irreversible, which is to say that there is no market for used machinery of any type; (b) capital equipment is infinitely durable; and (c) the recently introduced line of machines is the only major technological innovation relevant for the final goods-producing industry within the foreseeable future.<sup>26</sup>

### *2.6(b) Production technologies and the adopter's investment decision*

Each firm in the final goods sector must decide if and when to adopt the new technology, and will make this decision on the basis of the net present value of the investment represented by a newly installed automated plant. Suppose a firm has decided to invest at date  $T$ . It should then produce so as to maximize instantaneous profits at each point in time, using the old technology (which may be embodied in

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<sup>25</sup> This assumption is not a serious restriction upon the analysis, inasmuch as the extant firms are left free to vary their respective production scales; yet, it greatly simplifies matters, by equating the stock of the newest type of capital equipment (measured in standard machine units) with an index of the proportion of the firms that have adopted automation.

<sup>26</sup> Without radically altering the David-Olson (1984) model, the more glaring unrealism of simplifications (b) and (c) can be avoided, putting in their place the assumptions of stochastic depreciation of the "one-horse shay" variety, and stochastic technological obsolescence, both following exponential processes. Under the one horse shay assumption, depreciation occurs completely and instantaneously. It thus takes exactly the same form as technological obsolescence due to the sudden availability of a superior type of machine. If the stochastic processes governing these events yield exponential distributions of the depreciation and obsolescence dates, and if those distributions are independent, then the constant hazard rates for both events may be added to find the hazard rate for the termination of the benefit stream associated with a given piece of capital equipment. Assuming risk neutrality, the expected present value of the benefit stream may then be found simply by using the latter (constant) hazard rate as a "risk premium" added to the (riskless) time discount rate, leaving the analysis otherwise undisturbed. Ireland and Stoneman (1983), use this approach to model the effect of variations in obsolescence risks. The difficulty in treating physical depreciation the same way is that replacement demands break immediate correspondence between cumulative sales of the newest type of machine and the diffusion index.

existing capital goods) before date  $T$ , and the new technology thereafter. Assume that instantaneous net operating revenue functions are well defined for both technologies—at least over the range of output volumes considered here. For convenience the latter can be referred to as “profit functions”, remembering that the profits in this case are gross of fixed cost charges.

This specification of a profit function, denoted as  $R^i(\cdot)$  for the  $i$ -th technology, implies decreasing returns to scale in the utilization of variable inputs in the production processes. Let

$$R^i(p) = \max_x \{px - C^i(x)\} \quad i = 1, 2 \quad (2.1)$$

be the instantaneous profit functions for the old ( $i = 1$ ) and new ( $i = 2$ ) technologies, respectively, where  $C^i(\cdot)$  denotes the respective variable cost functions and  $p$  is the product price. Note that the firm’s optimal output  $x$  is now given for each technology by the derivative of the profit function:

$$x^i(p) = R_p^i(p) \quad i = 1, 2. \quad (2.2)$$

These cost functions remain stationary over time, by assumption. This implicitly imposes the simplifying assumption that factor input prices are time-stationary. For the sake of concreteness and convenience, let us refer to the new technology ( $i = 2$ ) as “computer automation.” Then, enhancements in the efficiency of automation equipment will be treated as equivalent reductions in the unit reproduction cost of “machines of a constant kind”, i.e., machines characterized by time-stationary variable costs of operation.

The automation technology can only be of interest to the firm if the profit difference

$$B(p) = R^2(p) - R^1(p) > 0 \quad (2.3)$$

for at least some range of future prices. This difference (the undiscounted gross benefit from adoption) is taken to be positive for all output prices considered here. It does no great violence to the engineering realities to posit also that this gross benefit function is increasing in  $p$ . The latter is equivalent to supposing that the marginal cost schedule for technology 2 lies everywhere below that for technology 1, and that the firm’s optimum supply (holding the market structure unaltered) thus is higher under the regime of automation. This last implication follows directly from



$$B_p(p) = x^2(p) - x^1(p) > 0. \quad (2.4)$$

As in all equilibrium models of diffusion *under perfect competition*, it is essential for this analysis that there be some objective, identifiable heterogeneity in the population of firms which results in the benefit function varying across the firms of the industry. (This proposition was demonstrated in David [1969]). Possible sources of heterogeneity will be mentioned shortly, but for the present it is sufficient simply to note that the profit function(s) will be indexed by a firm-specific characteristic,  $z$ .

Now consider the investment decision facing the firm of type  $z$  at date  $t$ , when the cost of installing a new, automated plant is  $k_t$ . The latter price is not indexed by  $z$ , since at any moment of time a uniform price prevails in the (competitive) market for capital equipment. The problem to be solved by the  $z$ -th firm therefore is to choose an adoption date  $T$  that will maximize the net present value function

$$V(T, z) = \int_T^{\infty} \{ B(p_t, z) e^{-rt} dt \} - k_T e^{-rT}, \quad (2.5)$$

in which  $r$  is the rate of time discount.

Assuming smooth price paths, the necessary first order condition is

$$V(T, z) \equiv -B(p_T, z) + r k_T - \dot{k}_T = 0. \quad (2.6)$$

This has a straightforward and familiar interpretation: the cost of marginally delaying the adoption (i.e., investment) date beyond  $T$  is the loss of instantaneous profits equal to  $B(\cdot)$ , whereas the marginal gain is the sum of the averted rental costs,  $rk$ , and the capital loss,  $-\dot{k}_T$ , that otherwise would be incurred from the instantaneous drop in the reproduction cost of the new, automated plant following date  $T$ .

Additional sufficient conditions for date  $T$  to be optimal for the  $z$ -th firm are plain enough:

$$v(t, z) > 0 \text{ or } < 0 \text{ for } t > \text{ or } < T \quad (2.7a)$$

$$v(T, z) \geq 0. \quad (2.7b)$$

This formulation points clearly to the potentially important role of expectations regarding the future real costs of the capital goods embodying the novel technology. Where incremental improvements can be anticipated, arising quite possibly as feedback from the diffusion process itself, further adoptions will depend upon the balance between the anticipated and the realized fall in the fixed input price. This formulation explicitly recognizes the role of developments on the supply side of the market for the embodied innovation, and in that respect bears a kinship with Griliches' original analyses of the hybrid corn case. But, there is a significant difference. In the latter situation, the sub-markets were spatially as well as temporally separated. Anticipations of eventual "improvements" in seed-quality would not exert the same delaying efforts, because they would not be pertinent to the conditions of the farmers in the initial regions where the innovation was introduced.

### *2.6(c) Heterogeneity and the "z-distribution" of firms*

If firms were identical, they all would choose the same adoption date, and at that date new plants would go into production as rapidly as they could be installed. Insofar as any diffusion path was observable, it would only reflect the sequence of temporary, "rationing equilibria" in the market for automation equipment. To create the possibility of market-clearing equilibrium diffusion paths in the present model, David and Olsen (1984) make a crucial assumption: firms in the final goods sector can be ordered according to a single parameter, or index,  $z$ , such that the gross profit difference  $B(\cdot)$  is a continuous and monotonically decreasing function of  $z$ :

$$B_z(p, z) < 0, \text{ for all prices } p \text{ in the relevant range.} \quad (2.8)$$

One possible interpretation of the  $z$ -parameter is that it indexes an intangible attribute affecting costs, such as managerial efficiency.<sup>27</sup> But  $z$  also may be taken to represent inter-firm differences in more objective, and directly verifiable conditions impinging upon operating profits, such as transport costs differences affecting the f.o.b. prices of their final product.<sup>28</sup> The firm-specific variate  $z$  also may be interpreted as the price of one of the inputs used in the firm's production process,

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<sup>27</sup> In this case we should call it "z-efficiency", by analogy with Harvey Leibenstein's famous "x-efficiency".

