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SIEPR Discussion Paper No. 03-02
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September 2003

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Private Capital Flows and Default Risk*

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Abstract

What has been the effect of the shift in emerging market capital flows toward private sector borrowers? Are emerging markets capital flows more efficient? If not, can government intervention improve welfare? This paper shows that the answers to these questions depend upon the form of default risk. When private loans are enforceable, but there is the risk of national default, constrained efficient capital flows can be decentralized with private borrowing subject to individual borrowing constraints. However, when private agents may individually default, private lending is always inefficient, and borrowing subsidies are potentially Pareto-improving.

JEL Classification: F21, F34, F41, O19.

Keywords: Capital flows, default, decentralization, borrowing constraints, capital subsidies.

*I thank, without implicating, Fernando Alvarez, Christine Groeger, Narayana Kocherlakota and Robert Martin for comments. Further comments welcome. An earlier version of this paper circulated under the title “Private Capital Flows, Capital Controls and Sovereign Risk.”

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1 Introduction

Capital flows to emerging markets have shifted markedly away from public sector and towards private sector borrowers. In the past decade, the private sector share of the stock of emerging market debt rose more than five-fold from around five per-cent in 1990 to more than one-quarter of the total by the end of the Century. Private non-debt capital flows have also increased rapidly, with gross foreign direct investment as a share of all private capital inflows more than doubling over the same period.

What has been the effect of this shift in emerging market capital flows? Are emerging market capital flows more efficient? And if not, can government intervention in the form of capital controls improve welfare? In this paper, we show that the form and efficiency of private capital flows depends upon the precise nature of the default risk facing foreign investors. We distinguish between two forms of default risk. When the government of a developing country provides for the enforcement of private contracts between residents and foreigners, but can default on the nations entire borrowings, we say that foreign investors face *national default risk*, and show that the constrained efficient allocations can be decentralized in a competitive equilibrium with private borrowing subject to individual borrowing constraints. That is, private borrowing is efficient and there is no scope for Pareto improving government intervention. However, when the countries legal system does not enforce private contracts with foreigners, so that foreign investors also face *resident default risk*, private borrowing is inefficient, and government intervention in the form of a borrowing subsidy is potentially Pareto improving.

Both forms of default risk appear to be important in practice. The presence of national default risk was historically a consequence of the doctrine of sovereign immunity which prohibited legal action against a sovereign state. Although this doctrine has been watered down over time to permit legal action against a sovereign, the process of attaching assets has proven difficult as most defaulting countries have little in the way of foreign assets. Foreign creditors also cannot typically use the domestic courts of the borrowing

country to pursue legal claims. Moreover, national default risk extends beyond the borrowings of the government itself to include default on private contracts either directly, by nationalizing private industries or taxing foreign remittances, or indirectly by imposing capital controls and other measures that prevent residents from honoring their obligations. Resident default risk, on the other hand, is a result of the fact that domestic contract enforcement institutions often will not uphold a foreigners claim on a domestic resident. In some cases, this is as a result of judicial corruption¹. However, even in developed countries the ability to use domestic courts to force fulfillment of a contract can be difficult, as illustrated by the Japan-Australia sugar dispute in which a group of Japanese sugar refiners refused to honor contracts after world sugar prices fell dramatically (see, for example, March [16]).

We begin by outlining a version of the standard consumption smoothing model of capital flows in the face of default risk, modified only to allow for the presence of heterogeneity in a countries resident population. As long as there are no constraints on enforcement within a country, we show that the constrained efficient allocations can be obtained from a standard social planning problem constrained by a sequence of participation constraints. We then show how the constrained efficient allocations can be decentralized as a competitive equilibrium in which individual residents borrow subject to their own borrowing constraint. The borrowing constraints are chosen in a way that echoes the approach used by Alvarez and Jermann [2] in their closed economy model of borrowing with solvency constraints. However, unlike that paper in which the constraints were set in order to deter default by an individual agent, the constraints here are determined so as to deter the economy as a whole from accumulating debt to the point where the national government would choose to default.

We then introduce the possibility of resident default risk, and show that the resulting competitive equilibria exhibit capital flows that are inefficiently low. The reason is that default can typically only be punished

¹Judicial partiality is commonly cited as an important component of corruption in emerging markets. For example, the World Bank's World Development Report of 1997 emphasized a lack of predictability in court rulings as a constraint on foreign investment. Al-Kilani [1] presents evidence that discrimination on the basis of nationality is common in middle eastern legal proceedings.

by excluding the offender from future international trade. If domestic contracts are enforceable, an agent in default will be able to access international markets using other domestic agents as intermediaries. This reduces the punishment to default which is then able to support less lending in equilibrium.

This inefficiency suggests a potential role for Pareto-improving government intervention. Presuming that it is not possible to directly strengthen the domestic contract enforcement institutions, or to indirectly do so via a mechanism such as a tax on default, we look for a mechanism that treats all agents symmetrically. Specifically, we show that the efficient allocations can be attained through a policy of *subsidizing access* to international financial markets. The key to this result is that a subsidy makes default less attractive by increasing the value of access to international capital markets. Importantly, this does not rely on the government being able to observe the default decision directly, but only to be able to observe the total flow of capital across its borders.

This paper builds on a substantial literature on trade in financial assets in the presence of enforcement frictions. One strand of this literature studies international debt in the presence of default risk, and has been surveyed by Eaton and Fernandez [10]. For the most part, this literature focuses on public sector lending, and assumes that a country is associated with a representative agent that makes all borrowing and default decisions on behalf of a country's residents². In contrast, in this paper we examine borrowing by private agents in the face of this risk, and show how the allocations obtained in this earlier literature might result out of the interaction of private agents in a competitive equilibrium. Another strand of this literature looks at the effects of enforcement frictions in closed economy debt markets, as in Alvarez and Jermann [2] and Kehoe and Levine [14]. Where the current paper differs is in drawing a distinction between the borrowing decision and the default decision (when private agents borrow subject to national repudiation risk), and in studying the interaction of enforcement frictions in international markets and the lack thereof in domestic markets (which allows residents who have defaulted to continue accessing international markets using other

²Exceptions include Cole and English [7] and [8], and Chang [9] who modifies the model of Bulow and Rogoff [5] in this direction.

residents as intermediaries).

In two recent papers, Jeske [12] and Kehoe and Perri [13] also study borrowing by private agents in environments subject to enforcement frictions. Jeske was the first to study resident default risk, and advocated the introduction of prohibitive controls on private capital flows, combined with some centralized mechanism for international borrowing as a vehicle for obtaining the constrained efficient level of borrowing. This has been interpreted as providing a case for the imposition of capital controls even in non-crisis periods. By contrast, this paper analyzes both national and resident default risk, and shows that the subsidization of capital flows, and not their prohibition, is capable of improving the efficiency of capital flows. Kehoe and Perri also study the decentralization of efficient allocations in the face of national repudiation risk. In their framework, private agents are induced to borrow at the efficient level by the presence of government taxes on capital flows (and not borrowing constraints), which the authors interpret as partial default.

The rest of this paper is organized as follows. Section 2 outlines the basic modeling environment, which is a version of the usual consumption smoothing model of capital flows under sovereign risk modified to allow for heterogeneity in a country's resident population. Section 3 characterizes the constrained efficient allocations in this context and establishes that, as long as individuals, but not the country as a whole, can commit to honor contracts with foreigners, these allocations can be decentralized in a framework where private individuals make their own borrowing decisions. Section 4 then studies the competitive equilibria of an economy in which individual agents both borrow and make their own default decisions and shows that equilibrium allocations are inefficient, and that a government borrowing subsidy can be used to increase welfare. Section 5 presents the results from a simple example, which can be computed by hand, and uses them to illustrate the differences between the various formulations, while Section 6 concludes.

2 Environment

The modeling environment is a version of the usual consumption smoothing model of capital flows under sovereign risk (for examples, see the survey of Eaton and Fernandez [10]), modified to allow for heterogeneity in a countries resident population. Consider a world economy populated by a finite number of countries denoted $m = 1, \dots, M$. Each country m is populated by N_m types of agents with the total number of types given by $N = \sum_{m=1}^M N_m$. We will use subscripts to denote types of individuals and superscripts to denote countries, so that $\lambda_n^m > 0$ denotes the measure of agents of type n in country m .

