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Measuring the Ex-Ante Social Cost of Aggregate Volatility

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Abstract: Ex-ante cost of aggregate fluctuations consist of all individual and social cost expanded by optimizing agents aiming to prevent or reduce fluctuations of consumption. These are measured by the cost of resources used to attain the level of consumption volatility currently observed. This paper proposes that the ex-ante component of cost is important and large. The unifying principle in measuring these cost arises from the fact that agents utilize resources for reserves used for self insurance against uninsurable aggregate fluctuations. Firms accumulate excess capacity in response to fluctuations in demand; our financial sector uses substantial resources to mitigate the effect of market fluctuations and to permit aggregate risk sharing. We estimate that the order of magnitude of the ex-ante social cost of aggregate fluctuations exceeds 5% of GNP.

Our formulation suggests that stabilization policy mitigates the effects of aggregate volatility by offering important insurance services. This complements the traditional view of stabilization policy as one designed to bolster the economy’s output so that it attains its potential by providing adequate demand. The insurance perspective of stabilization policy, which we offer here, suggests that without public policy each agent selects optimal private resources to be used for self insurance against aggregate volatility. Stabilization policy is then a public good which enables individual agents to reduce the private cost of self insurance.

JEL classification: D2; D21; D58; D92; E22; E32; E6.

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1. Introduction

Should aggregate market volatility, business cycles or financial crises be of concern to Economists? Using a single agent economy with a utility function $\sum_{t=1}^{\infty} \beta^{t-1} \frac{1}{1-\gamma} (c_t)^{1-\gamma}$, $0<\beta<1$, Lucas (1987) carried out a simple exercise which intended to answer this complex question. He characterized aggregate U.S. consumption by $c_t = A e^{\mu t} \left( e^{-\frac{1}{2} \sigma^2} \right) \varepsilon_t$, where $\log(\varepsilon_t) \sim N(0, \sigma^2)$, $E[e^{-\frac{1}{2} \sigma^2} \varepsilon_t] = 1$. Using post war data he estimated $\sigma = 0.032$. The exercise asked what fraction of consumption would the agent be willing to pay to keep $\mu$ unchanged but reduce consumption volatility to zero so that $\sigma = 0$. This amounts to computing the value $\lambda$ which satisfies

$$E\left[ \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left( (1+\lambda) e^{\mu t} \left( e^{-\frac{1}{2} \sigma^2} \right) \varepsilon_t \right)^{1-\gamma} \right] = \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left( e^{\mu t} \right)^{1-\gamma}.$$

The answer: practically zero. Lucas concluded that aggregate fluctuations should not be of concern since their elimination is not socially desirable. Given this result Lucas (2003) concluded that Macroeconomic policy should not aim to impact people’s demand flows but rather, focus only on gains from supply side policies which provide better incentives for growth.

The above argument has been reexamined in several papers, with mixed results. We review here a sample of these contributions. Imrohoroglu (1989) studied unemployment risk in an economy with indivisibilities and liquidity constraints but without asset markets. After calibration she found the welfare gains from elimination of exogenous shocks to be small. A similar analysis was carried out by Atkeson and Phelan (1994) who examined the effect of incomplete markets, but arrived at

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similar conclusions. Krusell and Smith (1999), (2002) focused on economies populated by agents with heterogenous wealth and preferences. They considered a more complex structure of shocks but found no gains from the elimination of business cycles. Alvarez and Jermann (2003) deduced the cost of volatility from asset prices. Assuming complete markets they computed the implied market valuation of the observed consumption stream and compared it to a riskless “trend.” These authors found the market valuation of removing all consumption risks is very large, around 30% of consumption. They then discounted the significance of this result by defining “trend” by a moving average and by focusing only on a fraction of consumption risks in business - cycle frequencies which are of duration eight years or less. So restricted they found the cost of consumption risk to be only 0.3%, which is very small. Barlevy (2001) found significant cost of business cycles in an endogenous growth process where a reduction in volatility can lead to higher growth rate of consumption. Dolmas (1998) and Tallarini (2000) found substantial cost of volatility in economies where preferences are of the Epstein-Zin type. Storesletten, Telmer and Yaron (2001) and Krebs (2003) showed that individual, idiosyncratic, shocks to labor productivity are amplified by business cycle shocks. With such structure the individual cost of fluctuations are much larger than the estimate of Lucas (1987). Finally, in an empirical study Ramey and Ramey (1995) found that countries with more volatile growth of GNP have slower growth rates.

Before proceeding we observe that an evaluation of the welfare cost of aggregate volatility should not distinguish between volatility at business cycle frequencies and volatility due to other aggregate and non diversifiable factors. Stabilization policy does not address only fluctuations viewed as “business cycles.” It takes equally seriously dislocations due to events such as the 1970's oil shocks or the East Asia crisis. Stabilization policy addresses the dislocation effects of all aggregate shocks: financial crises, technological shocks, variations in aggregate demand or others.

We first reexamine the question raised by Lucas (1987). Society clearly expends resources on attaining the observed level of consumption stability. Hence, it is obvious that the Lucas (1987) exercise measures the willingness to spend added resources for added stabilization above the level attained. His result may thus be interpreted to say that post war stabilization policy has been so successful that no added stabilization is desired. But, it is also clear that the crucial implicit issue debated is the desirability of stabilization policy and Lucas (2003) is explicit in his opposition to it.
Most papers cited above employ an RBC model where business cycles are exogenously caused by technological shocks. Given this RBC structure and Rational Expectations, there is no optimal policy that would have been advocated by Lucas or by authors of most of the papers cited had they concluded that market volatility is socially costly. Also, a conclusion that the welfare cost of market volatility is small is equivalent to an equity premium puzzle in the asset markets of the economy.

*It is our view that Lucas (1987) did not formulate the important question.* We start with a distinction between the *ex-post cost* of aggregate market volatility and the *ex-ante cost*. Ex-post cost consist of the reduction in social welfare due to the actual stochastic movement of aggregate consumption and output after all individual and social decisions dedicated to attain stabilization have been made. Ex-ante cost consist of all individual and social cost expanded in order to prevent the fluctuations of income and consumption, measured by the cost of resources used to attain the observed level of consumption volatility. Lucas (1987) measured ex-post cost but in our view the ex-ante cost are larger and more important. Indeed, under perfect stabilization the ex-post cost are zero but the ex-ante cost are high. We focus here on measuring the ex-ante cost of aggregate volatility. Our main argument is that aggregate, uninsurable, uncertainty causes wide distortions in the allocation of resources which result from the fact that households and firms protect themselves from the effects of market volatility. We show there is a unifying principle in measuring the effect of aggregate risk on individual economic units. It arises from the fact that such uninsurable risks create incentives for using resources to produce reserves for *self insurance against these risks*. Firms hold excess capacity as an optimal response to market fluctuations. Financial institutions use resources to mitigate the effect of aggregate market fluctuations. Society spends substantial resources on monitoring the behavior of agents with contract obligations designed to share aggregate risk and litigating against contract violators and defaulting agents. All agents hold riskless assets to permit self insurance against consumption volatility. Our central results evaluate the order of magnitude of excess capacity in our economy. In Section 4 we argue that a significant part of the value added in the financial sector reflect social cost of aggregate volatility and it is very large.

Our conclusion is simple. The main cost of aggregate volatility is the reduced efficacy in the use of social resources. Firms and households use resources to counter the risks they face and the largest component of these resources are the reserves and excess capacity used for self insurance. A
riskier economy uses more resources to address the effect of uninsurable market risks. We estimate that the cost of aggregate fluctuations in the U.S. economy exceed 5% of GNP.

**What is then the role of stabilization policy?** Aggregate volatility is a public “bad” which can be mitigated by individual or by collective actions. Without public policy each agent selects his optimal private resources to be employed for self insurance against market volatility. We show in this paper that such individual actions result in large social expenditures on ex-ante cost of aggregate volatility. Limited asset holdings by individuals may severely restrict the range and effectiveness of all self insurance private measures. When an effective stabilization policy is put in place it provides **insurance services as a public good** from which all benefit and hence reduce their private cost of self insurance. Stabilization policy is then an efficient way to save private resources. However, given the limits on individual asset holdings, public stabilization policy can attain higher level of benefits than can be attainable with the observed, unequal, distribution of asset holdings.

As our approach deviates from the RBC model we briefly outline two key methodological perspectives which we employed in this paper. The development below incorporates these ideas.

**A. What are Aggregate and Idiosyncratic Risks?**

What is “aggregate” uncertainty? Most writers treat such uncertainty $S_t$ as a grand uninsurable exogenous shock sustained uniformly by all agents. Idiosyncratic shocks are agent or firm specific shocks which are averaged out when aggregated and which can be insured against. In reality there are many different aggregate shocks with different effects in different sectors. A large shock to the home or office building industries could have a large primary effect in the construction and real estate sectors but would then have a weak economy wide effect. The shock to investment in the 2001 recession originated in the Information Technology sector where it had a depression size effect on output and employment when the rate of capacity utilization in the semiconductor industry fell from 100.046 in April 2000 to 67.303 in June 2001. But when spread to the rest of the economy it resulted in a very shallow recession. The East Asia financial crisis had a strong effect on foreign trade and high technology sectors but had only a mild secondary effects on the rest of the economy. In each example a different aggregate shock had a different effect across the economy. We thus define aggregate uncertainty as a vector $S_t = (s_t^1, s_t^2, s_t^3, ..., s_t^N)$ of correlated shocks. **None**
of these are idiosyncratic to an agent or a firm and they include market belief reflecting aggregate market expectations or market taste reflecting demand shocks. Their correlation does not imply there is a “common” component which we must then define as “the” aggregate shock.

Given the above, how do we interpret the low volatility of average consumption growth? The estimated long run growth rate of per capita consumption is $\log \left( \frac{C_{t+1}}{C_t} \right) = \mu + \log(\varepsilon_{t+1}) - \log(\varepsilon_t)$ with standard deviation of consumption growth, denoted $\sigma_{\Delta c}$, estimated to be $\sigma_{\Delta c} = 0.032$ annually. Evidence shows the volatility of individual consumption growth is much higher than 3.2% although there are limited panel data to study the problem. Brav et. al. (2002) report standard deviation of quarterly growth rates of durable goods consumption between 6% and 12% across households, with a mean of 9% (i.e., standard deviation of 18% at annual rate). The Bank of Italy Survey of Household Income and Wealth 1987-2002 reports an average standard deviation of annual family consumption growth of 19.82% and non-durables of 18.51%. Using income data of the PSID (1967-1992) the average standard deviation of annual income growth rate (computed over time), across 79 families with 26 years of data, is estimated to be 23.19%. Our view is that the high consumption volatility of individual households is not due just to idiosyncratic factors.

Under our conception of aggregate risk agents in each sector have idiosyncratic shocks but these are averaged out within that sector so that $S_t = (s_{1t}, s_{2t}, ..., s_{Nt})$ nets out all idiosyncratic shocks. However, sectoral outputs and consumptions (of agents who drive their income in that sector) are more volatile than the average and vary across sectors depending on how sectors are hit by $(s_{1t}, s_{2t}, ..., s_{Nt})$. For example, a vector of socks $S_t = (s_{1t}, s_{2t}, ..., s_{Nt})$ with primary effect in Information Technology causes mainly a reduction of consumption of agents in the information technology sector. More generally, $S_t$ cause volatility of output and consumption in individual sectors which are sharply different from the volatility of the aggregate economy and this difference has nothing to do with any idiosyncratic shocks of individuals or firms. With diverse aggregate shocks, the volatility of the aggregate growth rate of consumption is merely a statistical average. This means that $\sigma_{\Delta c} = 0.032$ is a statistical average over volatile consumption growth rates across sectors of the economy. The stability of the aggregate is then a consequence of the nature of the correlation among the $s_{jt}$, not of an averaging over independent idiosyncratic shocks.

