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Real Exchange Rate Fluctuations and Endogenous Tradability

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Abstract

This paper examines, empirically and theoretically, the sectoral decomposition of the volatility of real exchange rates. For the purpose of this decomposition, goods are classified as being traded or nontraded in international markets. The first part performs an empirical analysis for a broad cross section of countries. The relative price of nontraded goods to traded goods is found to be relatively more important in movements of real exchange rates of the country pairs that maintain stable nominal exchange rates. The paper goes on to construct a model with endogenous tradability to suggest an explanation for the evidence. The key features of the model are heterogeneous productivity, transport costs, and sticky wages. The nontraded sector arises from non-zero trade costs. The relative price of goods depends on productivity, transport costs, and in the short run, on the exchange rate regime. The calibration shows that the relative price of nontraded goods makes a much greater contribution to overall real exchange rate volatility under a fixed exchange rate regime than a flexible regime, as in the data.

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1 Introduction

This paper studies the sectoral decomposition of the volatility of real exchange rates. The real exchange rate between two countries is the relative price of a representative goods basket. The sectoral decomposition of real exchange rate fluctuations is important, because it has important implications for the dynamic adjustment of an open economy to exogenous shocks. For some countries, the movements in relative price levels come from the relative prices of internationally nontraded goods such as housing or construction, while for others from those of traded goods such as manufactures. The empirical part in the next section finds that, while in general the relative prices of traded goods are the most important in driving real exchange rate movements, the relative prices of nontraded goods are relatively more important for the country pairs that maintain stable nominal exchange rates.

To explain the empirical evidence, I construct a general equilibrium monetary model with heterogeneous productivity and endogenous tradability. The model shows how real exchange rate dynamics are connected to shifts in the tradability of goods through firms' price setting behavior. In some cases, the shifts and the simultaneous movements in nominal exchange rates lead to strong substitution effects effects among traded goods. In these cases, the relative prices of traded goods dictate the movements in real exchange rates. Therefore, limiting flexibility in nominal exchange rates can delay the adjustment in the relative prices of traded goods and raise the contribution to the volatility of the relative prices of nontraded goods, as observed in the data. My model offers a set of reasons why the relative price of nontraded to traded goods may be the major source of fluctuations in real exchange rates. The model can be extended to perform welfare analysis under various trade structures. Clearly, it also has strong implications for a design of exchange rate policy. These are the contributions of my paper.

In principle, we can decompose the fluctuations in real exchange rates into their traded and their nontraded goods components. The traded component is deviations from the law of one price for traded goods, while the nontraded component is fluctuations in the relative prices of nontraded to traded goods across countries. The traded component is by far dominant for the real exchange rates among the OECD countries that allow their currency to move freely (Engel 1999; Obstfeld 2001). To the contrary, the nontraded component is significantly more important in international data (Betts and Kehoe 2001a). The relative volatility of the nontraded real exchange rates and the overall volatility is 30 percent on average. The number becomes much higher as the volatility of real and nominal exchange rates falls. Such a pattern is consistent with what was found in the study on Mexico's real exchange rates by Mendoza (2000). As I present in the next section, the volatility of the nontraded real exchange rates even exceeds its traded counterpart and the overall volatility in some cases. In particular, the nontraded component of the real exchange rates between several member countries of the European Monetary System has displayed substantial volatility. In light of these empirical features, there seem to be linkages between the exchange rate regime and the sectoral decomposition of real exchange rate fluctuations.

In fact, the studies by Mussa (1986) and Baxter and Stockman (1989) have confirmed that the real exchange rate volatility is very different under different exchange rate regimes. The literature on real exchange rate fluctuations is precisely divided by the views regarding the source of these fluctuations. One strand of the literature puts an emphasis on the nontraded component by assuming nominal rigidities in the nontraded sector or in factor prices (see Dornbusch (1983) and Hau (2000), for example). The other asserts the importance of the traded component or deviations from the law of one price. (see Betts and Devereux (1996; 2000; 2001), Chari et al. (2002), Obstfeld (2000), and Rogoff (1996)). Few researchers have attempted to reconcile the literature using a theory of endogenous tradability. Betts and Kehoe (2001b) model endogenous tradability in a flexible price two-country framework. Bergin and Glick (2003) use a two-period small country model where firms take world prices as given. In both studies, the source of heterogeneity is product-specific transport costs. The mechanism therein is essentially the tradeoff between quantity and price adjustments in the nontraded sector. Once the nontraded sector can shrink or expand in response to shocks, the volatility of the relative price of nontraded to traded falls. The approach provides a rationale for cases where the nontraded component exhibits moderate to low volatility. Unfortunately, it fails to explain why we observe much higher volatility in the nontraded component for a significant number of country pairs.

In my view, the more important source of heterogeneity than product-specific transport costs is product-specific productivity, because it drives firm-specific pricing strategies and international trade, giving rise to fundamental differences in prices across countries. (See Baier and Bergstrand (2001) for evidence against the role of transport costs in the expansion of world trade.) Several empirical studies have documented differences in the frequency and the magnitude of price changes across product categories. (See the recent studies by Bils and Klenow (2002) and Campa and Goldberg (2002), for example. Taylor (1999) provides an excellent survey of the topic.) With the sectoral heterogeneity in price dynamics, shifts in the composition of import bundles in response to shocks must play a major role in the international shock transmission mechanism. I use this mechanism to study the volatility decomposition of real exchange rate between their traded and nontraded components. It should be noted that the heterogeneity in price dynamics has also attracted some researchers to explore its potential as an explanation for the persistence in real exchange rates (Imbs et al. 2002a; 2002b).

In order to address the issue of heterogeneous price dynamics in an open-economy context, production and trade patterns must be endogenous. The model is a dynamic and monetary version of the trade model with a continuum of goods by Dornbusch et al. (1977). There is a continuum of differentiated goods and goods markets are monopolistically competitive. Firms are differentiated by productivity and they set prices in their own currency. I abstract from local currency pricing because the main predictions of my model carry through regardless of the currency of denomination. I assume that prices are flexible and the source of nominal rigidities is households' wage-setting behavior. In fact, Huang and Liu (2002) find that wage stickiness is more powerful than price stickiness in explaining persistence of output. Although wage-stickiness can potentially give rise to weakly countercyclical movements in real wages, recent empirical studies do not suggest that real wages are systematically procyclical or countercyclical (Abraham and Haltiwanger 1995; Christiano, Eichenbaum, and Evans 1999). Hence, assuming away price stickiness helps simplify a framework where an analysis of monetary policy is possible (see Collard and Dellas (2002), for example). In my model, wage stickiness arises from convex adjustment costs *à la* Rotemberg (1982). Deviations from the law of one price take a form of iceberg-type transport costs and whether a good is traded is therefore endogenous.

In a free trade world, patterns of trade and production mostly follow the principle of comparative advantage. Trade is, however, not solely driven by comparative advantage, because of monopolistic competition in product and labor markets. Both countries produce exportables and nontradables, and import a range of goods from each other. By comparative advantage, both countries produce and export goods produced by relatively productive firms. The exportable firms of a country are therefore more productive than its nontradable ones. Heterogeneous productivity also leads to a heterogeneous price dynamics. However, there is no clear relationship between the sectoral productivity and price dynamics, because a shock can affect the productivity of different sector differently depending on how it changes the size of each sector. As a result, the relative volatility of prices of imports, exports and nontraded goods is ambiguous and depend on the type of shocks. The sectoral composition of real exchange rate volatility mainly depends on the size of exchange rate expenditure switching effects.

Given a flexible exchange rate regime, the nominal exchange rate endogenously responds to shocks. In a model with nominal rigidities, the exchange rate movement is the central adjustment mechanism, because it alters the relative prices of traded goods and causes the consumers to substitute between imports and domestically produced good. The expenditure switching effect can be captured by changes in expenditure share of export goods, import goods and nontraded goods, comparing to those under fixed exchange rate regime. When the expenditure switching effects is large, shutting down the nominal exchange rate channel can create large swings in the relative price of nontraded goods, which is an alternative adjustment mechanism. In that case, the relative price of nontraded to traded goods can partly account for the volatility of real exchange rates. So, the question here is, when do we observe a large expenditure switching effect?

My paper explores several answers to this question based on endogenous tradability. In my model, tradability or the size of traded goods sector is the share of consumption of all traded goods in a consumption basket. It partially determines the expenditure share of goods, because the expenditure share consisted of two component: The share of goods in a consumption basket and their price. Intuitively, the substitution among traded goods becomes an important adjustment channel when their consumption share is large. One key factor that influences the consumption share of traded goods is transport costs. Transport costs raise the relative price of imports and therefore lower the consumption share of traded goods and expand the size of the nontraded sector. The decrease in tradability is the mechanism through which a rise in transport costs decreases the expenditure switching effect. Besides transport costs, the intratemporal elasticity of substitution is also directly relevant. With a higher value of the elasticity of substitution, quantity becomes very responsive to a shock and an adjustment process can take place through small price changes. Therefore, the effect of the elasticity on the expenditure is ambiguous.

To investigate the relationships between the expenditure switching effect and transport costs, and the intratemporal elasticity of substitution, I calibrate the model based on a productivity shock and an interest rate shock, with several transport costs and elasticity parameters, under two different exchange rate regime. The impulse responses confirm that the nontraded component of real depreciation becomes more volatile under a fixed exchange rate regime as transport costs rise, regardless of type of shocks. However, there is no clear relationship between the elasticity of substitution and the sectoral decomposition of the real exchange rate volatility.

The gist of a theory of endogenous tradability lies in a temporary shift in patterns of trade as a result of an exogenous shock. In the short run, a positive productivity shock in the home country raises real wage in the home country, and cause some of the home exporters to become nontraded goods producers, while some of the foreign nontraded firms become new exporters. Since the new foreign exporters are initially not so productive, their goods are produced with relatively high cost and contributes to the rise in import prices in the home country. As a result, we observe real exchange rate appreciation. The impulse responses of under two exchange rate regimes are mostly identical, apart from the policy variables. They also comfirm that the nontraded component of real depreciation becomes more volatile under a fixed exchange rate regime as transport costs rise.

Interestingly, the shift of patterns of trade in response to a positive foreign interest rate shock is qualitatively identical that with a productivity shock, although it is from a different mechnism. A rise in foreign interest rate reduces foreign demand and output, and that raises terms of trade and real wage in the home country. As a result, we also observe real depreciation. The nontraded component of real appreciation also depicts higher volatility under a fixed exchange rate regime. Such a pattern becomes stronger as transport costs fall. When the expenditure switching effect of exchange rates is measured by the differences in impulse responses of expenditure share, it is found to be decreasing in transport costs, regardless of type of shocks. Its relationship with the intratemporal elasticity of substitution is ambiguous. Such findings support the reasoning that the component of the volatity of real exchange rate is influenced by exchange rate policy and degree of trade integration.

To summarize, my model predicts that the nontraded component of the real exchange rate of a pair of countries that are not highly integrated and maintain stable nominal exchange rates is not negligible. Such movements in real exchange rate can be understood better through a model where monetary nonneutrality coexists with a trade model.

The rest of the paper is organized as follows. The next section gives stylized facts of real exchange rate volatility. The model is developed in Section 3. Section 4 explains the equilibrium dynamics and the simulation results. I conclude the paper in Section 5.

2 Stylized Facts

This section gives empirical regularities about the volatility of real exchange rates, their sectoral composition and their connection with exchange rate policy. It illustrates that the volatility of real exchange rates of country pairs that maintain stable nominal exchange rates tends to come from the nontraded component.

The data are quarterly and cover the period from 1980:1 to 1998:4. The data set covers 35 countries and produces 595 pairs of bilateral real exchange rates. [1] The price data are from the International Financial Statistics (IFS). The exchange rate data are originally from the World Currency Report and provided by Reinhart and Rogoff (2004). They are viewed as market-determined exchange rates, unlike the conventional official exchange rates in the IFS database. In the case of emerging markets, there are large discrepancies between the exchange rate data from the two databases. I use the market exchange rates because they reflect the stance of monetary policy bettter and they are relevant to the equilibrium allocations.

I construct series of real exchange rates and their components using one of the methods in Engel (1999) and Betts and Kehoe (2001a). Define the real exchange rate as $Q_t = S_t P_t^\star / P_t$, where S_t is the nominal exchange rate, P_t and $P_t^\star$ are the home and foreign price level. Define the traded component as $Q_{t,T} = S_t P_{t,T}^\star / P_{t,T}$, where $P_{t,T}$ and $P_{t,T}^\star$ are traded goods price indices in the two countries. Then the nontraded component can be defined as

$$Q_{t,N} = Q_t / Q_{t,T}.$$

For instance, if we assume geometric price indices in both countries $P_t = P_{t,T}^{1-\delta} P_{t,N}^{\delta}$ and $P_t^\star = P_{t,T}^{\star 1-\delta^\star} P_{t,N}^{\star\delta^\star}$, then $Q_{t,N} = (P_{t,N}^\star / P_{t,T}^\star)^{\delta\star} / (P_{t,N}/P_{t,T})^{\delta}$. The weights δ and $\delta^\star$ are the consumption shares of nontraded goods in the two countries.

I use the consumer price index (CPI) as the measure of overall good prices and the producer price index (PPI) as the measure of traded goods prices. In logarithms,

$$q_t = s_t + \ln(CPI_t^\star) - \ln(CPI_t),$$

[1] The sample countries are Australia, Austria, Belgium, Brazil, Canada, Chile, Colombia, Denmark, Egypt, El Salvador, Finland, Germany, Greece, India, Indonesia, Ireland, Israel, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Singapore, Spain, Sri Lanka, Sweden, Switzerland, Thailand, United Kingdom, United States and Venezuela.

$$q_{t,T} = s_t + \ln(PPI_t^\star) - \ln(PPI_t),$$

$$q_{t,N} = \ln(CPI_t^\star) - \ln(PPI_t^\star) - (\ln(CPI_t) - \ln(PPI_t)).$$

The current method of decomposition has several problems and is better viewed as an imperfect approximation. First, the PPI does contain a large nontraded intermediate input (see Calvo and Kumhof (2003) who emphasize this issue). Second, the CPI and the PPI are constructed with different methodologies. Third, the traded and the nontraded components are negatively correlated by construction. Despite these drawbacks, the decomposition allows us to approximate both components of real exchange rates in a large sample, since the disaggregated price data are not available for most of the emerging markets in the sample. I apply the two methodologies to the real exchange rate series. First, I investigate their volatility based on relative standard deviations. Second, I decompose the variance of real exchange rates following the mothod used by Engel (1999).

2.1 Relative Standard Deviations

I detrend the series using a Baxter-King (1999) band-pass filter, with 8 leads and lags and a pass-band to 6 and 32 quarters. Table 1 reports the summary statistics.

The first three columns in Table 1 describe the volatility of the nontraded component relative to the overall volatility, as measured by the relative standard deviation $\sigma(q_{t,N})/\sigma(q_t)$. The volatility of nontraded real exchange rates varies from 7 to 123 percent of the overall volatility. The average is 29 percent and slightly lower than in Betts and Kehoe (2001a). The nontraded real exchange rate is more volatile in 1980s than 1990-98. Although I do not report the calculation based on official exchange rates, it should be noted that the standard deviation of the ratios approximately doubles.

Table 1. Summary Statistics of Volatility of Nontraded Real Exchange Rates

	$\sigma(q_{t,N})/\sigma(q_t)$			$\sigma(q_{t,N})/\sigma(q_{t,T})$		
Period	1980-98	1980s	1990-98	1980-98	1980s	1990-98
Average	0.29	0.30	0.31	0.30	0.30	0.31
Standard deviation	0.15	0.20	0.18	0.14	0.17	0.16
Maximum	1.23	2.27	1.46	0.90	1.12	1.07
Minimum	0.07	0.06	0.05	0.07	0.05	0.05
Sample size	595	595	595	595	595	595

The last three columns correspond to the relative volatility of the nontraded and the traded component, as measured by the relative standard deviation $\sigma(q_{t,N})/\sigma(q_{t,T})$. The volatility of the nontraded real exchange rates varies from 7 to 90 percent of its traded counterpart in the overall period. In both subperiods, there are cases where the volatility of the nontraded real exchange rate exceeds its traded counterpart. As the primary interest of this paper is the volatility of the nontraded real exchange rates relative to its traded counterpart, I focus on their relative variability and investigate their relationship with the volatility of nominal exchange rates.

Table 2. Volatility of Nontraded Real Exchange Rates and Volatility of Nominal Exchange Rates

	Average of $\sigma(s_t - s_{t-1})$		
Period	1980-98	1980s	1990-98
Group 1: Small nontraded component	0.12	0.15	0.09
Sample size	60	40	60
Group 2: Large nontraded component	0.06	0.07	0.05
Sample size	72	73	73

I divide the sample into two groups according to $\sigma(q_{t,N})/\sigma(q_{t,T})$. When the volatility measure is lower than its "average - standard deviation," I classify it as "small nontraded component." When the volatility measure exceeds its "average + standard deviation," I classify it as "large nontraded component." Table 2 reports the volatility of nominal exchange rates in each subgroups. Since the nominal exchange rates are volatile with high frequencies, it is appropriate to use a high-frequency filtering technique. Here, I use the log-differenced series, or the percentage changes. The standard deviation of percentage changes in nominal exchange rates of the small-nontraded-component group is approximately twice that of the large-nontraded-component group in all periods. In other words, a lower volatility of nominal exchange rate depreciation tend to accompany a higher contribution of the nontraded component of real exchange rate volatility. In fact, the European Monetary System (EMS) country pairs account for 21 percent of the sample of the large-nontraded-component group in 1980-98. The corresponding numbers for the subperiods 1980s and 1990-98 are 26 and 15 percent. On the other hand, they do not appear in the small-nontraded-component group at all. This clearly suggests a connection between exchange rate policy and the volatility of real exchange rates.

Table 3 divides the sample into two groups using the standard deviation of nominal exchange rate depreciations. When the standard deviation is less than 10 percent, I classify it as relatively "fixed" nominal exchange rate regime. Otherwise, I classify it as relatively "flexible" nominal exchange rate regime. The relative volatility of the nontraded real exchange rates is 1.5 times the relative volatility in the flexible rates pairs in the fixed-exchange-rate pairs in the overall period and the 1980s, and it is 14 percent higher in 1990-98. Similar to Table 2, Table 3 emphasizes the influence of exchange rate policy on real exchange rates.

Table 3. Exchange Rate Regime and Volatility of Nontraded Real Exchange Rates

	Average of $\sigma(q_{t,N})/\sigma(q_{t,T})$		
Period	1980-98	1980s	1990-98
Fixed exchange rate regime	0.33	0.36	0.32
Sample size	342	296	450
Flexible exchange rate regime	0.22	0.24	0.28
Sample size	253	299	145

2.2 Variance Decomposition

This section reports the variance decomposition following the methodology of Engel (1999). The results again confirm that the volatility of the nontraded component is high when nominal exchange rates are stable.

The movement of real exchange rates is measured by the mean-squared (MSE) error of their changes, or the sum of the variance and the squared drift,

$$MSE(q_t - q_{t-n}) = var(q_t - q_{t-n}) + [mean(q_t - q_{t-n})]^2.$$

The series are log-differenced at two horizons. One is one-quarter and the other is one year. The fraction of the mean-squared error of $q_t - q_{t-n}$ accounted for by the mean-squared error of the nontraded component $q_{t,N} - q_{t-n,N}$, can be computed by [2]

$$\frac{MSE(q_{t,N} - q_{t-n,N})}{MSE(q_{t,N} - q_{t-n,N}) + MSE(q_{t,T} - q_{t-n,T})}.$$

The results are in Table 4.

On average the nontraded component accounts for approximately 10 percent of the overall variance. However, there are cases where the variance exceeds that of the traded component. I divide samples into two groups and examine their nominal exchange rate volatility as done in the previous section. When the nontraded variance is smaller than 0.10, I classify it as "small nontraded component." When it exceeds 0.30, I classify it as "large nontraded component." The corresponding volatility of nominal exchange rates are reported in Table 5.

Table 4. Summary Statistics of Nontraded Component in Variance Decomposition

[2] I do not correct for the negative correlation between the traded and the nontraded real exchange rates, as the correlation is an artifact of the data.

	Nontraded Component					
Period	1980-1998		1980s		1990-98	
Horizon	n=1	n=4	n=1	n=4	n=1	n=4
Average	0.08	0.10	0.07	0.10	0.09	0.11
Standard deviation	0.05	0.06	0.05	0.07	0.08	0.08
Maximum	0.38	0.42	0.37	0.46	0.48	0.57
Minimum	0.01	0.01	0.01	0.01	0.00	0.01

In Table 5, the standard deviation of exchange rate depreciations in the small-nontraded-component group is 1 to 7 times of that in the large-nontraded-component group, depending on the time horizon. Table 5 presents similar facts to Tables 2 and 3. The volatility of real exchange rates of country pairs that experience low volatility in nominal exchange rates originates to a relatively larger extent in the nontraded real exchange rates. This finding is indeed consistent with what is found in the study on Mexico's real exchange rates by Mendoza (2000). He finds that during the fixed exchange rate period the nontraded component of U.S.- Mexico real exchange rates accounts for 29-71 percent of the overall variance, depending on the time horizon.

Table 5. Nontraded Component in Variance Decomposition and Volatility of Nominal Exchange Rates

	Average of $\sigma(s_t - s_{t-n})$					
Period	1980-1998		1980s		1990-98	
Horizon	n=1	n=4	n=1	n=4	n=1	n=4
Group 1: Small nontraded component	0.14	0.12	0.14	0.13	0.12	0.12
Sample size	470	372	483	353	394	291
Group 2: Large nontraded component	0.02	0.10	0.02	0.13	0.03	0.08
Sample size	4	6	2	13	18	20

To summarize, these stylized facts show that the traded-nontraded decomposition of volatility of real exchange rates is closely connected to exchange rate policy. When exchange rate is freely floating, the traded component dominates the nontraded component in almost all cases but signifiantly less so when the exchange rate is fixed. The model in the next section suggests a mechanism through which exchange rate policy may influence the movement in real exchange rates.

3 The Model

The basic setup follows a two-country model in the new open-economy macroeconomics literature. It is an extension of the trade model with a continuum of goods by Dornbusch et al. (1977) and the sticky wage model along the same line as Rotemberg (1982). This combination gives one new key feature: Heterogeneous and multi-period price dynamics.

The other main element is the explicit treatment of deviations from the law of one price in the form of ice-berg type transport costs, where a fraction τ $(0 < \tau < 1)$ of shipped goods is lost in transit. [3] The world economy consists of two open economies called home and foreign country. Let i denote the type of households and $i \in [0, 1]$. The home households are located in the range $[0, \alpha]$, where $0 < \alpha < 1$, and the rest are in foreign country. Each household i monopolistically supplies her labor to a competitive employment agency by setting a nominal wage, W_t^i. The employment agency sells aggregate labor services to all domestic firms taking the aggregate wage W_t as given. There is a continuum of differentiated goods indexed by z, where $z \in [0, 1]$. The goods markets are also monopolistically competitive. The home goods are located in $[0, z_t^h]$ while the foreign ones are in $[z_t^l, 1]$. The preference specification requires consumers to consume all goods. Hence the range of home imports and exports are $(z_t^h, 1]$ and $[0, z_t^l)$. Note that z_t^h and z_t^l are endogenous and the size of the nontraded sector $\delta_t = z_t^h - z_t^l$, $z_t^l < z_t^h$. I verify in a subsequent section that the condition $z_t^l < z_t^h$ always holds in equilibrium with $0 < \tau < 1$.

3.1 Employment Agency

A competitive employment agency buys labor services from all households at the wage W_t^i, $i \in [0, \alpha]$, and sells the aggregate labor to all domestic firms at the wage W_t. Let l_t^i denote labor services supplied by the household i. The aggregate labor L_t is defined by

$$L_t = \left[\left(\frac{1}{\alpha} \right)^{\frac{1}{\eta}} \int_0^\alpha l_t^{i\,\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}.$$

The agency faces the following cost minization problem.

$$min_{l_t^i} \int_0^\alpha W_t^i l_t^i di \qquad s.t. \qquad L_t = 1.$$

It chooses the stochastic processes $\{l_t^i\}_{t=0}^\infty$, $i \in [0, \alpha]$, that solve the minimization problem taking wages W_t^i and W_t as given. The technical appendix shows that the optimal demand for household i's labor is as follows.

$$l_t^i = \frac{1}{\alpha} \left[\frac{W_t^i}{W_t} \right]^{-\eta} L_t, \tag{1}$$

where

$$W_t = \left[\frac{1}{\alpha} \int_0^\alpha W_t^{i1-\eta} di \right]^{\frac{1}{1-\eta}}.$$

[3] See Brunner and Naknoi (2003) for modeling trade frictions in a framework of new open-economy macroeconomics.