Time is discrete and in each period $t = 0, 1, 2, \dots$ information about current and future endowments is indexed by the *state* θ_t , an element of the finite set Θ . Information about states forms a Markov chain, and the transition probability from θ to θ' is given by $\pi(\theta'|\theta)$ with the initial state θ_0 given. We let $\theta^t := (\theta_0, \theta_1, \dots, \theta_t) \in \Theta^t$ denote a history of states up to date t . The notation $\theta^s | \theta^t$ for $s > t$ refers to a history θ^s that *continues* θ^t in the sense that $\theta^s = (\theta^t, \theta_{t+1}, \theta_{t+2}, \dots, \theta_s)$. The probability of observing history θ^t is denoted by $\pi(\theta)^t$, and that of observing history θ^s conditional on having been in θ^t is $\pi(\theta^s | \theta^t)$.

There is one non-storable good available for consumption in each history. A state θ specifies a vector of endowments for each type of agents in each country. We denote by $e_n^m(\theta^t)$ the endowment of a type n agent in country m after history θ^t and by $c_n^m(\theta^t)$ the corresponding consumption level. It will be convenient to define, for all countries m and for all histories θ^t , the aggregate endowment of a country, m , as $e^m(\theta^t) = \sum_{n=1}^N \lambda_n^m e_n^m(\theta^t)$ and the aggregate consumption level of country m as $c^m(\theta^t) = \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t)$.

Individuals have preferences ordered by a time additively separable function

$$\sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^m(c(\theta^t)),$$

where $\beta \in (0, 1)$ is the discount factor of an agent, which is assumed the same over all types, while the function U_n^m is the period utility function which is strictly increasing and strictly concave.

We can envisage a number of trading regimes that differ according to the restrictions placed upon trade between agents within a country and also on trade between agents in different countries. In what follows we will study two types of enforcement frictions. The first case is *national default risk*, which refers to the risk that a nation as whole, as represented by its government, defaults on all of that nation's international obligations. This is to be distinguished from *resident default risk* in which residents individually can default on their own personal obligations to foreigners. National default risk is the result of national sovereignty, and the absence of supra-national institutions for contract enforcement. By contrast, resident default risk flows from a weakness in domestic contract enforcement institutions. This latter form can be interpreted as resulting from corruption in the judicial system that favors domestic residents over foreigners. To capture the idea that domestic residents find it easier to enforce contracts, we focus on the extreme case in which contracts between residents of the same country are perfectly enforceable.

3 National Default Risk

3.1 Constrained Efficient Allocations

In the presence of national repudiation risk, capital flows between countries must be such that they are self-enforcing. That is, we will require that an allocation deliver the residents of a nation sufficient utility to deter their national government from defaulting on their behalf. In doing so, it is necessary to specify the preferences of the nation's government that makes the default decision. We make two important simplifying assumptions in this regard. First, it is assumed that each national government is benevolent with respect to its own citizens and acts to maximize a weighted average of their lifetime utilities. Specifically, let $\phi_n^m > 0$ be the weight assigned to agents of type n by the government of country m . Second, and consistent with the maintained assumption that transfers between domestic agents are costlessly enforceable, we assume that

governments can pursue their welfare objectives by making date zero transfers of domestic securities between agents.

In this environment, a consumption allocation $c = \{c_n^m(\theta^t)\}_{n,m,\theta^t,t}$ is *resource feasible* if it is non-negative and satisfies

$$\sum_{m=1}^M \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) \leq \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m e_n^m(\theta^t), \quad (1)$$

for all t and all θ^t . A consumption allocation satisfies the sequence of *national participation constraints* if it satisfies

$$\sum_{n=1}^N \phi_n^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U_n^m(c_n^m(\theta^s)) \geq D^m(\theta^t, \phi^m), \quad (2)$$

for all m, t and for all θ^t . The constraint (2) states that the continuation of an allocation for a country m must, after each history, deliver at least as much weighted lifetime utility as default to autarky $D^m(\theta^t, \phi^m)$ for that country as a whole. Note that we have written this as a function of the vector $\phi^m \equiv \{\phi_n^m\}$ of Pareto weights to emphasize preferences of the benevolent national government. This value is given by

$$\begin{aligned} D^m(\theta^t, \phi^m) &= \max \sum_{n=1}^N \phi_n^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U_n^m(c_n^m(\theta^s)), \\ &\text{subject to } \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) \leq \sum_{n=1}^N \lambda_n^m e_n^m(\theta^t). \end{aligned}$$

In this environment, an allocation is *constrained efficient* if it satisfies resource and incentive feasibility, and there is no other allocation satisfying these constraints that is strictly Pareto-preferred. To characterize the set of constrained efficient allocations, let $\varphi^m > 0$ denote the Pareto weight attached to the welfare of country m by the international social planner, and for convenience normalize $\sum_n \phi_n^m = 1$. We can characterize the set of efficient allocations by solving the following convex planning problem

$$\max_c \sum_{m=1}^M \varphi^m \sum_{n=1}^N \phi_n^m \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^m(c_n^m(\theta^t)), \quad (\text{PP})$$

subject to the sequence of resource and incentive feasibility constraints in (1) and (2).

The strong separability embodied in this problem implies that the solution to the planning problem in (PP) can be found in two stages. In the second stage, aggregate country consumption is taken as given and is to be allocated between residents of a country to solve

$$\begin{aligned}
 V^m(c^m(\theta^t), \phi^m) &= \max \sum_{n=1}^N \phi_n^m U_n^m(c_n^m(\theta^t)), \\
 \text{subject to} & \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) \leq c^m(\theta^t).
 \end{aligned} \tag{3}$$

In the first stage, the planner allocates consumption between countries to

$$\begin{aligned}
 \max_c \sum_{m=1}^M \phi^m \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) V^m(c^m(\theta^t), \phi^m), \\
 \text{subject to} \quad \sum_{m=1}^M c^m(\theta^t) \leq \sum_{m=1}^M e^m(\theta^t), & \quad \text{for all } t, \text{ and} \\
 \phi^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) V^m(c^m(\theta^s), \phi^m) \geq D^m(\theta^t, \phi^m), & \quad \text{for all } m \text{ and all } t.
 \end{aligned} \tag{PP'}$$

The above result is useful because it allows us to solve a limited commitment problem at an international level to determine the efficient level of intertemporal trade between countries before later allocating these goods between residents of a country. Unfortunately, the solution of this problem requires knowledge of the appropriate country weights ϕ^m which determine the form of the period country welfare functions $V^m(c^m(\theta^t), \phi^m)$ as well as the default values $D^m(\theta^t, \phi^m)$. The following proposition shows that if preferences are homothetic and identical within a country (so that $U_n^m = U^m$ for all n), international capital flows will be independent of these weights. In particular, the solution of the two-stage problem can be computed by first solving the first stage for a representative agents with preferences given by U^m .

Proposition 1 *If preferences are homothetic and identical within a country, capital flows are independent of country welfare weights ϕ^m .*

Proof. Consider the two stage problem substituting U^m for $V^m(c^m(\theta^t), \widehat{\phi}^m)$. In the second stage, with preferences identically homothetic within the country, we have

$$c_n^m(\theta^t) = \alpha_n c^m(\theta^t),$$

for all histories θ^t and all types n where $\sum_{n=1}^N \alpha_n = 1$. Hence we have

$$\frac{u'(c^m((\theta^t, \theta_{t+1})))}{u'(c^m(\theta^t))} = \frac{u'(c_{n'}^m((\theta^t, \theta_{t+1})))}{u'(c_n^m(\theta^t))},$$

for all n, n' and t which implies the first order conditions of the planning problem.

Obviously, the solution to the two-stage problem satisfies resource feasibility for the planning problem.