The implication of the above is that the 3.2% standard deviation of aggregate consumption
growth does not measure the effect of aggregate risks on the consumption volatility of any agent even after disregarding all idiosyncratic risks. Volatility of individual consumption growth has two components. One is a sectoral and the second is an idiosyncratic component which we disregard in this paper. However, we observe that the sectoral component is larger than the volatility of the mean U.S. consumption growth rate. This distinction is made clear in Section 3.5.

B. Goods Price Volatility Represent Aggregate Risks

Standard equilibrium argument shows that price fluctuations are generated by fluctuations in state variables implying an equivalence between aggregate shocks and price fluctuations (e.g. Lucas (1978)). Hence, goods price uncertainty reflects aggregate uncertainty and for this reason we use the volatility of good prices as a proxy for aggregate uncertainty. Since individual good prices are free of agents’ idiosyncratic risks, these risks are then excluded from our models. In this paper we assume the volatility of good prices is as measured in the U.S. economy but we do not explain the assumed price volatility. This approach avoids the need to adhere to any particular theory of business fluctuations and allows us to consider all aggregate shocks, not only those defined as “business cycles,” as long as they are reflected in goods prices. We measure the cost of aggregate uncertainty to a competitive firm by the self insurance cost it pays in the form of excess capacity it uses, given the risky output and input prices and taking price volatility as exogenous. Followers of the RBC theory may simply assume good price fluctuations are caused by exogenous technological shocks. This is not the case under other theories of economic fluctuations. For example, Kurz, Motolese and Jin (2003a), (2003b), (2003c) show the dynamics of beliefs is central to propagation of investment and business fluctuations. From this perspective price fluctuations are consequences of demand fluctuations. Hence, treating prices as a proxy for aggregate fluctuations shifts the cause of economic fluctuations away from being only exogenous technological shocks. It adds variability of demand back as one of the forces which propagate aggregate fluctuations.

To understand some implications of the above suppose two equilibrium price functions are

\[ p_{t}^{1} = p^{1}(s_{1t}, s_{2t}, s_{3t}, ..., s_{Nt}) \quad \text{and} \quad p_{t}^{2} = p^{2}(s_{1t}, s_{2t}, s_{3t}, ..., s_{Nt}). \]

Hence the covariance between them would be approximated by

\[ \text{Cov}(p_{t}^{1}, p_{t}^{2}) \approx \sum_{i=1}^{N} \sum_{j=1}^{N} a_{i} a_{j} \text{Cov}(s_{it}, s_{jt}). \]
Four facts must then be clear from this way of thinking about price uncertainty:
(i) The variance of a price depends upon how it is affected by different shocks in S.
(ii) Without aggregate uncertainty \( \text{Var}(S) = 0 \) and hence zero price variability, \( \text{Var}(p_j) = 0 \) for all \( j \).
(iii) \( \text{Var}(p_i) \) and \( \text{Cov}(p_i, p_j) \) do not reflect shocks to sectors \( i \) or \( j \) which are “idiosyncratic” in any sense: the use of goods prices nets out all idiosyncratic effects.

The paper is develop in three stages. We focus first on optimal behavior of an isolated firm in static and then in dynamic setting with and without substitution but without sharing price risks. We next present empirical evidence about price volatility. In stage 3 we extend the results to a fully dynamic economy where risk sharing is feasible by trading in financial markets.

2. Optimal Firm Behavior and the Excess Capacity Principle

2.1 Optimal Excess Capacity without Risk Sharing and Short Run Substitution

We present first a simple model of a firm with fixed proportions in order to explain the excess capacity principle. Here “capacity” is naturally defined but is later generalized to more general economies. Thus, consider a firm who produces output \( Y \) by using a variable input \( X \) and a fixed capacity \( K \) - which is capital. In the short run the firm can change output only by changing the variable factor \( X \) when the price of this factor is known. Over the longer horizon capacity can be changed but at any date the firm must commit to a capital - capacity level without knowing the prices that will prevail in the future. Since the typical time it takes to add capacity is around six months, the unit of time in this paper is taken to be six months. Our notation is as follows:

- \( K \) - capital invested, defining capacity at \( t \);
- \( Y \) - firm’s output at \( t \);
- \( X \) - current input at \( t \);
- \( P_x \) - price of input \( x \) at \( t \);
- \( P \) - price of output at \( t \).

The short-run production function of the firm is assumed to have the simple form

\[
Y = \begin{cases} 
  aX^\sigma & \text{if } X \leq \alpha K \\
  a(\alpha K)^\sigma & \text{if } X > \alpha K 
\end{cases}
\]
where \( 0 < \sigma < 1 \) and \( \alpha > 0 \). (1) has rising short-run marginal cost and provides a clear
interpretation to the notion of “capacity.” To ensure the firm problem has a well defined long-term
solution we postulate that (1) exhibits decreasing returns. The profit function is defined by
\[
\Pi = PY - P_x X - RK , \quad R > 0.
\]

\( R \) is the known cost of capacity but prices are uncertain random variables. The firm selects \( X \) after
it knows both \( P \) and \( P_x \) but it must commit to a capacity level \( K \) before it knows output prices.

The firm is owned by agents who agree on a stochastic discount rate that assigns to a unit of
consumption in state \((P, P_x)\) the value \( \theta(P, P_x) \). It is then instructed to maximize the expected value
of profits evaluated at \( \theta(P, P_x) \), given the postulated information structure\(^3\). It thus aims to
\[
\text{Max}_{\{X,K\}} \int \theta(P, P_x) \Pi(P, P_x, X, K) f(P, P_x) \, dP dP_x.
\]
f(\( P, P_x \)) is the density of prices. Since the firm selects \( X \) after it knows prices, the solution for \( X \)
satisfies \( X \leq \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} \). Taking capacity into account, the solution for \( X \) is
\[
X = \begin{cases} 
\left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} \leq \alpha K \\
\alpha K & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} > \alpha K.
\end{cases}
\]

This solution reveals the main problem: if prices are favorable to the firm (high \( P \) or low \( P_x \)), it
cannot take advantage of them unless it has the capacity to produce output.

\[
\begin{align*}
\Pi &= \begin{cases} 
Pa\left[ \frac{P}{P_x} a \right]^{\frac{1-\sigma}{\sigma}} - P_x \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} - RK & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} \leq \alpha K \\
Pa(\alpha K)^{\sigma} - P_x \alpha K - RK & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} > \alpha K.
\end{cases}
\end{align*}
\]

Hence \( \Pi_K = \frac{\partial \Pi}{\partial K} \) is
\[
\Pi_K = \begin{cases} 
- R & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} \leq \alpha K \\
Pa \sigma a (\alpha K)^{\sigma-1} - P_x \alpha - R & \text{if } \left[ \frac{P}{P_x} a \right]^{\frac{1}{1-\sigma}} > \alpha K.
\end{cases}
\]

\(^3\) In a context of an intertemporal optimization where the owner has a utility \( E_0(\sum_{t=0}^{\infty} \beta^t u(C(P_t, P_{xt})) \) the firm would
optimize each period given the evaluation \( \theta(P_{t+1}, P_{xt+1}) = \beta \frac{u'(C(P_t, P_{xt}))}{u'(C(P, P_x))} \). In Section 3 we explicitly introduce the owner of
the firm and a standard decentralization argument shows that the firm uses this stochastic discount rate for optimization.
Consider now the case where \( P \) and \( P_x \) are not random but riskless constants \( P^o, P^o_x \). For this case the condition \( \Pi_K = 0 \) implies that the solution \( K^*(P^o, P^o_x) \) satisfies

\[
(6a) \quad K^*(P^o, P^o_x) = \frac{1}{\alpha} \left( \frac{P^o a \sigma}{P^o_x + \frac{R}{\alpha}} \right)^{\frac{1}{1 - \sigma}}.
\]

Expression (6a) specifies optimal capacity in the riskless economy in which prices are constant.

Denote by \( K(P^o, P^o_x) \) the boundary value of capacity relative to \( (P^o, P^o_x) \), at which \( X = \alpha K \). Thus

\[
(6b) \quad K(P^o, P^o_x) = \frac{1}{\alpha} \left( \frac{P^o a \sigma}{P^o_x} \right)^{\frac{1}{1 - \sigma}}.
\]

Comparing (6a) and (6b) and assuming \( R > 0 \) we see that \( K^*(P^o, P^o_x) < K(P^o, P^o_x) \).

**Conclusion:** For \( \alpha > 0, R > 0 \) and all \( (P, P_x) \), optimal capacity where \( \Pi_K = 0 \), is \( K^*(P, P_x) \). Since \( K^*(P, P_x) < K(P, P_x) \) the firm operates on the branch of the profit function where \( X = \alpha K \) and **capacity is an effective constraint** on output. Although \( \Pi_x < P_x \) the firm does not increase capacity since it is too costly.

Now consider the function \( \Pi_K \) in (5) for fixed values of \( K \) and \( P_x \) but for different values of \( P \). In figure 1 we draw this function.

**FIGURE 1: INSERT HERE**

We observe that \( \Pi_K \) is continuous since at \( X = \alpha K \), where \( \left( \frac{P}{P_x a \sigma} \right)^{\frac{1}{1 - \sigma}} = \alpha K \), we have that \( \Pi_K = -R \). This follows from the equality

\[
P a \sigma \left( \frac{P}{P_x a \sigma} \right)^{\frac{\alpha - 1}{1 - \sigma}} - P_x a - R = -R.
\]

*The key observation is that \( \Pi_K \) is a convex function of \( P \).*

Now we let \( P \) be a random variable with mean \( \overline{P} = E(P) \) and continue to assume that \( P_x \) is constant so the earlier analysis applies to \( (E(P), P_x) \). In this risky environment \( P \) is a **mean preserving spread**. We denote by \( K^{**} \) the optimal capacity for this risky problem and we know it
satisfies \( \int \Pi_K(P, X(P, P_x), K^+ +) \theta(P) \hat{f}(P) dP = 0 \). Now define the risk neutral probabilities
\[ \hat{f}(P) = \frac{\theta(P) f(P)}{\int \theta(P) f(P) dP} \]
and optimal capacity is defined by \( \int \Pi_K(P, X(P, P_x), K^+ +) \hat{f}(P) dP = 0 \). We now invoke the Rothschild and Stiglitz (1970) theorem on mean preserving spreads. Since \( \Pi_K \) is a convex in \( P \) and \( K \) is optimal in the riskless economy, \( \Pi_K(K'(E(P), P_x)) = 0 \). The theorem implies that
\[ K^+ + > K'(E(P), P_x). \]

**Excess Capacity result.** We have thus arrived at the conclusion that in the risky environment where \( P \) is a mean preserving spread, the firm will optimally invest in excess capacity defined by
\[ (7a) \text{ Excess Capacity} = K^+ + - K'(E(P), P_x). \]
Since \( K'(E(P), P_x) \) is optimal in a riskless economy, \( K^+ + > K'(E(P), P_x) \) is excess capacity used by the firm to insure against loss of profitable opportunities when demand is high and \( P \) is high.