3.2 Households

Each household i supplies her labor service to a competitive employment agency, which sells the aggregate labor service to all domestic firms, and set nominal wage W_t^i. The household i's utility function in the home country depends on a basket of all goods C_t^i, real money balances $m_t^i = M_t^i/P_t$ and labor supply l_t^i.

$$U_t^i = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{\sigma}{\sigma - 1} C_t^{i\,\frac{\sigma-1}{\sigma}} + \frac{\chi}{1-\epsilon} m_t^{i\,1-\epsilon} - \frac{1}{\mu} l_t^{i\,\mu} \right] \tag{2}$$

where $0 < \beta < 1$, $\mu < 1$, $\sigma > 0$, $\epsilon > 0$. The consumption aggregate C_t^i is defined as

$$C_t^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}.$$

$c_t^i(z)$ is the consumption of good z, and θ ($\theta > 1$) is the elasticity of substitution between goods. The price index P_t is [4]

$$P_t = \left[\int_0^1 p_t(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

Suppose there are complete domestic asset markets to insure idiosyncratic income risk from sticky wage-setting and incomplete international asset markets. There are two noncontingent bonds traded internationally. One is issued by the home households and denominated in the home currency, while the other is by the foreign households and in the foreign currency. The households have to incur a quadractic portfolio adjustment cost. The quadratic portfolio adjustment cost, which is first suggested by Neumeyer and Perri (2001), assures that the model has a unique steady state and stationary bond holdings. Let F_t^i and $F_t^{f,i}$ denote the stock of the home bond and the foreign bond owned by the household i. The functional form of the quadratic adjustment cost associated with the home bond is

$$\Phi(f_t^i) = \frac{1}{2}\phi(f_t^i - f_{ss}^i)^2,$$

where f_t is the real value of the home bond and defined as F_t^i/P_t. ϕ is a parameter. The cost is quadratic in deviation from the steady state level of bond holdings. The portfolio

[4] P_t is the cost associated with the solution of the cost minimization problem

$$min_{c_t^i(z)} \int_0^1 p_t(z) c_t^i(z) dz \qquad s.t. \qquad C_t^i = 1.$$

adjustment cost associated with the foreign bond is defined in a similar way.

$$\Phi^f(f_t^{f,i}) = \frac{1}{2}\phi^\star(f_t^{f,i} - f_{ss}^{f,i})^2$$

where $f_t^{f,i} = F_t^{f,i} S_t/P_t$ and S_t is nominal exchange rate. In general, $\phi^\star \neq \phi$.

The period-t budget constraint for households in the home country are:

$$\begin{aligned} P_t C_t^i + (M_t^i - M_{t-1}^i) + (F_t^i - F_{t-1}^i) + S_t(F_t^{f,i} - F_{t-1}^{f,i}) + P_t\Phi(F_t^i/P_t) + P_t\Phi^f(F_t^{f,i}S_t/P_t) \\ = P_t T_t^i + (1+\tau_w)W_t^i l_t^i - P_t h(\pi_t^{w^i}) + \Pi_t^i + i_{t-1}F_{t-1}^i + i_{t-1}^\star S_t F_{t-1}^{f,i} \end{aligned} \quad (3)$$

where T_t^i is the transfer from the government. Π_t^i is the nominal dividends distributed to the household

$$\Pi_t^i = \Pi_t = \int_0^{z_t^h} \frac{\Pi_t(z)}{\alpha} dz, \quad (4)$$

where $\pi_t(z)$ is the firm z's dividend. It is distributed evenly to all households through the complete domestic asset markets. τ_w is the rate of subsidy paid to the households to remove the steady state markup distortion and $\tau_w = 1/(\eta - 1)$. $h(\pi_t^{w^i})$ is the cost of changing nominal wages and it is a convex function of wage (gross) inflation $\pi_t^{w^i} = W_t^i/W_{t-1}^i$. The adjustment cost induces wage stickiness analogous to what generates price stickiness as pioneered by Rotemberg (1982). I assume that $h(\pi_t^{w^i})$ is quadratic in deviations from the deterministic steady state level of wage inflation $\pi_t^{w^i} - \pi_{ss}^{w^i}$, $h(\pi_t^{w^i}) = \phi^w\left(\pi_t^{w^i} - \pi_{ss}^{w^i}\right)^2/2$. ϕ^w is the wage adjustment cost parameter. i_{t-1} is the home nominal interest rate set in the period $t-1$. Let i_t and r_t denote the home nominal and real interest rate.

The household i chooses the set of stochastic processes $\{c_t^i(z), C_t^i, m_t^i, F_t^i, F_t^{f,i}, W_t^i\}_{t=0}^\infty$ to maximize (2) subject to (1), (3) and (4) and the transversality conditions

$$lim_{j\to\infty} E_t\left[F_{t+j}^i/\Pi_{s=0}^{j-1}(1+i_{t+s})\right] \geqslant 0, \quad lim_{j\to\infty} E_t\left[S_{t+j}F_{t+j}^{f,i}/\Pi_{s=0}^{j-1}(1+i_{t+s}^\star)\right] \geqslant 0,$$

taking as given the sequences $\{p_t(z), P_t, \Pi_t\}_{t=0}^\infty$ and the initial conditions ($M_{-1}^i, F_{-1}^i, F_{-1}^{f,i}, W_{-1}^i$). By the assumption of complete domestic equity markets and identical preferences, all household choices are symmetric, given by $\{c_t(z), C_t, m_t, F_t, F_t^f\, W_t\}$. The relevant first order conditions are as follows.

$$c_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t \quad (5)$$

$$1 + \Phi(f_t - f_{ss}) = \beta(1+i_t)E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\right] \tag{6}$$

$$1 + \Phi^f(f_t^f - f_{ss}^f) = \beta(1+i_t^\star)E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right] \tag{7}$$

$$m_t^\epsilon = \chi {C_t}^{1/\sigma}\left(\frac{1+i_t}{i_t}\right) \tag{8}$$

$$C_t^{-1/\sigma}\left(\eta\frac{l_t}{P_t} + \phi^w\frac{\pi_t^w - \pi_{ss}^w}{W_{t-1}} - \phi^w E_t\left[\frac{1}{1+r_t}\frac{(\pi_{t+1}^w - \pi_{ss}^w)W_{t+1}}{W_t^2}\right]\right) = \eta\frac{l_t^\mu}{W_t} \tag{9}$$

where

$$\frac{1}{1+r_t} = \beta E_t\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma}.$$

(5) is the intratemporal consumption decision. (6) and (7) are the intertemporal consumption decision. (8) is the optimal money demand function. (9) gives the optimal wage setting rule. Define the real wage $w_t = W_t/P_t$ and rewrite the wage setting rule.

$$C_t^{-1/\sigma}\left(\eta l_t w_t + \phi^w(\pi_t^w - \pi_{ss}^w)\pi_t^w - \phi^w E_t\left[\frac{(\pi_{t+1}^w - \pi_{ss}^w)}{(1+r_t)}\pi_{t+1}^w\right]\right) = \eta l_t^\mu. \tag{10}$$

The households stabilize their wage inflation at the inflation level in the steady state, $\pi_{ss}^w = \pi_{ss}$ where π_{ss} is the steady state rate of inflation of the nominal anchor. I assume identical preferences in the foreign country, and define the foreign variables in a similar way. As usual, the superscript star denotes the foreign variables. The analogous equations hold for the foreign households.

3.3 Interest Parity

I derive the relationship between the two interest rates from the two Euler equations.

$$1 + i_t = (1+i_t^\star)\frac{1+\Phi(f_t - f_{ss})}{1+\Phi^f(f_t^f - f_{ss}^f)}\left[E_t\left[\frac{S_{t+1}}{S_t}\right] + \frac{Cov_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\right]}{E_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{1/\sigma}\frac{P_t}{P_{t+1}}\right]}\right]. \tag{11}$$

With the portfolio adjustment cost, the uncovered interest rate parity does not hold. The deviation from the uncovered interest parity clearly depends on the portfolio adjustment cost parameters ϕ and $\phi^\star$. I assume $F_{ss} = F_{ss}^\star = 0$, thus the uncovered interest parity holds in the steady state.

3.4 International Trade and Aggregate Prices

International trade is costly. To be specific, I assume the iceberg-type transport costs, where a fraction τ $(0 < \tau < 1)$ of shipped goodss is lost in transit. An increase in τ implies more deviations from the law of one price. Let $p_t(z)$ denote the consumer price of good z in the home country charged by the home firm z. Similarly, $p_t^\star(z)$ denotes the consumer price in the foreign country charged by a foreign firm. The domestic price of home imports therefore becomes

$$p_t(z) = \frac{S_t p_t^\star(z)}{1-\tau} \qquad for \quad z \in (z_t^h, 1].$$

Similarly, the foreign consumer price of home exports is

$$p_t^\star(z) = \frac{p_t(z)}{S_t(1-\tau)} \qquad for \quad z \in [0, z_t^l).$$

Using the above relationships, I can rewrite the home CPI

$$P_t = \left[z_t^l P_{t,H}^{1-\theta} + \delta_t P_{t,N}^{1-\theta} + (1-z_t^h)P_{t,F}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{12}$$

where

$$P_{t,H} = \left[\frac{1}{z_t^l}\int_0^{z_t^l} p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

$$P_{t,N} = \left[\frac{1}{\delta_t}\int_{z_t^l}^{z_t^h} p_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}},$$

$$P_{t,F} = \left[\frac{1}{1-z_t^h}\int_{z_t^h}^{1} \left(\frac{S_t p_t^\star(z)}{1-\tau}\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}.$$

The price subindex $P_{t,j}$ $(j \in (F, H, N))$ is defined as the minimum expenditure required to obtain one unit of $C_{t,j}$ where C_j is the consumption subindex implicitly defined by

$$C_t = \left[z_t^l \frac{1}{\theta} C_{t,H}^{\frac{\theta-1}{\theta}} + \delta_t^{\frac{1}{\theta}} C_{t,N}^{\frac{\theta-1}{\theta}} + (1-z_t^h)^{\frac{1}{\theta}} C_{t,F}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$$

where

$$C_{t,H} = \left[\left(\frac{1}{z_t^l}\right)^{\frac{1}{\theta}} \int_0^{z_t^l} c_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}},$$

$$C_{t,N} = \left[\left(\frac{1}{\delta_t}\right)^{\frac{1}{\theta}} \int_{z_t^l}^{z_t^h} c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},$$

$$C_{t,F} = \left[\left(\frac{1}{1-z_t^h}\right)^{\frac{1}{\theta}} \int_{z_t^h}^{1} c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}.$$

Note that the traded price index $P_{t,T}$ is implicitly defined as

$$(1-\delta_t)P_{t,T}^{1-\theta} = z_t^l P_{t,H}^{1-\theta} + (1-z_t^h)P_{t,F}^{1-\theta}.$$

The CPI can be expressed in terms of the traded and nontraded prices as

$$P_t = \left[(1-\delta_t)P_{t,T}^{1-\theta} + \delta_t P_{t,N}^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

The foreign CPI can be obtained in a similar fashion.

$$P_t^\star = \left[z_t^l P_{t,H}^{\star 1-\theta} + \delta_t P_{t,N}^{\star 1-\theta} + (1-z_t^h)P_{t,F}^{\star 1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

where

$$P_{t,H}^\star = \left[\frac{1}{z_t^l} \int_0^{z_t^l} \left(\frac{p_t(z)}{S_t(1-\tau)}\right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$P_{t,N}^\star = \left[\frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} p_t^\star(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$P_{t,F}^\star = \left[\frac{1}{1-z_t^h} \int_{z_t^h}^{1} p_t^\star(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

Multiply the foreign price subindices $P_{t,H}^\star$, $P_{t,N}^\star$ and $P_{t,F}^\star$ by S_t.

$$S_t P_{t,H}^\star = \left[\frac{1}{z_t^l} \int_0^{z_t^l} \left(\frac{p_t(z)}{1-\tau}\right)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$S_t P_{t,N}^\star = \left[\frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} S_t p_t^\star(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}},$$

$$S_t P_{t,F}^\star = \left[\frac{1}{1-z_t^h} \int_{z_t^h}^{1} S_t p_t^\star(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}.$$

It is evident from the above equations that $P_{t,j} = S_t P_{t,j}^\star$ $(j = H, F)$ fails unless $\tau = 0$.

I show in a subsequent section that when $\tau = 0$, $z_t^h = z_t^l$ and the nontraded do not exist in the equilibrium. In short, the absolute purchasing power parity (PPP), $P_t = S_t P_t^\star$, or $Q_t = 1$, breaks down because of the presence of trade frictions.

3.5 Firms

Firms are price setters and prices are flexible. The technology is heterogeneous across firms and the production function of firm z is characterized by

$$y_{t,j}(z) = X_t a(z) l_{t,j}(z) \qquad j = H, N. \tag{14}$$

$y_{t,j}(z)$ ($j = H, N$) is the output of the good z. X_t is the stochastic component of productivity. $a(z)$ is the firm-specific productivity parameter, and $l_{t,j}(z)$ is the aggregate labor input used by the firm. Whether a good z is produced in the home country at all ($z < z_t^h$), and whether it is an exportable ($z < z_t^l$) or not is determined in equilibrium.

3.5.1 Exportable Firms

A firm $z \in [0, z_t^l]$ sells its products at the price $p_t^h(z)$ to the home demand $c_t^h(z)$ and to the foreign demand $c_t^{h\star}(z)$. Let $y_{t,H}(z)$ be total output of the home exportable z.

$$y_{t,H}(z) = c_t^h(z) + \frac{c_t^{h\star}(z)}{1-\tau} \tag{15}$$

As indicated by the denominator $1 - \tau$ in the second term, a fraction τ of the exportable is lost in transit and the foreign buyers incur the loss, by having to pay a higher price for an effective unit of the good. As I do not analyze the effects of fiscal shocks, I assume zero government consumption throughout. The aggregate demand in each country therefore equals total private consumption.

$$\begin{aligned} D_t &= \alpha C_t, \\ D_t^\star &= (1-\alpha) C_t^\star. \end{aligned}$$

Demand for products are given by the intratemporal consumption decision.

$$c_t^h(z) = \left[\frac{p_t^h(z)}{P_t}\right]^{-\theta} D_t, \tag{16}$$

$$c_t^{h\star}(z) = \left[\frac{p_t^h(z)}{S_t P_t^\star (1-\tau)}\right]^{-\theta} D_t^\star \tag{17}$$

The government subsidizes the production with the rate $\tau_y = 1/(\theta - 1)$ to eliminate the distortions arising from monopolistic competition. The profit function $\Pi_t^h(z)$ of the tradable firm z becomes

$$\Pi_t^h(z) = \left[(1+\tau_y)p_t^h(z) - MC_t(z)\right] \left(c_t^h(z) + \frac{c_t^{h\star}(z)}{1-\tau}\right) \tag{18}$$

The firm maximizes the present discounted value of its real profit stream

$$V_t = E_t \Pi_{s=0}^{\infty} R_{t,t+s} \frac{\Pi_{t+s}^h(z)}{P_{t+s}} \tag{19}$$

where $R_{t,t+s}$ is the s period ahead real discount factor,

$$R_{t,t+s} = \Pi_{j=1}^{s} \frac{1}{1+r_{t+j}}.$$

An exportable firm chooses the stochastic processes $\{p_t^h(z)\}_{t=0}^{\infty}$ to maximize (19) subject to (14)-(18) taking the sequences $\{P_t, P_t^\star, W_t, D_t, D_t^\star\}_{t=0}^{\infty}$ as given. The optimal price setting rule is

$$p_t^h(z) = MC_t(z) = \frac{W_t}{X_t a(z)}. \tag{20}$$

Deflate the nominal variables with P_t, $mc_t = MC_t/P_t$ and $\tilde{p}_t^h(z) = p_t^h(z)/P_t$. Rewrite the price setting rule.

$$\tilde{p}_t^h(z) = mc_t(z) = \frac{w_t}{X_t a(z)} \tag{21}$$

All exportable firms stabilize their prices at the marginal cost level, and in the steady state, $\tilde{p}_{ss}^h(z)a(z) = w_{ss}$.

3.5.2 Nonexportable Firms

For any $z \in (z_t^l, z_t^h)$, the firm z sells output $y_{t,N}(z)$ in the domestic market at the price $p_t^n(z)$.

$$y_{t,N}(z) = c_t(z) = \left[\frac{p_t^n(z)}{P_t}\right]^{-\theta} D_t. \tag{22}$$

They receive the production subsidy with the rate $1+\tau_y$ as well. The profit function for firm z becomes

$$\Pi_t^n(z) = \left[(1+\tau_y)p_t^n(z) - MC_t(z)\right] y_{t,N}(z). \tag{23}$$

The firm's objective function becomes

$$V_t = E_t \Pi_{s=0}^{\infty} R_{t,t+s} \frac{\Pi_{t+s}^n(z)}{P_{t+s}} \tag{24}$$

A nonexportable firm chooses the stochastic process $\{p_t^n(z)\}_{t=0}^{\infty}$ to maximize (24) subject to (14), (22) and (23) taking the sequences $\{P_t, W_t, D_t\}_{t=0}^{\infty}$ as given. The price setting rule is

$$p_t^n(z) = MC_t(z) = \frac{W_t}{X_t a(z)}. \tag{25}$$

We can normalize the nominal variables as in the previous subsection, $\tilde{p}_t(z,n) = p_t^n(z)/P_t$. The price setting rule for the nonexportable firm z becomes

$$\tilde{p}_t^n(z) = mc_t(z) = \frac{w_t}{X_t a(z)} \tag{26}$$

All nonexportable firms also stabilize their prices at the marginal cost level. In the steady state $\tilde{p}_{ss}^n(z)a(z) = w_{ss}$.

The foreign firms face the similar decision problems and the analogous equations hold. Also, I use the superscripts $f^\star$ and $n^\star$ to denote variables associated with the foreign exportable and nonexportable firms, respectively.

3.6 National Account

It is useful to define the gross output or GDP as Y_t and the net exports N_t.

$$P_t Y_t = \int_0^{z_t^l} p_t^h(z) y_{t,H}(z) dz + \int_{z_t^l}^{z_t^h} p_t^n(z) y_{t,N}(z) dz$$

$$P_t N_t = \int_0^{z_t^l} p_t^h(z) y_t^{h\star}(z) dz - \int_{z_t^h}^{1} \frac{S_t p_t^{f\star}(z)}{1-\tau} y_t^f(z) dz$$

The foreign real GDP and net exports can be defined in a similar way.

3.7 Government

For simplicity, I assume that the government rebates the seigniorage revenues net of the subsidy expense to the consumers in a lump-sum fashion. The home government budget constraint is

$$\alpha(M_t - M_{t-1}) = \alpha P_t T_t + \alpha \tau_w W_t L_t + \tau_y P_t Y_t \tag{27}$$

The analogous equation holds for the foreign government.

3.8 Monetary Policy Rule

3.8.1 Fixed Exchange Rate Regime

A fixed exchange rate regime is equivalent to targeting the rate of depreciation. Define exchange rate depreciation as $d_t = E_t[S_{t+1}/S_t]$. I define a fixed exchange rate regime as a policy that targets the path of d_t at the constant level d and that rules out any discrete jump in the path of S_t. The interest rate parity is therefore constrained by

$$d_t = d. \tag{28}$$

The monetary authority in the foreign country conducts an independent monetary policy. The foreign country follows a Taylor rule of the following form.

$$log(1+i_t^\star) = \lambda_i log(1+i_{t-1}^\star)+(1-\lambda_i)\left(\lambda_\pi E_t log(\frac{\pi_{t+1}^\star}{\pi_{ss}^\star}) + \lambda_y log(\frac{Y_t^\star}{Y_{ss}^\star})\right)+(1-\lambda_i)log(1+i_{ss}^\star)+v_t \tag{29}$$

v_t is foreign interest rate shock.

3.8.2 Flexible Exchange Rate Regime

A flexible exchange rate regime is consistent with a monetary target rule, an interest rule, an inflation target rule or a price target rule. I define the flexible exchange regime using the following price target rule in the home country.

$$\pi_t = \pi_{ss} \tag{30}$$

I also rule out any discrete jumps in the path of P_t. The foreign monetary authority implements the Taylor rule specified by Equation (29). The path of S_t is implied by the household optimality conditions.

3.9 Stochastic Processes of Shocks

I assume that the two countries are subject to productivity shocks of the following form.

$$log(X_t) = \rho_x log(X_{t-1}) + u_t, \tag{31}$$

$$log(X_t^\star) = \rho_x log(X_{t-1}^\star) + u_t^\star. \tag{32}$$

where u_t and $u_t^\star$ is a normally distributed shock with zero mean.

$$log(v_t) = \rho_v log(v_{t-1}) + v_t,$$

v_t is a normally distributed shock with zero mean.

3.10 Equilibrium

Before I define an equilibrium, I first outline the equilibrium pattern of trade as it is required for solving for the solution.

3.10.1 Equilibrium Pattern of Trade

The determination of the equilibrium pattern of trade follows the principle of comparative advantage as suggested by Dornbusch et al. (1977). In the short-run, trade is not driven by comparative advantage but imperfect competition in the labor and goods markets. Define the deterministic and the stochastic components of *relative productivity* of a home firm, respectively, as $A(z) = a(z)/a^\star(z)$ and $\chi_t = X_t/X_t^\star$. Without loss of generality, I can rank z such that $A'(z) < 0$. In short, the home country has comparative advantage in the low end of z. To be specific, I assume $a(z) = 2(1-z)$ and $a^\star(z) = 1$. Let ω_t denote the equilibrium relative wage $W_t/S_tW_t^\star$.

The world trade pattern depends on the relative price of imports and domestically produced products. A home firm will produce any variety z if and only if its price does not exceed the import price of the foreign product of the same variety,

$$\frac{S_t p_t^\star(z)}{1-\tau} \geqslant p_t(z).$$

The price-setting rules yield

$$\omega_t \leqslant \frac{\chi_t A(z)}{1-\tau}.$$

Otherwise a foreign firm will export the variety z to the home country. Similarly, a foreign firm will produce any variety z if and only if its price does not exceed the import price of the identical product.

$$\frac{p_t(z)}{S_t(1-\tau)} \geqslant p_t^\star(z)$$

The price-setting rules give

$$\omega_t \geqslant \chi_t A(z)(1-\tau).$$

Otherwise a home firm will export the variety z to the foreign country.

I can summarize the equilibrium pattern of production and trade as follows.

1. The home country produces any variety $z \in [0, z_t^h]$ and imports $z \in (z_t^h, 1]$ from the

foreign country where z_t^h satisfies

$$\omega_t = \frac{\chi_t A(z_t^h)}{1-\tau}. \tag{33}$$

2. The foreign country produces any variety $z \in [z_t^l, 1]$ and imports $z \in [0, z_t^l]$ from the home country where z_t^l satisfies

$$\omega_t = \chi_t A(z_t^l)(1-\tau). \tag{34}$$

3. $z_t^l \leq z_t^h$ and any variety $z \in (z_t^l, z_t^h)$ is produced in both countries but not traded internationally. It is easy to verify that (1) $z_t^l < z_t^h$ in the equilibrium with $0 < \tau < 1$, and (2) $z_t^l = z_t^h$ when $\tau = 0$. Suppose $z_t^l > z_t^h$, then by (33) there is a variety $z' \in (z_t^h, z_t^l)$ that a foreign tradable firm is willing to export to home country. But for z' to be produced for an exporting purpose in the foreign country, by (34) $z_t'^l$ must hold. This contradicts the definition of z'. Therefore $z_t^l < z_t^h$ must hold in any equilibrium with non-zero transport costs. If $\tau = 0$, then $A(z)/(1-\tau) = A(z)(1-\tau)$ for all z. In that case, the conditions (33) and (34) are equivalent and $z_t^l = z_t^h$.

Because of the monotonicity of $A(z)$ in z, z_t^h and z_t^l are unique.