<sup>28</sup> The differential transport cost interpretation of  $z$  appears to be quite germane in discussions of the diffusion of automated assembly technology, in view of the necessity of concentrating production in plants that will be intensively utilized through multiple shift-working.

thereby permitting recognition of factor market imperfections as a source of the heterogeneity among the population of potential adopters.<sup>29</sup>

It is of course a drastic simplification to suppose that firms can be thus ordered along a uni-dimensional scale. But while working with multivariate distributions is straightforward conceptually, it requires further specification of the covariances among the several sources of heterogeneity, and these soon begin to clutter up the analysis.

Finally, it is worth remarking that the  $z$ -parameter would turn out to be correlated with the scale of output under each technological regime. Hence, there could be positive rank correlations between the order of adoption among firms and their *ex ante* or *ex post* (output) size, just as in the scale-constrained models presented by numerous empirical studies.<sup>30</sup> Indeed, a closer examination of the results obtained in Mansfield's (1961, 1968) studies of the diffusion of industrial innovations suggests that the information-contagion rationale offered for his econometric specification notwithstanding, the statistically significant "profitability effects" on the rate of adoption that he reported have altogether different underlying causes. The significant effects in this statistical explanation of the estimated logistic slope coefficients were generated by the subset of industry cases where the innovations in question were fixed-capital using, and Mansfield's index of firm characteristics included measures that were in all likelihood positively correlated with differences in expected output scale.

In the model just presented, however,  $z$  is a more fundamental source of heterogeneity and *explains* observable differences in output scale. Moreover, the factor use bias of technological change may switch between one major wave of innovations and the next, so that the firms which enjoyed input-price advantages causing them to be largest in scale of output under the old technology would not necessarily be first to adopt the new.<sup>31</sup>

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<sup>29</sup> For this interpretation, note that when  $z$  is a factor's price, Hotelling's lemma tells us that  $B_z(p, z) = R_{z2}(p, z) - R_{z1}(p, z) = -[L_2(p, z) - L_1(p, z)]$ , where  $L_i(p, z)$  are the input demand functions for that particular factor under the alternative technologies. Thus, if the new technology results in the firm expanding its demand for the factor at the prevailing price  $z$  — at least over the relevant range of output price  $p$  — condition (A8) will indeed be satisfied.

<sup>30</sup> David (1966, 1972), Sargen (1979), Davies (1979), Stoneman and Ireland (1983) and Whatley (1983).

<sup>31</sup> Instead, the size ordering of firms in the industry may undergo non-monotonic transformation in the course of the diffusion process. To appreciate this, one would have to look more closely at the product market equilibrium conditions.

Under the more general conditions David and Olsen (1984, 1986 and 1992) derived for the existence of an rational foresight equilibrium diffusion path, the time-profile of the proportion of firms that have already installed equipment of the new type—the measure of the extent of diffusion—may exhibit the classic ogive, or S-shape. Nonetheless, there are circumstances in which the diffusion curve would be concave over its entire range.

With this class of models, complete diffusion is by no means a necessary, foreordained outcome that resembles the gradual but inevitable filling up of a bottle. There are solutions (dynamic equilibria) in which the diffusion and learning will get under way but be brought to a stop short of universal adoption. In addition, and of possibly greater interest, is the result that in some conjunctions of initial supply-side and demand-side conditions, the start of a diffusion-cum-learning process driven by perfect competition may remain blocked. This can be the case even where the positive-feedback process driven by learning effects could take over once the level of adoption and capital goods prices had been brought to a critical “take-off” point, presumably by non-market interventions. More generally still, under full-employment conditions, optimum social management of a new technology’s adoption in the presence of learning externalities may call for faster diffusion than would occur even with complete information (perfect foresight) and perfect competition prevailing in all the relevant markets.<sup>32</sup>

Complications of this nature begin to take on greater economic policy significance when one turns, as I do now, to consider the connection between the microeconomics of technology adoption and the macro- industry-level course of productivity growth.

### PART 3

#### **Diffusion and Productivity Growth: From Micro to Macro**

A portion of the comparative neglect of empirical research on the microeconomics of technology choice and innovations’ adoption histories may be attributed to the fact that the explicit connection between diffusion and productivity growth has not been developed formally. As I have pointed out elsewhere (David 1986; and also David & Foray 1995), the political economy of growth policy has

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<sup>32</sup> The implications of this latter point for patent policy as a second-best public strategy are examined in David and Olsen (1992).

promoted excessive attention to innovation as a determinant of technological change and productivity growth, to the neglect of attention to the role of conditions affecting access to knowledge of innovation and their adoption. The theoretical framework of aggregate production function analysis, whether in its early formulation or in the more recent genre of endogenous growth models, has simply reinforced that.

In this third Part I want to try to do what Zvi might have done, had he come back to his early interest in the diffusion of process innovations rather than working on the relationship between R&D efforts, patenting and their relationship of productivity growth: look explicitly at the connections between productivity growth and diffusion dynamics.<sup>33</sup>

### *3.1 Diffusion dynamics and productivity growth: a heuristic model*

A simple model serves to show the direct and indirect effects of the diffusion of a “radical” or “fundamental” process-innovation upon the measured growth of input productivity. The technical details establish some interesting and little recognized points of a rather general nature, concerning the relationship between the pace of productivity growth and the pace at which productivity-enhancing innovations are adopted.

But, perhaps the main value of this exercise is to re-focus greater attention upon the determinants of the dynamics of diffusion as the proximate factors governing the rate of aggregate productivity growth. There may be some service simply in resurrecting these ideas, with one or two new wrinkles that render their implications easier to grasp.

The model presented here envisages a discrete innovation that results in lower labor input requirements per unit of output, compared with a pre-existing technology. Hence, the level of average labor productivity in the industry, sector or economy into which it is introduced will be determined as the weighted average of

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<sup>33</sup> This is not the first time I have explored this, as will be recognized by the handful of those who actually studied the technical appendix to my OECD paper on “Computer and Dynamo.” The latter, David (1991), is a contribution far easier to cite than to read, as, in addition to being lengthy, it was from the outset hard to find in the publication *Technology and Productivity* (1991) – a bulky OECD volume that now is out of print (offprints may be obtained by writing to the author or contacting the Stanford Institute for Economic Policy Research at :<http://siepr.stanford.edu>, or (650) 725-1874, contact D. Baldwin and ask for the reprint.

the labor productivity levels characteristic of the new and old technologies, the weights being given by the extent of the innovation's diffusion. By "direct effect" is meant the impact upon the aggregate level of productivity of a redistribution of production from the old to the new-style process, the latter being more efficient in its use of inputs. By "indirect effects" are meant the whole range of (positive feedback) consequences that more widespread use of the new technology has upon its relative level of productivity — vis-à-vis the old technology — in all applications.