### 2.2 Comments on Fixed Proportions, Substitution and Excess Capacity

Our Excess Capacity result was proved without restrictions, but its limitations are clear: fixed proportions is extreme, the model is not dynamic and input prices are assumed constant. We note the result above is related to a literature which examined the effect of uncertainty on investment. Sandmo (1971) and Batra and Ullah (1974) studied effects of price uncertainty when decisions are made before any price is observed. They developed conditions under which a static competitive firm reduces inputs in response to uncertainty. If variable inputs are selected after prices are known, while capacity is selected before prices are observed, the results are different. A theorem showing that capital input increases due to uncertainty in a model with full substitution was first derived by Hartman (1976) and generalized by Epstein (1978). Other papers covering this problem include Hartman (1972), (1973), Pope (1980), Dalal (1990), Kon (1983), and Turnovsky (1973). Thus, suppose we replace the short-term production function (1) with
\[ (1') Y = [F(K, X)]^\mu. \]
F(K, X) is constant returns to scale and full substitution is allowed. \( \mu < 1 \) implies an optimal firm size exists and one may consider results as \( \mu \to 1 \). The authors cited developed conditions under
which increased uncertainty leads to increased capital input when (1') holds. Hartman (1976) gives conditions on the third derivatives of \( F \) to imply this result but shows the result holds for a CES production function if the elasticity of substitution is close to 1. Epstein (1978) generalizes these results to multiple commodities and a general structure of uncertainty.

Under a full substitution the concept of “excess capacity” is ambiguous since added capital contributes to output at all levels of variable inputs. Since output is increased by accumulated capital, a superficial view of excess capacity would insist that when substitution is possible any added capital is desirable. It would lead to a “conclusion” that economic volatility is desirable since it increases the long term average level of capital. This is the wrong conclusion, analogous to the discussion of Waugh (1944), Samuelson (1972a) (1972b), Oi (1972), Waugh (1972) and others. Placing the discussion in a General Equilibrium context Samuelson (1972a) showed that it is not possible to increase price uncertainty in the form of a mean preserving spread of prices. The budget constraints imply that an increased price volatility has to be associated with a change in mean prices and a corresponding decline in real income. We address this issue when we discuss the key question of the welfare cost of excess capacity.

In our static model the term \( K'' - K'(E(P)) \) has a natural meaning of excess capacity since some capacity is idle at most dates and the quantity \( K'' - K'(E(P)) \) measures the distortion in capital allocation caused by aggregate risk. When models like (1') are postulated and an optimal firm size exists, economic growth occurs by expanding the number of firms/plants and not by expanding the capacity of individual plants. Hence, if aggregate volatility affects optimal plant size we need to explore any distortions to the allocation of resources and define the cost of volatility in terms of these distortions. In the rest of this paper we assume that long term substitution of capital with variable resources is possible but the firm size is determinate. We then seek a quantity which is analogous to \( K'' - K'(E(P)) \) and define it to be the implied excess capacity due to volatility.

To measure the ex-ante cost of volatility we consider two economies: one riskless to provide baseline data on steady state capital, output and consumption. A second economy is subjected to price risk and a comparison between the two allows us to measure the effect of volatility on the demand of firms for capacity and the supply of households for labor. We show that increased price volatility leads firms to increase demand for long term capacity.
increased capacity is satisfied only if society makes the sacrifice of foregoing consumption and building the extra capacity. With changed long term level of capital, endogenous variables adjust, and we end up with changed long term level of average consumption and output, complicating welfare comparisons. To overcome these difficulties we shall define the *compensated economy* to be one which is analogous to the one proposed by Samuelson (1972a) by requiring it to have the same long run mean consumption as the level in the riskless economy. A compensated economy enables welfare comparisons which express the cost of volatility in terms of the added resources required for this compensated economy.

In a volatile economy excess capacity insures against fluctuations. As a result, high capacity is often employed with low variable inputs, producing low productivity and low consumption. If a compensated economy exhibits excess capacity with realistic consumption growth volatility, society reveals the cost of building and maintaining the larger capital stock without the benefits of higher consumption. In that case the *added capital is a pure social cost of volatility*. To arrive at such conclusion we compute the implied optimal capacity which depends upon the assumed price volatility parameters. Hence, to support the computational model, we need first to review the empirical evidence about price volatility.

### 2.3 Goods Price Volatility: the Empirical Evidence

To gain an empirical insight on actual U.S. price volatility we selected a random sample of 50 commodities from a list of BLS producer prices and estimated for each price the following Markov AR1 model for varying periods of available data in 1947-2004

\[
p_{t+1} - E[p] = \lambda_p (p_t - E[p]) + \rho_p^p t_{t+1} , \quad \rho_p^p \sim N(0, \sigma_p^2),
\]

where \( p_t = \log P_t \). Table 1 presents the estimated values of \((\lambda_p, \sigma_p^2)\) when the unit of time for the estimation is the same as the one we use in the two models developed below, which is six months. Since we do not use the specific estimates in the table, we avoid a detailed discussion of alternative ways of estimating the two parameters. We stress only two facts. First, price dynamics in a six month frequency is highly persistent; the \( \lambda_p \) are typically close to 1 and assuming \( \lambda_p = 0.90 - 0.95 \) in the simulations below is in accord with the empirical evidence. Second, the estimated standard deviations \( \sigma_p^2 \) exhibit large variations and for a six month model the average price volatility of
commodity prices in the U.S. is in the range of $\sigma_{p} \approx 5.0\%$ per period.

### Table 1: Estimated $(\hat{\lambda}_p, \hat{\sigma}_p)$ for 50 Random Prices; Six Month Time Unit

<table>
<thead>
<tr>
<th>Item</th>
<th>$\hat{\lambda}_p$</th>
<th>$\hat{\sigma}_p$</th>
<th>Other Item</th>
<th>$\hat{\lambda}_p$</th>
<th>$\hat{\sigma}_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Citrus Fruit</td>
<td>0.89</td>
<td>27.84</td>
<td>Passenger car radial tires</td>
<td>0.94</td>
<td>3.11</td>
</tr>
<tr>
<td>Synthetic Yarn</td>
<td>0.98</td>
<td>2.92</td>
<td>Rubber cement, 5 gal.</td>
<td>1.01</td>
<td>5.64</td>
</tr>
<tr>
<td>Leather</td>
<td>1.00</td>
<td>7.84</td>
<td>Plastic components of furniture</td>
<td>0.99</td>
<td>2.24</td>
</tr>
<tr>
<td>Natural Gas</td>
<td>0.98</td>
<td>23.30</td>
<td>Wood siding</td>
<td>0.98</td>
<td>7.25</td>
</tr>
<tr>
<td>Fats and Oils, inedible</td>
<td>0.84</td>
<td>14.22</td>
<td>Hardwood flooring</td>
<td>1.00</td>
<td>4.11</td>
</tr>
<tr>
<td>Basic organic chemicals</td>
<td>0.92</td>
<td>7.28</td>
<td>Shingles, Shakes, cooperage cork</td>
<td>0.93</td>
<td>6.76</td>
</tr>
<tr>
<td>Plastic products</td>
<td>0.98</td>
<td>3.50</td>
<td>Book paper, A grade, 100lb.</td>
<td>1.00</td>
<td>3.42</td>
</tr>
<tr>
<td>Hardwood lumber</td>
<td>0.99</td>
<td>5.21</td>
<td>Wrapping tissue</td>
<td>0.93</td>
<td>5.32</td>
</tr>
<tr>
<td>Newsprint</td>
<td>0.97</td>
<td>8.08</td>
<td>Corrugated paperboard in sheets</td>
<td>0.97</td>
<td>6.87</td>
</tr>
<tr>
<td>Iron and Steel</td>
<td>0.99</td>
<td>4.15</td>
<td>Heavy melting scrap</td>
<td>0.86</td>
<td>19.43</td>
</tr>
<tr>
<td>Furniture Hardware</td>
<td>0.99</td>
<td>2.46</td>
<td>Railroad wheels and specialties</td>
<td>0.99</td>
<td>2.91</td>
</tr>
<tr>
<td>Clay refractories</td>
<td>0.98</td>
<td>2.89</td>
<td>Motor vehicle hardware</td>
<td>0.97</td>
<td>1.81</td>
</tr>
<tr>
<td>White pan bread</td>
<td>0.98</td>
<td>2.79</td>
<td>Electronic computers</td>
<td>0.97</td>
<td>4.01</td>
</tr>
<tr>
<td>Beef/veal products, fresh/frozen</td>
<td>0.98</td>
<td>8.37</td>
<td>Drill industrial, 3/8 inch.</td>
<td>1.01</td>
<td>3.14</td>
</tr>
<tr>
<td>Salad and cooking oils</td>
<td>0.88</td>
<td>7.85</td>
<td>Mechanical presses</td>
<td>0.97</td>
<td>4.25</td>
</tr>
<tr>
<td>Dress or sportswear fabric yarn</td>
<td>0.84</td>
<td>4.74</td>
<td>Mobile vehicle refrigeration sys.</td>
<td>0.93</td>
<td>3.21</td>
</tr>
<tr>
<td>Fur products</td>
<td>0.98</td>
<td>4.98</td>
<td>Metal household dining sets</td>
<td>0.97</td>
<td>1.17</td>
</tr>
<tr>
<td>Textile fibers, yarns and fabrics</td>
<td>0.95</td>
<td>3.00</td>
<td>Dishwasher, undercounter</td>
<td>1.00</td>
<td>2.42</td>
</tr>
<tr>
<td>Casual footwear</td>
<td>0.77</td>
<td>3.94</td>
<td>Kitchen knife doz.</td>
<td>0.98</td>
<td>3.84</td>
</tr>
<tr>
<td>Unleaded regular gasoline</td>
<td>0.86</td>
<td>18.95</td>
<td>Portland cement</td>
<td>0.99</td>
<td>3.07</td>
</tr>
<tr>
<td>Automotive motor oil, retail</td>
<td>1.00</td>
<td>4.25</td>
<td>Sheet, plate and float glass</td>
<td>0.91</td>
<td>4.02</td>
</tr>
<tr>
<td>Jet fuel</td>
<td>0.92</td>
<td>16.80</td>
<td>Motor vehicle parts, new</td>
<td>0.97</td>
<td>0.62</td>
</tr>
<tr>
<td>Copper sulfate, 100lb.</td>
<td>0.99</td>
<td>7.16</td>
<td>Self propelled ships, nonmilitary</td>
<td>0.97</td>
<td>2.08</td>
</tr>
<tr>
<td>Soybean oil, lb.</td>
<td>0.92</td>
<td>21.46</td>
<td>Cigars</td>
<td>1.00</td>
<td>2.82</td>
</tr>
<tr>
<td>Automotive chemicals</td>
<td>0.82</td>
<td>11.27</td>
<td>Denture material</td>
<td>0.99</td>
<td>2.93</td>
</tr>
</tbody>
</table>

We shall now consider a fully dynamic firm optimization and proceed to our key question: what is the mean level of optimal excess capacity chosen by the firm in an economy which experiences an increased price volatility in the order of magnitude as in Table 1?