To show that it satisfies the participation constraints, note that by construction it satisfies for all θ^t

$$\sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(c^m(\theta^t)) \geq \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(e^m(\theta^t)).$$

The result will then follow if we can establish that the function

$$h(x) = \sum_{n=1}^N \phi_n^m U^m(\alpha_n (U^m)^{-1}(x)),$$

is affine and strictly increasing. For then

$$h\left(\sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(c^m(\theta^t))\right) \geq h\left(\sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(e^m(\theta^t))\right),$$

implies

$$\sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) h[U^m(c^m(\theta^t))] \geq \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) h[U^m(e^m(\theta^t))],$$

and hence

$$\begin{aligned}
\sum_{n=1}^N \phi_n^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(c_n^m(\theta^t)) &= \sum_{n=1}^N \phi_n^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(\alpha_n c^m(\theta^t)) \\
&\geq \sum_{n=1}^N \phi_n^m \sum_{s>t} \beta^{s-t} \sum_{\theta^s|\theta^t} \pi(\theta^s|\theta^t) U^m(\alpha_n e^m(\theta^t)) \\
&= D^m(\theta^t),
\end{aligned}$$

where the last equality comes from identical homotheticity.

To see that h is affine and strictly increasing, following the proof of Lemma 1 of Jeske [12] note that the derivative of h is

$$\begin{aligned}
h'(x) &= \sum_{n=1}^N \phi_n^m U^{m'}(\alpha_n (U^m)^{-1}(x)) \alpha_n \frac{1}{U^{m'}((U^m)^{-1}(x))} \\
&= \sum_{n=1}^N \phi_n^m \alpha_n \frac{U^{m'}(\alpha_n c)}{U^{m'}(c)},
\end{aligned}$$

where $c = (U^m)^{-1}(x)$. As U^m is homothetic, for all c_1 and c_2 in the domain of U^m ,

$$\frac{U^{m'}(\alpha c_1)}{U^{m'}(\alpha c_2)} = \frac{U^{m'}(c)}{U^{m'}(c)},$$

and hence

$$\frac{U^{m'}(\alpha_n c)}{U^{m'}(c)}$$

is a constant for all c . ■

3.2 Decentralization with Private Borrowing and National Default

The aim of this section is to establish that, as long as domestic courts enforce contracts with foreigners, the efficiency of private capital flows is constrained only by the ability of the national government to default. That is, the constrained efficient allocation can be decentralized. The key will lie in establishing a mechanism by which individual agent borrowing is constrained so that the nation as a whole, as represented by its government, has no incentive to default.

It is assumed that there exists a full set of state contingent securities, and we denote the one period prices as $q(\theta^t, \theta_{t+1})$, for securities purchased after history θ^t for payment in period $t + 1$ after observing event θ_{t+1} , while $b_n^m(\theta^t, \theta_{t+1})$ is the holdings of these securities by a type n agent in country m . The problem of a resident of type n of country m in this decentralization is to choose sequences for consumption and securities holdings to maximize

$$\sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^m(c(\theta^t))$$

subject to

$$c(\theta^t) + \sum_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) b_n^m(\theta^t, \theta_{t+1}) = e_n^m(\theta^t) + b_n^m(\theta^{t-1}, \theta_t),$$

for all $\theta^t = (\theta^{t-1}, \theta_t)$ for all t , and

$$b(\theta^t, \theta_{t+1}) \geq -\bar{B}_n^m(\theta^t, \theta_{t+1}) \tag{4}$$

for all θ^t for all t , with $b(\theta^0)$ given. Here, $\bar{B}_n^m(\theta^t, \theta_{t+1})$ is a borrowing constraint that is set so as to prevent agents from accumulating so much debt that their government is tempted to default on their behalf. A weaker constraint of this form would be necessary to rule out the possibility of Ponzi debt schemes, even in the absence of the possibility of national repudiation. Following Alvarez and Jermann [2], the constraint in (4) will be referred to as a *solvency constraint* and is type, country and history dependent. This leads to our definition of a competitive equilibrium with solvency constraints.

Definition 1 A *competitive equilibrium with solvency constraints* is a sequence of solvency constraints $\overline{B}_n^m(\theta^t, \theta_{t+1})$ for all countries m , types n , dates t and histories θ^t , a price system $\{q(\theta^t, \theta_{t+1})\}$ for all countries m , dates t , and histories θ^t , and an allocation $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1})\}$ such that for all countries m and types n , the allocation solves the residents problem given the price system, solvency constraints, and initial asset holdings, and markets clear. That is, for all dates t and all histories θ^t , goods markets clear

$$\sum_{m=1}^M \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) \leq \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m e_n^m(\theta^t),$$

and for all θ_{t+1} the markets for securities clear

$$\sum_{m=1}^M \sum_{n=1}^N \lambda_n^m b_n^m(\theta^t, \theta_{t+1}) = 0.$$

Let $W_n^m(b, \theta^t)$ be the value to a resident of type n in country m who begins at history θ^t with bond holdings b . We study the functional equation for this agent defined by

$$\begin{aligned} W_n^m(b, \theta^t) &= \max_{c, \{b', f'\}} \left\{ U(c) + \beta \sum_{\theta_{t+1}} W_n^m(e'(\theta^t, \theta_{t+1}) + b'(\theta^t, \theta_{t+1}), (\theta^t, \theta_{t+1})) \pi(\theta_{t+1} | \theta^t) \right\} \\ \text{subject to} \quad &c + \sum_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) b(\theta^t, \theta_{t+1}) = e_n^m(\theta^t) + b, \\ &b(\theta^t, \theta_{t+1}) \geq -\overline{B}_n^m(\theta^t, \theta_{t+1}) \end{aligned}$$

Our aim will be to find a sequence of solvency constraints for every agent that prevent default by not allowing agents to accumulate more debt than they would be willing to repay. At the same time, we do not want to set these constraints to be any tighter than needed. Following the spirit of Alvarez and Jermann, we will require that our solvency constraints bind if and only if the continuation utility of a country that is at its borrowing constraint equals the value to that country of defaulting. Note that such a constraint does

not prevent a country from choosing to default, but requires that in equilibrium they have no incentive to do so. In states where they do not bind, we will set them at levels such that, were the constraints to bind, a country would be indifferent to default. However, in contrast to the closed economy considered by Alvarez and Jermann, there will be many ways in which these constraints can be set that satisfy this requirement. Hence we will augment the definition to require that intra-national risk sharing is as large as possible.

Definition 2 *An equilibrium with solvency constraints that are **minimally constraining** is such that the solvency constraints satisfy, for all dates t and all histories θ^t*

$$\sum_{n=1}^N \phi_n^m W_n^m \left(-\bar{B}_n^m (\theta^{t-1}, \theta_t), \theta^t \right) = D^m (\theta^t)$$

and such that for all n, n'

$$\phi_n^m \frac{\partial W_n^m \left(-\bar{B}_n^m (\theta^{t-1}, \theta_t), \theta^t \right)}{\partial b} = \phi_{n'}^m \frac{\partial W_{n'}^m \left(-\bar{B}_{n'}^m (\theta^{t-1}, \theta_t), \theta^t \right)}{\partial b}$$

When solvency constraints are minimally constraining, they will bind if and only if the corresponding participation constraint in the planners problem binds. When these constraints bind, the marginal rates of substitution of all agents are not equalized across countries, although they are equalized within countries. Agents in constrained countries will accumulate assets according to their solvency constraints. In order to ensure that agents in unconstrained countries accumulate appropriately, it must be the case that equilibrium prices equal their marginal rate of substitution. This leads to our candidate equilibrium price sequence for bonds for all dates t and histories θ^t

$$q(\theta^t, \theta_{t+1}) \equiv \max_{n,m} \left\{ \beta \frac{u'(c_n^m((\theta^t, \theta_{t+1})))}{u'(c_n^m(\theta^t))} \pi(\theta_{t+1} | \theta^t) \right\} \quad (5)$$

Using these prices for one-period international Arrow securities we can define the time zero Arrow-Debreu price on international markets of a unit of consumption at time t after realization of history θ^t as

$$Q(\theta^t|\theta_0) = q(\theta_0, \theta_1) q(\theta^1, \theta_2) \cdots q(\theta^{t-1}, \theta_t)$$

This leads to the following definition.