For simplicity, the main relationships posited here are of a reduced form nature; they do not explicitly exhibit the microeconomic conditions governing decisions by producers to adopt the new technology, nor the decisions by suppliers of the new process-equipment to make available enhancements, nor the ways in which users acquire greater proficiency in application of the new technology. They are consistent, nonetheless, with a fully specified model of that kind. Consequently, the model showing how the rate of diffusion (and the extent of diffusion at a specific point in time) will be related to the aggregate productivity growth rate does not exhibit the complex interdependence that would exist between the pace of the new technology's diffusion and the rate of (endogenous) improvements stemming from experience with the new technology. On the other hand, were there other sources of change affecting user-costs of the new technology, in addition to experience-based improvements in input efficiency, it is quite plausible that the specifications employed here present a consistent picture of the aggregate productivity impacts of the diffusion process *per se*.

### 3.2 The general model of the labor productivity growth rate

The following notation refers to an industry, sector or economy producing a homogeneous output,  $V$ :

$\pi_j(t)$ : is output per unit of labor input using the  $j$ -th technique at time  $t$ ,  
 where  $j = 0$  represents the "old" technique and  $j = N$ , the "new" technique;  
 $\pi_N(t) \geq \pi_0(t)$  for all  $t$ .

$D(t)$ : is the proportion of aggregate output produced using technique  $N$ , at time  $t$ ;

$\pi(t)$ : is aggregate labor productivity at time  $t$ ;

$\dot{\pi}(t) \equiv \partial \ln \pi(t) / \partial t$  is the (proportional) rate of change of the variable  $\pi(t)$ ;

$$\pi(t) = [(1 - D(t)) / \pi_0(t) / \pi_N(t)]^{-1} . \quad (3.1)$$

*Assumption 1:*  $\pi_0(t) = \pi_0$  for all t.

This holds that the old technology undergoes no improvement or deterioration in its (fixed) unit labor input requirements. For simplicity we shall suppose the old technique uses only labor, so that  $\pi_0$  cannot be affected by factor substitution.

*Assumption 2:*  $\pi_N(t) = \pi_N \{D(t)\}, \frac{\partial \pi_N}{\partial D} > 0, \frac{\partial^2 \pi_N}{\partial D^2} < 0.$

This posits an “improvement function” for  $\pi_N$ , s.t. labor productivity with the new technique will increase as the process becomes more widely diffused, although such incremental enhancements predicated upon diffusion experience will be subject to diminishing marginal returns.

The general expression for the growth rate of labor productivity,  $\dot{\pi}$ , in terms of  $D(t)$  is found by first rewriting (3.1) as

$$\pi(t) = \left( \frac{\pi_0}{1 - [\beta(t)]D(t)} \right), \quad (3.2)$$

defining  $\beta(t) \equiv \left[ 1 - \frac{\pi_0}{\pi_N(t)} \right].$

Then, differentiating (3.2) with respect to t and multiplying through by  $[\pi(t)]^{-1}$ , we obtain:

$$\dot{\pi}_1(t) = \left( \frac{d\pi}{dt} \right) \frac{1}{\pi} = \left[ \frac{\beta(t)}{1 - [\beta(t)]D(t)} \right] \frac{dD(t)}{dt} + \left[ \frac{[1 - \beta(t)]\varepsilon(t)}{1 - [\beta(t)]D(t)} \right] \frac{dD(t)}{dt}, \quad (3.3)$$

defining  $\varepsilon(t) \equiv \frac{\partial \pi_N(t)}{\partial D(t)} \cdot \frac{D(t)}{\pi_N(t)}.$

In equation (3.3) the first term on the RHS gives the direct effect of diffusion, which is the total effect in the simplest case where neither the new nor the old technologies undergo any change in their respective unit labor input requirements, i.e. where  $\varepsilon(t) = 0$ , and  $\pi_N(t) = \pi_0$  for all t. The second item on the RHS, obviously, gives the indirect effect of a change in the extent of diffusion upon  $\dot{\pi}$  — via the induced incremental improvement of the new technique’s productivity in all uses.

### 3.3 The impact of diffusion in the absence of indirect learning effects

Imposing the restrictions  $\epsilon(t) = 0$  and  $\pi_N(t) = \pi_N(o)$ , so that  $\beta(t) = \beta > 0$  for all  $t$ , we obtain from (3.3) the expression for the labor productivity growth rate where only the direct effect of diffusion is operating:

$$\dot{\pi}_1(t) = \left( \frac{\beta}{1 - \beta D(t)} \right) \frac{dD(t)}{dt}, \quad \beta > 0. \quad (3.4)$$

From this,  $\dot{\pi}_1$  evidently is not simply proportional to the change in the extent of diffusion ( $dD$ ), and so does not reach a maximum when  $dD/dt$  reaches its maximum. This is readily shown by differentiating  $\dot{\pi}_1(t)$  with respect to time, whence we obtain

$$\frac{d\dot{\pi}_1(t)}{dt} = \left( \frac{\beta}{1 - \beta D(t)} \right) \left( \frac{d^2 D}{dt^2} \right) + \left( \dot{\pi}_1 \right)^2, \quad (3.5)$$

from which it follows that

$$\text{at } \max \frac{dD}{dt}, \quad \frac{d^2 D}{dt^2} \rightarrow 0, \quad \text{and} \quad \frac{d\dot{\pi}_1(t)}{dt} \rightarrow \left( \dot{\pi}_1 \right)^2 > 0.$$

For the typical case,  $[\max(dD)]$  occurs in the interval  $(0,1)$ , which implies that  $\left[ \dot{\pi}_1 | \max(dD) \right]$  cannot be at a maximum. Since the term in brackets ( ) on the RHS of equation (3.4) is increasing monotonically in  $D(t)$ , the  $\max \{ \dot{\pi}_1(t) \}$ -point will occur at a time after  $\max(dD)$  occurs.

### 3.4 Model specifications and the total factor productivity residual

For eventual computational convenience, we can make the following assumptions in specifying the model:

*Assumption 3(a):* We suppose that the stationary underlying distribution of the critical variate  $z$  in the population of potential adopters has a log-logistic distribution; the threshold value for agents to select the new technique is  $z^*(t)$  at time  $t$ , and declines at the exponential rate  $\lambda$ .

*Assumption 3(b):* For heuristic convenience we assume, further, that the new technique is embodied in a fixed discrete input-bundle, only one unit of which is



acquired by each adopting agent. Firms working with a unit of the innovative technology all will have identical and constant flow output capacity  $k_N$ , whereas non-adopting firms will have constant flow output capacity  $k_o$ .<sup>34</sup>

This pair of assumptions leads immediately to two results that are useful in simplifying the following exposition. From Assumption 3(a) and the derivation in the preceding sections we may state that an index of the extent of diffusion at time  $t$ ,  $D(t)$ , defined as proportion of the population that has adopted the innovation, will be a logistic function in the  $t$ -domain, with asymptotic saturation at  $D(\infty)=1$ . This implies the following expressions for the level, the absolute and the proportional changes in  $D(t)$ .

First, we have the form already familiar from the derivation in section 2.4:

$$D(t) = (1 + \{z^*(0)\}^\gamma e^{-(\lambda\gamma)t})^{-1}, \quad \lambda > 0, \gamma > 0; \quad (3.6)$$

where  $\{z^*(0)\}^\gamma = \Phi$  is a constant reflecting the initial position of the threshold variable in the  $z$ -distribution at  $t = 0$ .