3.10.2 The Definition of Equilibrium

An equilibrium of the world economy is defined as the stochastic processes of allocation $\{c_t(z), C_t, c_t^\star(z), C_t^\star, m_t, m_t^\star, l_{t,H}(z), l_{t,N}(z), L_t, l_{t,F}(z), l_{t,N}^\star(z), L_t^\star, \Pi_t, \Pi_t^\star, T_t, T_t^\star, F_t, F_t^\star, F_t^f, F_t^{f\star}, y_{t,H}(z), y_t^h(z), y_t^{h\star}(z), y_{t,N}(z), y_{t,F}(z), y_t^f(z), y_t^{f\star}(z), y_{t,N}^\star(z), \Pi_t(z), \Pi_t^\star(z)_{z\in[0,1]}\}_{t=0}^\infty$, the price system $\{p_t^h(z), p_t^n(z), P_t, p_t^\star(z,f), p_t^{n\star}(z), P_t^\star, W_t, W_t^\star, i_t, r_t\}_{t=0}^\infty$ for a fixed exchange rate regime, the price system $\{p_t^h(z), p_t^n(z), P_t, p_t^\star(z,f), p_t^{n\star}(z), P_t^\star, W_t, W_t^\star, i_t, r_t, S_t\}_{t=0}^\infty$ for a flexible exchange rate regime, the world production pattern $\{z_t^h, z_t^l\}_{t=0}^\infty$, a pair of government policy (28), (29) under a fixed exchange rate regime or (29), (30) under a flexible exchange rate regime, and the exogenous shocks $\{X_t, X_t^\star\}_{t=0}^\infty$ that satisfy the following.

1. The household's maximization problem: Equations (2)-(9) and their foreign analogues.

2. The firm's profit maximization problem: Equations (14), (18), (20), (23), (25) and their foreign analogues.

3. The labor market clearing condition.

$$\int_0^{z_t^l} l_{t,H}(z)dz + \int_{z_t^l}^{z_t^h} l_{t,N}(z)dz = L_t. \tag{35}$$

$$\int_{z_t^l}^{z_t^h} l_{t,N}^{\star}(z)dz + \int_{z_t^h}^{1} l_{t,F}(z)dz = L_t^{\star}. \tag{36}$$

4. The goods market clearing conditions: Equations (15)-(17), (22) and their foreign analogues.

5. The international bond markets clear.

$$\alpha F_t + (1-\alpha)F_t^{\star} = 0 \tag{37}$$

$$\alpha F_t^f + (1-\alpha)F_t^{f\star} = 0 \tag{38}$$

6. The world production pattern follows Equations (33) and (34).

7. The exogenous shocks follow the stochastic processes (31)-(32).

The total number of variables excluding the shock variables is 44.

4 Equilibrium Dynamics

This section presents the key dynamic equations which describe the adjustment mechanism in the model. First, I outline the wage inflation dynamics. Next, I derive the aggregate price dynamics. Finally, I close the section with the discussion on the real exchange rate dynamics. The detailed derivation of the linear approximation of the model can be found in the technical appendix.

Let $\hat{x}_t = dx_t/x_{ss}$ denote the deviation of x_t from its deterministic steady state level x_{ss}. In the steady state, $r_{ss} = (1-\beta)/\beta$. The CPI inflation rates $\pi_{ss}, \pi_{ss}^{\star}$ and the depreciation rate d_{ss} depend on the monetary policy rules. Relative purchasing power parity holds in steady state, therefore $d_{ss} = \pi_{ss} - \pi_{ss}^{\star}$. I assume $F_{ss} = F_{ss}^{\star} = F_{ss}^f = F_{ss}^{f\star} = 0$.

4.1 Wage Inflation Dynamics

The linear approximation of the wage equation gives the following wage inflation dynamics.

$$\hat{\pi}_{t+1}^w = \frac{1}{\beta}\hat{\pi}_t^w - B_w\left[(\mu-1)\hat{l}_t + \frac{1}{\sigma}\hat{C}_t - \hat{w}_t\right], \tag{39}$$

$B_w = \eta l_{ss} w_{ss} (\phi^w \beta \pi_{ss}^2)^{-1}$. Note that the definition of w_t implies

$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t. \tag{40}$$

The corresponding equation in the foreign country is

$$\hat{\pi}^{w\star}_{t+1} = \frac{1}{\beta}\hat{\pi}^{w\star}_t - B^\star_w\left[(\mu-1)\hat{l}^\star_t + \frac{1}{\sigma}\hat{C}^\star_t - \hat{w}^\star_t\right], \tag{41}$$

where $B^\star_w = \eta l^\star_{ss} w^\star_{ss}(\phi^{w\star}\beta {\pi^\star_{ss}}^2)^{-1}$ and

$$\hat{w}^\star_t = \hat{w}^\star_{t-1} + \hat{\pi}^{w\star}_t - \hat{\pi}^\star_t. \tag{42}$$

4.2 Price Dynamics of Export Sector

Define the average productivity of the home export sector as

$$a_{t,H} = \left(\frac{1}{z^l_t}\int_0^{z^l_t} a(z)^{\theta-1}dz\right)^{\frac{1}{\theta-1}}.$$

The price dynamics corresponding to the home export sector are summarized by two variables, $\hat{\tilde{P}}_{t,H}$, $\hat{a}_{t,H}$ and $\hat{\pi}_{t,H}$. From the optimal price setting rule and the definition of $P_{t,H}$,

$$\hat{\tilde{P}}_{t,H} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,H}. \tag{43}$$

$\hat{a}_{t,H}$ can be derived from its definition above.

$$\hat{a}_{t,H} = \frac{z^l_{ss}(a(z^l_{ss})/a_H)^{\theta-1} - 1}{\theta-1}\hat{z}^l_t, \tag{44}$$

where a_H denote the steady state level productivity. Evidently, the equation above indicates that an expansion of the export sector may raise or lower its productivity, depending on the productivity structure. The dynamics of sectoral inflations can be derived from their definition. By definition of $\pi_{t,H}$,

$$\pi_{t,H} = \frac{\tilde{P}_{t,H}}{\tilde{P}_{t-1,H}}\pi_t.$$

Therefore,

$$\hat{\pi}_{t,H} = \hat{\tilde{P}}_{t,H} - \hat{\tilde{P}}_{t-1,H} + \hat{\pi}_t. \tag{45}$$

In the foreign country, the export sector price dynamics are characterized by two variables, $\hat{\tilde{P}}^{\star}_{t,F}$ and $\hat{\pi}^{\star}_{t,F}$. The dynamics of these variables are similar to those in the home country.

$$\hat{\tilde{P}}^{\star}_{t,F} = \hat{w}^{\star}_t - \hat{X}^{\star}_t \tag{46}$$

$$\hat{\pi}^{\star}_{t,F} = \hat{\tilde{P}}^{\star}_{t,F} - \hat{\tilde{P}}^{\star}_{t-1,F} + \hat{\pi}^{\star}_t \tag{47}$$

4.3 Price Dynamics of Nontraded Sector

Define the average productivity of the home export sector as

$$a_{t,N} = \left(\frac{1}{\delta_t} \int_{z^l_t}^{z^h_t} a(z)^{\theta-1} dz \right)^{\frac{1}{\theta-1}}.$$

Let a_N denote the steady state level productivity. The dynamics of the nontraded sector price are governed by the following equations.

$$\hat{\tilde{P}}_{t,N} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,N} \tag{48}$$

$$\hat{a}_{t,N} = \frac{z^h_{ss}(a(z^h_{ss})/a_N)^{\theta-1} - z^h_{ss}/\delta_{ss}}{\theta - 1} \hat{z}^h_t - \frac{z^l_{ss}(a(z^l_{ss})/a_N)^{\theta-1} - z^l_{ss}/\delta_{ss}}{\theta - 1} \hat{z}^l_t \tag{49}$$

$$\hat{\pi}_{t,N} = \hat{\tilde{P}}_{t,N} - \hat{\tilde{P}}_{t-1,N} + \hat{\pi}_t \tag{50}$$

Equation (44) and (49) indicate that, in general, a shock can affect the productivity in the export and nontraded sector differently, depending on how its alter the size of each sector. Similar equations hold for the foreign nontraded sector.

$$\hat{\tilde{P}}^{\star}_{t,N} = \hat{w}^{\star}_t - \hat{X}^{\star}_t \tag{51}$$

$$\hat{\pi}^{\star}_{t,N} = \hat{\tilde{P}}^{\star}_{t,N} - \hat{\tilde{P}}^{\star}_{t-1,N} + \hat{\pi}^{\star}_t \tag{52}$$

4.4 Price Dynamics of Import Sector

For the home import sector,

$$\hat{\tilde{P}}_{t,F} = \hat{Q}_t + \hat{w}^{\star}_t - \hat{X}^{\star}_t \tag{53}$$

$$\hat{\pi}_{t,F} = \hat{\tilde{P}}_{t,F} - \hat{\tilde{P}}_{t-1,F} + \hat{\pi}_t. \tag{54}$$

For the foreign import sector,

$$\hat{\pi}^{\star}_{t,H} = \hat{\tilde{P}}^{\star}_{t,H} - \hat{\tilde{P}}^{\star}_{t-1,H} + \hat{\pi}^{\star}_t \tag{55}$$

$$\hat{\tilde{P}}^{\star}_{t,H} = \hat{w}_t - \hat{Q}_t - \hat{X}_t - \hat{a}_{t,H}. \tag{56}$$

4.5 CPI Inflation

The path of CPI inflation follows its definition. In the home country,

$$\begin{aligned}
\hat{\pi}_t &= z^l_{ss}(\pi_{ss}\frac{w_{ss}}{a_H})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + \hat{\pi}_{t,H} + \hat{\hat{P}}_{t-1,H}) \\
&+\delta_{ss}(\pi_{ss}\frac{w_{ss}}{a_N})^{1-\theta}(\frac{1}{1-\theta}(\frac{z^h_{ss}}{\delta_{ss}}\hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}}\hat{z}^l_t) + \hat{\pi}_{t,N} + \hat{\hat{P}}_{t-1,N}) \\
&+(1-z^h_{ss})(\pi_{ss}\frac{w^\star_{ss}Q_{ss}}{1-\tau})^{1-\theta}(\frac{-z^h_{ss}}{1-z^h_{ss}}\frac{1}{1-\theta}\hat{z}^h_t + \hat{\pi}_{t,F} + \hat{\hat{P}}_{t-1,F}) \qquad (57)
\end{aligned}$$

The foreign CPI inflation is derived in a similar fashion.

$$\begin{aligned}
\hat{\pi}^\star_t &= z^l_{ss}(\pi^\star_{ss}w^\star_{ss})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + \hat{\pi}^\star_{t,H} + \hat{\hat{P}}^\star_{t-1,H}) \\
&+\delta_{ss}(\pi^\star_{ss}{w_{ss}}^\star)^{1-\theta}(\frac{1}{1-\theta}(\frac{z^h_{ss}}{\delta_{ss}}\hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}}\hat{z}^l_t) + \hat{\pi}^\star_{t,N} + \hat{\hat{P}}^\star_{t-1,N}) \\
&+(1-z^h_{ss})(\pi^\star_{ss}\frac{w_{ss}}{Q_{ss}(1-\tau)})^{1-\theta}(\frac{-z^h_{ss}}{1-z^h_{ss}}\frac{1}{1-\theta}\hat{z}^h_t + \hat{\pi}^\star_{t,F} + \hat{\hat{P}}^\star_{t-1,F}) \qquad (58)
\end{aligned}$$

4.6 Real Exchange Rate Dynamics

In this subsection, I decompose the movements in real exchange rates and show the linkages with endogenous tradability. Using the price indices in Equation (12) and (13), I can decompose the deviation of the real exchange rate from its steady-state level as follows.

$$\begin{aligned}
\hat{Q}_t - \hat{Q}_{t-1} &= \hat{q}_{t,T} + \hat{q}_{t,N} &(59)\\
\hat{q}_{t,T} &= \left[\hat{d}_t + \pi_{ss}^{\star 2(1-\theta)}(\hat{\pi}^\star_{t,F} + \hat{\tilde{P}}^\star_{t-1,F}) - \pi_{ss}^{2(1-\theta)}(\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})\right] \\
&+\pi_{ss}^{\star 1-\theta} z^l_{ss}(\pi^\star_{ss}\frac{w_{ss}}{Q_{ss}(1-\tau)a_H})^{1-\theta}\left[\hat{\pi}^\star_{t,H} - \hat{\pi}^\star_{t,F} + \hat{\tilde{P}}^\star_{t-1,H} - \hat{\tilde{P}}^\star_{t-1,F}\right] \\
&-\pi_{ss}^{1-\theta} z^l_{ss}(\pi_{ss}\frac{w_{ss,H}}{a_H})^{1-\theta}\left[\hat{\pi}_{t,H} - \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,H} - \hat{\tilde{P}}_{t-1,F}\right] \\
&+\frac{1}{1-\theta}\pi_{ss}^{\star 2(1-\theta)}\left[z^l_{ss} w_{ss}^{\star 1-\theta}\hat{z}^l_t - z^h_{ss} w_{ss}^{\star 1-\theta}\hat{z}^h_t\right] \\
&-\frac{1}{1-\theta}\pi_{ss}^{2(1-\theta)}\left[z^l_{ss}\left(\frac{w_{ss}}{a_H}\right)^{1-\theta}\hat{z}^l_t - z^h_{ss}\left(\frac{Q_{ss}w^\star_{ss}}{1-\tau}\right)^{1-\theta}\hat{z}^h_t\right] \\
&+0.5\frac{(\pi^\star_{ss}w^\star_{ss})^{1-\theta} - (\pi_{ss}w_{ss}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right] &(60)\\
\hat{q}_{t,N} &= \delta_{ss}\left[(\pi^\star_{ss}w^\star_{ss})^{1-\theta}(\hat{\pi}^\star_{t,N} - \hat{\pi}^\star_{t,T} + \hat{\tilde{P}}^\star_{t-t,N} - \hat{\tilde{P}}^\star_{t-1,T})\right] \\
&-\delta_{ss}\left[(\pi_{ss}\frac{w_{ss}}{a_N})^{1-\theta}(\pi_{t,N} - \pi_{t,T} + \hat{\tilde{P}}_{t-1,T} - \hat{\tilde{P}}_{t-1,N})\right] \\
&+0.5\frac{(\pi^\star_{ss}w^\star_{ss})^{1-\theta} - (\pi_{ss}w_{ss,N}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right] &(61)
\end{aligned}$$

$\hat{q}_{t,T}$ and $\hat{q}_{t,N}$ are the traded and nontraded component of the deviation of the real depreciation from its steady state level, respectively. The traded component comes from four channels: the substitution between the imports and domestically produced traded goods in the two countries (the first, second and third terms) and endogenous tradability (the last three terms). For the nontraded component, its deviation from the steady state level depends on the international difference in inflation differentials of the nontraded and the traded sector (the first and second terms), and endogenous tradability (the last term). The first and second terms actually capture the substituation between nontraded and traded goods consumption. The home bias in consumption is essentially represented by the first, second and third terms in $\hat{q}_{t,T}$ and the first and second terms in $\hat{q}_{t,N}$.

The transport costs in my model lead to two important characteristics of the real exchange rate dynamics. First, they create the wedge between the prices of domestically produced goods and those of import goods. It is precisely what drives the substitution between domestically produced and import goods. It is precisely what Rogoff (1996) refers as the absence of arbitrage and claimed to be an explanation for the persistent deviations of real exchange rates from the purchasing power parity. Second, endogenous tradability affects the real exchange rate dynamics through non-homotheticity of the preferences specification. The utility function is non-homothetic over the traded and nontraded goods consumption. It therefore broadens the scope of substitution between the two types of goods by allowing their consumption share to vary in the transition dynamics. As the size of traded and nontraded sector also captures its share in total consumption expenditure, the endogenous tradability terms represent the direct effect of changes in consumption share on real exchange rates.

The movements of these components of real exchange rates largely depend on the expenditure switching effect of exchange rates. However, under fixed exchange rate regime, the effect entirely disappears. In the face of real shocks, fixing exchange rates delays the movements of terms of trade and the substitution between domestically produced goods and imports. In that case, both countries have to rely on adjustment in the relative prices of nontraded to traded goods, as indicated by inflation differentials in $\hat{q}_{t,N}$. When the expenditure switching effect is large, limiting the movements of nominal exchange rates can create high volatility in the relative prices of nontraded goods.

4.7 Expenditure Switching Effect of Exchange Rates

5 Calibration

I calibrate the model under two exchange rate regimes as described by the two different monetary rules. I first calibrate the benchmark model and then vary some parameters of interest, namely the transport cost parameter, country size, intertemporal and intratemporal elasticity of substitution. Table 6 summarizes the parameter values.

In the benchmark model, the size of home country is 0.5. The benchmark value for the transport costs parameter is 25 percent. The transport cost parameters are from the study by Brunner and Naknoi (2003). Transport costs are found to be as high as 30 percent, taking into account other trade costs that cannot be easily measured such as language and other information barriers. The resulting output share of the nontraded goods in the home country in the steady state is 50 percent and 40 percent in the foreign country in the benchmark model. The rate of time preferences gives 4 percent of the real interest rate. The elasticity of substitution across goods follows Obstfeld (2000). The first order-correlation of productivity shock is assumed to be 0.5. The Taylor rule specification follows Clarida et al. (2000).

The wage adjustment coefficients are the same in both countries. The resulting elasticity of wage inflation with respect to marginal rate of substitution between labor and consumption are 0.25 in the home country, and 0.20 in the foreign country.

Table 6. Parameter Values

	Parameters
Transport costs	$\tau = 0.15 - 0.3$
Country size	$\alpha = 0.5$
Preferences	$\theta = 6$, $\sigma = 0.5$
	$\beta = 0.99$, $\mu = 1.4$, $\epsilon = 9$
Labor demand	$\eta = 1$
Nominal rigidities	
Wage stickiness	$\phi^w = \phi^{w\star} = 50$
Portfolio adjustment cost	$\phi = \phi^\star = 0.2$
Technology	$\rho_x = 0.5$
Monetary rule	
Home country	$d = 1$ or $\pi_t = 0.02$
Foreign country	$\lambda_i = 0.79$, $\lambda_\pi = 2.15$, $\lambda_y = 0.2325$,$\rho_v = 0.5$

I discuss the steady state relationship in the next subsection. Then, I solve the log-linearized system using the algorithm developed by Uhlig (1999). The later subsection reports the impulse responses.

5.1 Short-run Dynamics

This subsection shows the impulse responses with respect to a one percent increase in the home productivity shock and the foreign interest rate under the two exchange rate regimes. The impulse responses should be interpreted as qualitative predictions of the model, since the underlying variance of the noise term is not explicited addressed.

To investigate the relationships between the expenditure switching effect and transport costs, and the intratemporal elasticity of substitution, I calibrate the model based on a productivity shock and an interest rate shock, with several transport costs and elasticity parameters, under two different exchange rate regime. The impulse responses confirm that the nontraded component of real depreciation becomes more volatile under a fixed exchange rate regime as transport costs rise, regardless of type of shocks. However, there is no clear relationship between the elasticity of substitution and the sectoral decomposition of the real exchange rate volatility.

The impulse responses corresponding to the productivity shock are in Figures 1.1, 1.2 for a flexible exchange rate regime and in Figures 2.1 and 2.2 for a fixed exchange rate regime. The gist of a theory of endogenous tradability lies in a temporary shift in patterns of trade as a result of an exogenous shock. In the short run, a positive productivity shock in the home country raises real wage in the home country, and cause some of the home exporters to become nontraded goods producers, while some of the foreign nontraded firms become new exporters. Since the new foreign exporters are initially not so productive, their goods are produced with relatively high cost and contributes to the rise in import prices in the home country. As a result, we observe real exchange rate appreciation. The impulse responses of under two exchange rate regimes are mostly identical, apart from the policy variables. They also comfirm that the nontraded component of real depreciation becomes more volatile under a fixed exchange rate regime as transport costs rise (Figure 7.1).

Interestingly, the shift of patterns of trade in response to a positive foreign interest rate shock is qualitatively identical that with a productivity shock, although it is from a different mechnism. (See Figures 3.1, 3.2, 4.1 and 4.2.) A rise in foreign interest rate reduces foreign demand and output, and that raises terms of trade and real wage in the home country. As a result, we also observe real depreciation. The nontraded component of real appreciation also depicts higher volatility under a fixed exchange rate regime. Such a pattern becomes stronger as transport costs fall (Figure 7.2). When the expenditure switching effect of exchange rates is measured by the differences in impulse responses of expenditure share, it is found to be decreasing in transport costs, regardless of type of shocks (See Figures 5.1 and 5.2). Its relationship with the intratemporal elasticity of substitution is ambiguous (See Figures 8.1 and 8.2). Such findings support the reasoning that the component of the volatity of real exchange rate is influenced by exchange rate policy and degree of trade integration.

6 Concluding Remarks

This paper takes a new look at an old issue: Is the relative price of nontraded to traded goods a source of real exchange rate fluctuations? The first part of the paper establishes stylized facts about the volatility of real exchange rates. The answer to the question is yes, for some countries. These countries are found to share one common characteristic, namely, stable bilateral nominal exchange rates.

Taking the evidence into account, I construct a model that makes an analysis of exchange rate regime and pattern of trade possible. The model highlights the role of the expenditure switching effect of exchange rates as the central price adjustment mechanism. The effect increases as transport cost falls or as the two economies are more integrated. When the effect is large, limiting the movements in nominal exchange rate can increase the volatility of the relative price of nontraded to traded goods. Such a pattern is confirmed by the impulse responses in my calibration exercise. In almost all cases, the relative price of nontraded to traded goods is more volatile under a fixed exchange rate regime, as predicted. The calibration results are, however, preliminary and should be interpreted as qualitative predictions.

The most natural extension of this piece of research is to extend it to an optimal policy anlysis. In addition, incorporating nontraded intermediate inputs is an interesting avenue that might further explains the short-run volatility arising from shifts in patterns of trade.