Definition 3 *Given a sequence of candidate international Arrow-Debreu prices $\{Q(\theta^t|\theta_0)\}$ defined for the allocation $\{c_n^m(\theta^t)\}$, we say that **implied international interest rates are high** if the date zero value of aggregate consumption at these prices is finite:*

$$\sum_{t=0}^{\infty} \sum_{\theta^t} Q(\theta^t|\theta_0) \left(\sum_{m=1}^M \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) \right) < +\infty$$

Using these results, we are able to state a version of the second welfare theorem for our economy.

Proposition 2 *Given an allocation $\{c_n^m(\theta^t)\}$ that solves the planning problem above and such that implied international interest rates are high, there exists a sequence of solvency constraints $\{\bar{B}_n^m(\theta^t, \theta_{t+1})\}$, an initial wealth allocation defined by $\{b_n^m(\theta^0)\}$, and a sequence of asset holdings $\{b_n^m(\theta^t, \theta_{t+1})\}$ such that the plan $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1})\}$ is a competitive equilibrium for the solvency constraints and initial wealth allocation. Moreover, the sequence of solvency constraints can be chosen so that they are minimally constraining.*

Proof. The proof is constructive, and adapts that of Alvarez and Jermann [2] to our framework. Given an allocation $\{c_n^m(\theta^t)\}$ and the implied sequences of Arrow $\{q(\theta^t, \theta_{t+1})\}$ and Arrow-Debreu $\{Q(\theta^t|\theta_0)\}$ prices, we can compute the bond holdings of an agent of type n in country m after history θ^t as the value of the difference between consumption and endowments from that history onwards. To do this, define $Q(\theta^s|\theta^t)$

as the history θ^t value of a unit of consumption in history θ^s . Then

$$b_n^m(\theta^t) = [c_n^m(\theta^t) - e_n^m(\theta^t)] + \sum_{s \geq 1} \sum_{\theta^{t+s}} Q(\theta^s | \theta^t) [c_n^m(\theta^{s+t}) - e_n^m(\theta^{s+t})]$$

where this sum is well defined by the assumption that implied interest rates are high. By construction of the Arrow prices, the sequences $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1})\}$ satisfy the agents' flow budget constraints.

As an intermediate step, construct a first candidate sequence of solvency constraints $\tilde{B}_n^m(\theta^t, \theta_{t+1})$. For all histories θ^t such that the participation constraint of country m binds, set

$$\tilde{B}_n^m(\theta^t, \theta_{t+1}) = -b_n^m(\theta^t, \theta_{t+1})$$

for all agents in that country. For all other histories, we pick a candidate solvency constraint at such a level that, if it is satisfied, the agent cannot enjoy positive consumption in any future state. Specifically, pick

$$\tilde{B}_n^m(\theta^t, \theta_{t+1}) = -\sum_{s \geq 1} \sum_{\theta^{t+s}} Q(\theta^s | \theta^t) e_n^m(\theta^{s+t})$$

That $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1})\}$ attains the maximum in the agents problem for prices $\{q(\theta^t, \theta_{t+1})\}$ and solvency constraints $\{\tilde{B}_n^m(\theta^t, \theta_{t+1})\}$ can be verified by checking the necessary and sufficient Euler and transversality conditions for the agents problem. From the definition of Arrow prices in (5), we have that

$$u'(c_n^m(\theta^t)) q(\theta^t, \theta_{t+1}) \geq \beta u'(c_n^m(\theta^t, \theta_{t+1})) \pi(\theta_{t+1} | \theta^t)$$

with equality if $-b_n^m(\theta^t, \theta_{t+1}) > -\tilde{B}_n^m(\theta^t, \theta_{t+1})$ which verifies that the Euler equation is satisfied. To see

that the transversality condition holds, note that the requirement of high implied interest rates implies

$$\lim_{T \rightarrow \infty} \sum_{\theta^T} Q(\theta^T | \theta_0) \sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) e(\theta^{s+T}).$$

Therefore

$$\begin{aligned} & \lim_{T \rightarrow \infty} \sum_{\theta^T} \beta^T u'(c_n^m(\theta^T)) \left[b_n^m(\theta^T) + \tilde{B}_n^m(\theta^T) \right] \pi(\theta^T | \theta_0) \\ & \leq \lim_{T \rightarrow \infty} \sum_{\theta^T} \beta^T u'(c_n^m(\theta^T)) \left[\sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) c_n^m(\theta^{s+T}) \right] \pi(\theta^T | \theta_0), \end{aligned}$$

from the fact that $b_n^m(\theta^T) + \tilde{B}_n^m(\theta^T) = 0$ if the participation constraint for country m binds at θ^T , or equals $\sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) c_n^m(\theta^{s+T}) \geq 0$, by construction of our candidate solvency constraints, otherwise.

But as the allocation is feasible we have

$$\begin{aligned} & \lim_{T \rightarrow \infty} \sum_{\theta^T} \beta^T u'(c_n^m(\theta^T)) \left[\sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) c_n^m(\theta^{s+T}) \right] \pi(\theta^T | \theta_0) \\ & \leq \lim_{T \rightarrow \infty} \sum_{\theta^T} \beta^T u'(c_n^m(\theta^T)) \left[\sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) e(\theta^{s+T}) \right] \pi(\theta^T | \theta_0) \\ & \leq u'(c_n^m(\theta_0)) \lim_{T \rightarrow \infty} \sum_{\theta^T} Q(\theta^T | \theta_0) \sum_{s=1}^{\infty} \sum_{\theta^{s+T}} Q(\theta^{s+T} | \theta^T) e(\theta^{s+T}) = 0, \end{aligned}$$

where the last inequality follows from the definition of the Arrow-Debreu and Arrow prices, and the last equality follows from above.

Finally, to show that we can set these solvency constraints so that they are minimally constraining, define the value functions $\{W_n^m\}$ that solve the agents problem for all n and m . As the plan $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1})\}$ satisfies the Euler and transversality conditions, it constitutes a solution for these value functions as long as the initial conditions are given by $\{b_n^m(\theta^0)\}$. For all θ^t such that the participation constraint of country m

binds, define our final sequence of solvency constraints $\{\bar{B}_n^m(\theta^t, \theta_{t+1})\}$ such that

$$\bar{B}_n^m(\theta^t, \theta_{t+1}) = \tilde{B}_n^m(\theta^t, \theta_{t+1}).$$

For all other θ^t , define them such that

$$\sum_{n=1}^N \phi_n^m W_n^m(-\bar{B}_n^m(\theta^{t-1}, \theta_t), \theta^t) = D^m(\theta^t)$$

and

$$\phi_n^m \frac{\partial W_n^m(-\bar{B}_n^m(\theta^{t-1}, \theta_t), \theta^t)}{\partial b} = \phi_{n'}^m \frac{\partial W_{n'}^m(-\bar{B}_{n'}^m(\theta^{t-1}, \theta_t), \theta^t)}{\partial b}.$$

Using the envelope condition, it is easy to verify that this latter condition is also satisfied at states at which the participation constraint binds. ■

This framework has two desirable features. First, individual residents retain access to international financial markets, and it is to these markets that the burden of ensuring that default does not occur is devolved. Second, the decentralization does not rule out the possibility of default: default remains feasible for agents. However, default does not occur because the solvency constraints leave agents with no *incentive* to default.

However, there are three obvious criticisms of this approach. First, there is a sense in which the burden of ruling out default that has been placed upon financial markets is very large. As countries must have no incentive to default, in setting these solvency constraints it is necessary for financial markets to have knowledge of the preferences of a nation's government, which in turn depends upon the preferences of its residents and the vector of national Pareto weights. Similarly, financial markets must have knowledge of the aggregate endowment process of a nation, as it is this process that determined the worst that can happen to a country in default. The criticism that this informational burden is incredibly large has been made, in

the context of the closed economy model, by Browning, Hansen and Heckman [4].

Second, and in contrast to the closed economy model, the individual solvency constraints can take unappealing forms. Note that complete markets within a country serve to ensure that marginal rates of substitution are equalized within a country. Hence, when one agent is borrowing constrained, all agents must be constrained. In general, the asset positions of agents will depend upon their initial wealth levels, and at any moment in time some agents within a country may be creditors even when a nation as a whole is a debtor. Consequently, it may be necessary that for some types B_n^m be negative so that some agents are constrained to *save* a minimum amount in bonds.