Consequently, for all  $\gamma\lambda > 0$ , we obtain

$$\frac{dD(t)}{dt} = (\gamma\lambda)[1 - D(t)]D(t); \quad (3.7a)$$

$$\dot{D}(t) = (\gamma\lambda)[1 - D(t)]. \quad (3.7b)$$

*Assumption 4:* The endogenous “improvement function” for the new technology is characterized by a constant (less-than unitary) elasticity of response to the increased extent of diffusion. This specification is expressed by:

$$\pi_N(t) = [\pi_N(o)] \left[ \frac{\{D(t)\}^\theta}{\kappa} \right], \quad 0 < \theta < 1, \quad (3.8)$$

where  $\kappa$  is an arbitrary normalization constant.

It may be remarked that equation (3.8) gives the “improvement function” the classic learning curve or “progress function” form suggested by Hirsch (1952) and

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<sup>34</sup> For present purposes, the relationship between the two per period capacity rates need not be restricted. What will matter is the two techniques’ relative intensities of fixed vs. variable inputs, and their relative fixed input-output coefficients.

Arrow (1962) – when we interpret the extent of diffusion as an index of experience gained with the new technology. Such an interpretation would be straightforward enough when the innovation came embodied in infinitely durable machines of a fixed capacity. As was noted above, the proportion of output represented by the capacity of the new machine stock then would vary directly with the cumulated output of the industry supplying such equipment, and also with the cumulated volume of gross investment represented by those machines. The interpretation of the reduced form improvement function is meant, however, to be more general and more comprehensive than the usual learning-by-doing and learning-by-using formulations.

The foregoing specifications, in conjunction with equation (3.3), lead to the following simulation equations for the direct and indirect effects combined:

$$\dot{\pi}_2(t) = \left( \frac{\beta(t)(1-\theta) + \theta}{1 - [\beta(t)]D(t)} \right) (D(t)[1 - D(t)](\gamma\lambda)), \quad (3.9)$$

$$\beta(t) = 1 - \alpha[D(t)]^{-\theta},$$

where  $\alpha = \frac{\pi_0}{\pi_N(o)}(\kappa).$

In the special case in which there are no “learning effects”, i.e.  $\epsilon(t) = \theta = 0$ , the simulation equations reduce to:

$$\dot{\pi}_1(t) = \left( \frac{[1 - \pi_0 / \pi_N(0)]}{1 - [1 - \pi_0 / \pi_N(0)]D(t)} \right) [D(t)[1 - D(t)](\gamma\lambda). \quad (3.10)$$

The expressions in equations (3.9) and (3.10) are quadratic in form, leading us to anticipate that the monotonic rise of the logistic diffusion curve generates a single-peaked wave in the growth rate of labor productivity. This may be seen directly from Figure 3, which graphs three alternative diffusion paths on the left-side of the panel, and shows on the right-side the corresponding waves that are induced in the growth rate of labor productivity.<sup>35</sup> From the positive value of the parameter  $\theta$

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<sup>35</sup> From the top panel of Figure 4 (below) it also may be seen that although the behavior of the average labor productivity growth rate is non-monotonic, the underlying diffusion rate (i.e., the proportional growth of  $D$ , is undergoing continuous retardation along the logistic path. A good bit of surprise, and some confusion on this point stems from the casual supposition that the rate of productivity growth

that appears in the notes beneath Figure 3, it can be seen that these growth rate simulations are based on equation (3.9), which allows learning effects with the new technology to affect the growth of aggregate labor productivity.

[Figure 3 here]

Figure 3 exhibits a second point upon which Part 2 remarked: the influence upon diffusion dynamics, and hence upon aggregate productivity growth in the industry, of the shape of the distribution of underlying population heterogeneity. Other things being equal, the lower the value of the logistic parameter  $\gamma$ , the greater is the variance (and the lower is the Kurtosis) of the frequency distribution of the population characteristic ( $z$ ) that enters the micro-level choice of technique decisions. Thus, with the “break-even” assumed to be falling exponentially at the same (fast) rate in all three situations, it is seen that lower values of  $\gamma$  stretch out the diffusion process, lower the productivity growth profiles and displace the (attenuated) peak substantially into the future.<sup>36</sup>

Two implications follow immediately from this. First, one is only seeing half the picture by focusing on the determinants of the pace at which the threshold point  $z^*(t)$  is pushed downward through the  $z$ -distribution. Putting this more concretely, the essentially neoclassical factor-substitution story that economists like to tell about the way that a new form of capital raises aggregate capital-intensity and thereby raises labor productivity, more often than not is an inadequate “representative agent” tale. All the emphasis is placed on the forces causing the falling real user-costs of fixed capital inputs – such as computer equipment—as the determinant of the growth rate of labor productivity. But if the  $z$ -distribution differed from one sector of the economy to the next, there would be quite different patterns of diffusion and correspondingly different labor productivity performance – for which the hypothesized representative agent “model” would have, at best, only *ad hoc* explanations.<sup>37</sup>

should reflect immediately reflect the rate of diffusion, whereas it is the absolute rate of change in  $D$  that matters.

<sup>36</sup> Note that the absolute values of  $\gamma$  and  $\lambda$  used in this simulation are rather arbitrary; the same results could be obtained if the annual rate of decline in  $\lambda$  were taken to be half as fast – approximating the 15% per annum trend in the hedonic prices of computer and communications equipment – if the underlying heterogeneity distribution was half as spread out (i.e.,  $\gamma = \{0.6, 0.9 \text{ and } 1.2\}$ ).

<sup>37</sup> Is the resemblance of this picture to the line of interpretation of the computer-revolution’s contribution to aggregate productivity that appears in the influential work of Jorgenson (2000) and his co-authors, purely coincidental?

Secondly, it is worth noticing that the measured pace of diffusion and the dynamics of productivity may well be affected by the alteration of the underlying  $z$ -distribution, as a result of the economic pressures emanating from the adoption of the innovation by some firms in the industry. It is quite conceivable that competitive pressures on the non-adopting remnant of the industry would force out firms at the low  $z$  end of the distribution, thereby tending to raise the parameter  $\gamma$  over the course of the process. The result would no longer be a strictly logistic diffusion path. To preserve the latter form, it would be necessary for the  $z$ -distribution to be transformed by a  $\gamma$ -preserving upward shift in its mean. Suppose that evolution of the first moment of  $z$  proceeded at a constant proportional rate,  $\mu$ . It is simple enough to show that the slope coefficient of the resulting logistic diffusion path would then become  $\{\gamma(\lambda+\mu)\}$ . Consequently, the working of competitive forces at the industry level can quite neatly be formally assimilated into this richer account of long-run productivity growth dynamics.

#### *3.4(a) Aggregation, diffusion and the productivity growth residual*

Explicit modeling of the microeconomics of diffusion decisions also may help shed some further and different light upon the sources of the “productivity residual.” Quite clearly this cannot be the whole picture, because the diffusion process resembles the evolutionary process of selection in being “a fire that consumes its own fuel.” When the new technique finds its way into all the available niches of use, the impetus imparted to productivity improvement is exhausted. Further progress will depend upon the generation of further innovations.