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Figure 1.1 Productivity shock - Flexible exchange rate

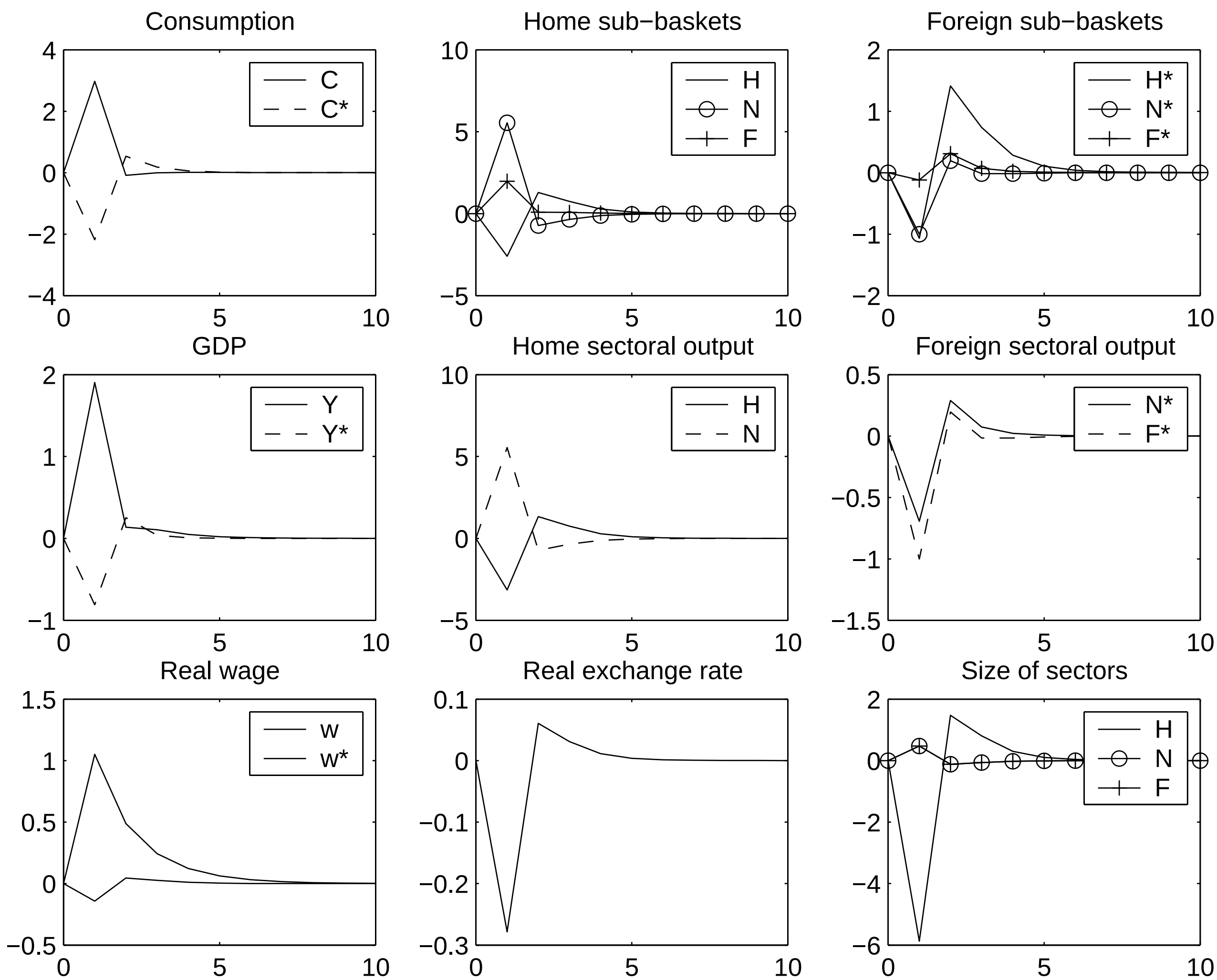


Figure 1.2 Productivity shock - Flexible exchange rate

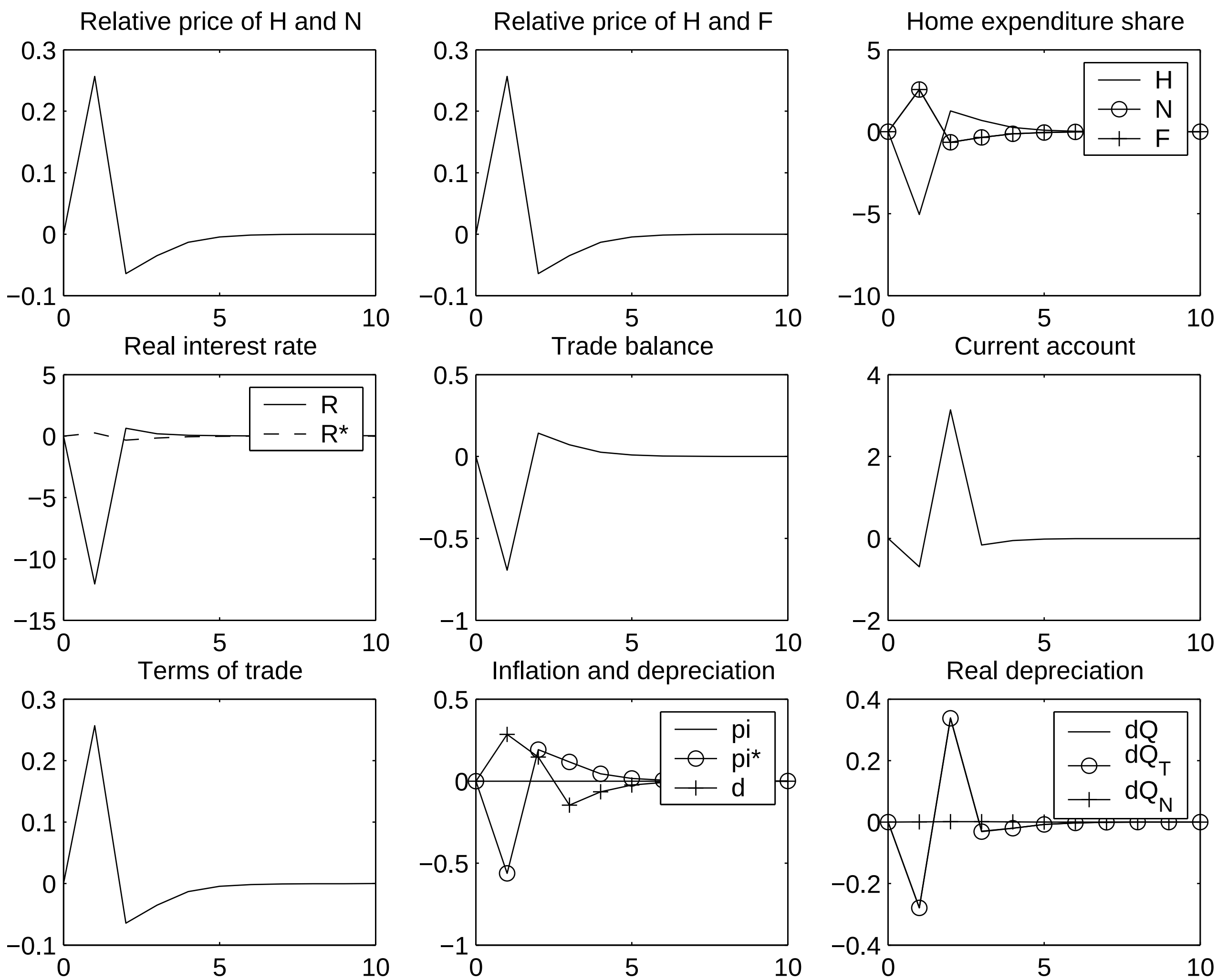


Figure 2.1 Productivity shock - Fixed exchange rate

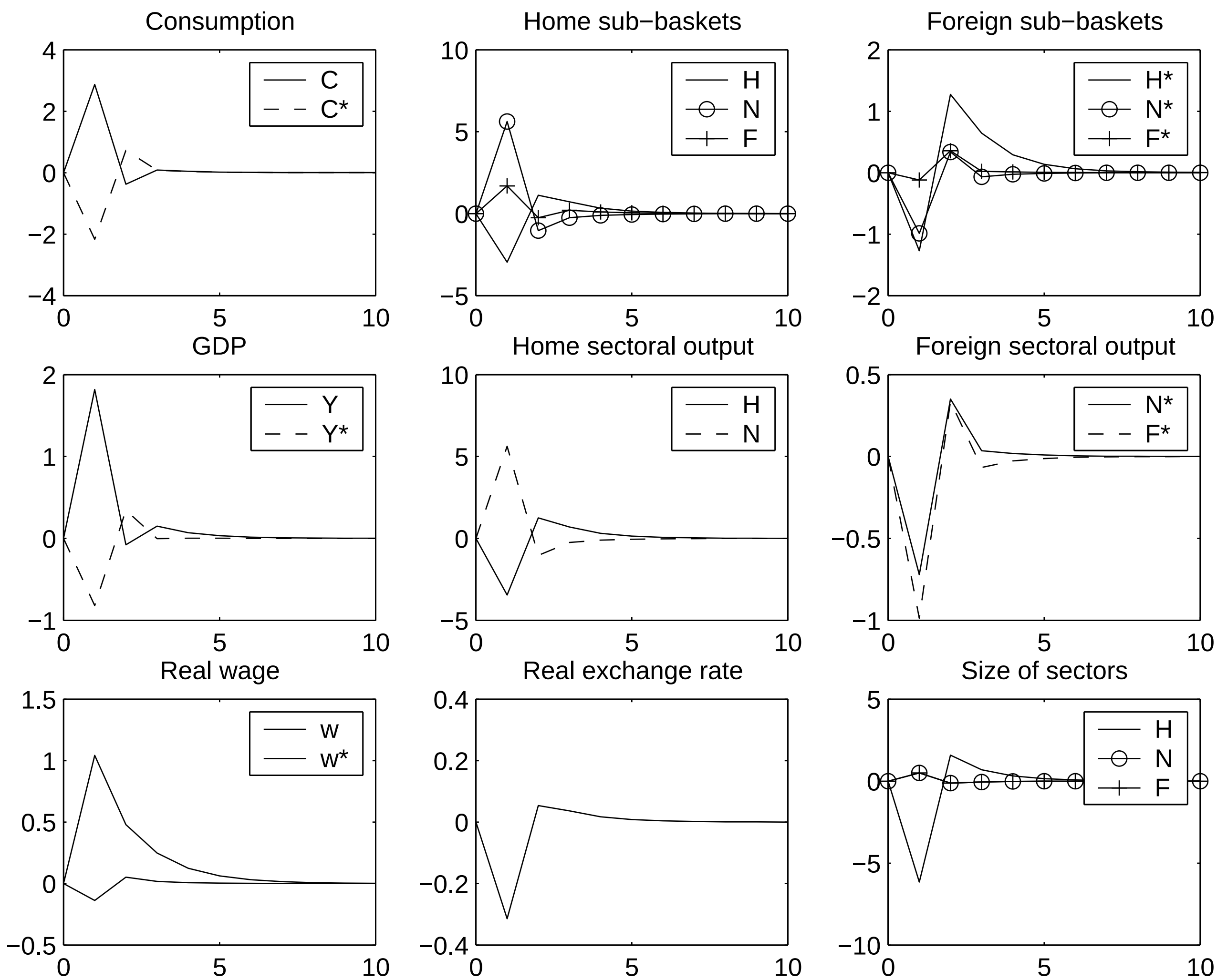


Figure 2.2 Productivity shock - Fixed exchange rate

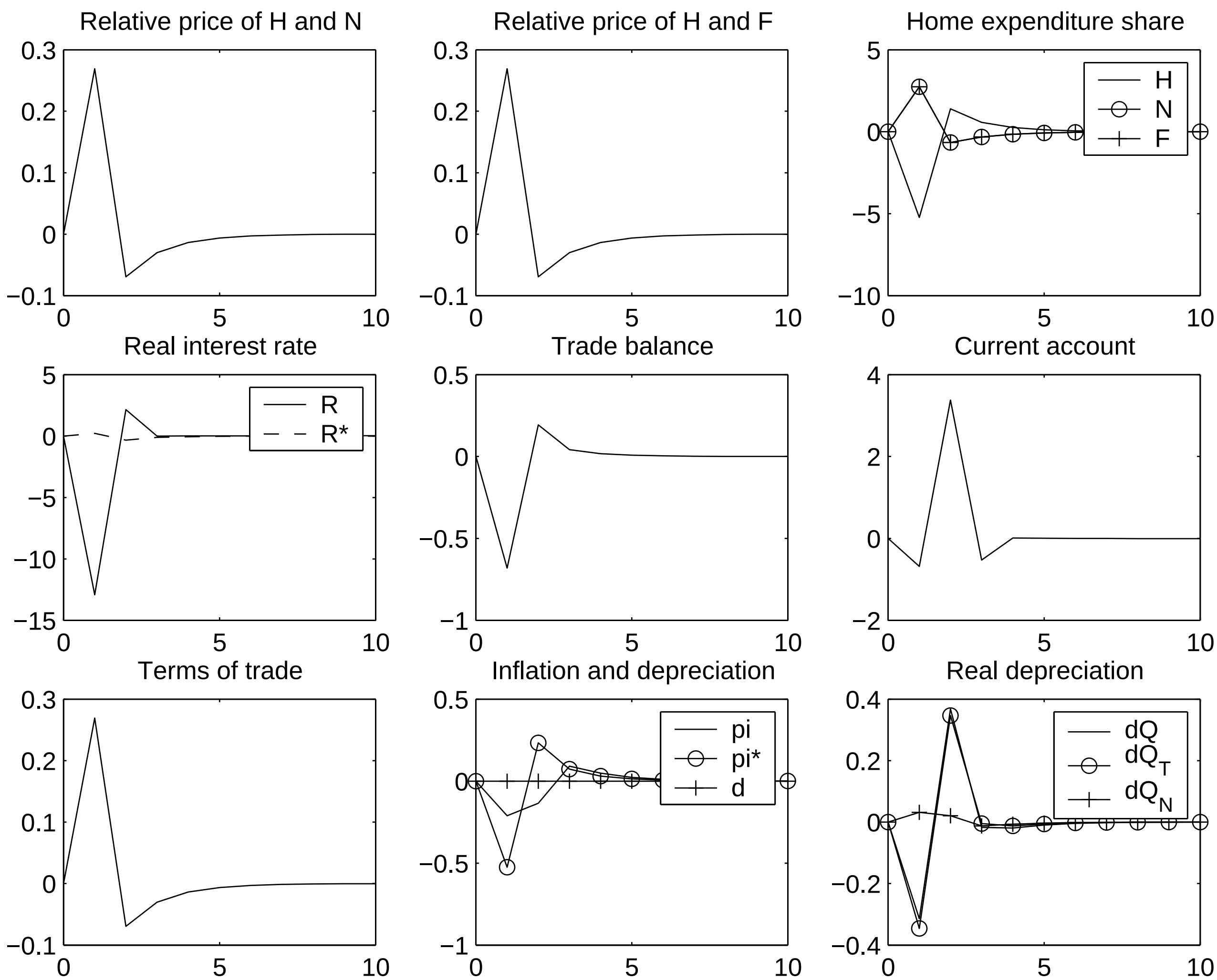


Figure 3.1 Interest rate shock - Flexible exchange rate

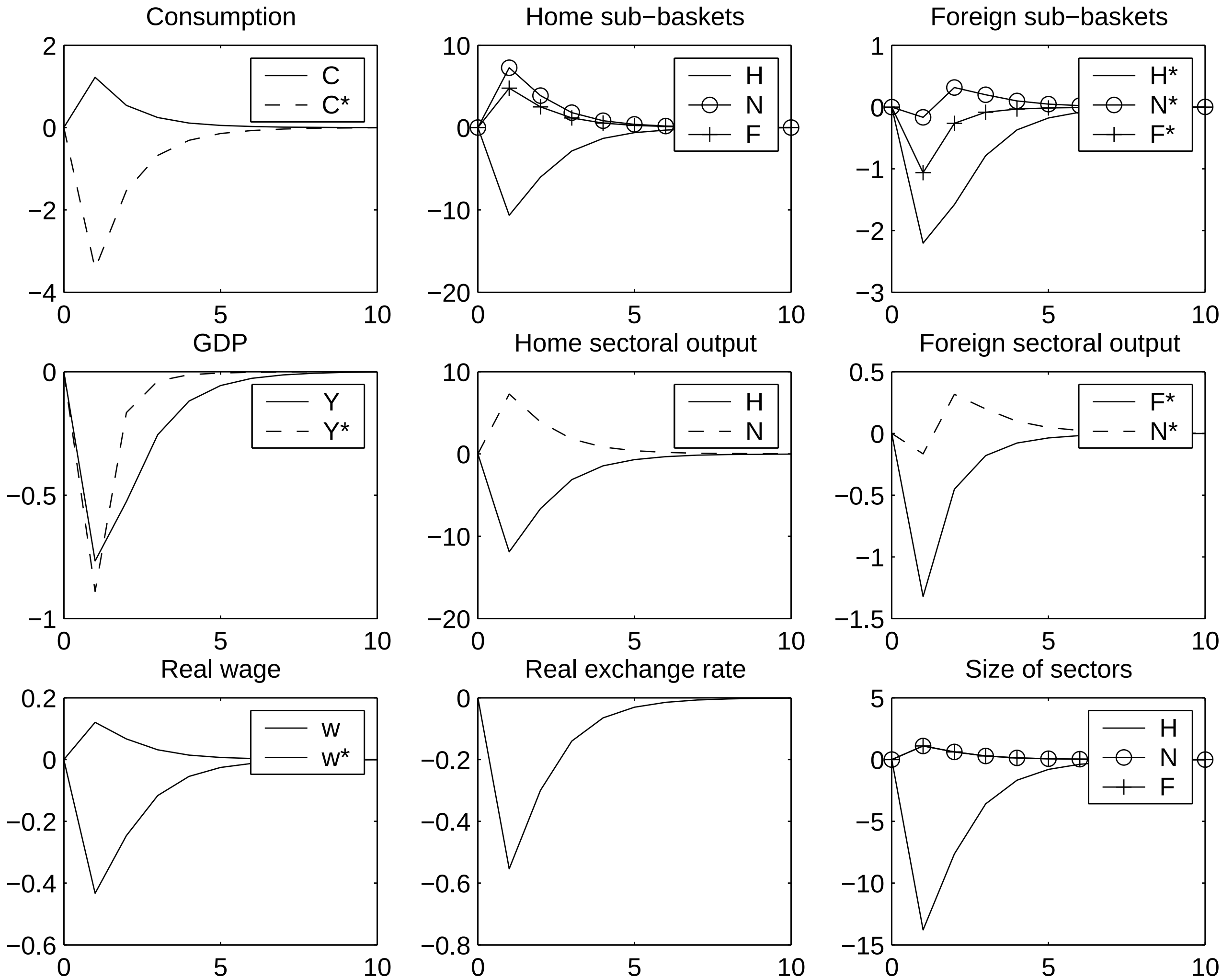


Figure 3.2 Interest rate shock - Flexible exchange rate

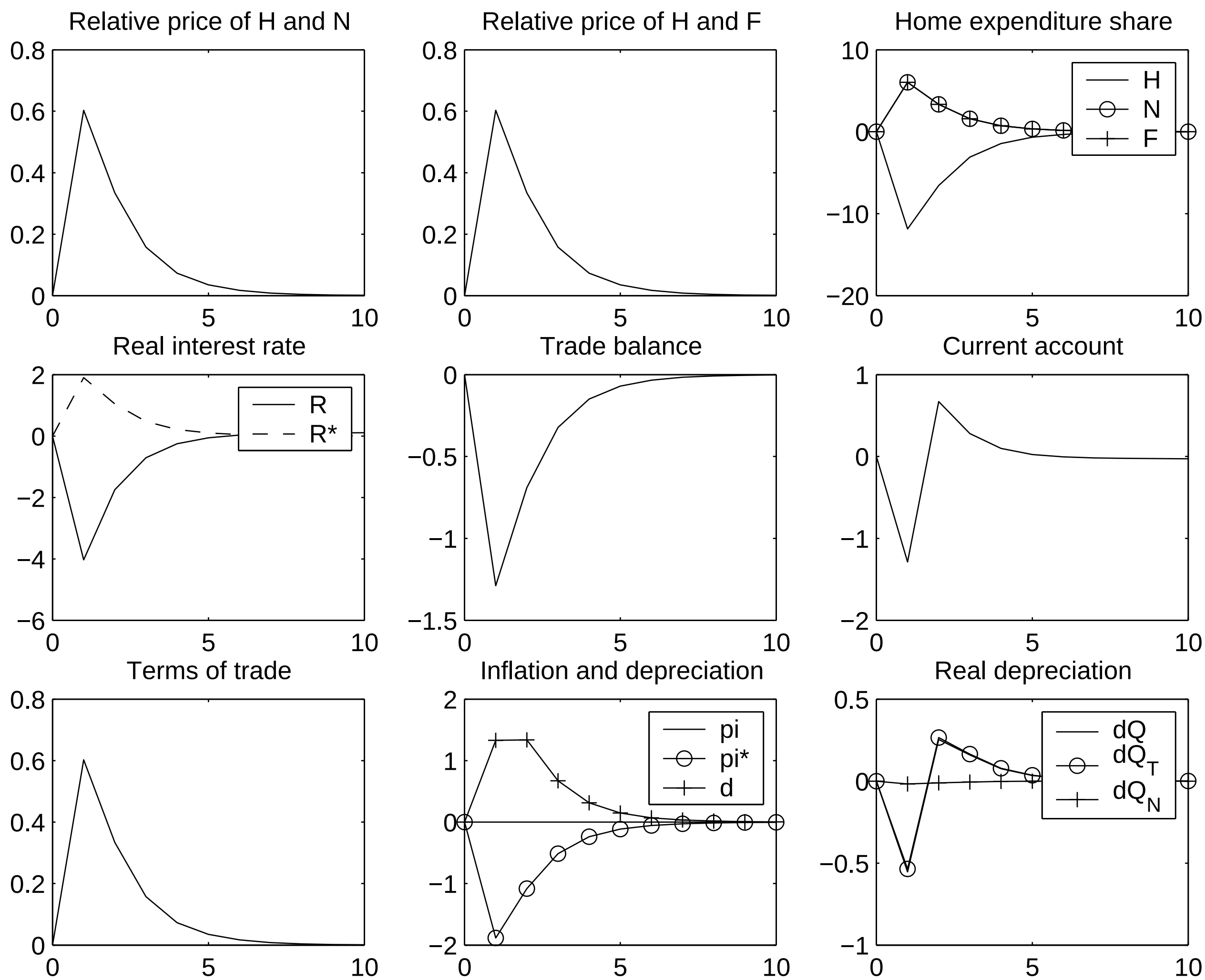


Figure 4.1 Interest rate shock - Fixed exchange rate

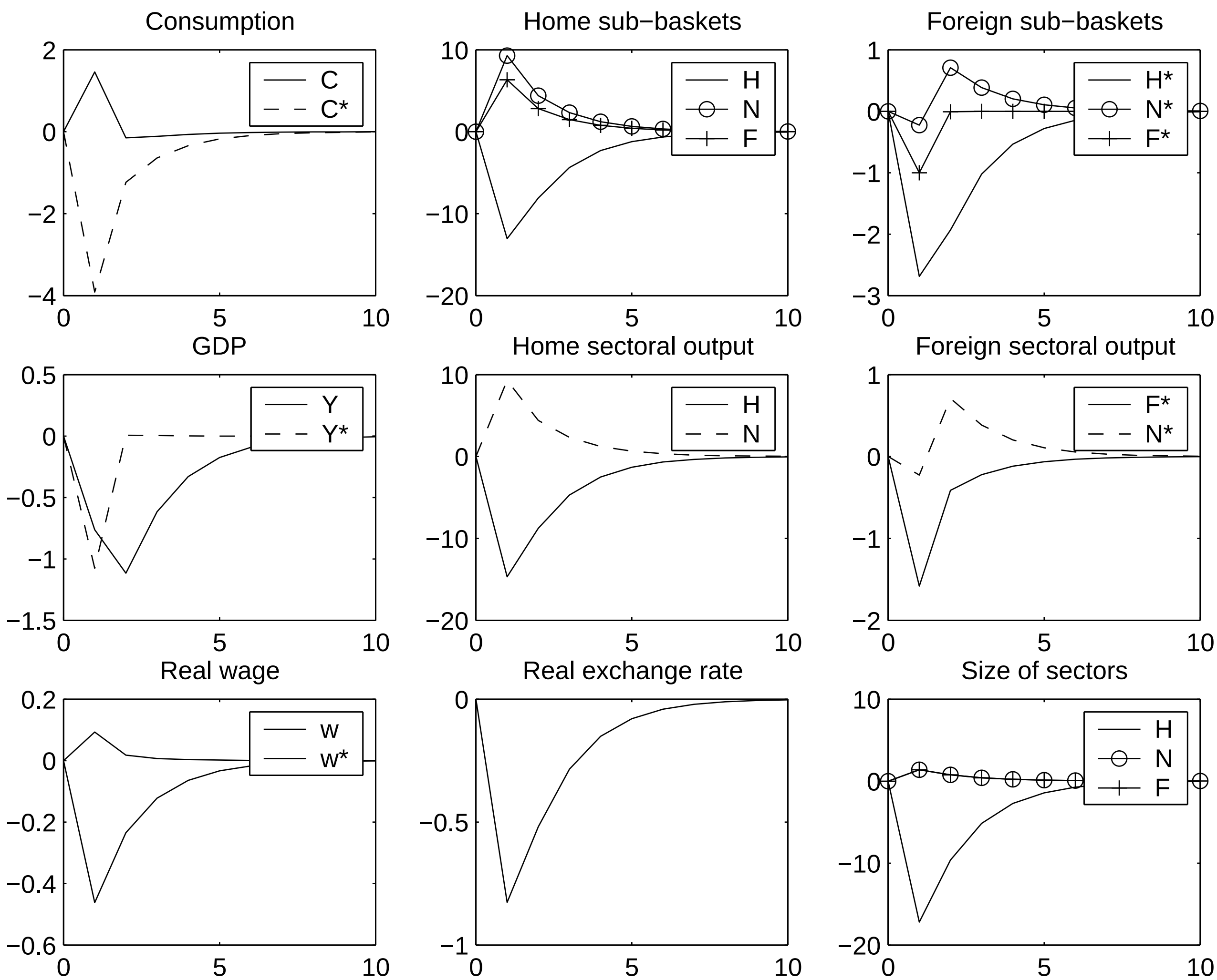


Figure 4.2 Interest rate shock - Fixed exchange rate

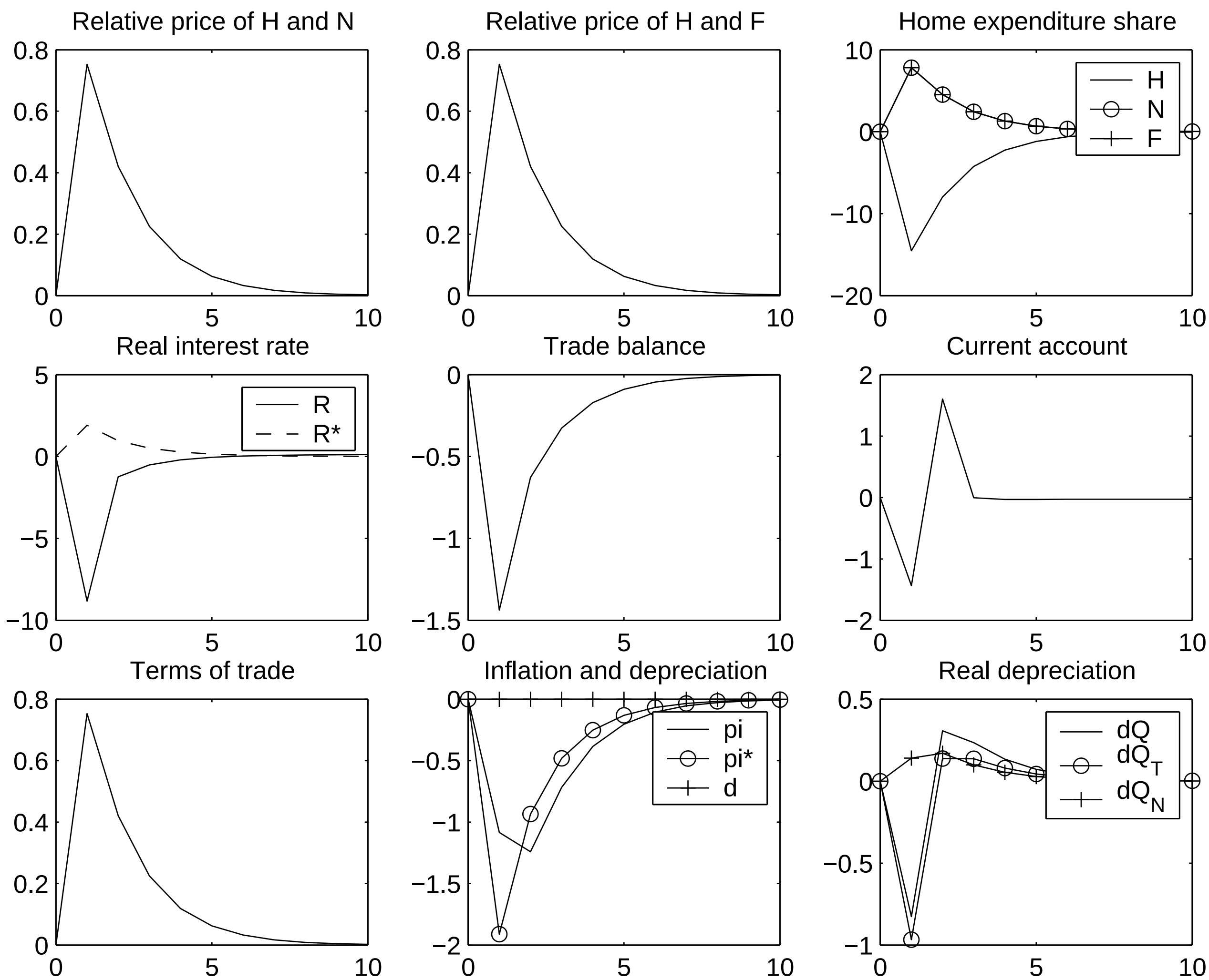


Figure 5.1 Expenditure switching effect of exchange rates and transport costs: Productivity shock