### 2.4 Optimal Excess Capacity in a Dynamic Model with Substitution. No Risk Sharing

In this section we continue to study the firm in isolation so that trading or sharing of risk is not possible as yet. Such trading will be permitted in the next section. However, we now assume that substitution is possible so that output is given by a production function of the form

\[
Y_t = [K_t^{\sigma} X_t^{1-\sigma}]^\mu, \quad \sigma = 0.40, \quad \mu = 0.90.
\]

We continue to assume that the firm selects $X$ after observing prices but selects $K$ before knowing prices. The firm is owned by one agent and consumption is exactly equal to the cash flow the owner receives from the firm. Price volatility incorporates technology shocks. This agent has a utility function $u(D)$ over consumed cash flow, which is defined by

\[
D_t = P_t Y_t - P X_t - I_t.
\]

Note that $P_t$ fixes the exchange rate between the product which the firm produces and units of a
consumption good which is kept in the background. The owner\firm owns a capital stock $K_t$ which is measured in units of consumption and evolves according to

$$K_{t+1} = (1 - \delta) K_t + I_t, \quad \delta = 0.05.$$  

The objective is then to solve the optimization problem

$$\text{Max} \quad E \left\{ \sum_{t=0}^{T} \frac{\beta^t}{1 - \gamma} D_t^{1 - \gamma} \right\}, \quad \beta = 0.98, \quad \gamma = 2$$

subject to (8a), (8b), (8c), the information structure postulated above and transversality condition $\lim_{t \to \infty} E[ \beta^t K_t ] = 0$. In fact, we consider only solutions with bounded capital stock. A depreciation rate $\delta = 0.05$ and discount rate $\beta = 0.98$ reflect the model's six month time unit which is the typical length of time needed to expand capacity, while $\gamma = 2$ is empirically reasonable.

The Euler equations of this optimization problem are standard:

$$(10a) \quad u_{D_t} = \beta E_t \{ u_{D_{t+1}} \left( \frac{\partial Y_t + 1}{\partial K_{t+1}} \right) + (1 - \delta) \}$$

$$(10b) \quad P_t \frac{\partial Y_t}{\partial X_t} = P_{x_t}.$$  

The final question is the agent\firm's belief about future prices. Heterogenous beliefs is central to our model in Section 3. There we distinguish between the empirical distribution of a random variable which all agents discover from data, and the probability distribution an agent believes to be the truth. For simplicity we assume here that our firm accepts the empirical distribution as the truth.

To specify this dynamics denote $p_t = \log P_t$, $p_{x_t} = \log P_{x_t}$. The evidence shows that the empirical distribution of prices is well approximated by a Markov process with transition of the form

$$p_{t+1} - p^* = \lambda_p (p_t - p^*) + \rho_p p_{t+1}, \quad \rho_p \sim N \left( p^*, \sigma_p^2 \right) \sim N \left( 0, \sigma_p^2 \right).$$

$$(11) \quad p_{x_{t+1}} - p_{x^*} = \lambda_{p_x} (p_{x_t} - p_{x^*}) + \rho_{p_x} p_{x_{t+1}}, \quad \rho_{p_x} \sim N \left( 0, \sigma_{p_x}^2 \right).$$

The evidence suggests a parameter range of $\lambda_p \in [0.90, 0.95]$ and $\sigma_p \in [0.04, 0.05]$. $\sigma_{p_x}$ varies across industries but since wages are included in the variable cost index we set it at a modest level of $\rho(p, p_x) = \frac{\sigma_{pp_x}}{\sigma_p \sigma_{p_x}} = 0.3 - 0.5$. Prices are hypothetical and their mean levels $(p^*, p_{x^*})$ are modeling devices explained later. Once $(p^*, p_{x^*})$ are set, the firm's optimization problem is specified. This random economy is populated by many optimizing firms and we denote it by $\mathcal{E}(u, p^*, p_{x^*})$. The problem is how to think of $(p^*, p_{x^*})$, which is the basis of our Compensated Economy.
Welfare Comparisons and the Compensated Economy

\((p^*, p_x^*)\) are important since they fix the exchange rate between output, input and units of consumption over which utility is defined. We start by fixing the mean levels \((p^*, p_x^*)\) of the riskless economy at \(p^* = 1\) and \(p_x^* = 2.0\), implying that in the riskless reference economy a unit of output equals a unit of consumption good. This fixes the riskless steady state solutions \((K^*, Y^*, X^*, D^*)\). Now introduce price risk. Samuelson’s (1972a) General Equilibrium considerations show that in a risky economy populated by optimizing agents price volatility cannot be a mean preserving spread of \((p^*, p_x^*)\) since mean incomes must change. Since firm’s optimum alters the desired level of capital holdings, the long term average of \(\{(K_t, Y_t, X_t, D_t), t = 1, 2, 3,...\}\) changes, making welfare comparisons difficult. We seek a criterion with which we can establish a lower bound on the welfare cost of introducing market volatility, and based on this we propose the following concept.

**Definition:** A Compensated Economy is a risky economy \(\mathcal{E}(u, p^{**}, p_x^{**})\) which is the same as the economy \(\mathcal{E}(u, p^*, p_x^*)\) except that the mean value in (11) is set so that the mean cash flow in \(\mathcal{E}(u, p^{**}, p_x^{**})\) satisfies \(\bar{D} = D^*\). This means that \(p^{**}\) guarantees that the mean consumption in the compensated economy is the same as the consumption level in the riskless steady state.

To show the Compensated Economy \(\mathcal{E}(u, p^{**}, p_x^{**})\) is useful, suppose in this economy \(\bar{K} > K^*\) and \(\bar{D} = D^*\). Agents in this economy are worse off than in the riskless steady state, for three reasons:

(i) they accumulated and now maintain a larger capital stock;

(ii) Although larger capacity, their mean consumption \(\bar{D}\) equals the riskless level \(D^*\);

(iii) their consumption is risky.

We thus claim the mean excess capacity \(\bar{K} - K^*\) is a low estimate of the actual cost of economic volatility in the risky economy \(\mathcal{E}(u, p^{**}, p_x^{**})\). To explain note that in this economy agents choose optimally their consumption which ends up having the same mean as the steady state values in \(\mathcal{E}(u, p^*, p_x^*)\). But they are worse off relative to the riskless steady state since they incur the cost of accumulating added capital for insurance with which they attain the optimal consumption. Despite the excess capacity insurance, they end up with risky consumption.

Once \(p^{**}\) is set, we compute the solution in \(\mathcal{E}(u, p^{**}, p_x^{**})\) and generate via simulations 400
samples, each runs over 1,000 time periods which we use to compute the change in the means of \( \{ (K_t, Y_t, X_t) : t = 1, 2, 3, ... \} \) relative to \( (K^*, Y^*, X^*) \). We use this procedure since we employ perturbation methods to compute the solutions. Due to the underlying volatility, some of the simulated samples leave the orbit of convergence of the model. We report only the means over those samples whose runs remain for 1,000 units of time within the invariant set.

The results in Table 2 are based on our thinking of \( X \) as labor input with \( \sigma_{px} = 0.01 \) as the standard deviation of the wage rate. The table reports the results for persistence parameters \( \lambda_p = \lambda_{px} \) of 0.85, 0.90 and 0.95; for correlation between \( p \) and \( p_x \) of either +0.3 or +0.6 and for the standard deviations \( \sigma_p \) in the 4%-5% range. In the table we use the notation:

\[
%\Delta K = \frac{1}{N} \sum_{j=1}^{N} \frac{K_t - K^*}{K^*} - 1, \quad %\Delta Y = \frac{1}{N} \sum_{j=1}^{N} \frac{Y_t - Y^*}{Y^*} - 1, \quad %\Delta X = \frac{1}{N} \sum_{j=1}^{N} \frac{X_t - X^*}{X^*} - 1
\]

### Table 2: Excess Capacity in the Compensated Economy vs. Riskless Economy

<table>
<thead>
<tr>
<th>( \lambda_p = \lambda_{px} )</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
<th>0.85</th>
<th>0.90</th>
<th>0.95</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_p = 0.040, \sigma_{px} = 0.01 )</td>
<td>%\Delta K</td>
<td>1.55</td>
<td>8.14</td>
<td>26.90</td>
<td>2.51</td>
<td>8.72</td>
</tr>
<tr>
<td></td>
<td>%\Delta Y</td>
<td>0.30</td>
<td>3.44</td>
<td>10.10</td>
<td>0.70</td>
<td>4.21</td>
</tr>
<tr>
<td></td>
<td>%\Delta X</td>
<td>0.08</td>
<td>3.44</td>
<td>7.84</td>
<td>0.78</td>
<td>4.29</td>
</tr>
<tr>
<td>( \sigma_p = 0.045, \sigma_{px} = 0.01 )</td>
<td>%\Delta K</td>
<td>5.01</td>
<td>10.48</td>
<td>14.89</td>
<td>5.78</td>
<td>10.80</td>
</tr>
<tr>
<td></td>
<td>%\Delta Y</td>
<td>2.23</td>
<td>4.08</td>
<td>10.80</td>
<td>1.54</td>
<td>5.33</td>
</tr>
<tr>
<td></td>
<td>%\Delta X</td>
<td>2.46</td>
<td>3.82</td>
<td>8.68</td>
<td>1.65</td>
<td>5.13</td>
</tr>
<tr>
<td>( \sigma_p = 0.050, \sigma_{px} = 0.01 )</td>
<td>%\Delta K</td>
<td>7.44</td>
<td>14.89</td>
<td>31.27</td>
<td>6.34</td>
<td>14.59</td>
</tr>
<tr>
<td></td>
<td>%\Delta Y</td>
<td>3.59</td>
<td>5.78</td>
<td>12.00</td>
<td>2.79</td>
<td>6.03</td>
</tr>
<tr>
<td></td>
<td>%\Delta X</td>
<td>3.85</td>
<td>5.07</td>
<td>10.30</td>
<td>2.93</td>
<td>5.47</td>
</tr>
</tbody>
</table>

Table 2 provides a first glance at the order of magnitude of an optimizing firm’s choice of excess capacity due to market volatility and how sensitive such increases in capacity are to the economic environment. The key theoretical conclusions which can be drawn from Table 2 are as follows:

(I) Without alternative vehicles for risk sharing, excess capacity could be very large.

(II) Excess capacity increases with \( \sigma_p \) and with \( \lambda_p = \lambda_{px} \) but is not sensitive to \( \rho_{ppx} \) around 0.50. Hence, higher price volatility and higher persistence increase the cost of volatility.

(III) A \( \sigma_p = 5.0\% \) is close to mean price volatility while \( \lambda_p = 0.95 \) is reasonable. Under such conditions a firm may hold excess capacity in the range of 30%. This is very large.

To sum up, increased market volatility increases the riskiness of a firm’s investments and leads to increased distortions in the allocation of capital induced by the fact that capacity input is committed in advance. Table 2 suggests that without risk sharing in financial markets the level of excess capacity can be very large. Examination of the role of risk sharing via financial markets is our next major task.
3. Excess Capacity in Equilibrium Dynamics with Risk Sharing via Financial Markets

3.1 General Formulation

In Section 2 we studied the optimization of individual firms but these models did not allow trading or risk sharing and did not incorporate wage income. We now introduce wage income and study the impact of market risk sharing. Computational feasibility allows us only two types of agents. Hence, our model is not a full General Equilibrium model that benefits from natural averaging which aggregate economic variables possess. Our aim is to examine the effects and complications which trading and sharing of uncertainty entails. Hence, we introduce a bond market where agents trade risk. The model consists of two types of household/firms in two industries with different prices. We assume that in each industry there is a large number of identical competitive firms, each operating at constant returns to scale. A firm is owned by a household who earns labor income from work and capital income from the ownership of its capital. Although in this section agents trade securities and share uncertainty in a competitive equilibrium context, goods prices continue to express aggregate uncertainty and are assumed exogenous. Our equilibrium is thus not entirely “general.”