This transparent consideration certainly is sufficient warrant for the attention that Zvi Griliches’ empirical research program devoted to the nexus between firm-level R&D investment and multi-factor productivity growth. But perhaps something was lost by working at that low level of aggregation: it suppressed attention to the industry- and sector-level productivity effects that depended upon the diffusion of the novelties created in company laboratories, and by publicly funded research in universities and government mission-agencies.

We can follow the conventional Solow (1957) residual computation to find the total factor productivity growth measure or obtain the TFP growth rate as the share-weighted average of the average labor productivity and capital productivity growth rates:

$$\dot{A} = [\omega_L(t)] \dot{\pi}(t) + [1 - \omega_L(t)] (\dot{v}(t)) . \quad (3.11)$$

Given the expressions for the labor productivity growth rate in the foregoing section, we can derive corresponding expressions for the proportional growth rate of total factor productivity,  $\dot{A}$ , once we have expressions for the time rates of change of output per unit of capital input, denoted  $(\dot{v})$ ; and also for the share of labor in the aggregate output of the sector in question,  $\omega_L(t)$ . The latter is needed for the calculation in equation (3.11), on the assumption that competitive equilibrium in factor and product markets leads to the labor share being a close approximation to the elasticity of output with respect to labor inputs.

### 3.4(b) Labor's share

*Assumption 5:* The share of labor (elasticity of output with respect to labor input) characteristic of the new technology is a constant,  $\omega_N$ .

Since, as suggested by the remarks on Assumption 1, the share of labor in the old technology-sector of the economy is taken to be unity, we can write

$$\omega_L(t) = 1 - D(t)[1 - \omega_N] \quad (3.12)$$

### 3.4(c) The growth rate of capital productivity

The aggregate capital productivity growth rate obviously depends upon the rate of change in the extent of diffusion, and the level and changes occurring in the productivity of capital used in the new technology segment of the industry (or sector). Denoting the latter by  $v_n(t)$  the aggregate capital productivity is simply:

$$v(t) = (D(t)/v_N(t))^{-1} = v_N(t)/D(t) , \quad (3.13)$$

because no capital is used in the old technology sector (see Assumption 1, for discussion.)

From equation (3.13), by differentiation, and multiplication of both sides of the resulting expression by  $1/v(t)$ , we obtain:

$$\dot{v}(t) = \dot{v}_N(t) - \dot{D}(t) \quad (3.14)$$

There are two alternative special assumptions of interest in regard to  $\dot{v}_N(t)$ :

*Assumption 6a:* Improvements in the efficiency of the new technology due to endogenous, diffusion-dependent changes are Harrod-neutral, i.e. they raise  $\pi_N(t)$ , but, leave  $\dot{v}_N(t) = 0$  for all  $t$ . Thus  $v_N(t) = v_N(0)$  for all  $t$ .

Assumption 6a implies that:

$$\left[ \dot{v}(t) \mid \text{HaN} \right] = -\dot{D}(t) \quad (3.15)$$

Alternatively, we may consider:

*Assumption 6b:* Improvements in the efficiency of the new technology due to endogenous, diffusion-dependent changes are Hicks-neutral, i.e. they result in  $\dot{v}_N(t) = \pi_N(t)$  for all  $t$ .

Making use of eq. (3.8), Assumption 6b implies:

$$\left[ \dot{v}(t) \mid \text{HiN} \right] = \theta \left[ \dot{D}(t) \right] - \dot{D}(t) = -(1-\theta)\dot{D}(t) \quad (3.16)$$

### 3.5 Simulation equations for the TFP growth rate

Combining the results given by equations (3.11) and (3.15) and (3.16) alternatively, we find for the Harrod-neutrality and Hicks-neutrality cases, respectively:

$$\left[ \dot{A}(t) \mid \text{HaN} \right] = \left[ \dot{\pi}_2(t) \right] \omega_L(t) - [1 - \omega_L(t)] \dot{D}(t) \quad (3.17)$$

and

$$\left[ \dot{A}(t) | HiN \right] = \left[ \dot{\pi}_2(t) \right] \omega_L(t) - [1 - \omega_L(t)](1 - \theta) \dot{D}(t) \quad (3.18)$$

Substituting for  $\omega_L(t)$  from equation (3.12), for  $\dot{D}(t)$  from (3.7b), these expressions may be rewritten in the form:

$$\left[ \dot{A}(t) | HaN \right] = \dot{\pi}_2(t) [1 - (1 - \omega_N) D(t)] - (\gamma\lambda)(1 - \omega_N) D(t) [1 - D(t)], \quad (3.19)$$

and

$$\left[ \dot{A}(t) | HiN \right] = \left[ \dot{A}(t) | HaN \right] + \theta(\gamma\lambda)(1 - \omega_N) [1 - D(t)]. \quad (3.20)$$

The measured growth of rates of total factor productivity (TFP) or multi-factor productivity (MFP) growth rate for the economy (and of sectoral real value-added productivity growth for industries and sectors) are given alternatively by equations (3.19) or (3.20). The difference between the cases stems from the restriction of diffusion-driven learning effects to raising the efficiency of labor inputs alone, under the Harrod-neutrality assumption. Other things being equal, simulations of equation (3.20) produce TFP growth rates that lie everywhere above those from equation (3.19), since the second of the right-hand terms in the first of these equations is positive.

From equations (3.18) and (3.9), one readily can find the first-order condition for the peak TFP growth rate,  $d \left[ \dot{A}(t) | HiN \right] = 0$ . The positive value of  $D(t)^{**}$  which satisfies that condition is a function of the four parameters  $(\alpha, \theta, \lambda, \omega_N)$  and the normalizing constant,  $\kappa$ . Given  $D^{**}(t)$ , and the parameter  $\Phi$  defined in equation (3.6), it is straightforward to solve for a general expression giving the date  $t^{**}$  at which the peak growth rate of TFP occurs. The numerical simulations displayed in Figure 4, however, convey the essential points of the story rather more immediately.

[Figure 4 here]

The top panel in Figure 4 presents alternative diffusion paths generated by variant specifications regarding the rate of decline in the “break-even” threshold level  $z^*$ , and the corresponding time profiles of the proportionate rate of diffusion,

which is seen to undergo continuous retardation. The latter is more pronounced when the process is being driven by a comparatively fast decline in  $z^*(t)$ .

Simulation results for the growth rate of average labor productivity and the multi-factor productivity residual, appear on left- and right hand side of the lower panel, respectively. These calculations have been made under three alternative specifications regarding the strength of the effects of learning from diffusion experience on the incremental improvement of the new technology's relative efficiency. In the base case, condition  $\theta = 0$  signifies the absence of such learning effects. Under the assumption of a fast rate of decline in the threshold value  $z^*(t)$ , the inflection point of the diffusion path occurs at  $D = 0.5$ , indicated by the dotted vertical line at the  $t=30$  date. One can see that the peaks in the labor productivity growth rate are displaced to the right of that, the 'delay' being more pronounced the stronger are the endogenous learning effects.

The results show that the peak of the MFP growth rate is similarly displaced in time beyond the date of the inflection point of the diffusion path. This displacement is more pronounced than that observed in the case of the growth rate of labor productivity. The latter reflects the strong contributions of increasing fixed input (capital) deepening during the phase when the absolute changes in the extent of diffusion become large.