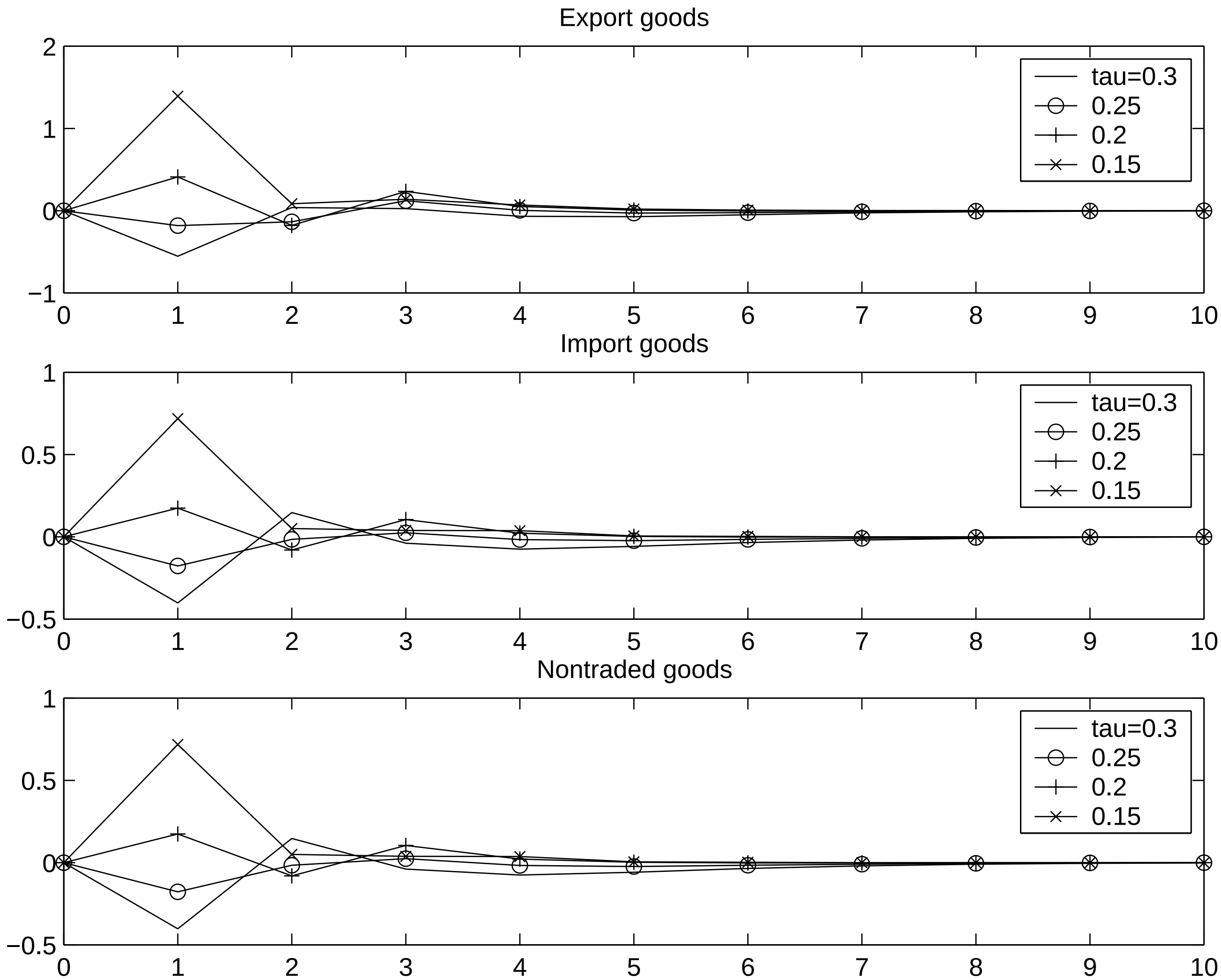


Figure 5.2 Expenditure switching effect of exchange rates and transport costs: Interest rate shock

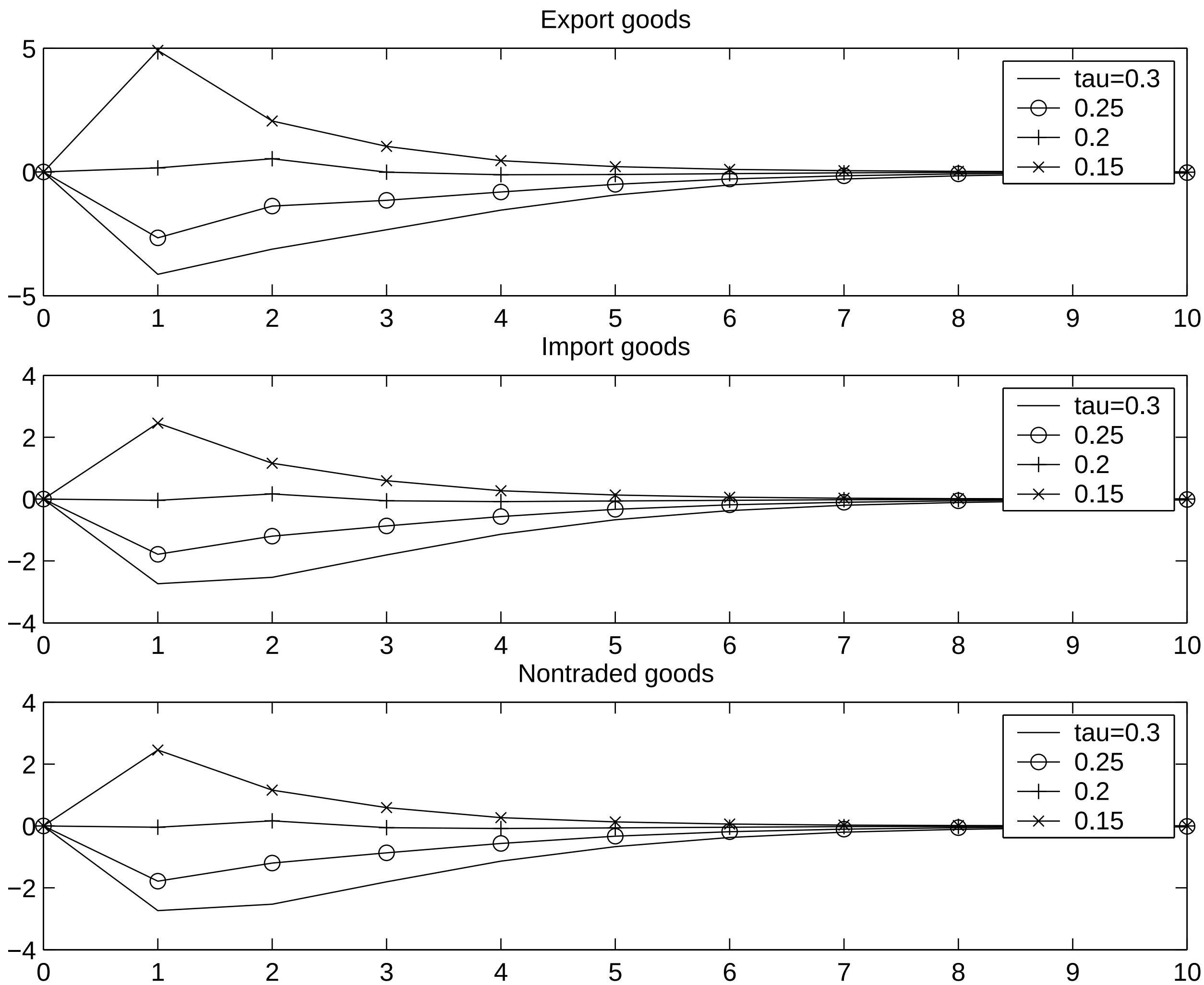


Figure 6.1 Expenditure switching effect of exchange rates and intratemporal elasticity of substitution: Productivity shock

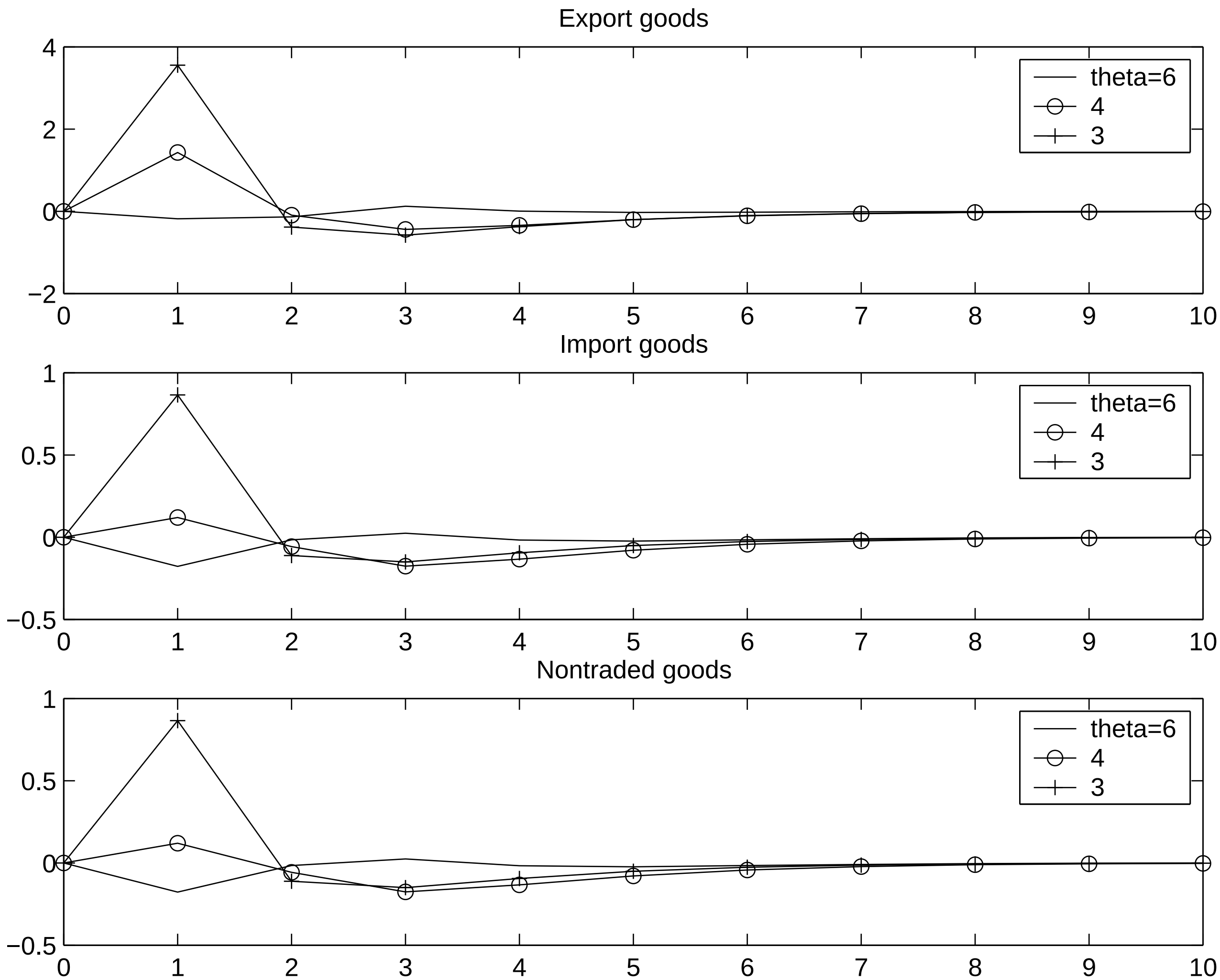


Figure 6.2 Expenditure switching effect of exchange rates and intratemporal elasticity of substitution: Interest rate shock

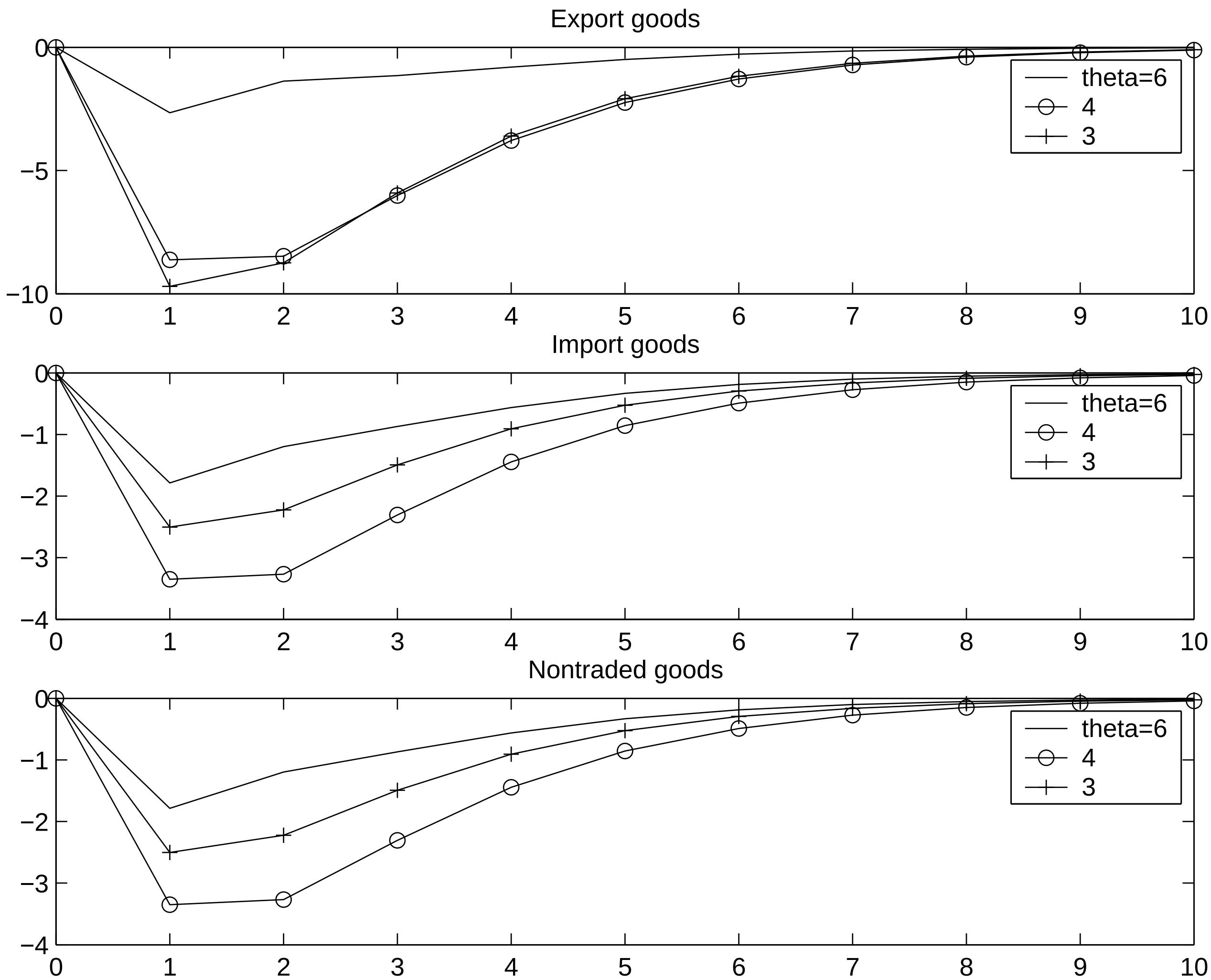


Figure 7.1 Real exchange rate decomposition and transport costs: Productivity shock

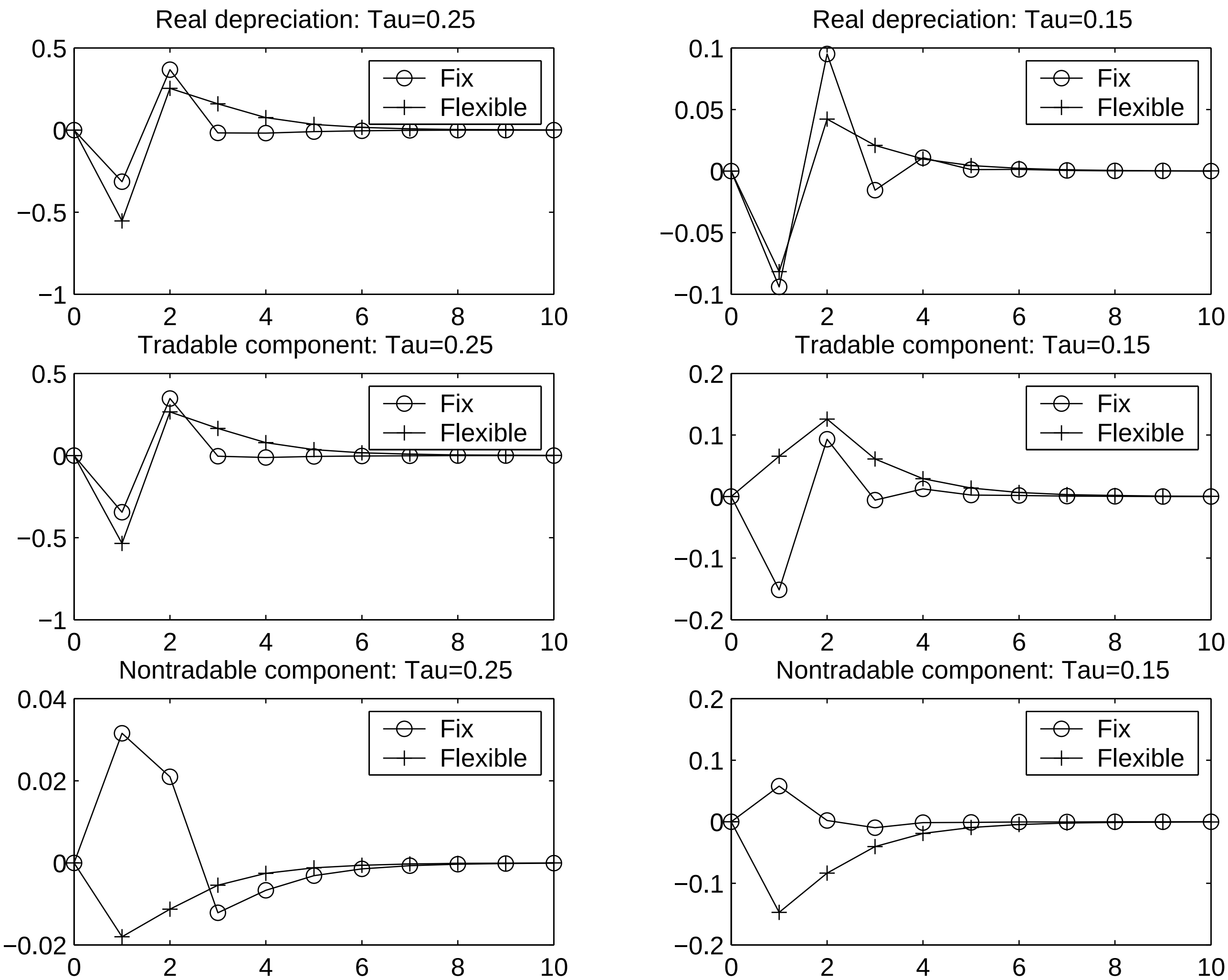


Figure 7.2 Real exchange rate decomposition and transport costs: Interest rate shock

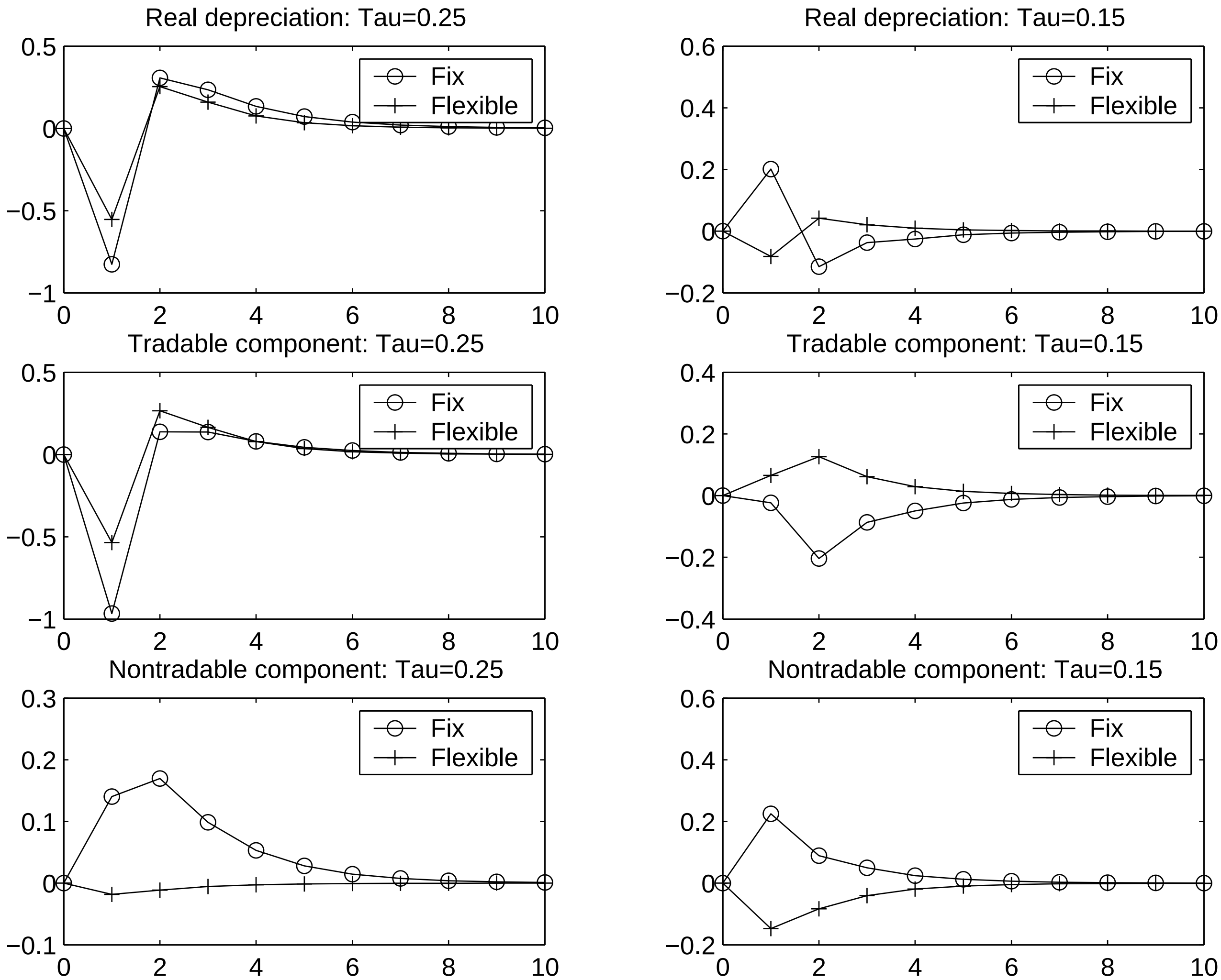


Figure 8.1 Real exchange rate decomposition and intratemporal elasticity of substitution: Productivity shock