For simplicity we mostly ignore the index of the firms. The production function of a firm is

\[ Y_t = K_t^\sigma (\xi N_t)^{1-\sigma}, \quad \sigma = 0.40, \quad \xi = 1.009. \]

\( K_t \) is capital owned and employed, \( N_t \) is labor input selected when the wage rate is known and \( \xi \) is a deterministic technology growth factor. The Random technology shock are absorbed into the random exogenous output price which we specify later. The capital stock \( K_t \) evolves according to

\[ K_{t+1} = (1 - \delta) K_t + I_t, \quad \delta = 0.05. \]

Parameter values \( \xi = 1.009 \) and \( \delta = 0.05 \) reflect realistic productivity growth and depreciation for a six month time unit. A household/firm can borrow or lend in a competitive bond market. Let \( B_t \) be the amount of riskless bond a household buys at \( t \) and let \( Q_t^b = \frac{1}{1+r_t} \) be the price of the bond. The household’s utility function \( u(\cdot) \) is defined over consumption and leisure. Finally, let \( L_t \) be labor supply of the household and hence the budget constraint of the household/firm is

\[ C_t = W_t L_t + [P_t Y_t - W_t N_t] - I_t + [B_{t-1} - Q_t^b B_t]. \]

The firm is competitive in the labor market and adjusts its labor input every period hence we have

\[ P_t (1 - \sigma) \left( \frac{K_t}{\xi N_t} \right)^\sigma \xi^t = W_t. \]
Human capital cannot be transferred with ease from one industry to another. Over the short period of six months an automobile worker cannot become a computer programmer. Mobility cost could be handled with retraining cost a worker must pay in order to move between industries but we prefer to simplify and assume no labor mobility in the short run and set \( N = L \) as the short term equilibrium condition. This limitation also determines the optimal size of the firm and we do not assume decreasing returns. To accommodate long term labor mobility we postulate the industries are symmetric with the same production technology, the same labor supply and the same marginal distribution of the price processes. Hence, the long run average wage rates in the two industries are the same and \textit{there is no labor demand for long term mobility across the two industries.}

Denote by \( \ell_t \) the amount of leisure a household enjoys then its objective is defined by

\[
\text{Max} \quad \mathbb{E} \left[ \sum_{t=0}^{\infty} \frac{\beta}{1-\gamma} \left( C_t (\ell_t) \right)^{1-\gamma} \right], \quad \ell_t = (1-L_t)
\]

subject to (7), (8), (9), the information structure assumed above and the transversality condition

\[
\lim_{t \to \infty} \mathbb{E}[B_t] = 0.
\]

We set \( \zeta = 3.00 \) so the steady state fraction of labor time is around 0.225. The Euler equations of this optimization problem are standard:

\[
\begin{align*}
(16a) & \quad C_t^\gamma \delta_t \mathbb{E} \left[ C_{t+1}^\gamma \right] = \beta \mathbb{E} \left[ C_{t+1}^\gamma \right] \\
(16b) & \quad C_t^\gamma \delta_t \mathbb{E} \left[ C_{t+1}^\gamma \right] = \beta \mathbb{E} \left[ C_{t+1}^\gamma \right] (P_{t+1} \sigma \left( \frac{N_{t+1} K_{t+1}}{F_{t+1}} \right)^{1-\sigma}) + (1 - \delta)) \\
(16c) & \quad C_t = \frac{1}{\zeta} \delta_t W_t.
\end{align*}
\]

Goods prices are exogenous but wage rates and interest rates are endogenous, determined in equilibrium. Denote a household/firm type by an index \( j \), then market clearing conditions are

\[
\begin{align*}
(16d) & \quad N_t^j = L_t^j, \quad j = 1, 2 \\
(16e) & \quad B_t^1 + B_t^2 = 0.
\end{align*}
\]

Once we specify agents’ beliefs about future prices, we use (16a)-(16e) to derive the equilibrium solution of the interest rate, wage rates, consumptions, labor inputs and firm capacity. We turn now to the beliefs of agents about future price distributions.

3.2 \textit{Why Does Belief Heterogeneity Matter?}

Diversity of beliefs is a reality of economic life and standard models such as the Arrow-
Debreu model permit diverse probabilities over states. It is very well documented that agents make diverse forecasts of macroeconomic variables (e.g. GDP growth) about which there is no asymmetry in information. This diversity makes financial markets so much more central for trading uncertainty. However, diversity of beliefs has an effect which is also essential to the argument of this paper.

In the absence of adverse selection or moral hazard, trading of risks via insurance markets is minimally affected by market beliefs. All that matters is that a law of large numbers holds even if agents hold diverse beliefs about their idiosyncratic components. When we open asset markets for sharing aggregate risks, the problem is altered by the dynamics of beliefs. When beliefs are identical an asset price is a function of an aggregate state like \( q(s_t) \) hence to forecast \( q(s_{t+1}) \) one needs to forecast only \( s_{t+1} \). Since diverse market beliefs have their own dynamics, prices are functions of the distribution of conditional probability functions of the agents, denoted by \( z_t \). Prices are then functions of the form \( q(s_t, z_t) \) and forecasting \( s_{t+1} \) is not the same as forecasting \( q(s_{t+1}, z_{t+1}) \). Incomes, consumptions and other endogenous variables become functions of an expanded state \((s_t, z_t)\) and market risks expand to the risk of what the market belief will be at \( t+1 \). In earlier papers (see Kurz (1994) ,(1997), Kurz and Wu (1996) ) we called this belief externality “Endogenous Uncertainty.”

Arrow’s (1953) conception of asset markets being used by agents to trade uncertainty works well if agents’ beliefs do not create belief externalities which affects price dynamics. When belief externalities turn prices into functions of an expanded state, asset markets lose some of their ability to facilitate risk sharing since opening of an asset market creates a channel through which a new risk is introduced about the future price of this asset. Indeed, this is the risk of what market belief will be tomorrow. This risk is confounded with the initial risk of exogenous shocks which agents wanted to share to begin with. Hence, diverse and dynamic belief structures have an impact on the efficacy of risk sharing via asset markets. In the context of our excess capacity problem, we allow agents to share risk via financial markets in order to reduce the need for excess capacity. But when we open a bond market for agents to trade the risk of their \( t+1 \) income, they find these same financial markets present a new risk that was not there before and this limits the ability of financial markets to carry out risk sharing of income. This has an effect on optimal excess capacity.

Diversity of belief is also important for consistency between our theory of the cost of aggregate volatility, the assumed high volatility of good prices and a high equity premium. In earlier
papers we show (see Kurz and Motolesse (2001), and Kurz, Jin and Motolesse (2003c)) that the dynamics of market expectations are central to asset market volatility and a high equity premium is explained by the structure of market beliefs. High volatility of good prices is also implied.

But why do agents disagree about future state variables? Because the dynamics of our economy is a time dependent process, varying with technology, resource supplies and society’s institutions. Diversity of beliefs is thus a result of diversity of models used to forecast the future. In considering such heterogeneity we distinguish between what agents deduce from past data and what they believe about the distribution of future observable variables at any date \( t \).

The true dynamics is complex and unknown, with a time dependent non-stationary good price process. Such dynamics is compatible with the existence of an empirical distribution of observable variables. Kurz (1994) calls this property “statistical stability” and shows the implied empirical distribution is expressed as a stationary process. Agents do not know the true non-stationary process of good prices but have ample past data. They interpret with subjective models. A statistical summary of the past is represented by the empirical distribution of all observables and this includes all moments. Since it is computed from public data it is known to all. Here we assume the empirical distribution of prices is specified, as in (11a)-(11b), by a Markov process of the form

\[
\begin{align*}
\rho_1^{t+1} - \rho_1^t &= \lambda_p^1 (\rho_1^{t+1} - \rho_1^t) + \rho_1^{t+1} \\
\rho_2^{t+1} - \rho_2^t &= \lambda_p^2 (\rho_2^{t+1} - \rho_2^t) + \rho_2^{t+1},
\end{align*}
\]

For this economy \( \rho_1^{\cdot} = \rho_2^{\cdot} = 1 \) are hypothetical parameters which fix the exchange rates between the goods produced and consumption. Based on data in Table 1 the parameters in (18) are estimated by

\[
\begin{align*}
\rho_1^1 = 0.95, \quad \sigma_1 = \sigma_2 = 0.050.
\end{align*}
\]

\( \sigma_{12} \) reflects shocks which affect the two prices similarly (see comment B in Section 1). The relevant case is \( \sigma_{12} > 0 \) but if \( \rho_{pp} = \frac{\sigma_{12}}{\sigma_1 \sigma_2} = 1.00 \) then both agent types are subjected to exactly the same risk and risk sharing is not possible. As a model strategy we assume \( \rho_{pp} > 0.30 \) but examine the effect of varying this parameter. Agents know (18) but may not agree on the “correct” model that generated this empirical evidence. Indeed, one expects that different agents looking at the same evidence will have different theories to explain it and hence have different models to forecast prices. But then, one may ask, how do agents arrive at these beliefs? Since they do not hold Rational Expectations, how
do they rationalize these beliefs? These important questions are not addressed here. Since we study the cost of volatility we focus on a narrow but operational question which we have addressed elsewhere\(^4\). To explain it note that (16a)-(16c) require only a statement of the agents’ conditional probabilities. Since their models are time dependent we need a tractable way to describe differences among beliefs as well as time variability of conditional probability functions without writing down complete models to justify them. From our perspective what matters is that beliefs are diverse and time dependent; the reasoning which lead to the agents’ subjective models are secondary. The tool we developed for this goal is the individual and the market “states of belief” which we now explain.

3.3 Describing Equilibria with Diverse Beliefs: Market States of Belief and Anonymity

In this section we outline a modeling strategy of economies with diverse and dynamically changing beliefs. This formulation does not depend upon the particular theory one uses to explain this heterogeneity. Once we complete this formulation we present a theorem which shows exactly what are the restrictions which are imposed on the model by the theory of Rational Beliefs (see Kurz (1994), (1997)) which we adopt in the computational model in Section 3.5.