The alternative cases presented by Figure 4 display the time profile of the multi-factor productivity residual under the assumption that there are positive *Hicks-neutral* efficiency improvements in the new technology that proceed *pari-passu* with the widening of experience in the use of the new technology (i.e., with the extent of diffusion). Intuition is satisfied by observing that the greater is the elasticity of these learning effects on efficiency with respect to the extent of diffusion, the stronger the upward effect on the profile of the MFP growth rate.

Were the learned improvements in factor efficiency to be confined to enhancements in the efficiency of labor, as would be the case under Harrod-neutrality, the general level of the MFP profile would be lower; also its peak would be reached still later in the diffusion process than the simulation results show the Hicks-neutrality specification. The intuition for this is quite direct: under Harrod neutrality improvements there is no source of capital efficiency improvements to offset the decline in the sector-wide average productivity of capital as diffusion proceeds.



Note that some special conditions are required in order for the foregoing assumption of incremental improvement in the relative efficiency of the new technique to be consistent with the (unchanged) specification of the diffusion path. The effects upon either the heterogeneity distribution, or the movement of the adoption threshold must change in an offsetting manner and so keep the threshold falling over time at a constant exponential rate, as the simulations posit. Alternatively, one may suppose that the distribution of critical heterogeneities in the population is displaced at a rate that offsets the declining pace of growth of labor- and capital-input efficiency due of experience-based learning. It is not implausible that the pace of upward shift in the expected output size distribution could be accelerating over time in such a fashion. Similarly, it is quite conceivable that the relative user-cost of the new capital in the industry might fall at a quickening pace, either because the economy-wide level was being forced upwards or because scale effects were lowering the real costs of producing the new fixed inputs. Of course, the constant rate of fall in the adoption threshold has been assumed here simply for expositional convenience.

## **PART 4**

### **Conclusion: How May We Go Forward from Here?**

The second part of this essay pointed to the explicit connection between diffusion and the phenomenon of distributed lags in aggregate industry or sectoral investment behavior. It is quite natural to see the two as dual, particularly where investment decisions reflect the successive commitments of different agents to the adoption of a new technology, or cluster of complementary techniques that are embodied in durable production goods.

The model of diffusion based upon the underlying heterogeneity of the potential adopter population affecting micro-level adoption decisions is very general, and therefore capable of accounting for distributed lag structures that are very protracted, as well as those which are highly attenuated in time. That observation serves to “connect the dots” formed by the first- and the second-most cited among Zvi’s early publications. The essays’ third part explicitly joined the micro-level diffusion model with the macro-level process of productivity growth, thereby completing the “3-dot” picture.

This much can be done at the conceptual level, and made more concrete by simple modeling exercises of the sort presented here. The real challenge, which the

economics profession is late in addressing, is whether and how it is possible to put empirical meat on the bones of this analytical framework.

Consequently, I wish to close, albeit it very briefly, on a theme that I know Zvi would have liked, however he felt about everything that has led me to invoke it here: We really do need to invest in developing systematic data collection systems to enable empirical studies of the diffusion of new processes, and also of new products into consumer budgets. This has been a neglected area, both in regard to process innovations, and to the early diffusion of new products which now account for an increasing share of households' expenditures. Pursuing that line of inquires would extend the connections that I have sought to expose here. It would connect studies of the determinants of diffusion with the issues of the appropriate treatment of new goods in price deflators, and thereby close the circle of Zvi's concerns with the difficult problems of the real output measurement deficiencies that increasingly cloud our view of the actual course of productivity growth in modern economies.<sup>38</sup>

In this vein, I can do no better than to recall that gentle but insistent reproach to the economics profession that Zvi included in his 1994 paper on "R&D, Productivity and the Data Constraint":<sup>39</sup>

"We ourselves do not put enough emphasis on the value of data and data collection in our training of graduate students and in the reward structure of our profession. It is the preparation skill of the econometric chef that catches the professional eye, not the quality of the raw materials in the meal, or the effort that went into procuring them.

### Coda

I did not set out to write an essay about "Zvi and Me." But, finding myself having returned to the topic of the conversation that never quite happened between

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<sup>38</sup> I have made a brief foray in this direction (in David 2001), looking at the way that the problem of "splicing in" new products into price deflators became an increasingly large drag on the growth rate of real output and measured productivity in the U.S. during the period from the early 1970s through the early 1990's. The proximate mechanism identified in that preliminary exploration was the "mass customization" movement, and the growing acceptance of novel, differentiated product versions in household budgets. Unfortunately, the paucity of systematic data proved a serious obstacle, confirming my sense that Griliches' achievements rested in good part on his readiness to devote time and effort to data development.

<sup>39</sup> Reprinted from the *AER* in Griliches, (1998):p. 364.

us, I am missing his presence on this occasion all the more keenly. Composing this contribution has left me with an acute sense of loss — in not having been able to really re-engage Zvi in critical discussions about the theory and history of the diffusion of innovations. It is an emotion considerably more intense than the mingled feelings of relief and disappointment that I experienced 30 years ago.

Had things been otherwise, I am sure that I would have arrived much sooner at a proper understanding of the relationship between his seminal research on the economics of diffusion and the approach that I was exploring. Even had we argued about it, as would have been likely at the time, I cannot imagine that this would have jeopardized the warm cordiality of the relationship we came to enjoy over the decades that followed. Indeed, just the opposite, for a good academic argument is a bond.

Such comfort as I can derive now must come from the presence at this wonderful conference of so many keen spirits and sharp intellects who are carrying forward the traditions of warm debate, penetrating, constructive criticism, and unrelenting commitment to the idea of empirical economics as a quantitative science. For me, these form the enduring and most cherished part of Zvi Griliches' legacy.

### **Acknowledgements**

John Gabriel Goddard furnished swift and accurate assistance with the simulations and the production of the graphics based upon them. Bronwyn Hall's useful comments extended beyond the points noted in the footnotes. I am grateful to them both. The present version has benefited from Wesley Cohen's comments as the discussant of the conference presentation.