Third, an equilibrium with solvency constraints is in general associated with a commitment by the government to make very large transfers between residents out of equilibrium. Specifically, the act of default, which never occurs in equilibrium, implies potentially large transfers between agents. On the one hand, default benefits debtors who no longer must repay their debts, and hurts creditors who lose their existing claims on foreign agents as well as possibly losing some future access to credit markets. On the other hand, there is a tendency for prices within a country to vary creating capital gains in the assets of creditors. Although these effects may cancel out, in general large transfers between agents will still be necessitated. To the extent that large transfers are implausible, this decentralization is problematic. In Section 5 below we present a simple numerical example which serves to illustrate these last two criticisms.

In summary, we have shown that when domestic courts enforce contracts symmetrically between both residents and foreigners, it is possible to decentralize the constrained efficient allocations in an economy in which individuals borrow subject to their own solvency constraint. These constraints do not rule out default, but make it suboptimal for the government in equilibrium. In the next section we consider the alternative case in which domestic courts do not enforce contracts with foreigners, and hence individual residents can default.

4 Resident Default Risk

4.1 Competitive Equilibrium

The previous section showed that, in the presence of national default risk, private capital flows will be constrained efficient as long as the contracts made by domestic residents with foreigners are enforced in domestic courts. In practice, however, court systems are often partial towards their own citizens. In this section we study the impact on capital flows that results from allowing domestic residents to make their own default decisions under the assumption that the only recourse of foreign investors is to refuse to trade with the domestic residents in default. As has been emphasized by Jeske [12], the fact that other domestic residents who are not in default can continue to access international markets means that an agent in default can still trade internationally using other agents as intermediaries. This substantially weakens the penalty to default and leads to inefficiently low levels of capital flows.

Consider a world economy of the Kehoe-Levine [14] type in which there are $M + 1$ sets of state contingent securities. The first M of these securities are country specific trading only within country m while the last refers to securities traded on world markets. We denote the prices of these securities as $q(\theta^t, \theta_{t+1})$ for international securities purchased after history θ^t for payment in period $t + 1$ after observing event θ_{t+1} , and, analogously for securities in country m , by $p^m(\theta^t, \theta_{t+1})$. Holdings of foreign securities are denoted $f(\theta^t, \theta_{t+1})$ and of domestic securities in country m , by $b^m(\theta^t, \theta_{t+1})$.

It is assumed that an agent that defaults does so assuming that prices in domestic markets stay unchanged resulting from the fact that they are small relative to the market. Given a domestic price vector p^m the problem of an agent that defaults at a given point in time s after history θ^s with domestic bond holdings b

is represented as

$$\begin{aligned}
D_n^m(b, \theta^s, \{p^m\}) &= \max_{c, \{b^t\}} \sum_{t=s}^{\infty} \beta^{t-s} \sum_{\theta^t | \theta^s} \pi(\theta^t | \theta^s) U_n^m(c(\theta^t)), \\
\text{subject to} & \quad c + \sum_{\theta_{t+1}} p^m(\theta^t, \theta_{t+1}) b(\theta^t, \theta_{t+1}) = e(\theta^t) + b, \\
& \quad b(\theta^t, \theta_{t+1}) \geq -\bar{B},
\end{aligned}$$

for all θ^t that continue θ^s with b given, and where in the last constraint \bar{B} is set sufficiently large so that it never binds, and serves to rule out Ponzi schemes.

The value $D_n^m(b, \theta^t, \{p^m\})$ plays an important role in determining the set of self-enforcing allocations. Specifically, the residents problem (RP) is to choose sequences for consumption and domestic and international securities holdings to maximize

$$\sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^m(c(\theta^t)), \tag{RP}$$

subject to

$$\begin{aligned}
& c(\theta^t) + \sum_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) f(\theta^t, \theta_{t+1}) + \sum_{\theta_{t+1}} p^m(\theta^t, \theta_{t+1}) b(\theta^t, \theta_{t+1}) \\
& \leq e(\theta^t) + f(\theta^{t-1}, \theta_t) + b(\theta^{t-1}, \theta_t),
\end{aligned} \tag{6}$$

for all $\theta^t = (\theta^{t-1}, \theta_t)$ for all t ,

$$\sum_{t=s}^{\infty} \beta^{t-s} \sum_{\theta^t | \theta^s} \pi(\theta^t | \theta^s) U_n^m(c(\theta^t)) \geq D_n^m(b(\theta^{s-1}, \theta_s), \theta^s, \{p^m\}), \tag{7}$$

for all θ^s , for all s ,

$$b(\theta^t, \theta_{t+1}) \geq -\bar{B}, \quad (8)$$

$$f(\theta^t, \theta_{t+1}) \geq -\bar{F}.$$

for all θ^t for all t , and $b(\theta^0)$ given. Here, (6) is the usual flow budget constraint, and (7) is the participation constraint which guarantees that a resident cannot choose a path for consumption that would leave them preferring to default after any history θ^s . The final constraints (8) serve to rule out Ponzi schemes in both the domestic and foreign bonds. Note that (7) assumes that default is not feasible, which is much stronger than the framework of the previous section in which it is feasible, but never optimal, for an agent to default. Note also that, given the preferences of the national government, the vector of initial domestic securities holdings is assumed to be chosen to maximize the governments objectives.

In this framework, we allow the domestic and international state contingent securities to trade at different prices. This is despite the fact that such securities have the same payoffs and are free of default risk (domestic securities by virtue of domestic enforcement mechanisms, and international by virtue of the infeasibility of default implied by constraint (7)). Indeed, it will emerge that unless the interest rate on domestic borrowing is higher than that on international borrowing, there is no effective penalty for default. Arbitrage opportunities are ruled out by the participation constraint (7).

This leads to the definition of an equilibrium in this economy. To distinguish it from the equilibrium with solvency constraints considered above, we will refer to it as a *debt constrained equilibrium*.

Definition 4 A *debt constrained equilibrium* is an allocation $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1}), f_n^m(\theta^t, \theta_{t+1})\}$, and a price system $\{q(\theta^t, \theta_{t+1}), p^m(\theta^t, \theta_{t+1})\}$ for all countries m , such that for all countries m and types n , the allocation solves the residents problem given the price system and initial asset holdings, and markets

clear:

$$\begin{aligned} \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m c_n^m (\theta^t) &\leq \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m e_n^m (\theta^t), & \text{for all dates } t \text{ and all histories } \theta^t \\ \sum_{n=1}^N \lambda_n^m b_n^m (\theta^t, \theta_{t+1}) &= 0, & \text{for all } \theta_{t+1} \text{ and all } m, \\ \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m f_n^m (\theta^t, \theta_{t+1}) &= 0. & \text{for all } \theta_{t+1}. \end{aligned}$$

The characterization of this equilibrium is inhibited by the fact that the consumption set of a resident is not convex. This follows from the fact that $D_n^m (b, \theta^s, \{p^m\})$ is typically strictly concave in b for all θ^s and $\{p^m\}$. Jeske [12] shows how this problem can be surmounted. For any history θ^s , and given a price system, define the time-zero domestic price of a unit of the consumption good in after history $\theta^s = (\theta^{s-1}, \theta_s)$ by

$$P^m (\theta^s) = P^m (\theta^{s-1}) p^m (\theta^{s-1}, \theta_s),$$

with $p^m (\theta^0) = 1$. For a resident of type n in country m , the transfer they make to foreigners after any history θ^s is equal to

$$nx_n^m (\theta^s) = \sum_{\theta_{s+1}} q (\theta^s, \theta_{s+1}) f_n^m (\theta^s, \theta_{s+1}) - f_n^m (\theta^s),$$

which we denote by $nx_n^m (\theta^s)$ as the net exports of a resident of type n .