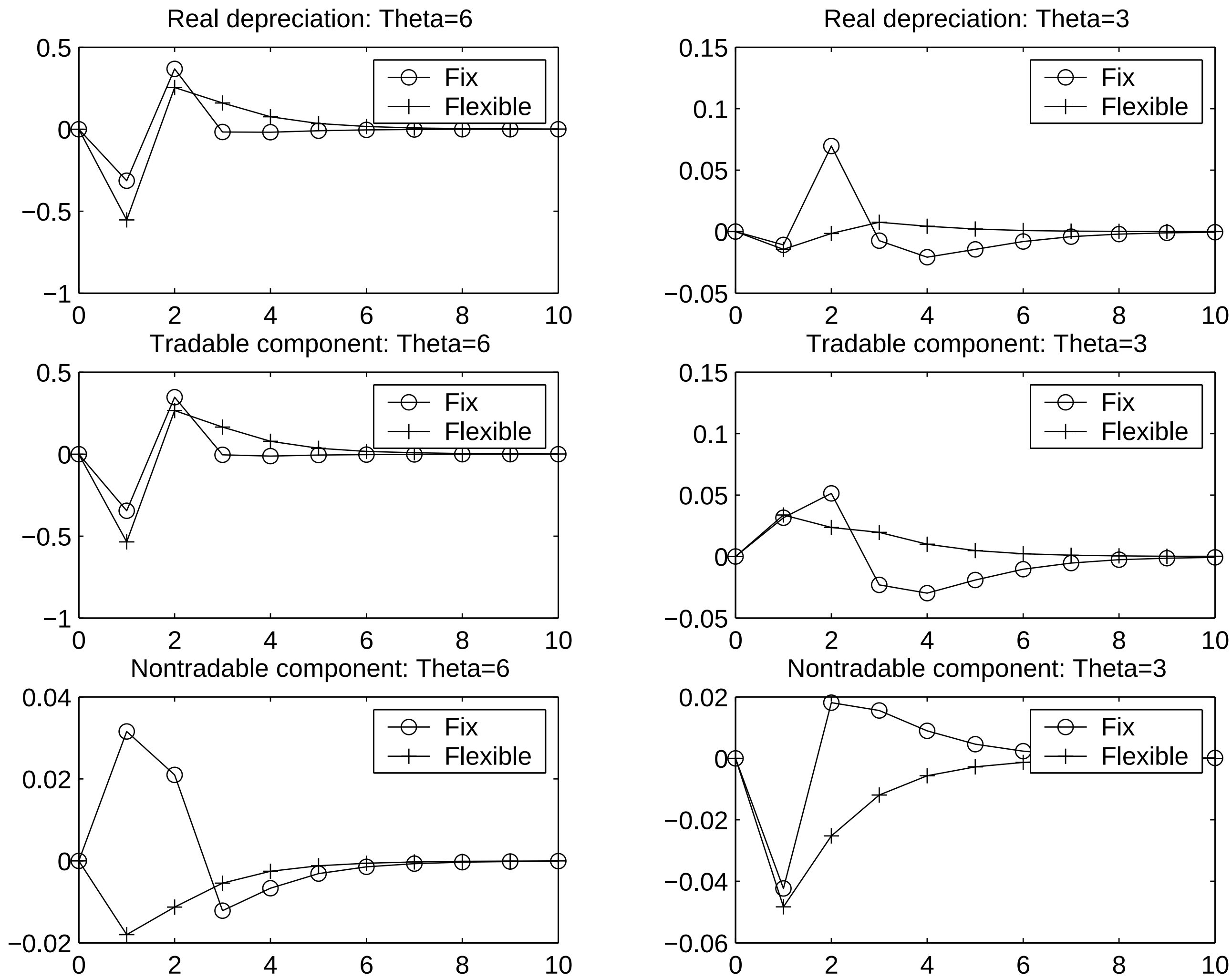
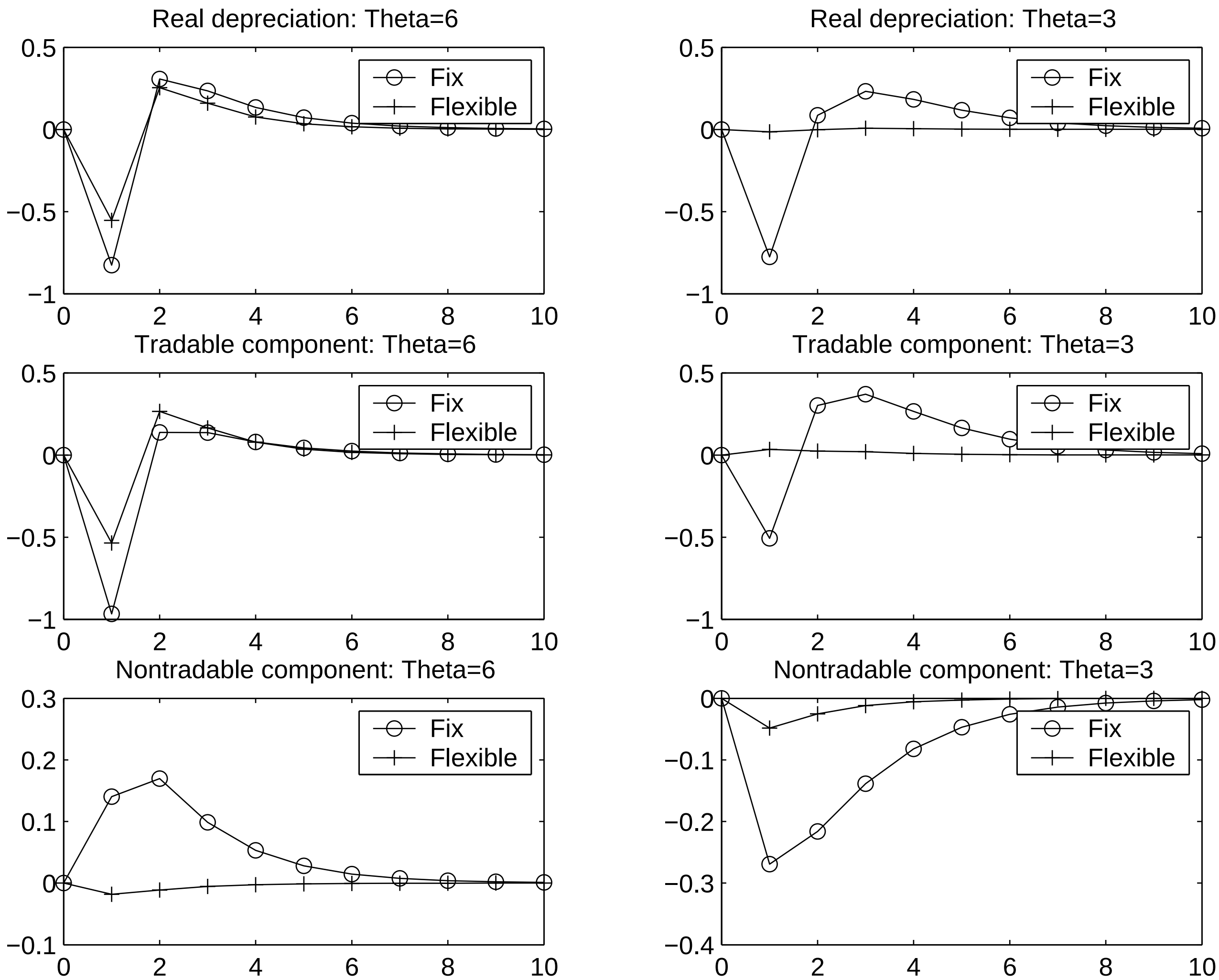


Figure 8.2 Real exchange rate decomposition and intratemporal elasticity of substitution: Interest rate shock



Technical Appendix

The technical appendix gives the detailed derivation of the first order conditions, the log-linearized system and the decomposition of the real exchange rate.

A Optimization Problems

A.1 Employment Agency

The employment agency's problem can be written as the Lagrangean,

$$\mathscr{L} = \int_0^\alpha W_t^i l_t^i di - \mu_t \left[L_t - 1 \right]. \tag{A.1}$$

The employment agency chooses the stochastic process $\{l_t^i\}_{t=0}^\infty$ to minimize (A.1) taking as given W_t^i and W_t. The first order condition with respect to l_t^i is

$$W_t^i = \mu_t \frac{dL_t}{dl_t^i}. \tag{A.2}$$

The function form of L_t indicates that

$$\frac{dL_t}{dl_t^i} = \left(\frac{1}{\alpha} L_t \right)^{\frac{1}{\eta}} \left(l_t^i \right)^{-\frac{1}{\eta}}. \tag{A.3}$$

(A.2) and (A.3) reduce to

$$W_t^i = \mu_t \left(\frac{1}{\alpha} L_t \right)^{\frac{1}{\eta}} l_t^{i-\frac{1}{\eta}} \tag{A.4}$$

Multiply (A.4) with l_t^i and aggregate over i. It is equivalent to W_t when $L_t = 1$.

$$W_t = \int_0^\alpha W_t^i l_t^i di \bigg|_{L_t=1} = \mu_t \left(\frac{1}{\alpha} L_t \right)^{\frac{1}{\eta}} \int_0^\alpha l_t^{i\frac{\eta-1}{\eta}} di \bigg|_{L_t=1} = \mu_t \tag{A.5}$$

(A.4) and (A.5) give the labor demand function

$$l_t^i = \frac{1}{\alpha} \left(\frac{W_t^i}{W_t} \right)^{-\eta} L_t. \tag{A.6}$$

A.2 Households

The household i's problem can be written as the Lagrangean

$$
\begin{aligned}
\mathscr{L} = & E_t \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{\sigma}{\sigma-1} C_t^{i\frac{\sigma-1}{\sigma}} + \frac{\chi}{1-\epsilon} m_t^{i1-\epsilon} - \frac{1}{\mu} l_t^{i\mu}(W_t^i) \\
& -\lambda_t \Bigg(C_t^i + m_t^i - m_{t-1}^i \frac{P_{t-1}}{P_t} + \frac{F_t^i}{P_t} - (1+i_{t-1}) \frac{F_{t-1}^i}{P_t} + \frac{S_t F_t^{f,i}}{P_t} + \Phi(\frac{F_t^i}{P_t}) \\
& + \Phi^f(\frac{F_t^{f,i} S_t}{P_t}) - (1+i_{t-1}^{\star}) \frac{S_t F_{t-1}^{f,i}}{P_t} - T_t^i - (1+\tau_w) \frac{W_t^i}{P_t} l_t^i(W_t^i) \\
& + h(\pi_t^{w^i}) - \frac{\Pi_t^i}{P_t} \Bigg) \Bigg], \qquad \text{(A.7)}
\end{aligned}
$$

where the labor demand function l_t^i is given by (A.6). The wage inflation $\pi_t^{w^i}$ is defined as $W_t^i/W_{t-1}^i - 1$. The wage adjustment cost $h(\pi_t^{w^i})$ is quadratic in deviations from the deterministic steady state level of wage inflation $\pi_t^{w^i} - \pi_{ss}^{w^i}$, $h(\pi_t^{w^i}) = \phi^w \left(\pi_t^{w^i} - \pi_{ss}^{w^i}\right)^2/2$.

The household i chooses the set of stochastic processes $\{c_t^i(z), C_t^i, m_t^i, F_t^i, F_t^{f,i}, W_t^i\}_{t=0}^{\infty}$ to maximize (A.7) subject to the transversality conditions

$$
lim_{j\to\infty} E_t \left[F_{t+j}^i / \Pi_{s=0}^{j-1}(1+i_{t+s})\right] \geqslant 0, \quad lim_{j\to\infty} E_t \left[S_{t+j} F_{t+j}^{f,i} / \Pi_{s=0}^{j-1}(1+i_{t+s}^{\star})\right] \geqslant 0.
$$

The first order conditions with respect to $F_t^i, F_t^{f,i}, C_t^i, m_t^i$ and W_t^i are as follows.

$$
\frac{\lambda_t}{P_t}(1+\Phi'(\frac{F_t^i}{P_t})) = \beta(1+i_t) E_t \left[\frac{\lambda_{t+1}}{P_{t+1}}\right] \qquad \text{(A.8)}
$$

$$
\frac{\lambda_t S_t}{P_t}(1+\Phi'(\frac{F_t^{f,i} S_t}{P_t})) = \beta(1+i_t^{\star}) E_t \left[\frac{\lambda_{t+1} S_{t+1}}{P_{t+1}}\right] \qquad \text{(A.9)}
$$

$$
C_t^{i-1/\sigma} = \lambda_t \qquad \text{(A.10)}
$$

$$
\chi m_t^{i-\epsilon} = \lambda_t - \beta E_t \left[\lambda_{t+1} \frac{P_t}{P_{t+1}}\right] \qquad \text{(A.11)}
$$

$$
\frac{\eta l_t^{i\mu}}{W_t^i} = \lambda_t \left(\frac{(1+\tau_w)}{P_t}((\eta-1) l_t^i) + \frac{h'(\pi_t^{w^i})}{W_{t-1}^i} \right) - \beta E_t \lambda_{t+1} \frac{h'(\pi_{t+1}^{w^i}) W_{t+1}^i}{W_t^{i2}} \qquad \text{(A.12)}
$$

The demand $c_t^i(z)$ is derived from the following Lagrangean.

$$
\mathscr{L} = \int_0^1 c_t^i(z) p_t(z) dz - \omega_t \left[C_t^i - 1\right]. \qquad \text{(A.13)}
$$

The household i chooses the stochastic process $\{c_t^i(z)\}_{t=0}^{\infty}$ to minimize (A.13) taking given $p_t(z)$ and P_t. The first order condition with respect to $c_t^i(z)$ is

$$\int_0^1 p_t(z)dz = \omega_t \frac{dC_t}{dc_t^i(z)}. \tag{A.14}$$

The function form of C_t indicates that

$$\frac{dC_t^i}{dc_t^i(z)} = C_t^{i\frac{1}{\theta}} c_t^i(z)^{-\frac{1}{\theta}}. \tag{A.15}$$

(A.14) and (A.15) reduce to

$$p_t(z) = \omega_t C_t^{i\frac{1}{\theta}} c_t^i(z)^{-\frac{1}{\theta}} \tag{A.16}$$

Multiply (A.16) with $c_t^i(z)$ and aggregate over z. It is equivalent to P_t when $C_t^i = 1$.

$$P_t = \int_0^1 p_t(z)c_t^i(z)dz\bigg|_{C_t^i=1} = \omega_t C_t^{\frac{1}{\theta}} \int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}}dz\bigg|_{C_t^i=1} = \phi_t \tag{A.17}$$

(A.16) and (A.17) give the demand function

$$c_t^i(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t^i. \tag{A.18}$$

Since every household faces the same decision problem, I can remove the superscript i and simplify the first order conditions as Equation (5)-(9) in the text.

A.3 Firms

A.3.1 Exportable firms

The objective function for the exportable firm z is the present discounted value of its real profit stream

$$\begin{aligned} V_t &= E_t \Pi_{s=0}^{\infty} R_{t,t+s} \left[\frac{(1+\tau_y)p_{t+s}^h(z) - MC_{t+s}(z)}{P_{t+s}} \left(\left[\frac{p_{t+s}^h(z)}{P_{t+s}}\right]^{-\theta} \alpha C_{t+s} \right.\right. \\ &\quad \left.\left. + \left[\frac{p_{t+s}^h(z)}{S_{t+s}P_{t+s}^{\star}(1-\tau)}\right]^{-\theta} (1-\alpha)\,\frac{C_{t+s}^{\star}}{1-\tau}\right)\right]. \end{aligned} \tag{A.19}$$

$R_{t,t+s}$ denotes the s period ahead real discount factor based on the the real interest rate.

An exportable firm chooses the stochastic processes $\{p_t^h(z)\}_{t=0}^{\infty}$ to maximize (A.19) taking the sequences $\{P_t, P_t^\star, W_t, D_t, D_t^\star\}_{t=0}^{\infty}$ as given. The optimal price setting rule is

$$0 = \left[\frac{1+\tau_y}{P_t} - \frac{\theta}{P_t}(1+\tau_y - \frac{MC_t(z)}{p_t^h(z)})\right]\left(\left[\frac{p_t^h(z)}{P_t}\right]^{-\theta}\alpha C_t + \left[\frac{p_t^h(z)}{S_t P_t^\star(1-\tau)}\right]^{-\theta}\frac{1-\alpha}{1-\tau}C_t^\star\right) \tag{A.20}$$

Rewrite the first order condition.

$$p_t^h(z) = MC_t(z) \tag{A.21}$$

A.3.2 Nonexportable Firms

The objective function for the nonexportable firm z is the present discounted value of its real profit stream

$$V_t = E_t \Pi_{s=0}^{\infty} R_{t,t+s} \frac{(1+\tau_y)p_{t+s}^n(z) - MC_{t+s}(z)}{P_{t+s}}\left[\frac{p_{t+s}^n(z)}{P_{t+s}}\right]^{-\theta}\alpha C_{t+s}. \tag{A.22}$$

A nonexportable firm chooses the stochastic processes $\{p_t^n(z)\}_{t=0}^{\infty}$ to maximize (A.22) taking the sequences $\{P_t, P_t^\star, W_t, D_t, D_t^\star\}_{t=0}^{\infty}$ as given. The optimal price setting rule is

$$0 = \left[\frac{1+\tau_y}{P_t} - \frac{\theta}{P_t}(1+\tau_y - \frac{MC_t(z)}{p_t^n(z)})\right]\left[\frac{p_t^n(z)}{P_t}\right]^{-\theta}\alpha C_t. \tag{A.23}$$

Rewrite the first order condition.

$$p_t^n(z) = MC_t(z) \tag{A.24}$$

B Log-Linearized System

Define the gross real and nominal interest rate as $R_t = 1 + r_t$ and $I_t = 1 + i_t$, respectively. In the steady state, $R_{ss} = 1/\beta$. Since $I_t = R_t E_t \pi_{t+1}$, the gross nominal interest rate becomes

$$I_{ss} = R_{ss}\pi_{ss} = \pi_{ss}/\beta.$$

I assume

$$F_{ss} = F_{ss}^f = 0 = F_{ss}^f = F_{ss}^{f\star}$$

As a result, the household budget constraint gives

$$C_{ss} = w_{ss} l_{ss}.$$

From the wage setting rule,

$$w_{ss} = l_{ss}^{\mu-1} C_{ss}^{1/\sigma}.$$

Also, the relative purchasing power parity holds in the steady state. It implies

$$d_{ss} = \pi_{ss} - \pi_{ss}^{\star}.$$

Let $\hat{x}_t$ denote the log-linearization around steady state level $\hat{x}_t = dlog(x_t) = dx_t/x_{ss}$.

B.1 Households

The real value of the domestic and foreign bond are defined as $f_t = F_t/P_t$ and $f_t^f = F_t^f S_t/P_t$. I define their deviation from the steady state as $\hat{f}_t = f_t$ and $\hat{f}_t^f = f_t^f$. The households' first order conditions with respect to F_t, F_t^f and m_t can be log-linearized as follows.

$$\phi \hat{f}_t = \hat{I}_t + \frac{1}{\sigma}(\hat{C}_t - \hat{C}_{t+1}) - \hat{\pi}_{t+1} \tag{B.1}$$

$$\phi^{\star} \hat{f}_t^f = \hat{I}_t^{\star} + \frac{1}{\sigma}(\hat{C}_t - \hat{C}_{t+1}) - \hat{\pi}_{t+1} + \hat{d}_{t+1} \tag{B.2}$$

$$\epsilon \hat{m}_t = \frac{1}{\sigma}\hat{C}_t - \frac{\beta}{\pi_{ss} - \beta}\hat{I}_t \tag{B.3}$$

Log-linearizing the wage equation gives the following wage inflation dynamics.

$$\hat{\pi}_{t+1}^w = \frac{1}{\beta}\hat{\pi}_t^w - B_w \left[(\mu - 1)\hat{l}_t + \frac{1}{\sigma}\hat{C}_t - \hat{w}_t \right], \tag{B.4}$$

$B_w = \eta l_{ss} w_{ss} (\phi^w \beta \pi_{ss}^2)^{-1}$. The dynamics of the real wage can be derived from the definition of π_w,

$$\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{w_t}{w_{t-1}}\pi_t. \tag{B.5}$$

Hence,

$$\hat{w}_t = \hat{w}_{t-1} + \hat{\pi}_t^w - \hat{\pi}_t. \tag{B.6}$$

The corresponding equations for the foreign households are as follows.

$$\phi^{\star}\hat{f}_t^{f\star} = \hat{I}_t^{\star} + \frac{1}{\sigma}(\hat{C}_t^{\star} - \hat{C}_{t+1}^{\star}) - \hat{\pi}_{t+1}^{\star} \quad \text{(B.7)}$$

$$\phi\hat{f}_t^{\star} = \hat{I}_t + \frac{1}{\sigma}(\hat{C}_t^{\star} - \hat{C}_{t+1}^{\star}) - \hat{\pi}_{t+1}^{\star} - \hat{d}_{t+1} \quad \text{(B.8)}$$

$$\epsilon\hat{m}_t^{\star} = \frac{1}{\sigma}\hat{C}_t^{\star} - \frac{\beta}{\pi_{ss}^{\star} - \beta}\hat{I}_t^{\star} \quad \text{(B.9)}$$

$$\hat{\pi}_{t+1}^{w\star} = \frac{1}{\beta}\hat{\pi}_t^{w\star} - B_w^{\star}\left[(\mu - 1)\hat{l}_t^{\star} + \frac{1}{\sigma}\hat{C}_t^{\star} - \hat{w}_t^{\star}\right], \quad \text{(B.10)}$$

$B_w^{\star} = \eta l_{ss}^{\star} w_{ss}^{\star} (\phi^{w\star}\beta\pi_{ss}^{\star 2})^{-1}$,

$$\hat{w}_t^{\star} = \hat{w}_{t-1}^{\star} + \hat{\pi}_t^{w\star} - \hat{\pi}_t^{\star}. \quad \text{(B.11)}$$

B.2 Goods Markets Clearing Conditions

B.2.1 Home Goods Markets

Gross domestic output (GDP) in the home country is defined as

$$P_t Y_t = \int_0^{z_t^l} p_t^h(z) y_{t,H}(z) dz + \int_{z_t^l}^{z_t^h} p_t^n(z) y_{t,N}(z) dz$$

Define the aggregate output in the export and nontraded sectors as $Y_{t,H}$ and $Y_{t,N}$ such that

$$P_t Y_{t,H} = \int_0^{z_t^l} p_t^h(z) y_{t,H}(z) dz$$

$$P_t Y_{t,N} = \int_{z_t^l}^{z_t^h} p_t^n(z) y_{t,N}(z) dz.$$

Hence,

$$Y_t = Y_{t,H} + Y_{t,N}. \quad \text{(B.12)}$$

It is useful to define the world demand in terms of the home consumption basket as

$$C_t^w = \alpha C_t + (1 - \alpha)(1 - \tau)^{\theta - 1} Q_t^{\theta} C_t^{\star}.$$

Substitute the export goods markets clearing condition into $y_{t,H}(z)$ in the definition of $Y_{t,H}$.