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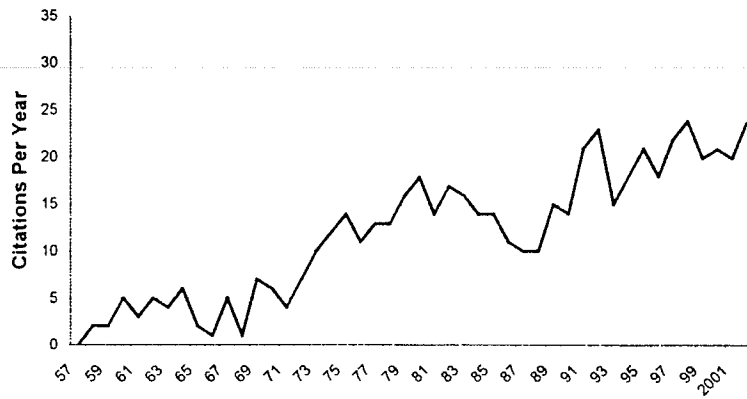
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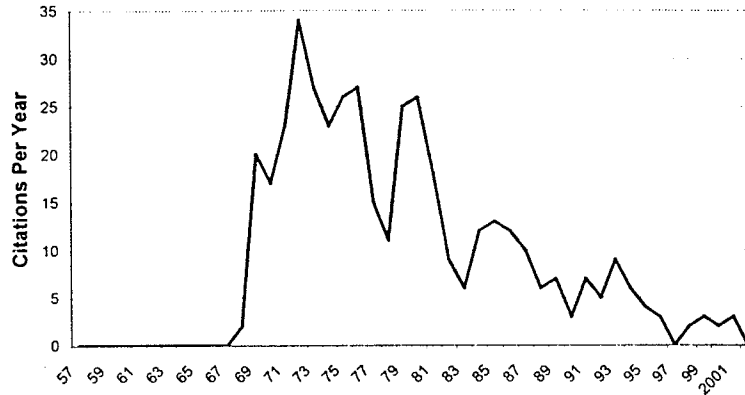
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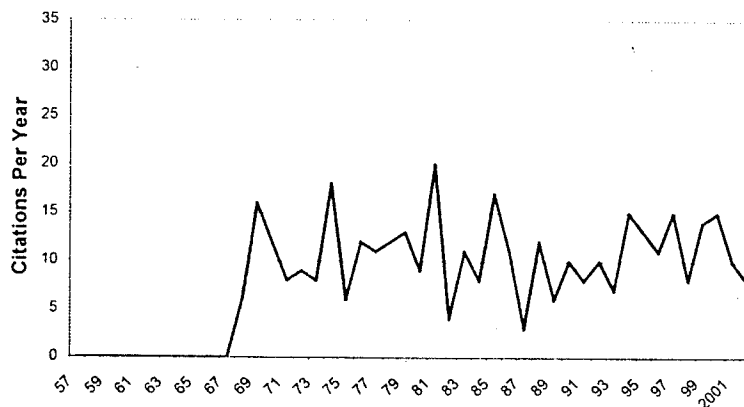
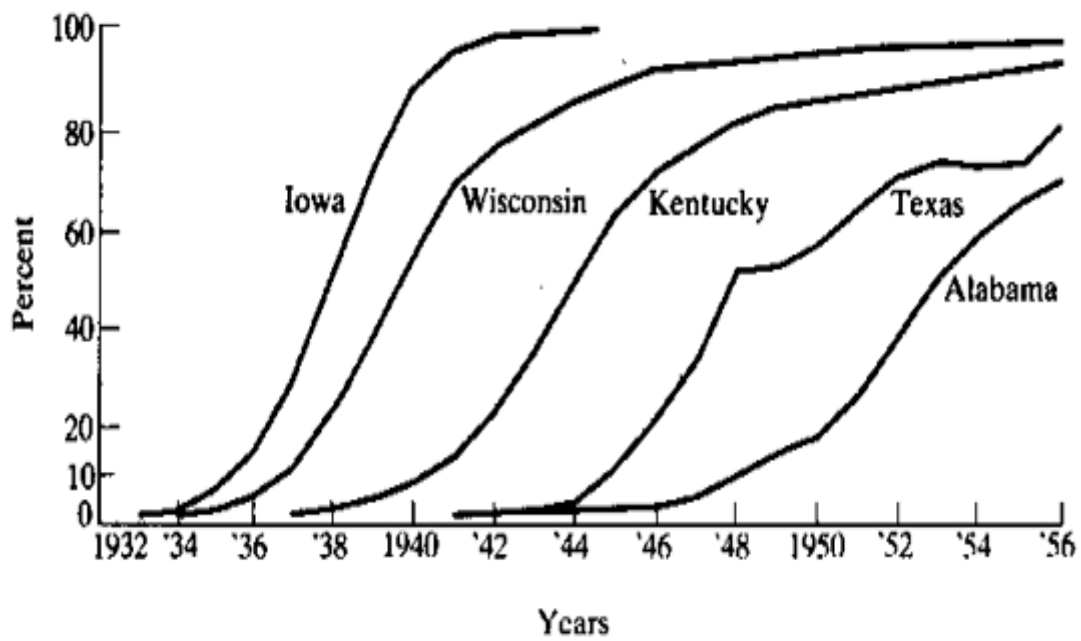


Figure 1

Annual Citations of Zvi Griliches' Four Most Frequently Journal Articles (Among those published before 1971)

Source: Arthur M. Diamond, Jr., "Zvi Griliches' Contributions to the Economics of Technology and Growth," *Economics of Innovation and New Technology*, forthcoming in 2004. [Available at: <http://cba.unomaha.edu/faculty/adiamond/web/DiamondPDFs/GrilProofs.pdf>]





**Figure 2**

**Percentage of total corn acreage planted with hybrid seed, derived from  
 USDA, Agricultural Statistics, various years.**

**Source:** Zvi Griliches, "Hybrid Corn: An Exploration in the Economics of Technological Change," *Econometrica* 25(4), 1957: 501-522. (Figure 2.1)

Extent of diffusion

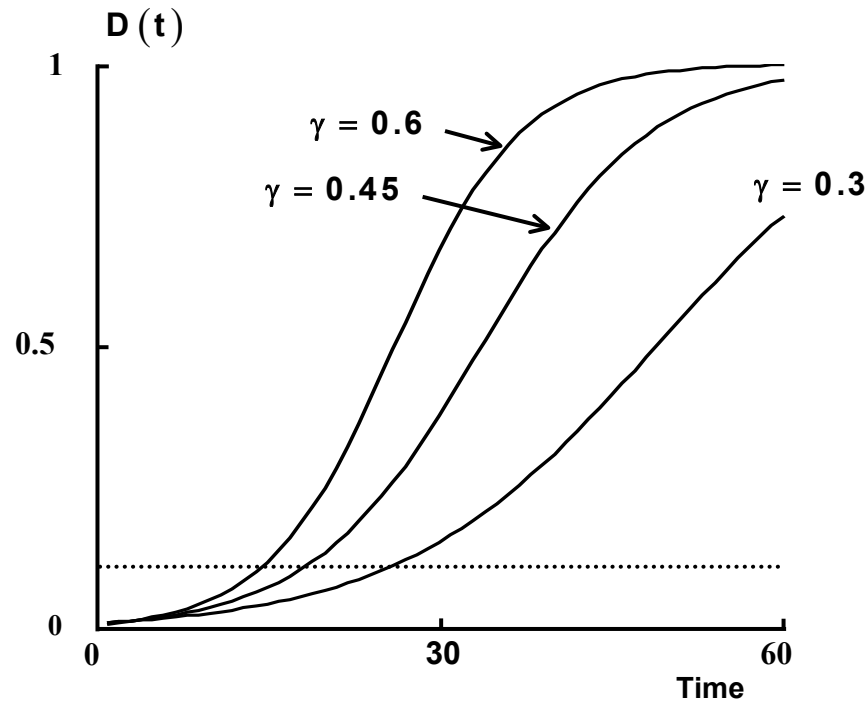


Figure 3 (a)

Effect on diffusion of greater variance (smaller  $\gamma$ ) of the underlying heterogeneity distribution.