The usual state space for agent \(j\) is denoted by \(S^j\) but in order to describe possible changes in belief over time we introduce an additional state variable called “agent \(j\) state of belief.” It is a parameter generated by agent \(j\), expressing his date \(t\) view of the future and denoted by \(g^j_t\). It has the property that once specified, the conditional probability function of an agent is fully specified hence it takes has form \(\Pr(s^j_{t+1}, g^j_{t+1} | s^j_t, g^j_t)\). Changes in \(j\)’s conditional probability function are pinned down by \(j\)’s state of belief hence \(g^j_t\) is actually a proxy for \(j\)’s conditional probability function. In the model of this paper agents forecast prices, which are the exogenous variables, hence \(g^j_t\) describe agent \(j\) conditional probability of prices at \(t+1\). We interpret \(g^j_t\) in the following way:

- If \(g^j_t = 0\) \(j\)’s model agrees with the empirical distribution and makes price forecasts as in (18);

\(^4\) The subjective models of agents used in the computational models are rational in accord with the theory of Rational Beliefs (see Kurz [1994], [1997]). This means that each subjective model can be simulated and the simulated data must reproduce the empirical distribution of the observables in the economy which include (18) but will also include the market states of belief, explained in this section. To specify the implied restrictions on the firm’s belief, we do not need to specify the full model of the firm. As explained in detail in Kurz, Jin and Motoles (2003b) (2003c), we only need to specify how the structural model of the firm is restricted by the empirical distribution of the observables.
• If $g_j^t > 0$, j’s model disagrees with the empirical distribution: when $g_j^t > 0$ j’s model makes price forecasts which are higher than as in (18) and when $g_j^t > 0$ it makes lower forecasts.

It is a common practice among forecasters to use the pure statistical forecast only as a benchmark. Given such benchmark, a forecaster uses his own model to add a component reflecting an evaluation of the circumstances at a date $t$ that call for a deviation at $t$ from the benchmark. In short, $g_j^t$ is a description of how the model of agent $j$ deviates from the statistical forecast implied by (18). In this paper we assume that at any date the state of belief is a realization of a process of the form

$$g_{t+1}^j = \lambda_z g_t^j + \lambda^x_{p1}(p_t^1 - p_t^1) + \lambda^x_{p2}(p_t^1 - p_t^1) + \delta_{g_{t+1}}^j, \quad \delta_{g_{t+1}}^j \sim N(0, \sigma_g^2).$$

The dynamics of the state of beliefs about future prices have persistence and it depends upon current prices. In this sense (19) resembles the posterior distribution of an unknown parameter except that we add the term $\delta_{g_{t+1}}^j$ to reflect the judgment of an agent. To keep our model below simple we shall generally ignore the effect of date $t$ prices and assume $\lambda_z^x = \lambda_z^x = 0$.

When agents with diverse beliefs make optimal choices, equilibrium variables become functions of the distribution of the $g_t^j$ hence they are functions of $g_t = (g_1^t, g_2^t, \ldots, g_N^t)$ which is the agents’ vector of conditional probabilities. But then, should agent $j$ be allowed to recognize that his $g_t^j$ is the $j$th coordinate of $g_t$ and give him some market power? The principle of anonymity introduced in Kurz, Jin and Motolese (2003b), (2003c) requires agents to be competitive and not assume they can affect endogenous variables. To do that we define the “market state of belief” as a vector $z_t = (z_1^t, z_2^t, \ldots, z_N^t)$ with a model consistency condition $z_t = g_t$ which is not taken into account by optimizing agents. This fact turns the market state of belief into a macroeconomic state variable and all endogenous variables are actually functions of $z_t$, not functions of $g_t$. Agents know market expectations have an effect on endogenous variables but they do not assume their own expectations have an effect on markets. Agent $j$ views $z_t$ as “market expectation” which is, in fact, the combined state of belief of other agents in the market. In small economies equilibrium variables depend upon the entire distribution $z_t = (z_1^t, z_2^t, \ldots, z_N^t)$ but in many applications only a few moments of this distribution matter. In some models of diverse beliefs writers simplify by considering only the average, and define the market state of belief by the mean $\bar{z}_t = \frac{1}{N} \sum_{j=1}^N z_{t,j}^j$.

The introduction of individual and market states of belief has two important effects which are

---

5 See for example Woodford (2003), Morris and Shin (2002), Allen Morris and Shin (2003) and others
crucial to the way an equilibrium works in economies with diverse beliefs:

(i) Endogenous variables are defined on an expanded state space that includes the state of belief $z_t$. Hence diverse beliefs create new uncertainty an agent perceives: the uncertainty of what others may do. This adds a new component of economic volatility which cannot be explained by fundamental exogenous shocks.

(ii) To forecast future endogenous variables agents must forecast future market states of beliefs. This central characteristic reminds us of the Keynes Beauty Contest: to forecast future equilibrium variables you must forecast the beliefs of “other” agents. However, the date $t+1$ market belief is not a probability about your date $t$ belief state.

We now simplify further by assuming the market state of belief $(z_{1t}^1, z_{1t}^2)$ is observable. This assumption is reasonable since there is a vast amount of public data on the distribution of market forecasts. We comment below on data of the Blue Chip Economic Indicators and on the Survey of Professional Forecasters. Note, the fact that $(z_{1t}^1, z_{1t}^2)$ is observable requires us now to modify the description of the empirical distribution (18a) - (18b) and include $(z_{1t}^1, z_{1t}^2)$ in it. In accord with (19) and symmetry, the expanded empirical distribution is assumed an AR1 process of the form

$$
\begin{align*}
\rho_{1t}^1 - p^1_1 &= \lambda_p (p_{t-1}^1 - p^1) + \rho_{1t-1}^1, \\
\rho_{1t}^2 - p^2_1 &= \lambda_p (p_{t-1}^2 - p^2) + \rho_{1t-1}^2, \\
\rho_{z1t}^1 &= \lambda_{z1} z_{1t}^1 + \rho_{z1t-1}^1, \\
\rho_{z2t}^2 &= \lambda_{z2} z_{2t}^2 + \rho_{z2t-1}^1 \\
\end{align*}
$$

$$
\begin{pmatrix}
\rho_{1t}^1 \\
\rho_{1t}^2 \\
\rho_{z1t}^1 \\
\rho_{z2t}^2
\end{pmatrix}
\sim N
\begin{pmatrix}
0 \\
0 \\
0 \\
0
\end{pmatrix}
\begin{pmatrix}
0 & \sigma^2 & \sigma_{12} & 0 & 0 \\
\sigma_{12} & \sigma^2 & 0 & 0 \\
0 & 0 & 1 & \sigma_{z1z2} & 0 \\
0 & 0 & \sigma_{z1z2} & 1
\end{pmatrix}
\text{i.i.d.}
$$

$\lambda_p = 0.95, \sigma = 0.050$ have been estimated and we stress the assumed symmetry in the price process. To estimate parameters of the $z^j$ equations we used forecasts of the Blue Chip Economic Indicators and purged them of information in current observables. By estimating principal components we handle multiple variables forecasted (for details, see Fan (2004)). The extracted belief indexes

---

6 Allen, Morris and Shin (2003) seem to suggest the Beauty Contest is associated with the failure of the market belief to satisfy the law of iterated expectations (see title of their paper). It is clear the average market probability beliefs does not satisfy the law of iterated expectations since the average conditional probability is not a proper conditional probability. However, this fact is independent of the problem defined by the Keynes Beauty Contest which requires agents in an economy with diverse beliefs to forecast the future average market state of belief. Also, in Woodford (2003) and in Morris and Shin (2002) the higher order beliefs arise only due to contemporaneous strategic interaction among the players, a phenomena which is not present in a competitive market setting in which traders satisfy anonymity.
exhibit autocovariances of 0.4 - 0.7. We selected the modest value $\lambda_z = 0.40$ and simplify by assuming $\lambda_z = \lambda_{z,1} = \lambda_{z,2} = 0.40$. The data exhibit high correlations across agents and 0.9 is a good estimate of $\sigma_{z_1z_2}$. In sum, parameters of (20) are set in empirically realistic range. Finally, to write (20) in a compact notation let $x_t = (p_{t,1}^1, p_{t,2}^2, z_{t,1}^1, z_{t,2}^2)$, $\rho_{t} = (\rho_{t,1}^i, \rho_{t,2}^i, \rho_{t,1}^z, \rho_{t,2}^z)$ and denote by $A$ the 4x4 matrix of parameters in (20). We then rewrite (20) in a matrix notation

\begin{align}
    x_{t+1} &= Ax_t + \rho_{t+1}, \quad \rho_{t+1} \sim N(0, \Sigma) \\
\end{align}

where $\Sigma$ is the covariance matrix in (20). We refer to the probability on sequences implied by (20) as $m$ and we write \( E_m(x_{t+1} \mid H_t) = Ax_t \) where $H_t$ is the history at $t$. 

### 3.4 The Belief Structure and Belief Parameters

A perception model is a set of transition functions of the state variables, reflecting the agent’s belief about date $t+1$ transition probability. We first explain the general form of a perception model, using (20), providing details later. Let $x_{t+1}^j = (p_{t,1}^{j,1}, p_{t,1}^{j,2}, z_{t,1}^{j,1}, z_{t,1}^{j,2})$ be date $t+1$ variables as perceived by agent $j$ and let $\Psi_{t+1}(g_t^j)$ be a 4 dimensional vector of date $t+1$ random variables conditional upon parameter $g_t^j$.

**Definition:** A perception model in the economy under study has the general form

\begin{align}
    x_{t+1}^j &= Ax_t + \Psi_{t+1}(g_t^j) \quad \text{together with} \quad (19). \\
\end{align}

Since \( E_m(x_{t+1} \mid H_t) = Ax_t \), we write (21a) in the simpler form

\begin{align}
    x_{t+1}^j - E_m(x_{t+1} \mid H_t) = \Psi_{t+1}(g_t^j). \\
\end{align}

(21b) reveals that \( E_j[\Psi_{t+1}(g_t^j)] \) is the deviation of $j$’s forecast of $x_{t+1}$ from $E_m(x_{t+1} \mid H_t)$ and we shall see that, in general, \( E_j[\Psi_{t+1}(g_t^j)] \neq 0 \). We also note that $j$’s forecasts change with $g_t^j$. If $\Psi_{t+1}(g_t^j) = \rho_{t+1}$ as in (20), $j$ uses the empirical probability $m$ as his belief.

In modeling $\Psi_{t+1}(g_t^j)$ we observe that in an economy with diverse beliefs agents may be over-confident by being optimistic or pessimistic relative to the empirical forecasts. However, to maintain simplicity we specify the random sequence $\Psi_{t+1}(g_t^j)$ to take the simple form
where $\hat{p}_{t+1}^j = (\hat{p}_{t+1}^{x1j}, \hat{p}_{t+1}^{x2j}, \hat{p}_{t+1}^{z1j}, \hat{p}_{t+1}^{z2j})$. A perception model includes $g_t^j$ as a fifth dimension with an innovation $\hat{g}_{t+1}^j$ and a covariance matrix denoted by $\Omega$, reflecting the vector $r_t^j = \text{Cov}(x_t^j, g_t^j)$ for $i = 1, 2, 3, 4$. To keep the model simple we use only one random variable $\eta_{t+1}^j(g_t^j)$ to define all components of $\Psi_{t+1}(g_t^j)$. The parameters $\lambda_g = (\lambda_g^{p1}, \lambda_g^{p2}, \lambda_g^{z1}, \lambda_g^{z2})$ are central in describing how the agent’s forecasts vary with his beliefs. To keep the model simple we assume symmetry and ignore the effect of an agent’s belief on forecasting the belief of others and set $\lambda_g^{z1} = \lambda_g^{z2} = 0$, $\lambda_g^{p1} = \lambda_g^{p2} = \lambda_g^p$.