Clearly, if the value of the future stream of such transfers, evaluated at domestic prices, was positive, a resident would be better off by defaulting on their debts. That is, a necessary condition for repayment is that foreign bond holdings satisfy

$$\sum_{\theta^s \geq \theta^t} P^m (\theta^s) nx_n^m (\theta^s) \leq 0, \tag{9}$$

for all histories θ^t , where the notation $\theta^s \geq \theta^t$ is used to denote that the summation is over all histories that continue θ^t . Note that this constraint defines a convex subset of the set of affordable allocations. The following proposition shows that the solution to the *auxiliary residents problem*, which is simply problem

(PP) modified in that the sequence of participation constraints (7) is replaced by the sequence in (9), is also the solution to the original problem.

Proposition 3 *A solution to the auxiliary residents problem is also a solution to the residents problem (RP).*

Proof. Jeske [12] Propositions 3 and 4. ■

As domestic markets are complete, and all contracts are enforceable, it must be the case that in equilibrium we have

$$p^m(\theta^{t-1}, \theta_t) = \beta \pi(\theta_t | \theta^{t-1}) \frac{U_n^{m'}(c(\theta^{t-1}, \theta_t))}{U_n^{m'}(c(\theta^{t-1}))} = \beta \pi(\theta_t | \theta_{t-1}) \frac{U_n^{m'}(c(\theta^{t-1}, \theta_t))}{U_n^{m'}(c(\theta^{t-1}))}. \quad (10)$$

Importantly, all residents of a country have the same marginal rate of substitution. This also implies that, in any state of the world θ^t , either all residents are on their participation constraints (7) or none are constrained. The intuition is that, if some residents were constrained while others were not, the unconstrained residents could borrow from international markets and lend to the constrained residents. In this sense, we can refer to a country as a whole as being either constrained or unconstrained. Further, if a country m is unconstrained, it must be that the international bond price equals the marginal rate of substitution of all residents in that country. If the residents of a country are constrained, they would like to borrow more, and their marginal rates of substitution must be lower than this price. This implies that

$$q(\theta^{t-1}, \theta_t) = \max_{m=1, \dots, M} p^m(\theta^{t-1}, \theta_t). \quad (11)$$

That is, domestic interest rates are always at least as high as international interest rates. It is this fact that acts as a deterrent to default; a resident who defaults would only have access to a market in which interest rates are higher in the periods in which an individual wants to borrow.

Note that if we use these values for p^m , we get

$$P^m(\theta^t) = \beta^t \pi(\theta^t) \frac{U_n^{m'}(c(\theta^t))}{U_n^{m'}(c(\theta^0))}.$$

Substituting into the relaxed constraint (9) for θ^0 and rearranging gives

$$\sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^{m'}(c(\theta^t)) [e_n^m(\theta^t) - c_n^m(\theta^t)] \leq 0.$$

This equation looks very similar to the participation constraint in the default model of Wright [20]. The difference is that, in the present model, the agent chooses its own consumption taking the prices, and hence the $U_n^{m'}(c(\theta^t))$, as given. In Wright [20], the optimizing government takes into account the fact that its actions affect these prices.

In our analysis so far, we have concentrated solely on the residents decision to default. A natural question that arises is whether it is possible for residents to voluntarily enter into agreements that their government would like to repudiate? As agents cannot coordinate in default, is it possible that they could receive allocations worse than autarky in some states of the world? Under our maintained assumption that governments redistribute domestic claims in accordance with their welfare weights, the following proposition shows that this is never the case.

Proposition 4 *In the debt constrained equilibrium, the $\{\phi_n^m\}$ -weighted average of residents utilities, in the continuation of the allocation after any history θ^t , is always no less than $D^m(\theta^t, \phi^m)$.*

Proof. We show the result for $t = 0$; the argument is analogous after any other history. As the government is able to set transfer to maximize the weighted average of residents utilities, the weighted

average of residents utilities in country m can be written as the solution to the problem

$$\max_{c^m} \sum_{n=1}^N \phi_n^m \sum_{t=0}^{\infty} \beta^t \sum_{\theta^t} \pi(\theta^t) U_n^m(c_n^m(\theta^t)),$$

subject to the date zero budget constraint

$$\sum_{t=0}^{\infty} \sum_{\theta^t} P^m(\theta^t) \sum_{n=1}^N c_n^m(\theta^t) \leq \sum_{t=0}^{\infty} \sum_{\theta^t} P^m(\theta^t) \sum_{n=1}^N [e_n^m(\theta^t) - nx_n^m(\theta^t)],$$

given sequences of prices and net exports. By the result of Proposition 3, the autarky allocation remains affordable at these prices. But as it was not chosen, it cannot yield a greater value for $\{\phi_n^m\}$ -weighted resident utilities. ■

As the debt constrained allocations are feasible and, by the result of Proposition 4, give the country at least the autarkic utility level, we have also established that they are no better than the constrained efficient allocations. It is also straightforward to produce examples, such as the one examined in Section 5, in which the debt constrained equilibrium is strictly worse than the constrained efficient equilibrium. This suggests that government intervention could be Pareto improving. The obvious remedy would be for the government to reform the domestic law enforcement system to allow domestic residents to commit to honoring contracts with foreigners. Capital flows would still be constrained because the government itself has no means to commit itself to retaining these enforcement institutions; that is, national repudiation risk would remain. Indeed, Section 3 above established that if such reform was possible, allocations would be efficient.

However, in practice such reform may not be possible. Current efforts at reducing corruption in the developing world, of which removing partiality towards domestic residents in judicial systems is one part, have run into a number of difficulties. National governments are often unable to effect reform of domestic institutions. However, they typically do retain control over their borders, and hence also over flows of capital. Could capital controls be welfare improving? We show in the next subsection that, in fact, subsidized capital

flows, and not capital flow restrictions, can be used to effect a Pareto improvement.

4.2 Efficient Borrowing Subsidies

The inefficiency of capital flows when residents may individually default results from the fact that the penalty from default is too mild; defaulters may access international capital markets in effect using other residents as intermediaries. This suggests that there is room for efficiency improving government intervention aimed at increasing the penalty to default. One possibility would be for the government to directly tax default. However, this would require that the government observe the identities of individuals who default, and essentially amounts to directly enforcing contracts. As argued above, this is not feasible in many less developed economies. Instead we consider an alternative form of intervention that does not require that the government observe the identities of residents in default. Specifically, we allow governments to subsidize international financial market access. In this way, a resident who defaults and is excluded from financial markets, is penalized by being denied access to the subsidy.

Consider the constrained efficient allocation $\{c_n^m(\theta^t)\}_{n,m,\theta^t,t}$ constructed in Section 2 above. Given this allocation, we can construct sequences of domestic and international bond prices using the formulae (10) and (11). By the results above, we know that this allocation evaluated at these prices will in general violate the sequence of participation constraints (9). Moreover, the equality of marginal rates of substitution over residents of a country implies that this constraint will be violated for all residents of a country at the same time. Knowledge of the allocation also allows us to calculate the contribution of each type to net exports, as $nx_n^m(\theta^t) = e_n^m(\theta^t) - c_n^m(\theta^t)$, which given an international price for bonds in this new equilibrium $\hat{q}(\theta^t, \theta_{t+1})$ (which will in general differ from the q calculated from equation (11)), implies asset holdings of

$$nx_n^m(\theta^t) = \sum_{\theta_{t+1}} \hat{q}(\theta^t, \theta_{t+1}) f_n^m(\theta^t, \theta_{t+1}) - f_n^m(\theta^t).$$

Our approach will be to introduce a subsidy on international lending so that the rents gained from access to international markets are larger, and the penalties from being excluded are greater. This will have the effect of reducing the transfer overseas made by a resident in periods when they would be tempted to default. Attaching the subsidy to new lending is notationally simpler, but equivalent to, a policy of subsidizing a residents interest payments on past borrowing.

In order to introduce such a subsidy, it will be necessary to distinguish between before-tax and after-tax international bond prices. Specifically, denote by $q^m(\theta^t, \theta_{t+1})$, the price of an international state contingent security, after taxes and subsidies, in country m . This will be determined by the international price $\hat{q}(\theta^t, \theta_{t+1})$ and any tax (or subsidy if negative) $\tau^m(\theta^t, \theta_{t+1})$, according to

$$q^m(\theta^t, \theta_{t+1}) = (1 + \tau^m(\theta^t, \theta_{t+1})) \hat{q}(\theta^t, \theta_{t+1}),$$

where for the moment we are allowing this subsidy to be country specific. A *government policy* is then a sequence of taxes and subsidies on foreign lending $\tau^m(\theta^t)$, and a sequence of lump-sum taxes $T_n^m(\theta^t)$, which for the moment we allow to be resident specific.