$$\begin{aligned}
Y_{t,H} &= \int_0^{z_t^l} \left(\frac{p_t^h(z)}{P_t}\right)^{1-\theta} C_t^w dz \\
&= z_t^l \frac{1}{z_t^l} \int_0^{z_t^l} \left(\frac{p_t^h(z)}{P_t}\right)^{1-\theta} dz C_t^w \\
&= z_t^l \left(\frac{P_{t,H}}{P_t}\right)^{1-\theta} C_t^w \\
\therefore Y_{t,H} &= z_t^l \tilde{P}_{t,H}^{1-\theta} C_t^w. \qquad \text{(B.13)}
\end{aligned}$$

Substitute the nontraded goods markets clearing condition into $y_{t,N}(z)$ in the definition of $Y_{t,N}$.

$$\begin{aligned}
Y_{t,N} &= \int_{z_t^l}^{z_t^h} \left(\frac{p_t^n(z)}{P_t}\right)^{1-\theta} \alpha C_t dz \\
&= \delta_t \frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} \left(\frac{p_t^n(z)}{P_t}\right)^{1-\theta} dz \alpha C_t \\
&= \delta_t \left(\frac{P_{t,N}}{P_t}\right)^{1-\theta} \alpha C_t \\
\therefore Y_{t,N} &= \delta_t \tilde{P}_{t,N}^{1-\theta} \alpha C_t \qquad \text{(B.14)}
\end{aligned}$$

Define the average productivity of the home export sector in the steady state as

$$a_H = \left(\frac{1}{z_{ss}^l} \int_0^{z_{ss}^l} a(z)^{\theta-1} dz\right)^{\frac{1}{\theta-1}}.$$

The corresponding productivity of the home nontraded sector is

$$a_N = \left(\frac{1}{\delta_{ss}} \int_{z_{ss}^l}^{z_{ss}^h} a(z)^{\theta-1} dz\right)^{\frac{1}{\theta-1}}.$$

In the steady state, $\tilde{p}_{ss}^j(z) = w_{ss}/a(z)$ for $j \in (h, n)$. Then,

$$\tilde{P}_{ss,H} = \frac{w_{ss}}{a_H}, \qquad \text{(B.15)}$$

$$\tilde{P}_{ss,N} = \frac{w_{ss}}{a_N}. \qquad \text{(B.16)}$$

As a result, the output in the steady state becomes

$$Y_{ss} = Y_{ss,H} + Y_{ss,N}, \tag{B.17}$$

where

$$Y_{ss,H} = z^l_{ss} \left(\frac{w_{ss}}{a_H} \right)^{1-\theta} C^w_{ss},$$
$$Y_{ss,N} = \delta_{ss} \left(\frac{w_{ss}}{a_N} \right)^{1-\theta} \alpha C_{ss}.$$

Define γ as the share of domestic demand for the home export goods,

$$\gamma = \frac{\alpha C_{ss}}{C^w_{ss}}.$$

Then the output share of the export and nontraded sectors in the steady state can be defined as follows.

$$\Gamma_H = \frac{Y_{ss,H}}{Y_{ss}} = \frac{z^l_{ss} {a_H}^{\theta-1}}{z^l_{ss} {a_H}^{\theta-1} + \delta_{ss} a_N^{\theta-1} \gamma}$$
$$\Gamma_N = \frac{Y_{ss,N}}{Y_{ss}} = \frac{\delta_{ss} {a_N}^{\theta-1} \gamma}{z^l_{ss} {a_H}^{\theta-1} + \delta_{ss} a_N^{\theta-1} \gamma}$$

The log-linearized version of the home GDP becomes

$$\hat{Y}_t = \Gamma_H \hat{Y}_{t,H} + \Gamma_N \hat{Y}_{t,N} \tag{B.18}$$

where $\hat{Y}_{t,H}$ and $\hat{Y}_{t,N}$ are given by the following expressions.

$$\hat{Y}_{t,H} = \hat{z}^l_t + (1-\theta) \hat{\tilde{P}}_{t,H} + \hat{C}^w_t \tag{B.19}$$
$$\hat{Y}_{t,N} = \hat{\delta}_t + (1-\theta) \hat{\tilde{P}}_{t,N} + \hat{C}_t, \tag{B.20}$$

where

$$\hat{\delta}_t = \frac{z^h_{ss}}{\delta_{ss}} \hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}} \hat{z}^h_t$$

From the definition of C^w_t,

$$\hat{C}^w_t = \gamma \hat{C}_t + (1-\gamma)(\hat{C}^\star_t + \theta \hat{q}_t). \tag{B.21}$$

Therefore $\hat{Y}_{t,H}$ can be written as

$$\hat{Y}_{t,H} = \hat{z}^l_t + (1-\theta) \hat{\tilde{P}}_{t,H} + \gamma \hat{C}_t + (1-\gamma)(\hat{C}^\star_t + \theta \hat{q}_t). \tag{B.22}$$

To summarize, the dynamics of the home aggregate outputs are characterized by $\hat{Y}_t$, $\hat{Y}_{t,N}$ and $\hat{Y}_{t,H}$, as indicated in Equations (B.18), (B.20) and (B.22).

B.2.2 Foreign Goods Markets

The dynamics of the foreign goods markets can be derived in a similar way. By definition,

$$P_t^\star Y_t^\star = \int_{z_t^h}^1 p_t^{f\star}(z) y_{t,F}^\star(z) dz + \int_{z_t^l}^{z_t^h} p_t^{n\star}(z) y_{t,N}^\star(z) dz.$$

Define the aggregate output in the export and nontraded sectors as

$$P_t^\star Y_{t,F}^\star = \int_{z_t^h}^1 p_t^{f\star}(z) y_{t,F}^\star(z) dz,$$

and

$$P_t^\star Y_{t,N}^\star = \int_{z_t^l}^{z_t^h} p_t^{n\star}(z) y_{t,N}^\star(z) dz.$$

Hence,

$$Y_t^\star = Y_{t,F}^\star + Y_{t,N}^\star. \tag{B.23}$$

Define $C_t^{w\star}$ as the world demand in terms of the foreign consumption basket,

$$C_t^{w\star} = \alpha(1-\tau)^{\theta-1} Q_t^{-\theta} C_t + (1-\alpha) C_t^\star.$$

Substituting the export goods markets clearing condition into $y_{t,F}^\star(z)$ in the definition of $Y_{t,F}^\star$ gives

$$Y_{t,F}^\star = (1-z_t^h) \tilde{P}_{t,F}^{\star 1-\theta} C_t^{w\star} \tag{B.24}$$

Substitute the nontraded goods markets clearing condition into $y_{t,N}^\star(z)$ in the definition of $Y_{t,N}^\star$. Hence,

$$Y_{t,N}^\star = \delta_t \tilde{P}_{t,N}^{\star 1-\theta} (1-\alpha) C_t^\star. \tag{B.25}$$

In the steady state, $\tilde{p}_{ss}^{j\star}(z) = w_{ss}^\star$ for $j \in (f, n)$. Then,

$$\tilde{P}_{ss,F}^\star = \tilde{P}_{ss,N}^\star = w_{ss}^\star.$$

As a result,

$$Y_{ss,F}^\star = (1-z_{ss}^h) w_{ss}^{\star 1-\theta} C_{ss}^{w\star},$$

$$Y^\star_{ss,N} = \delta_{ss} w^{\star 1-\theta}_{ss} (1-\alpha) C^\star_{ss}.$$

Define $\gamma^\star$ as the share of domestic demand for the foreign export goods,

$$\gamma^\star = \frac{(1-\alpha)C^\star_{ss}}{C^{w\star}_{ss}}.$$

Then the output share of the export and nontraded sectors in the steady state can be defined as follows.

$$\Gamma^\star_F = \frac{Y^\star_{ss,F}}{Y^\star_{ss}} = \frac{1-z^h_{ss}}{1-z^h_{ss}+\delta_{ss}\gamma^\star}$$

$$\Gamma^\star_N = \frac{Y^\star_{ss,N}}{Y^\star_{ss}} = \frac{\delta_{ss}\gamma^\star}{1-z^h_{ss}+\delta_{ss}\gamma^\star}$$

The log-linearized version of the foreign GDP becomes

$$\hat{Y}^\star_t = \Gamma^\star_F \hat{Y}^\star_{t,F} + \Gamma^\star_N \hat{Y}^\star_{t,N} \tag{B.26}$$

where $\hat{Y}^\star_{t,F}$ and $\hat{Y}^\star_{t,N}$ are given by the following expressions.

$$\hat{Y}^\star_{t,F} = -\frac{z^h_{ss}}{1-z^h_{ss}}\hat{z}^h_t + (1-\theta)\hat{\tilde{P}}^\star_{t,F} + \hat{C}^{w\star}_t \tag{B.27}$$

$$\hat{Y}^\star_{t,N} = \hat{\delta}_t + (1-\theta)\tilde{P}^\star_{t,N} + \hat{C}^\star_t, \tag{B.28}$$

where

$$\hat{\delta}_t = \frac{z^h_{ss}}{\delta_{ss}}\hat{z}^h_t - \frac{z^l_{ss}}{\delta_{ss}}\hat{z}^l_t.$$

From the definition of $C^{w\star}_t$,

$$\hat{C}^{w\star}_t = (1-\gamma^\star)(\hat{C}_t - \theta\hat{q}_t) + \gamma^\star \hat{C}^\star_t. \tag{B.29}$$

Therefore $\hat{Y}^\star_{t,F}$ can be written as

$$\hat{Y}^\star_{t,F} = -\frac{z^h_{ss}}{1-z^h_{ss}}\hat{z}^h_t + (1-\theta)\hat{\tilde{P}}^\star_{t,F} + (1-\gamma^\star)(\hat{C}_t - \theta\hat{q}_t) + \gamma^\star\hat{C}^\star_t. \tag{B.30}$$

To summarize, the dynamics of the foreign aggregate outputs are characterized by $\hat{Y}^\star_t$, $\hat{Y}^\star_{t,N}$ and $\hat{Y}^\star_{t,F}$, as indicated in Equations (B.26), (B.28) and (B.30).

B.3 Aggregate Resource Constraint

Multiply the agent budget constraint by the number of population.

$$\begin{aligned}\alpha P_t C_t &+ \alpha(M_t - M_{t-1}) + \alpha(F_t - I_{t-1}F_{t-1}) + \alpha(S_t F_t^f - I_{t-1}^{\star} S_t F_{t-1}^f) \\ &+ \alpha P_t \Phi(F_t/P_t) + \alpha P_t \Phi^f(F_t^f S_t/P_t) \\ &= \alpha P_t T_t + \alpha(1+\tau_w) W_t l_t - \alpha P_t h(\pi_t^w) + \alpha \Pi_t \end{aligned} \tag{B.31}$$

Substitute the government budget constraint,

$$\alpha(M_t - M_{t-1}) = \alpha P_t T_t + \alpha \tau_w W_t L_t + \tau_y P_t Y_t, \tag{B.32}$$

into the resource constraint above.

$$\begin{aligned}\alpha P_t C_t &+ \alpha(F_t - I_{t-1}F_{t-1}) + \alpha(S_t F_t^f - I_{t-1}^{\star} S_t F_{t-1}^f) + \alpha P_t \Phi(F_t/P_t) \\ &+ \alpha P_t \Phi^f(F_t^f S_t/P_t) = \alpha W_t l_t - \alpha P_t h(\pi_t^w) + \alpha \Pi_t - \tau_y P_t Y_t \end{aligned} \tag{B.33}$$

From the profit functions, the aggregate profit dividends must be equal to the total outputs net of labor costs, plus the production subsidy.

$$\alpha \Pi_t = P_t Y_t - \alpha W_t l_t + \tau_y P_t Y_t \tag{B.34}$$

Substitute the expression above into (B.33).

$$\begin{aligned}\alpha P_t C_t &+ \alpha(F_t - I_{t-1}F_{t-1}) + \alpha(S_t F_t^f - I_{t-1}^{\star} S_t F_{t-1}^f) + \alpha P_t \Phi(F_t/P_t) + \alpha P_t \Phi^f(F_t^f S_t/P_t) \\ &+ \alpha P_t h(\pi_t^w) = P_t Y_t \end{aligned} \tag{B.35}$$

The log-linearized version of the above equation is

$$\alpha C_{ss} \hat{C}_t = \alpha(\hat{f}_t - R_{ss}\hat{f}_{t-1}) + \alpha(\hat{f}_t^f - R_{ss}\hat{f}_{t-1}^f) + Y_{ss}\hat{Y}_t. \tag{B.36}$$

The corresponding equation for the foreign country is

$$(1-\alpha) C_{ss}^{\star} \hat{C}_t^{\star} = (1-\alpha)(\hat{f}_t^{\star} - R_{ss}\hat{f}_{t-1}^{\star}) + (1-\alpha)(\hat{f}_t^{f\star} - R_{ss}\hat{f}_{t-1}^{f\star}) + Y_{ss}^{\star}\hat{Y}_t^{\star}. \tag{B.37}$$

B.4 Labor Markets

B.4.1 Home Labor Market

The labor market clearing condition in the home country is

$$\alpha l_t = l_{t,H} + l_{t,N}, \tag{B.38}$$

where

$$l_{t,H} = \int_0^{z_t^l} \frac{y_{t,H}(z)}{X_t a(z)} dz, \tag{B.39}$$

$$l_{t,N} = \int_{z_t^l}^{z_t^h} \frac{y_{t,N}(z)}{X_t a(z)} dz. \tag{B.40}$$

Substitute $y_{t,H}(z)$ into $l_{t,H}$.

$$\begin{aligned} l_{t,H} &= \int_0^{z_t^l} \frac{\tilde{p}_t^h(z)^{-\theta}}{X_t a(z)} C_t^w dz \\ &= \int_0^{z_t^l} \frac{\tilde{p}_t^h(z)^{1-\theta}}{\tilde{p}_t^h(z) X_t a(z)} dz C_t^w \\ &= \int_0^{z_t^l} \frac{\tilde{p}_t^h(z)^{1-\theta}}{w_t} dz C_t^w \\ \therefore l_{t,H} &= \frac{Y_{t,H}}{w_t} \end{aligned} \tag{B.41}$$

The log-linearized version of the aggregate labor demand in the home export sector is as follows.

$$\hat{l}_{t,H} = \hat{Y}_{t,H} - \hat{w}_t \tag{B.42}$$

For the home nontraded sector, I can derive the aggregate labor demand in a similar fashion.

$$\hat{l}_{t,N} = \hat{Y}_{t,N} - \hat{w}_t \tag{B.43}$$

The dynamics of the aggregate labor demand in the home country is derived from (B.38).

$$\hat{l}_t = \frac{l_{ss,H}}{\alpha} \hat{l}_{t,H} + \frac{l_{ss,N}}{\alpha} \hat{l}_{t,N} \tag{B.44}$$

where

$$l_{ss,H} = z_{ss}^l \left(\frac{w_{ss}}{a_H} \right)^{1-\theta} \frac{C_{ss}^w}{w_{ss}},$$

$$l_{ss,N} = \delta_{ss} \left(\frac{w_{ss}}{a_N} \right)^{1-\theta} \frac{\alpha C_{ss}}{w_{ss}}.$$

To summarize, (B.42)-(B.44) summarizes the dynamics of the labor demand in the home country.

B.4.2 Foreign Labor Market

The labor market clearing condition in the home country is

$$(1-\alpha)l_t^\star = l_{t,F}^\star + l_{t,N}^\star, \tag{B.45}$$

where

$$l_{t,F}^\star = \int_{z_t^h}^{1} \frac{y_{t,F}^\star(z)}{X_t^\star a^\star(z)} dz, \tag{B.46}$$

$$l_{t,N}^\star = \int_{z_t^l}^{z_t^h} \frac{y_{t,N}^\star(z)}{X_t^\star a^\star(z)} dz. \tag{B.47}$$

Since $a^\star(z) = 1$ for all z, all firms in a sector choose the same price. I can derive the log-linearized version of the labor demand schedule by following the similar steps as done in the case of home labor market.

$$\hat{l}_t^\star = \frac{l_{ss,F}^\star}{1-\alpha}\hat{l}_{t,F}^\star + \frac{l_{ss,N}^\star}{1-\alpha}\hat{l}_{t,N}^\star \tag{B.48}$$

where

$$l_{ss,F}^\star = (1-z_{ss}^h) w_{ss}^{-\theta} C_{ss}^{w\star},$$

$$l_{ss,N}^\star = \delta_{ss} w_{ss}^{-\theta}(1-\alpha) C_{ss}^\star.$$

The dynamics of the sectoral demand schedule in the foreign export and nontraded sector are given by the following equations.

$$\hat{l}_{t,F}^\star = \hat{Y}_{t,F}^\star - \hat{w}_t^\star \tag{B.49}$$

$$\hat{l}_{t,N}^\star = \hat{Y}_{t,N}^\star - \hat{w}_t^\star \tag{B.50}$$

B.5 Aggregate Price Dynamics

This subsection shows the derivation of the paths of sectoral inflation $\pi_{t,H}, \pi_{t,N}, \pi_{t,F}^\star$ and $\pi_{t,N}^\star$, and the paths of CPI inflation π_t and $\pi_t^\star$.

B.5.1 Sectoral Prices

Define the average productivity of the home export sector as

$$a_{t,H} = \left(\frac{1}{z_t^l} \int_0^{z_t^l} a(z)^{\theta-1} dz \right)^{\frac{1}{\theta-1}}.$$

The corresponding productivity of the home nontraded sector is

$$a_{t,N} = \left(\frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} a(z)^{\theta-1} dz \right)^{\frac{1}{\theta-1}}.$$

I can rewrite the equilibrium sectoral prices using the optimal price-setting rules.

$$P_{t,H} = \frac{W_t}{X_t a_{t,H}} \tag{B.51}$$

$$P_{t,N} = \frac{W_t}{X_t a_{t,N}} \tag{B.52}$$

$$P_{t,F} = \frac{S_t W_t^\star}{X_t^\star (1-\tau)} \tag{B.53}$$

Deflate the above three equations by P_t.

$$\tilde{P}_{t,H} = \frac{w_t}{X_t a_{t,H}} \tag{B.54}$$

$$\tilde{P}_{t,N} = \frac{w_t}{X_t a_{t,N}} \tag{B.55}$$

$$\tilde{P}_{t,F} = \frac{Q_t w_t^\star}{X_t^\star (1-\tau)} \tag{B.56}$$

Hence,

$$\hat{\tilde{P}}_{t,H} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,H} \tag{B.57}$$

$$\hat{\tilde{P}}_{t,N} = \hat{w}_t - \hat{X}_t - \hat{a}_{t,N} \tag{B.58}$$

$$\hat{\tilde{P}}_{t,F} = \hat{Q}_t + \hat{w}_t^\star - \hat{X}_t^\star. \tag{B.59}$$

The dynamics of the productivity in the home export and nontraded sectors are as follows.

$$\hat{a}_{t,H} = \frac{z_{ss}^l (a(z_{ss}^l)/a_H)^{\theta-1} - 1}{\theta - 1} \hat{z}_t^l \tag{B.60}$$

$$\hat{a}_{t,N} = \frac{z_{ss}^h (a(z_{ss}^h)/a_N)^{\theta-1} - z_{ss}^h/\delta_{ss}}{\theta - 1} \hat{z}_t^h - \frac{z_{ss}^l (a(z_{ss}^l)/a_N)^{\theta-1} - z_{ss}^l/\delta_{ss}}{\theta - 1} \hat{z}_t^l \tag{B.61}$$

The dynamics of sectoral inflations can be derived from their definition. By definition of $\pi_{t,H}$,

$$\pi_{t,H} = \frac{\tilde{P}_{t,H}}{\tilde{P}_{t-1,H}} \pi_t.$$

Therefore,

$$\hat{\pi}_{t,H} = \hat{\tilde{P}}_{t,H} - \hat{\tilde{P}}_{t-1,H} + \hat{\pi}_t. \tag{B.62}$$

Similarly,

$$\hat{\pi}_{t,N} = \hat{\tilde{P}}_{t,N} - \hat{\tilde{P}}_{t-1,N} + \hat{\pi}_t, \tag{B.63}$$

$$\hat{\pi}_{t,F} = \hat{\tilde{P}}_{t,F} - \hat{\tilde{P}}_{t-1,F} + \hat{\pi}_t. \tag{B.64}$$

For the foreign country,

$$P^\star_{t,H} = \frac{W_t}{S_t(1-\tau)X_t a_{t,H}} \tag{B.65}$$

$$P^\star_{t,N} = \frac{W^\star_t}{X^\star_t} \tag{B.66}$$

$$P^\star_{t,F} = \frac{W^\star_t}{X^\star_t}. \tag{B.67}$$

Deflate the sectoral prices with $P^\star_t$.

$$\tilde{P}^\star_{t,H} = \frac{w_t}{Q_t(1-\tau)X_t a_{t,H}} \tag{B.68}$$

$$\tilde{P}^\star_{t,N} = \frac{w^\star_t}{X^\star_t} \tag{B.69}$$

$$\tilde{P}^\star_{t,F} = \frac{w^\star_t}{X^\star_t}. \tag{B.70}$$

The dynamics of the sectoral price relative to consumer price index in the foreign country are as follows.

$$\hat{\tilde{P}}^\star_{t,H} = \hat{w}_t - \hat{Q}_t - \hat{X}_t - \hat{a}_{t,H} \tag{B.71}$$

$$\hat{\tilde{P}}^\star_{t,N} = \hat{w}^\star_t - \hat{X}^\star_t \tag{B.72}$$

$$\hat{\tilde{P}}^\star_{t,F} = \hat{w}^\star_t - \hat{X}^\star_t. \tag{B.73}$$

The dynamics of the sectoral inflations are derived in a similar fashion to those in the home country.

$$\hat{\pi}^\star_{t,H} = \hat{\tilde{P}}^\star_{t,H} - \hat{\tilde{P}}^\star_{t-1,H} + \hat{\pi}^\star_t \tag{B.74}$$

$$\hat{\pi}^\star_{t,F} = \hat{\tilde{P}}^\star_{t,F} - \hat{\tilde{P}}^\star_{t-1,F} + \hat{\pi}^\star_t \tag{B.75}$$

$$\hat{\pi}^\star_{t,N} = \hat{\tilde{P}}^\star_{t,N} - \hat{\tilde{P}}^\star_{t-1,N} + \hat{\pi}^\star_t \tag{B.76}$$

To summarize, (B.57)-(B.64) and (B.71)-(B.76) characterize the sectoral price and inflation dynamics in the home and foreign country, respectively.