We study the symmetric case where the two industries have the same technology, the same labor supply and the same marginal price distribution. They are different in two respects: they may have different date $t$ prices and agents in the two industries may have different beliefs. The construction of the random variable $\eta_{t+1}^j(g_t^j)$ is developed in an Appendix. We note it has one parameter $b$. We now state the final form of the subjective perception model of agent $j$:

$$\begin{align*}
\Psi_{t+1}(g_t^j) &= \left( \lambda_g^{p1} \eta_{t+1}^{j1}(g_t^j) + \hat{p}_{t+1}^{x1j}, \\
\lambda_g^{p2} \eta_{t+1}^{j2}(g_t^j) + \hat{p}_{t+1}^{x2j}, \\
\lambda_g^{z1} \eta_{t+1}^{z1}(g_t^j) + \hat{p}_{t+1}^{z1j}, \\
\lambda_g^{z2} \eta_{t+1}^{z2}(g_t^j) + \hat{p}_{t+1}^{z2j} \right), \\
\hat{p}_{t+1}^j &\sim \mathcal{N}(0, \Omega_{pp}) \text{ i.i.d.}
\end{align*}$$

(22)

The vector $(\hat{p}_{t+1}^j, \hat{g}_{t+1}^j) = (\hat{p}_{t+1}^{x1j}, \hat{p}_{t+1}^{x2j}, \hat{p}_{t+1}^{z1j}, \hat{p}_{t+1}^{z2j}, \hat{p}_{t+1}^{g1j})$ is distributed i.i.d. Normal with mean zero and covariance matrix $\Omega^j$ which is written in the form

$$\Omega^j = \begin{pmatrix}
\Omega_{pp}^{jj}, \\
\Omega_{pg}, \\
\Omega_{gx}^{jj}, \\
\sigma_g^{2j}
\end{pmatrix},$$

where $\Omega_{gx} = \text{Cov}(\hat{p}_{t+1}^{x1j}, \hat{g}_{t+1}^j), \text{Cov}(\hat{p}_{t+1}^{x2j}, \hat{g}_{t+1}^j), \text{Cov}(\hat{p}_{t+1}^{z1j}, \hat{g}_{t+1}^j), \text{Cov}(\hat{p}_{t+1}^{x2j}, \hat{g}_{t+1}^j)$. (23) shows that given our simplifications, $\lambda_g^p = 0.00$ characterizes an economy where all agents believe the empirical
distribution is the truth. This case has the no diverse beliefs property of an REE. In this case the volatility of equilibrium quantities is determined by the volatility of the state variables as in (18).

**Summary of Belief Parameters.** Given the assumed symmetry, the parameters specifying belief of \( j \) are then \((\lambda_g^j, \Omega^j, b)\) where \( b \) is a parameter defining the random variable \( p^j_{t+1}(g_t^j) \) explained in the Appendix. A theory with unrestricted beliefs would specify these parameters and some may think of such a theory as one of bounded rationality. Note that *anonymity* requires the idiosyncratic component of an agent’s belief not to be correlated with market beliefs. This is translated to require

\[
\begin{align*}
\Omega^{Z^1}_{x^jg} &= \text{Cov}(\tilde{\rho}^{Z^1}_{t+1}, \tilde{\rho}^{g^j}_{t+1}) = 0 \\
\Omega^{Z^2}_{x^jg} &= \text{Cov}(\tilde{\rho}^{Z^2}_{t+1}, \tilde{\rho}^{g^j}_{t+1}) = 0
\end{align*}
\]

which then restricts two components of \( \Omega_{xg} \) even within a theory without restrictions on belief.

### 3.5 Effect of Risk Sharing and Diverse Rational Beliefs on the Cost of Volatility

In this section we present our computational model which utilizes the restrictions imposed by the theory of Rational Beliefs (in short RB) due to Kurz (1994), (1997). We first define RB.

**Definition:** A belief as in (23) above is said to be an RB if the agent’s model \( x^j_{t+1} = A x_t + \Psi_{t+1}(g_t^j) \) together with (19) has the same empirical distribution as \( x^j_{t+1} = A x_t + \rho_{t+1} \).

The definition requires that \( \Psi_{t+1}(g_t^j) \) together with (19) have the same empirical distribution as \( \rho_{t+1} \) which is \( N(0, \Sigma) \). An RB is a model with the property that *when simulated, it reproduces the empirical distribution of market data hence the model matches all moments of observables.* An RB is not a theory which calls for rational agents to adopt a specific belief. The theory formulates restrictions which the belief of a rational agent should satisfy in an economy where agents cannot learn the true process since it changes too rapidly relative to the data. It proposes a simple principle of rationality which says that if an agent’s model does not reproduce a known empirical distribution, the model is irrational. To “reproduce” an empirical distribution a model must match all moments. Agents holding rational beliefs may make “incorrect” forecasts at date \( t \) but must be correct, on average. Hence, date \( t \) forecasts may deviate from forecasts implied by the empirical distribution.
Since RB rationality requires the time average of an agent’s forecasts to agree with forecast based on the empirical frequencies, it follows as a theorem that agents who hold rational beliefs have forecast functions that vary over time. So, what are the restrictions of the RB principle on our model?

**Theorem:** Let the beliefs of an agent satisfy the RB rationality principle. Then the agent’s belief is restricted as follows:

(i) For any feasible pair of parameters \((\lambda^p_g, b)\) the Variance-Covariance matrix \(\Omega^j\) is fully pinned down and is not subject to choice;

(ii) The condition that \(\Omega^j\) is a positive definite matrix establishes a region of feasibility for the two parameters \((\lambda^p_g, b)\). In particular we have the simple restriction \(|\lambda^p_g| \leq \sigma\).

**Proof:** See Kurz, Jin and Motolese (2003b), Appendix.

Under the RB restrictions we can select only \((\lambda^p_g, b)\) with \(|\lambda^p_g| \leq 0.050\) and \(|b| \leq 10\). Here we assume, as before, \(\lambda_p = 0.95\), \(\sigma_p = 0.05\) and examine equilibria for \(b = -5\). Given these assumptions we vary \((\rho_{pp}, \lambda^p_g)\) in a range allowed by the RB restrictions. Since the exogenous price dynamics reflects business fluctuations, a realistic value for \(\rho_{pp}\) should be at least \(\rho_{pp} = 0.30\). We select four values of \(\lambda^p_g\). In the case \(\lambda^p_g = 0.00\) all agents believe the empirical distribution is the truth. It may be considered REE since beliefs play no role in it. We then study \(\lambda^p_g = 0.020, 0.030\) and \(0.040\), reflecting increased intensity of beliefs. For each \(\lambda^p_g\), we study the effect of aggregate shocks as expressed by the correlation \(\rho_{pp}\). These values of \((\rho_{pp}, \lambda^p_g)\) span the full range permitted by the RB restrictions hence the results in Table 3 cover all possible equilibria under the RB restrictions.

Without restrictions on short sales the volatility in bond trading may increase the number of cases which leave the model’s radius of convergence. We thus conduct the simulations by taking 800 random samples, each run over 400 model dates. Random sequences which leave the model’s radius of convergence are ignored. This means that although each estimate in the table is based on 320,000 points, sampling errors remain. Also, computations of the parameter value equalizing the mean consumption in the compensated economy to the steady state value is only an approximation. One can measure this error by rerunning the procedure for may different seeds but this is very time consuming. Our experimentation shows the error can be as much as 0.25 of a percentage point.
Table 3: Effect of Correlation and Diverse Beliefs on Excess Capacity in the Compensated Economy

<table>
<thead>
<tr>
<th>$\rho_{pp}$</th>
<th>$\lambda_g^p$</th>
<th>0.00</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
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<tbody>
<tr>
<td></td>
<td>$% \Delta K$</td>
<td>4.49</td>
<td>8.09</td>
<td>12.03</td>
<td>17.73</td>
</tr>
<tr>
<td></td>
<td>$% \Delta Y$</td>
<td>1.18</td>
<td>2.24</td>
<td>3.29</td>
<td>4.93</td>
</tr>
<tr>
<td></td>
<td>$% \Delta L$</td>
<td>0.23</td>
<td>0.89</td>
<td>1.71</td>
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<td>$\rho_{pp} = 0.90$</td>
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<td>4.16</td>
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<td>5.56</td>
</tr>
<tr>
<td></td>
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<td></td>
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<td>2.16</td>
<td>3.08</td>
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<td>0.95</td>
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</tr>
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<td></td>
<td>$\sigma_{\Delta c}$</td>
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</tr>
<tr>
<td>$\rho_{pp} = 0.70$</td>
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<td>7.62</td>
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<td>16.51</td>
</tr>
<tr>
<td></td>
<td>$% \Delta Y$</td>
<td>0.18</td>
<td>2.33</td>
<td>3.91</td>
<td>4.53</td>
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<td>-0.32</td>
<td>1.17</td>
<td>2.62</td>
<td>2.88</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\Delta c}$</td>
<td>2.36</td>
<td>2.84</td>
<td>3.34</td>
<td>3.93</td>
</tr>
<tr>
<td>$\rho_{pp} = 0.50$</td>
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<td>6.63</td>
<td>10.49</td>
<td>16.06</td>
</tr>
<tr>
<td></td>
<td>$% \Delta Y$</td>
<td>0.53</td>
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<td>2.47</td>
<td>4.56</td>
</tr>
<tr>
<td></td>
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<td>1.25</td>
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<tr>
<td></td>
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<td>1.52</td>
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<td>2.10</td>
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</tbody>
</table>

The first three numbers for each case in Table 3 are results for the compensated economy relative to the riskless steady state of the original economy: $\% \Delta K$ is excess capacity, measured by the percentage change in mean capital employed; $\% \Delta Y$ and $\% \Delta L$ are percentage changes in the mean output and mean labor employed. These three are the direct measures of the changes in resources used by the compensated economy while mean consumption in this economy is the same as the constant consumption in the riskless economy. The change in these resources used is pure social cost of volatility. We now need to explain the fourth number in Table 3.

Volatility of consumption growth. We report in Table 3 the value $\sigma_{\Delta c}$ which is the implied standard deviation of aggregate consumption growth rate of in this economy. To explain how $\sigma_{\Delta c}$ is computed note that price volatility reflect aggregate risk and there is no idiosyncratic uncertainty in the model. Differences between two agent types arise since they get hit differently by aggregate shocks and hence risk sharing is possible through financial markets. There is a limit to risk sharing since the volatility they face is correlated. Suppose now we have a large number of agent types with a large number of different industries in which they would invest. In that case $\sigma_{\Delta c}$ is the standard deviation of individual consumption growth and the stability of the variance of mean consumption growth is a result of statistical averaging. It takes the form
\[ \log \frac{c_i}{c'} = \frac{1}{N} \sum_{j=1}^{N} \log \frac{c_j}{c'} \]  

(c' and c'j are the steady state values).

Hence

\[ \sigma_{\Delta c}^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \text{Cov}(\Delta c_i, \Delta c_j). \]

As a reasonable approximation we assume

(I) \( \sigma_{\Delta c_i}^2 = \text{The same for all } i, \) and  

(II) \( \text{Cov}(\Delta c_i, \Delta c_j) = \rho_{pp} \sigma_{\Delta c_i}^2 \text{ all } i \text{ and } j. \)

Hence, we have that for large N

\[ \sigma_{\Delta c}^2 = \frac{1}{N} \sigma_{\Delta c_i}^2 + \frac{N(N-1)}{N^2} \rho_{pp} \sigma_{\Delta c_i}^2 = \rho_{pp} \sigma_{\Delta c_i}^2. \]

The fourth item in Table 3, denoted \( \sigma_{\Delta c}, \) reports the implied standard deviation of aggregate consumption growth which is defined by

\[ \sigma_{\Delta c} = \sqrt{\rho_{pp}}. \]

Table 3 then show that the volatility of aggregate consumption growth in our economy is close to the volatility in Lucas’ (1987) economy. Hence, individual agents in the model have opportunities for risk sharing via financial markets which are similar to the risk sharing available in the real economy.