Our task is, given a consumption allocation, to find a government policy and a sequence of before-tax prices $\{\hat{q}(\theta^t, \theta_{t+1})\}$ such that the implied securities holdings can be supported in a debt-constrained equilibrium. This leads to the following definition.

Definition 5 *A debt constrained equilibrium with taxes is an allocation $\{c_n^m(\theta^t), b_n^m(\theta^t, \theta_{t+1}), f_n^m(\theta^t, \theta_{t+1})\}$, a government policy $\{\tau^m(\theta^t), T_n^m(\theta^t)\}$, and a price system for international bonds $\{\hat{q}(\theta^t, \theta_{t+1})\}$, and domestic bonds $\{p^m(\theta^t, \theta_{t+1})\}$ for all countries m , such that for all countries m and types n , the allocation solves the residents problem given the price system, government policy and initial asset holdings, each*

government's budget balances,

$$\sum_{t=0}^{\infty} \sum_{\theta^t} \sum_{n=1}^N P(\theta^t) [\tau^m(\theta^t) \widehat{q}(\theta^t, \theta_{t+1}) f_n^m(\theta^t, \theta_{t+1}) - T_n^m(\theta^t)] = 0,$$

and markets clear:

$$\begin{aligned} \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m c_n^m(\theta^t) &\leq \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m e_n^m(\theta^t), && \text{for all dates } t \text{ and all histories } \theta^t \\ \sum_{n=1}^N \lambda_n^m b_n^m(\theta^t, \theta_{t+1}) &= 0, && \text{for all } \theta_{t+1} \text{ and all } m, \\ \sum_{m=1}^M \sum_{n=1}^N \lambda_n^m f_n^m(\theta^t, \theta_{t+1}) &= 0. && \text{for all } \theta_{t+1}. \end{aligned}$$

The following proposition establishes the main result of this section.

Proposition 5 *If implied international interest rates are high, any constrained efficient consumption allocation $\{c_n^m(\theta^t)\}$ can be supported as a debt constrained equilibrium with taxes.*

Proof. The proof is constructive. Given an allocation, and a first estimate of prices from (10) and (11), we can check whether at the efficient allocation a residents participation constraint is satisfied. Let H be the (countable) set of all possible histories θ^t , and let NV be that subset of H , such that for all $\theta^t \in NV$ no residents participation constraints are violated at θ^t . For all $\theta^t \in NB$ set the international prices $\widehat{q}(\theta^t, \theta_{t+1}) = q(\theta^t, \theta_{t+1})$ so that for all m , $\tau^m(\theta^t) = 0$.

Given the efficient allocation, we can form net exports for a country after any history as $nx^m(\theta^t) = e^m(\theta^t) - c^m(\theta^t)$. For all $\theta^t \in H \setminus NB$, we can use these values to construct an estimate of the extent to which a country as a whole is violating its participation constraint (noting that if some residents of a country have violated their constraints, then all residents of that country have, and we can without loss focus on country aggregates)

$$- \sum_{\theta^s \geq \theta^t} P^m(\theta^s) nx^m(\theta^s)$$

which are negative (as the constraints are violated) and finite if international interest rates are high (as $P^m(\theta^t) \leq Q(\theta^t)$).

In equilibrium, at the world pre-tax bond price, we have

$$nx^m(\theta^t) = \sum_{\theta_{t+1}} \hat{q}(\theta^t, \theta_{t+1}) f^m(\theta^t, \theta_{t+1}) - f^m(\theta^t).$$

Our aim is to find a sequence of values for extra surplus $S^m(\theta^t) \geq 0$ for each country and each $\theta^t \in H \setminus NB$ for which its participation constraints are violated, such that when the surplus is added, these participation constraints are then satisfied. In equilibrium, we will distribute this surplus in terms of a subsidy on lending, so that

$$S^m(\theta^t) = -\tau(\theta^t) \sum_{\theta_{t+1}} \hat{q}(\theta^t, \theta_{t+1}) f^m(\theta^t, \theta_{t+1})$$

(where if $\tau < 0$ we have a subsidy). This is a countable sequence of affine equations in a countable number of unknowns which we can index by j . It is convenient to view this as a mapping from the closed e ball in l_∞ into itself, where e is defined as the largest possible value of the aggregate endowment. Then, as the Arrow-Debreu prices are absolutely summable, the mapping is a bijection and there is a unique solution for the surplus sequence.

Given initial asset holdings, we can then solve for future asset holdings by iterating on the following. Given $f^m(\theta^t)$, we can find $f^m(\theta^t, \theta_{t+1})$ from, for countries whose constraints bind at θ^t ,

$$\begin{aligned} nx^m(\theta^t) - S^m(\theta^t) &= (1 + \tau(\theta^t)) \sum_{\theta_{t+1}} \hat{q}(\theta^t, \theta_{t+1}) f^m(\theta^t, \theta_{t+1}) - f^m(\theta^t) \\ &= \sum_{\theta_{t+1}} q(\theta^t, \theta_{t+1}) f^m(\theta^t, \theta_{t+1}) - f^m(\theta^t), \end{aligned}$$

where $q(\theta^t, \theta_{t+1})$ is the level of after-tax international bond prices, which have been set equal to the price level constructed in (11). Given estimates of bond holdings for countries whose constraints are binding at

θ^t , we can construct the aggregate holdings for countries whose constraints are not binding from the market clearing condition. If the set of such countries at θ^t is given by $M(\theta^t)$, as these countries set no subsidy, and trade at $\hat{q}(\theta^t, \theta_{t+1})$, we can use

$$\sum_{m \in M(\theta^t)} nx^m(\theta^t) = \sum_{m \in M(\theta^t)} \left[\sum_{\theta_{t+1}} \hat{q}(\theta^t, \theta_{t+1}) f^m(\theta^t, \theta_{t+1}) - f^m(\theta^t) \right],$$

to get an estimate of the $\hat{q}(\theta^t, \theta_{t+1})$ (and hence $\tau(\theta^t)$) where we have noted that $\tau(\theta^t)$ is common over all assets at this history, and all countries whose constraints bind.

The precise sequence of lump-sum taxes across residents is indeterminate. However, its present value, along with initial domestic bond transfers, can be determined from residents lifetime budget constraints. ■

In Section 5 below, in the context of a simple numerical example that can be computed by hand, we show how to construct such a government policy. That example is tractable because it exploits symmetry over agents and stationarity over time to simplify the construction of the optimal sequence of subsidies. More generally, the following method, which relies on the fact that interest rates are high and hence histories that are distant in time are significantly discounted, appears to work well in practice. For each $n = 1, 2, 3, \dots$, set all but the first n elements of the surplus sequence to zero, and solve the first n equations for the participation constraint. By construction, these equations are linearly independent, and have a unique solution. Repeating for each n gives us, for each j , a monotone decreasing sequence of values that are bounded below by zero and hence converge. This is our $\{S^m(\theta^t)\}$ for all $\theta^t \in H \setminus NB$.

In assessing the informational advantages of such a policy, the only direct requirement is that the government observe the level of capital flows across its borders. However, in order to determine the appropriate level of the subsidy, the government requires a large amount of information about the structure of the world economy. Of course, in order to attain its welfare objectives, the government also requires a lot of information about residents identities in order to ensure that initial transfers, as well as the lump-sum taxes required

to pay for this subsidy, are allocated correctly.