Growth rate of labor productivity

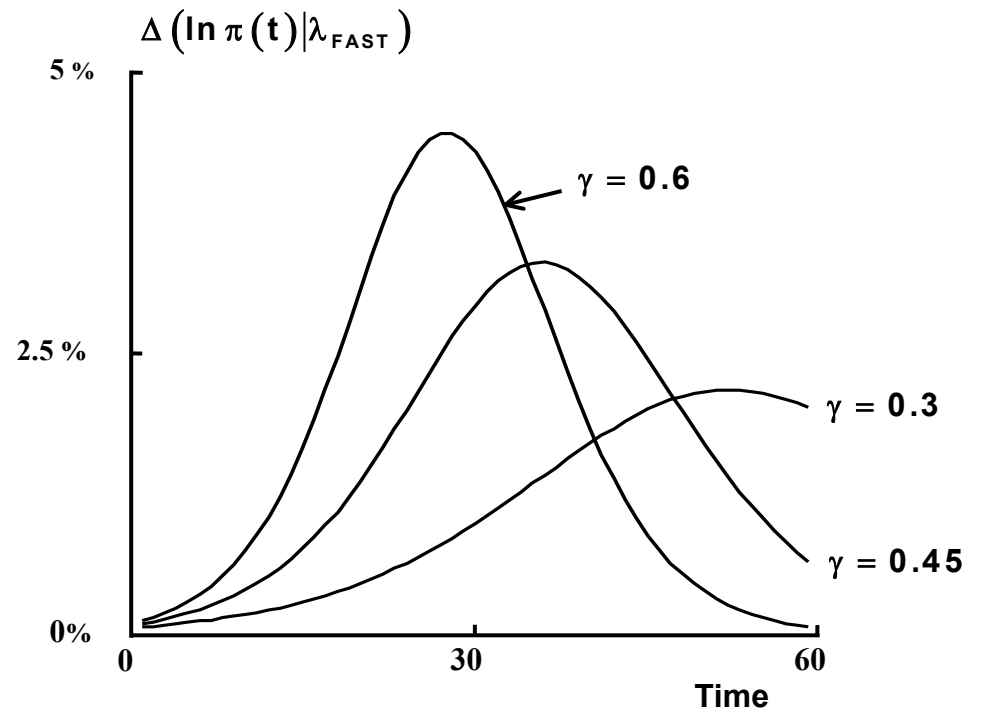


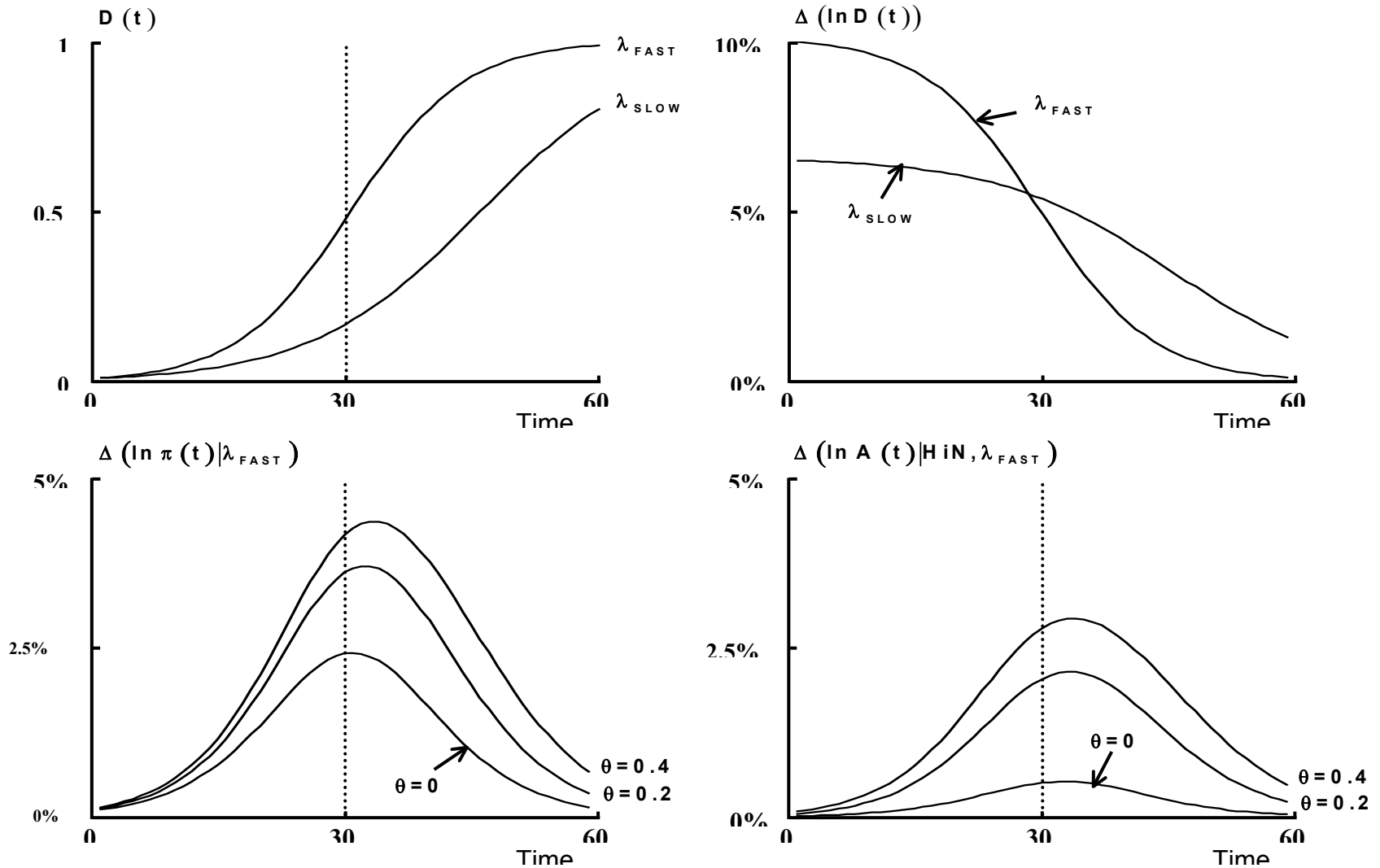
Figure 3 (b)

Effect on aggregate labor productivity growth of greater variance (smaller  $\gamma$ ) of the underlying heterogeneity distribution.

Figure 3: Simulation parameter values

$\lambda = \lambda_{FAST} = 0.3$ ;  $\theta = 0.2$ ;  $\omega_N = 0.5$ ;  $\gamma = \{0.3, 0.45, 0.6\}$ ;  $\frac{\pi_o}{\pi_N(0)} = 0.4$ ;  $\phi = 60$ ; where  $D(t) = (1 + \phi e^\gamma e^{-(\gamma\lambda)t})^{-1}$ . (Note:  $\phi$  normalizes the initial value of  $D(t)$  to 0.01.)

**Figure 4 : Effects of alternative values for  $\lambda$  -- the rate of fall of the adoption ‘threshold level  $z^*(t)$  – on the diffusion path, and on the growth rates of labor productivity ( $\Delta \ln \pi(t)$ ) and multifactor productivity ( $\Delta \ln A(t)$ ), given alternative diffusion-driven (Hicks-neutral) ‘learning effects’ the relative productivity of the new technology**



**Figure 4 Simulation parameter values:**  $\lambda_{SLOW} = 0.2$ ;  $\lambda_{FAST} = 0.3$ ;  $\theta = \{0, 0.2, 0.4\}$ ;  $\omega_N = 0.5$ ;  $\gamma = 0.5$ ;  $\frac{\pi_o}{\pi_N(0)} = 0.4$ ;  $\phi = 60$ ; where  $D(t) = (1 + \phi e^\gamma e^{-(\gamma\lambda)t})^{-1}$ .

(Note:  $\phi$  normalizes the initial value of  $D(t)$  to 0.01.)