B.5.2 CPI Dynamics

By definition,

$$P_t = [z_t^l P_{t,H}^{1-\theta} + \delta_t P_{t,N}^{1-\theta} + (1-z_t^h) P_{t,F}^{1-\theta}]^{\frac{1}{1-\theta}}. \tag{B.77}$$

Divide it through by P_{t-1}.

$$\pi_t = [z_t^l (\pi_{t,H} \tilde{P}_{t-1,H})^{1-\theta} + \delta_t (\pi_{t,N} \tilde{P}_{t-1,N})^{1-\theta} + (1-z_t^h)(\pi_{t,F} \tilde{P}_{t-1,F})^{1-\theta}]^{\frac{1}{1-\theta}} \tag{B.78}$$

Hence,

$$\pi_t^{1-\theta} = z_t^l (\pi_{t,H} \tilde{P}_{t-1,H})^{1-\theta} + \delta_t (\pi_{t,N} \tilde{P}_{t-1,N})^{1-\theta} + (1-z_t^h)(\pi_{t,F} \tilde{P}_{t-1,F})^{1-\theta} \tag{B.79}$$

Therefore,

$$\begin{aligned}(1-\theta)\hat{\pi}_t &= z_{ss}^l (\pi_{ss,H} \tilde{P}_{ss,H})^{1-\theta} (\hat{z}_t^l + (1-\theta)(\hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H})) \\ &+ \delta_{ss} (\pi_{ss,N} \tilde{P}_{ss,N})^{1-\theta} (\hat{\delta}_t + (1-\theta)(\hat{\pi}_{t,N} + \hat{\tilde{P}}_{t-1,N})) \\ &+ (1-z_{ss}^h)(\pi_{ss,F} \tilde{P}_{ss,F})^{1-\theta} (\frac{-z_{ss}^h}{1-z_{ss}^h} \hat{z}_t^h + (1-\theta)(\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})).\end{aligned} \tag{B.80}$$

$$\begin{aligned}\hat{\pi}_t &= z_{ss}^l (\pi_{ss} \frac{w_{ss}}{a_H})^{1-\theta} (\frac{1}{1-\theta} \hat{z}_t^l + \hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H}) \\ &+ \delta_{ss} (\pi_{ss} \frac{w_{ss}}{a_N})^{1-\theta} (\frac{1}{1-\theta} (\frac{z_{ss}^h}{\delta_{ss}} \hat{z}_t^h - \frac{z_{ss}^l}{\delta_{ss}} \hat{z}_t^l) + \hat{\pi}_{t,N} + \hat{\tilde{P}}_{t-1,N}) \\ &+ (1-z_{ss}^h)(\pi_{ss} \frac{w_{ss}^\star Q_{ss}}{1-\tau})^{1-\theta} (\frac{-z_{ss}^h}{1-z_{ss}^h} \frac{1}{1-\theta} \hat{z}_t^h + \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})\end{aligned} \tag{B.81}$$

The foreign CPI inflation is derived in a similar fashion.

$$\begin{aligned}\hat{\pi}_t^\star &= z_{ss}^l (\pi_{ss}^\star w_{ss}^\star)^{1-\theta} (\frac{1}{1-\theta} \hat{z}_t^l + \hat{\pi}_{t,H}^\star + \hat{\tilde{P}}_{t-1,H}^\star) \\ &+ \delta_{ss} (\pi_{ss}^\star {w_{ss}}^\star)^{1-\theta} (\frac{1}{1-\theta} (\frac{z_{ss}^h}{\delta_{ss}} \hat{z}_t^h - \frac{z_{ss}^l}{\delta_{ss}} \hat{z}_t^l) + \hat{\pi}_{t,N}^\star + \hat{\tilde{P}}_{t-1,N}^\star) \\ &+ (1-z_{ss}^h)(\pi_{ss}^\star \frac{w_{ss}}{Q_{ss}(1-\tau)})^{1-\theta} (\frac{-z_{ss}^h}{1-z_{ss}^h} \frac{1}{1-\theta} \hat{z}_t^h + \hat{\pi}_{t,F}^\star + \hat{\tilde{P}}_{t-1,F}^\star)\end{aligned} \tag{B.82}$$

B.6 Real Exchange Rate

Define π_t^q as the gross rate of real depreciation, $\pi_t^q = Q_t/Q_{t-1}$. Then the law of motion of $\hat{Q}_t$ follows

$$\hat{Q}_t = \hat{Q}_{t-1} + \hat{\pi}_t^q, \tag{B.83}$$

where $\hat{\pi}_t^q$ is derived from the definition of π_t^q above.

$$\pi_t^q = \frac{S_t/S_{t-1}P_t^\star/P_{t-1}^\star}{P_t/P_{t-1}} = \frac{d_t\pi_t^\star}{\pi_t} \tag{B.84}$$

Hence,

$$\hat{\pi}_t^q = \hat{d}_t + \hat{\pi}_t^\star - \hat{\pi}_t. \tag{B.85}$$

In sum, the dynamics of Q_t becomes

$$\hat{Q}_t = \hat{Q}_{t-1} + \hat{d}_t + \hat{\pi}_t^\star - \hat{\pi}_t. \tag{B.86}$$

B.7 Bond Markets

Deflate the home bond market clearing condition by P_t.

$$\begin{aligned} 0 &= \alpha f_t + (1-\alpha)\frac{F_t^\star}{P_t} \\ &= \alpha f_t + (1-\alpha)Q_t f_t^\star \end{aligned}$$

Find its total derivatives and collect terms. The log-linearized version is as follows.

$$0 = \alpha \hat{f}_t + (1-\alpha)Q_{ss}\hat{f}_t^\star \tag{B.87}$$

Similarly, dividing the foreign bond market clearing condition by $P_t^\star$ gives,

$$0 = \alpha\frac{f_t^f}{Q_{ss}} + (1-\alpha)f_t^{f\star}$$

Hence,

$$0 = \alpha\frac{1}{Q_{ss}}\hat{f}_t^f + (1-\alpha)\hat{f}_t^{f\star}. \tag{B.88}$$

B.8 Pattern of Trade

The equilibrium conditions for z_t^h and z_t^l are

$$\begin{aligned} p_t^n(z_t^h) &= \frac{p_t^{f\star}(z_t^h)S_t}{1-\tau}, \\ \frac{p_t^h(z_t^l)}{S_t(1-\tau)} &= p_t^{n\star}(z_t^l). \end{aligned}$$

The price comparison is between the import goods price and the nontraded good price because the two cutoff points separate the nontraded and import sector. Deflate both equation with P_t.

$$\tilde{p}_t^n(z_t^h) = \frac{\tilde{p}_t^{f\star}(z_t^h)Q_t}{1-\tau}, \tag{B.89}$$

$$\frac{\tilde{p}_t^h(z_t^l)}{Q_t(1-\tau)} = \tilde{p}_t^{n\star}(z_t^l). \tag{B.90}$$

The log-linearized version of the above equations are

$$\hat{w}_t - \hat{X}_t - \frac{a'(z_{ss}^h)z_{ss}^h}{a(z_{ss}^h)}\hat{z}_t^h = \hat{w}_t^\star + \hat{Q}_t - \hat{X}_t^\star, \tag{B.91}$$

$$\hat{w}_t - \hat{X}_t - \frac{a'(z_{ss}^l)z_{ss}^l}{a(z_{ss}^l)}\hat{z}_t^l = \hat{w}_t^\star + \hat{Q}_t - \hat{X}_t^\star. \tag{B.92}$$

B.9 Interest Parity

The Euler equations ((B.1) and (B.2)) in the home country gives the log-linearized UIP condition.

$$\hat{I}_t = \hat{I}_t^\star + \hat{d}_{t+1} + \phi\hat{f}_t - \phi^\star\hat{f}_t^f \tag{B.93}$$

B.10 Monetary Policy

Under fixed exchange rate regime, the monetary authority in the home country follows the exchange target rule,

$$d_t = d_{ss}. \tag{B.94}$$

Hence,

$$\hat{d}_t = 0. \tag{B.95}$$

Under flexible exchange rate regime, the home monetary authority follows the price target rule,

$$\pi_t = \pi_{ss}. \tag{B.96}$$

Therefore,

$$\hat{\pi}_t = 0. \tag{B.97}$$

The monetary authority in the foreign country follows the following Taylor rule regardless of the exchange rate regime.

$$\hat{I}_t^\star = \lambda_i \hat{I}_{t-1}^\star + (1-\lambda_i)(\lambda_\pi E_t \hat{\pi}_{t+1}^\star + \lambda_y \hat{Y}_t^\star). \tag{B.98}$$

B.11 Productivity Shocks

$$\hat{X}_t = \rho_x \hat{X}_{t-1} + u_t \tag{B.99}$$
$$\hat{X}_t^\star = \rho_x \hat{X}_{t-1}^\star + u_t^\star \tag{B.100}$$

There are 46 variables excluding the shock variables: $\hat{C}_t, \hat{C}_t^\star, \hat{m}_t, \hat{m}_t^\star, \hat{l}_t, \hat{l}_{t,H}, \hat{l}_{t,N}, \hat{l}_t^\star, \hat{l}_{t,F}^\star, \hat{l}_{t,N}^\star, \hat{f}_t, \hat{f}_t^\star, \hat{f}_t^f, \hat{f}_t^{f\star}, \hat{Y}_t, \hat{Y}_{t,H}, \hat{Y}_{t,N}, \hat{Y}_t^\star, \hat{Y}_{t,F}^\star, \hat{Y}_{t,N}^\star, \hat{I}_t, \hat{I}_t^\star, \hat{d}_t, \hat{Q}_t, \hat{\pi}_t, \hat{\pi}_t^\star, \hat{\pi}_{t,H}, \hat{\pi}_{t,N}, \hat{\pi}_{t,F}, \hat{\pi}_{t,H}^\star, \hat{\pi}_{t,N}^\star, \hat{\pi}_{t,F}^\star, \hat{\tilde{P}}_{t,H}, \hat{\tilde{P}}_{t,N}, \hat{\tilde{P}}_{t,F}, \hat{\tilde{P}}_{t,H}^\star, \hat{\tilde{P}}_{t,N}^\star, \hat{\tilde{P}}_{t,F}^\star, \hat{\pi}_t^w, \hat{\pi}_t^{w\star}, \hat{w}_t, \hat{w}_t^\star, \hat{a}_{t,H}, \hat{a}_{t,N}, \hat{z}_t^h$ and $\hat{z}_t^l$. The system of equations is consisted of 46 equations: (B.1), (B.3), (B.4), (B.6), (B.7), (B.9)-(B.11), (B.18), (B.20), (B.22), B(26), B(28), (B.30), (B.36), (B.37), (B.42)-(B.44), (B.48)-(B.50), (B.57)-(B.64), (B.71)-(B.76), (B.81), (B.82), (B.86)-(B.88), (B.91)-(B.93), (B.95) and (B.98) for fixed exchange rate regime. For flexible exchange rate regime, I replace (B.95) with (B.97). The exogenous shock processes are (B.99)-(B.100).

C Real Exchange Rate Dynamics

In this section I decompose the movements in real exchange rates into the traded and nontraded components. From the definition of Q_t,

$$\left(\frac{Q_t}{S_t}\right)^{1-\theta} = \left(\frac{P_t^\star}{P_t}\right)^{1-\theta} = \frac{(1-\delta_t)P_{t,T}^{\star 1-\theta} + \delta_t P_{t,N}^{\star 1-\theta}}{(1-\delta_t)P_{t,T}^{1-\theta} + \delta_t P_{t,N}^{1-\theta}}, \tag{C.1}$$

where

$$P_{t,T} = \left[z_t^l P_{t,H}^{1-\theta} + (1-z_t^h)P_{t,F}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{C.2}$$
$$P_{t,T}^\star = \left[z_t^l P_{t,H}^{\star 1-\theta} + (1-z_t^h)P_{t,F}^{\star 1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{C.3}$$

Divide (C.1) through by its one-period lag.

$$\left(\frac{Q_t}{Q_{t-1}d_t}\right)^{1-\theta} = \frac{(1-\delta_t)(\pi^\star_{t,T}\tilde{P}^\star_{t-1,T})^{1-\theta} + \delta_t(\pi^\star_{t,N}\tilde{P}^\star_{t-1,N})^{1-\theta}}{(1-\delta_t)(\pi_{t,T}\tilde{P}_{t-1,T})^{1-\theta} + \delta_t(\pi_{t,N}\tilde{P}_{t-1,N})^{1-\theta}} \tag{C.4}$$

Find the total derivatives and collect terms.

$$\begin{aligned}(1-\theta)(\hat{Q}_t - \hat{Q}_{t-1} - \hat{d}_t) \;=\; & (1-\delta_{ss})(\pi^\star_{ss}\tilde{P}^\star_{ss,T})^{1-\theta}\left((1-\theta)(\hat{\pi}^\star_{t,T} + \hat{\tilde{P}}^\star_{t-1,T}) - \frac{z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t}{1-\delta_{ss}}\right) \\ & +\delta_{ss}(\pi^\star_{ss}\tilde{P}^\star_{ss,N})^{1-\theta}\left((1-\theta)(\hat{\pi}^\star_{t,N} + \hat{\tilde{P}}^\star_{t-1,N}) + \frac{z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t}{\delta_{ss}}\right) \\ & -(1-\delta_{ss})(\pi_{ss}\tilde{P}_{ss,T})^{1-\theta}\left((1-\theta)(\hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,T}) - \frac{z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t}{1-\delta_{ss}}\right) \\ & -\delta_{ss}(\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}\left((1-\theta)(\hat{\pi}_{t,N} + \hat{\tilde{P}}_{t-1,N}) + \frac{z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t}{\delta_{ss}}\right)\end{aligned} \tag{C.5}$$

Since $P_t = \left((1-\delta_t)P^{1-\theta}_{t,T} + \delta_t P^{1-\theta}_{t,N}\right)^{\frac{1}{1-\theta}}$, then

$$(1-\delta_{ss})(\pi_{ss}\tilde{P}_{ss,T})^{1-\theta} = \pi^{1-\theta}_{ss} - \delta_{ss}(\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}. \tag{C.6}$$

Substitute (C.6) into (C.5).

$$\begin{aligned}\hat{Q}_t - \hat{Q}_{t-1} \;=\; & \left[\pi^{\star 1-\theta}_{ss}(\hat{\pi}^\star_{t,T} + \hat{\tilde{P}}^\star_{t-1,T}) + \hat{d}_t - \pi^{1-\theta}_{ss}(\hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,T})\right] \\ & +\delta_{ss}\left[(\pi^\star_{ss}\tilde{P}^\star_{ss,N})^{1-\theta}(\hat{\pi}^\star_{t,N} - \hat{\pi}^\star_{t,T} + \hat{\tilde{P}}^\star_{t-1,N} - \hat{\tilde{P}}^\star_{t-1,T})\right] \\ & -\delta_{ss}\left[(\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}(\hat{\pi}_{t,N} - \hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,N} - \hat{\tilde{P}}_{t-1,T})\right] \\ & +\frac{(\pi^\star_{ss}\tilde{P}^\star_{ss,N})^{1-\theta} - (\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right].\end{aligned} \tag{C.7}$$

$$\hat{Q}_t - \hat{Q}_{t-1} = \hat{q}_{t,T} + \hat{q}_{t,N} \tag{C.8}$$

where

$$\begin{aligned}
\hat{q}_{t,T} &= \pi_{ss}^{\star 1-\theta}(\hat{\pi}^{\star}_{t,T} + \hat{\tilde{P}}^{\star}_{t-1,T}) + \hat{d}_t - \pi_{ss}^{1-\theta}(\hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,T}) \\
&\quad +0.5\frac{(\pi^{\star}_{ss}\tilde{P}^{\star}_{ss,N})^{1-\theta} - (\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right] \qquad \text{(C.9)} \\
\hat{q}_{t,N} &= \delta_{ss}\left[(\pi^{\star}_{ss}\tilde{P}^{\star}_{ss,N})^{1-\theta}(\hat{\pi}^{\star}_{t,N} - \hat{\pi}^{\star}_{t,T} + \hat{\tilde{P}}^{\star}_{t-t,N} - \hat{\tilde{P}}^{\star}_{t-1,T})\right] \\
&\quad -\delta_{ss}\left[(\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}(\pi_{t,N} - \pi_{t,T} + \hat{\tilde{P}}_{t-1,T} - \hat{\tilde{P}}_{t-1,N})\right] \\
&\quad +0.5\frac{(\pi^{\star}_{ss}\tilde{P}^{\star}_{ss,N})^{1-\theta} - (\pi_{ss}\tilde{P}_{ss,N})^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right] \qquad \text{(C.10)}
\end{aligned}$$

I can further decompose $\hat{q}_{t,T}$ using the definition of $P_{t,T}$ and $P^{\star}_{t,T}$ in (C.2) and (C.3). Deflate (C.2) by $P_{t-1,T}$.

$$\pi_{t,T} = \left[z^l_t\left(\frac{\pi_{t,H}\tilde{P}_{t-1,H}}{\tilde{P}_{t-1,T}}\right)^{1-\theta} + (1-z^h_t)\left(\frac{\pi_{t,F}\tilde{P}_{t-1,F}}{\tilde{P}_{t-1,T}}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad \text{(C.11)}$$

Hence,

$$(\pi_{t,T}\tilde{P}_{t-1,T})^{1-\theta} = z^l_t(\pi_{t,H}\tilde{P}_{t-1,H})^{1-\theta} + (1-z^h_t)(\pi_{t,F}\tilde{P}_{t-1,F})^{1-\theta}. \qquad \text{(C.12)}$$

$$\begin{aligned}
(1-\theta)(\hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,T}) &= z^l_{ss}(\pi_{ss,H}\tilde{P}_{ss,H})^{1-\theta}(\hat{z}^l_t + (1-\theta)(\hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H})) \\
&\quad +(1-z^h_{ss})(\pi_{ss,F}\tilde{P}_{ss,F})^{1-\theta}(\frac{-z^h_{ss}}{1-z^h_{ss}}\hat{z}^h_t + (1-\theta)(\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})). \qquad \text{(C.13)}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\hat{\pi}_{t,T} + \hat{\tilde{P}}_{t-1,T} &= z^l_{ss}(\pi_{ss}\tilde{P}_{ss,H})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + (\hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H})) \\
&+(1-z^h_{ss})(\pi_{ss}\tilde{P}_{ss,F})^{1-\theta}(\frac{-z^h_{ss}}{(1-\theta)(1-z^h_{ss})}\hat{z}^h_t + (\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F})). \qquad \text{(C.14)}
\end{aligned}$$

Applying similar steps to (C.3) gives

$$\begin{aligned}
\hat{\pi}^{\star}_{t,T} + \hat{\tilde{P}}^{\star}_{t-1,T} &= z^l_{ss}(\pi^{\star}_{ss}\tilde{P}^{\star}_{ss,H})^{1-\theta}(\frac{1}{1-\theta}\hat{z}^l_t + (\hat{\pi}^{\star}_{t,H} + \hat{\tilde{P}}^{\star}_{t-1,H})) \\
&+(1-z^h_{ss})(\pi^{\star}_{ss}\tilde{P}^{\star}_{ss,F})^{1-\theta}(\frac{-z^h_{ss}}{(1-\theta)(1-z^h_{ss})}\hat{z}^h_t + (\hat{\pi}^{\star}_{t,F} + \hat{\tilde{P}}^{\star}_{t-1,F})). \qquad \text{(C.15)}
\end{aligned}$$

Substituting (C.14) and (C.15) into (C.9) gives the tradable component of real depreciation in terms of sectoral prices.

$$\begin{aligned}
\hat{q}_{t,T} &= \hat{d}_t + \pi_{ss}^{\star 1-\theta} z_{ss}^l (\pi_{ss}^\star \tilde{P}_{ss,H}^\star)^{1-\theta} \left(\frac{1}{1-\theta} \hat{z}_t^l + \hat{\pi}_{t,H}^\star + \hat{\tilde{P}}_{t-1,H}^\star \right) \\
&+ \pi_{ss}^{\star 1-\theta} (1 - z_{ss}^h) (\pi_{ss}^\star \tilde{P}_{ss,F}^\star)^{1-\theta} \left(\frac{-z_{ss}^h}{(1-\theta)(1-z_{ss}^h)} \hat{z}_t^h + \hat{\pi}_{t,F}^\star + \hat{\tilde{P}}_{t-1,F}^\star \right) \\
&- \pi_{ss}^{1-\theta} z_{ss}^l (\pi_{ss} \tilde{P}_{ss,H})^{1-\theta} \left(\frac{1}{1-\theta} \hat{z}_t^l + \hat{\pi}_{t,H} + \hat{\tilde{P}}_{t-1,H} \right) \\
&- \pi_{ss}^{1-\theta} (1 - z_{ss}^h) (\pi_{ss} \tilde{P}_{ss,F})^{1-\theta} \left(\frac{-z_{ss}^h}{(1-\theta)(1-z_{ss}^h)} \hat{z}_t^h + \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F} \right) \\
&+ 0.5 \frac{(\pi_{ss}^\star \tilde{P}_{ss,N}^\star)^{1-\theta} - (\pi_{ss} \tilde{P}_{ss,N})^{1-\theta}}{(1-\theta)(1-\delta_{ss})} \left(z_{ss}^h \hat{z}_t^h - z_{ss}^l \hat{z}_t^l \right) \qquad \text{(C.16)}
\end{aligned}$$

Since $P_{t,T} = \left(z_t^l P_{t,H}^{1-\theta} + (1 - z_t^h) P_{t,F}^{1-\theta} \right)^{\frac{1}{1-\theta}}$, then

$$(1 - z_{ss}^h)(\pi_{ss} \tilde{P}_{ss,F})^{1-\theta} = \pi_{ss}^{1-\theta} - z_{ss}^l (\pi_{ss} \tilde{P}_{ss,H})^{1-\theta}. \qquad \text{(C.17)}$$

Substitute (C.17) into (C.16).

$$\begin{aligned}
\hat{q}_{t,T} &= \hat{d}_t + \pi_{ss}^{\star 2(1-\theta)} (\hat{\pi}_{t,F}^\star + \hat{\tilde{P}}_{t-1,F}^\star) - \pi_{ss}^{2(1-\theta)} (\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F}) \\
&+ \pi_{ss}^{\star 1-\theta} z_{ss}^l (\pi_{ss}^\star \tilde{P}_{ss,H}^\star)^{1-\theta} \left(\hat{\pi}_{t,H}^\star - \hat{\pi}_{t,F}^\star + \hat{\tilde{P}}_{t-1,H}^\star - \hat{\tilde{P}}_{t-1,F}^\star \right) \\
&- \pi_{ss}^{1-\theta} z_{ss}^l (\pi_{ss} \tilde{P}_{ss,H})^{1-\theta} \left(\hat{\pi}_{t,H} - \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,H} - \hat{\tilde{P}}_{t-1,F} \right) \\
&+ \frac{1}{1-\theta} \pi_{ss}^{\star 2(1-\theta)} \left(z_{ss}^l \tilde{P}_{ss,H}^{\star 1-\theta} \hat{z}_t^l - z_{ss}^h \tilde{P}_{ss,F}^{\star 1-\theta} \hat{z}_t^h \right) \\
&- \frac{1}{1-\theta} \pi_{ss}^{2(1-\theta)} \left(z_{ss}^l \tilde{P}_{ss,H}^{1-\theta} \hat{z}_t^l - z_{ss}^h \tilde{P}_{ss,F}^{1-\theta} \hat{z}_t^h \right) \\
&+ 0.5 \frac{(\pi_{ss}^\star \tilde{P}_{ss,N}^\star)^{1-\theta} - (\pi_{ss} \tilde{P}_{ss,N})^{1-\theta}}{(1-\theta)(1-\delta_{ss})} \left(z_{ss}^h \hat{z}_t^h - z_{ss}^l \hat{z}_t^l \right) \qquad \text{(C.18)}
\end{aligned}$$

Substitute $\tilde{P}_{ss,j}$ and $\tilde{P}^{\star}_{ss,j}$ where $j \in (F, H, N)$ into (C.18) and (C.10).

$$\begin{aligned}
\hat{q}_{t,T} &= \hat{d}_t + \pi_{ss}^{\star 2(1-\theta)}(\hat{\pi}^{\star}_{t,F} + \hat{\tilde{P}}^{\star}_{t-1,F}) - \pi_{ss}^{2(1-\theta)}(\hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,F}) \\
&\quad + \pi_{ss}^{\star 1-\theta} z^l_{ss}(\pi^{\star}_{ss}\frac{w_{ss}}{Q_{ss}(1-\tau)a_H})^{1-\theta}\left(\hat{\pi}^{\star}_{t,H} - \hat{\pi}^{\star}_{t,F} + \hat{\tilde{P}}^{\star}_{t-1,H} - \hat{\tilde{P}}^{\star}_{t-1,F}\right) \\
&\quad - \pi_{ss}^{1-\theta} z^l_{ss}(\pi_{ss}\frac{w_{ss,H}}{a_H})^{1-\theta}\left(\hat{\pi}_{t,H} - \hat{\pi}_{t,F} + \hat{\tilde{P}}_{t-1,H} - \hat{\tilde{P}}_{t-1,F}\right) \\
&\quad + \frac{1}{1-\theta}\pi_{ss}^{\star 2(1-\theta)}\left(z^l_{ss} w_{ss}^{\star 1-\theta}\hat{z}^l_t - z^h_{ss} w_{ss}^{\star 1-\theta}\hat{z}^h_t\right) \\
&\quad - \frac{1}{1-\theta}\pi_{ss}^{2(1-\theta)}\left(z^l_{ss}\left(\frac{w_{ss}}{a_H}\right)^{1-\theta}\hat{z}^l_t - z^h_{ss}\left(\frac{Q_{ss}w^{\star}_{ss}}{1-\tau}\right)^{1-\theta}\hat{z}^h_t\right) \\
&\quad + 0.5\frac{(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta} - (\pi_{ss}w_{ss}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left(z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right) \qquad \text{(C.19)}
\end{aligned}$$

$$\begin{aligned}
\hat{q}_{t,N} &= \delta_{ss}\left[(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta}(\hat{\pi}^{\star}_{t,N} - \hat{\pi}^{\star}_{t,T} + \hat{\tilde{P}}^{\star}_{t-t,N} - \hat{\tilde{P}}^{\star}_{t-1,T})\right] \\
&\quad - \delta_{ss}\left[(\pi_{ss}\frac{w_{ss}}{a_N})^{1-\theta}(\pi_{t,N} - \pi_{t,T} + \hat{\tilde{P}}_{t-1,T} - \hat{\tilde{P}}_{t-1,N})\right] \\
&\quad + 0.5\frac{(\pi^{\star}_{ss}w^{\star}_{ss})^{1-\theta} - (\pi_{ss}w_{ss,N}/a_N)^{1-\theta}}{(1-\theta)(1-\delta_{ss})}\left[z^h_{ss}\hat{z}^h_t - z^l_{ss}\hat{z}^l_t\right] \qquad \text{(C.20)}
\end{aligned}$$