5 An Illustrative Example

Consider the following example which can be solved by hand. There are two agents each with identical logarithmic felicity functions. There are two states of the world $\theta = 1, 2$, with type θ agents receiving the high endowment $1 + y$ for some $y \in (0, 1)$ when the state is θ , and the low endowment $1 - y$ otherwise. The initial state of the world determined by a coin flip, after which it alternates deterministically, or $\pi(2|1) = \pi(1|2) = 1$. The total population of each country is normalized to one, and total measure of each type throughout the world is set to one, so that the world endowment is fixed at two for all periods in all states. However, the distribution of each type within each country varies, with type one agents being of measure $1 - \omega$ in country one for some $\omega < 1/2$. The aggregate endowment of country one is $e^1(1) = 1 + y(1 - 2\omega) > 1$ in state one and $e^1(2) = 1 - y(1 - 2\omega) < 1$ in state two meaning that state one is the “good state” for country one.

5.1 National Default

When nations, but not individual residents, default and preferences are isoelastic we can solve for efficient allocations using a completely standard limited commitment model. Restricting attention to the symmetric stationary efficient allocation, consumption alternates between $1 + x$ and $1 - x$ where

$$x = \min_{z \geq 0} \{ \log(1 + z) + \beta \log(1 - z) \geq \log(1 + y(1 - 2\omega)) + \beta \log(1 - y(1 - 2\omega)) \}.$$

The critical values of β are give by

$$\beta_{FI} \equiv -\frac{\log(1 + y(1 - 2\omega))}{\log(1 - y(1 - 2\omega))}, \quad \text{and} \quad \beta_A \equiv \frac{1 - y(1 - 2\omega)}{1 + y(1 - 2\omega)}.$$

If $\beta \geq \beta_{FI}$, the solution to this problem is $x = 0$ and full insurance is achievable. If $\beta \leq \beta_A$, the expression $\log(1+z) + \beta \log(1-z)$ reaches a maximum at $z = y$ and only autarky is feasible.

To determine aggregate capital flows, note that world interest rates are determined by the marginal rate of substitution of the country that is unconstrained tomorrow. Then we must have $q = \beta(1+x)/(1-x) > \beta$, and so world interest rates are lower than under complete markets. To determine the borrowing constraints on nations consistent with this allocation, the country budget constraints combined with symmetry and stationarity over time give $f^* = (y(1-2\omega) - x)/(1+q)$. Hence, the solvency constraints for countries expecting the high shock next period must be set to this f^* .

To see how these allocations can be decentralized, note that in the stationary allocation individual bond holdings must satisfy $f_t^i = (e_t^i - c_t^i)/(1+q)$. In general, this will be positive for types with the high endowment and negative for types with the low endowment. However, the solvency constraints will bind for all types at the same dates. Consequently, these solvency constraints will enforce minimum positive holdings of these assets for some agents. This illustrates our second criticism of this decentralization.

To illustrate the third criticism consider a subgame that begins in period t and in which a country, say country one, has defaulted. From the perspective of agents of type two, their international claims have been extinguished, while the debts of agents of type one have been extinguished. Offsetting this is the fact that type two agents will now gain from having a higher endowment when the aggregate endowment of the country is small. As these transfers are unlikely to exactly offset in welfare terms, it will be necessary for the national government to transfer resource between types in the event of a default.

5.2 Individual Resident Default

We now consider the case in which enforcement institutions discriminate in favor of domestic residents. To characterize the debt constrained allocations, once again we will focus on symmetric allocations, and in the

light of Proposition 4, we will consider the case in which $\beta > \beta_A$. Summing the lifetime budget constraints of individuals starting from a date in which the country has the high shock, we get

$$\frac{1 + x^m + q(1 - x^m)}{1 - pq} \leq \frac{1 + y(1 - 2\omega) + q(1 - y(1 - 2\omega))}{1 - pq} + \frac{f - qf' + q(f' - qf)}{1 - pq},$$

where f denotes bond holdings that pay off in the high state, and f' bonds that pay off in the low state. By Proposition 3, the last term which represents the rents earned from access to international markets, must be zero, as the participation constraint (9) binds in the high state. Rearranging this, and substituting for the value of net exports, we get $x^m(1 - q) = y(1 - 2\omega)(1 - q)$. There are two solutions to this equation. The first is autarky, or $x^m = y(1 - 2\omega)$, while the second requires $q = 1$. Substituting this into the expression for q we get $x^m = (1 - \beta) / (1 + \beta)$, and $p = \beta^2$, and note that $x^m < y(1 - 2\omega)$ as long as $\beta > \beta_A$. International interest rates are zero so that international debt is a bubble asset; it is a similar mechanism that generates the existence of debt in Hellwig and Lorenzoni [11]. Domestic interest rates are positive, which provides enough surplus from the access to international financial markets to deter default. Exploitation of the apparent arbitrage opportunity is prevented by the existence of the debt constraints.

Under symmetry, the (absolute value) of trade at any point in time is equal to $y(1 - 2\omega) - x^m$, which given that $q = 1$ implies that the level of bond trade is equal to $(y(1 - 2\omega) - x^m) / 2$. Note that this is due to the assumption of symmetry; equivalent allocations could have been obtained by trading a non-interest paying token of value $y(1 - 2\omega) - x^m$ in each period. This equivalence highlights the similarity of these allocations to those derived from the turnpike monetary model of Townsend [19]. And as in that model, the payment of interest on this token, or in our case the subsidization of capital flows, can lead to a welfare improvement.

Let \hat{q} denote the pre-tax (or subsidy) price of an international claim. We will consider a policy of taxing or subsidizing international capital flows at rate τ , so that the after tax price is $(1 + \tau)\hat{q}$. We will set this τ such that the after tax price of such a claim in a country that is not constrained is equal to the efficient

international price q derived above, or $q = (1 + \tau)\hat{q}$, and we will show that in equilibrium, this τ is negative and thus represents a subsidy, so that $\hat{q} < q$. We impose within period balanced government budgets so that, at least in aggregate, the total cost of providing this subsidy, τqf , is paid for by lump-sum taxes T . This ensures that the government does not borrow internationally; that is, all capital flows are private. Under this constraint, at the efficient domestic prices, and given appropriate initial bond holdings, the efficient allocation still satisfies the lifetime budget constraint of a resident. Given that the after tax prices are at the efficient levels, the marginal conditions for optimality are satisfied. The only thing that remains to be checked is that the participation constraint (9) is satisfied or our choices of T and τ .

The relevant participation constraint for an agent, starting in a period in which it binds, is given by

$$0 = P_0 f_0 - P_0 q_1 f_1 + P_1 f_1 - P_1 q_2 f_2 + P_2 f_2 - \dots$$

As the agent is not constrained in period one, $P_0 q_1 = P_0 p_1 = P_1$, while as the agent is constrained in period two we have that all the terms starting with $P_2 f_2$ onwards sum to zero. Hence we have $0 = P_0 f_0 - P_1 q_2 f_2$.

As the agent is constrained in period two, we have $q_2 = \hat{q} = q / (1 + \tau)$, and normalizing $P_0 = 1$ and imposing $f_0 = f_2$ by symmetry we get $\tau = q^2 - 1$, which is negative.

6 Conclusion

The shift in emerging market capital flows away from sovereign towards private borrowers has led to much speculation about changes in the pattern of future capital flows, their efficiency, and the likelihood of future debt repudiations. In this paper we have shown that the answer to these questions depends on the enforcement institutions of a country and the extent to which they discriminate between domestic residents and foreigners in the enforcement of contracts. If enforcement is symmetric, the pattern of private capital flows

is identical to the pattern of sovereign capital flows in a traditional model of sovereign debt. If, however, enforcement favors domestic residents, private capital flows will tend to be inefficiently low, generating the possibility for welfare improving government regulation. Optimal regulation may take on many forms, but is unlikely to include restrictions on capital flows. In fact, we have shown above that subsidizing access to international financial markets can improve efficiency as it increases the benefits to accessing international markets, and thus increases the penalty associated with exclusion from these markets.

A subsidy on capital flows may be preferable to some alternative schemes on the basis of the information required of the government: the government need not know the identity of the borrower or whether they have defaulted. Nonetheless, the information requirements for a subsidy are still large. In the present perfect information environment, it is impossible to definitively investigate the superiority of a particular scheme in this regard. To investigate the optimal regulation of capital flows in a private information regulation environment will necessitate further work on the decentralization of optimal allocations with imperfect information. Some work in this regard has been undertaken by Prescott and Townsend [17] and [18], and Bisin and Gottardi [3].

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