To explain this conclusion recall that in the Lucas’ (1987) economy the standard deviation of consumption growth, \( \sigma_{\Delta c}, \) is annually 3.2% and hence 2.3% at the model time unit of six months. We also showed that individuals have \( \sigma_{\Delta c_i} \) which are much larger, and for a six month unit of time are above 10%. A good fraction of this volatility of individual \( \sigma_{\Delta c_i} \) is truly idiosyncratic which is averaged out and is never reflected in price volatility. We mark in bold letters those cases in Table 3 where the implied \( \sigma_{\Delta c} \) is in the range between 2.1% and 2.9% and suggest this is the range which is compatible with the volatility of aggregate consumption growth in the Lucas (1987) economy.

Which of the bold faced cases are then the applicable results to the U.S. economy? We cannot be entirely certain of the answer. We reject the REE case \( \lambda_g = 0.00 \) for the simple reason that it contradicts the assumed good price volatility and implies an equity premium puzzle and the contradictions it entails. We have already indicated that the empirical evidence in support of belief heterogeneity is persuasive. Hence the applicable cases are those with \( \lambda_g > 0.00. \) We then conclude that the first key result in Table 3 says that the level of excess capacity which theory predicts for the economy is in the range of 6%-10%.

**Risk sharing and the bond market.** Our second key result in Table 3 is the difference between resources used in this table and resources used in the third column of Table 2 (applicable to
\( \lambda_p = 0.95 \). It shows a drastic reduction in excess capacity and variable inputs due to the great impact of risk sharing via the bond market. Indeed, the essence of the results in Table 3 is that risk sharing of aggregate uncertainty via asset markets is a very powerful tool but it is not a sufficient tool, leaving the need for added self insurance. This risk sharing arises from the fact that aggregate shocks impact different markets in different ways and at different times, leading prices to be only partly correlated. It is surely true that the risk of aggregate shocks could be fully shared if prices were negatively correlated since then the bond market would permit agents to attain virtually constant consumption streams. The limitation of risk sharing via financial markets arises from two facts. First, since price shocks are positively correlated, an agent faces a component of uninsurable aggregate risk that cannot be traded. Second, in a market with diverse beliefs, correlation among beliefs distorts the ability of agents to trade the aggregate risks reflected in goods prices. Hence, even if goods prices have low correlation and thus offer some risk sharing, a positive correlation of beliefs reduces the correlation between bond prices and goods prices. That is, interest rates in our model have their own dynamics which does not reflect only movement of goods prices.

**The social cost of excess capacity.** Table 3 shows that excess capacity and resource cost of volatility increase with higher correlation \( \rho_{pp} \) and higher intensity of beliefs \( \lambda_g^p \). To explain the meaning of the results in Table 3 consider the case of \( \rho_{pp} = 0.30 \) and \( \lambda_g^p = 0.03 \). The table shows this compensated economy has business fluctuations with the following features:

(I) it has the same long term level of average consumption as the riskless steady state;

(II) this risky economy has long term average level of excess capacity of 10.24%;

(III) the long term average level of hours of work is higher by 1.52%

(IV) the long term average level of output is higher by 2.34%.

The additional resources are used to support higher capacity and higher depreciation, and finance higher investments needed to support the larger capacity. **These resources are never used to provide additional consumption.** Even if we consider the REE in column 1 where \( \lambda_g^p = 0.00 \), we see that excess capacity is in the range of 3.2% (for \( \rho_{pp} = 0.50 \)) but without an increase in labor input. However, if we consider entries with diverse beliefs \( (\lambda_g^p > 0) \) we find

(i) excess capacity in the range of 6% - 10%;

(ii) additional labor effort of 0.5% - 1.5%;

(iii) added level of output (with no added consumption) of 1.5% - 2.5%.
What are the social cost of 8% excess capacity and a 1% added labor input? An annual cost of capital (including depreciation) of 15% and a capital\(\times\)annual output ratio of 2.8 imply that the cost of 8% excess capacity is 3.36% of GDP (i.e. 0.0336 = (0.08)\times(0.15)(2.8)). Since consumption is 0.78 of GDP, 8% excess capacity cost 4.31% of consumption. In addition, a 1% increased labor input is worth .60% of GDP or .77% of aggregate consumption. Adding the two we find that 8% excess capacity and 1% added labor input imply that the excess capacity cost of market volatility is of an order of magnitude of 4% of GDP.

4. Some Empirical Evidence

4.1 Excess Capacity

Excess capacity as defined in Sections 2-3 is unobservable: it is measured relative to the optimal level of capacity in a theoretical riskless steady state economy. In order to gain an empirical insight into the magnitude of capacity used as a hedge against aggregate uncertainty we need to use an approximation. The most useful data for this purpose is the capacity utilization data reported by the Federal Reserve Board for major economic sectors and for a detailed list of industries (see [http://www.economagic.com/frbg17.htm](http://www.economagic.com/frbg17.htm)). It is well known that “rated capacity” is motivated in part by engineering considerations, aiming to capture a maximal level of output attainable by the equipment under some specified but demanding technical conditions. As a result, the actual level of capacity utilization does not have a direct economic interpretation and has fluctuated around 80%. However, there are two facts which make the capacity utilization data interesting and very useful:

(i) The rate of capacity utilization is not too volatile. Figure 2 exhibits it for 1967-2004.
(ii) Changes in capacity utilization are highly correlated with business conditions and reflect well the changes in the intensity levels of capital employment.

**FIGURE 2: INSERT HERE**

Observation (i) implies that when a utilization rate rises to a high level it remains there for some time, reflecting a sustainable production rate. Observation (ii) implies a declining utilization rate reflects declining market demand and hence an implied under-employment of capital. Given the assumption of substitution one may argue capital is always fully employed and we have addressed this question in Section 2. Based on the these considerations we interpret the data as follows:
(I) The time average of capacity utilization is approximately proportional to the level that would prevail at the riskless steady state;

(II) The ratio of the highest level of capacity utilization over an interval and the average level over that time interval is a proxy for the average level of excess capacity in that industry.

Figure 2 exhibits time series of capacity utilization 1967-2004. There is a difference in behavior between the earlier and later periods. The utilization rate is more volatile in the earlier period, reflecting the well documented post WWII decline in aggregate volatility of real variables. Also, the utilization rate is somewhat higher in the 1960's. To evaluate present day excess capacity, which we measure by the Max/Mean ratio proposed in (II) above, we computed it using only data from April 1975 to March of 2004. The 1973 - 1975 recession ended on April 1975 and we use this start date since there is some evidence the oil shocks of 1973 - 1975 caused a structural break in productivity.

Table 4: Capacity Utilization and Excess Capacity
Selected Sectors (1975:4 - 2004:3)

<table>
<thead>
<tr>
<th>Industrial Category</th>
<th>Mean Utilization</th>
<th>Max/Mean Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Total Index</td>
<td>80.64%</td>
<td>1.07</td>
</tr>
<tr>
<td>Durable Manufacturing</td>
<td>77.60</td>
<td>1.12</td>
</tr>
<tr>
<td>Non Durable Manufacturing</td>
<td>81.68</td>
<td>1.06</td>
</tr>
<tr>
<td>Mining</td>
<td>86.39</td>
<td>1.09</td>
</tr>
<tr>
<td>Oil and Gas Extraction</td>
<td>91.74</td>
<td>1.04</td>
</tr>
<tr>
<td>Electric and Gas Utilities</td>
<td>86.30</td>
<td>1.13</td>
</tr>
<tr>
<td>Computers and Peripheral Equipment</td>
<td>77.41</td>
<td>1.20</td>
</tr>
<tr>
<td>Semiconductors and Related Equipment</td>
<td>80.06</td>
<td>1.25</td>
</tr>
<tr>
<td>Primary Semifinished Processing</td>
<td>81.51</td>
<td>1.08</td>
</tr>
<tr>
<td>Finished Processing</td>
<td>77.92</td>
<td>1.07</td>
</tr>
<tr>
<td>Food Beverages and Tobacco</td>
<td>81.84</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Table 4 presents the result of computing excess capacity for a sample of major sectors using the Fed’s capacity utilization data (www.federalreserve.gov/releases/g17/caputl.htm) The data for a long list of diverse industrial categories has a similar character and is not presented here. The interesting fact about Table 4 is that it reports estimates of excess capacity for 1975 - 2004 which are very close to those attained in the theoretical model.

4.2 The Cost of Financial Intermediation

Apart from excess capacity in production of goods, most sectors of the economy utilize other resources which constitute cost of aggregate fluctuations. Financial institutions are perhaps the most
distinct in this category and we examine this sector more as an example of the added cost of aggregate fluctuations rather than as an exhaustive summary of all such cost.

Imagine the changes that would occur in our environment if our economy had no aggregate fluctuations or risks. Imagine that all risks are idiosyncratic and, apart from moral hazard or adverse selection, they are all insurable. All random technological shocks are idiosyncratic and the aggregate impact of technology is thus deterministic. Imagine that investment returns can be fully diversified so that the equity premium is zero. In addition, all aggregate indexes of asset prices are deterministic. Government bond prices are all fully deterministic. All aggregate economic magnitudes and aggregate price indexes are deterministic. Without aggregate risks and adequate private insurance bank runs do not occur and with a fully diversified loan portfolio banks need virtually no reserves. All defaults arise from idiosyncratic sources which, by and large, can be insured against. Under such conditions, how would you expect our financial markets and financial institutions to change?

Table 5: Percent of Value Added by Economic Sector, 2003

<table>
<thead>
<tr>
<th>SECTOR</th>
<th>Percent of GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banking, Credit Intermediation and related activity</td>
<td>3.2%</td>
</tr>
<tr>
<td>Securities, Commodity contracts, and investments</td>
<td>1.7%</td>
</tr>
<tr>
<td>Funds, Trusts and other Financial Vehicles</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

Table 5 reports that the proportion of 2003 GDP used by the financial sector was 5.1%. The table does not cover items such as (i) cost of Federal and State regulatory agencies, (ii) private cost of litigation related to borrowing and default, (iii) cost of monitoring by accounting firms required by loan covenants and default contracting. The question is then: what fraction of this 5.1% would we still need to spend if there was no aggregate risk in our economy? We do not answer this question here but believe a large proportion of this activity would not be needed and hence represents real cost of aggregate risk. Even if we assume that only 1%-2% of this 5.1% is real cost of aggregate risk and add it to the 4% estimated before, we find the order of magnitude of the cost of aggregate risk exceeds 5% of GDP. We hope to make more precise estimates in future research.

5. Concluding Comment: on the Role of Stabilization Policy

All measures of real aggregate fluctuations in the U.S. has been lower in the post war period than in the interwar period and this increased stability is the main reason for the observation that the
ex-post cost of volatility is low. In this paper we have shown that this stability is purchased at a very high ex-ante social cost of aggregate fluctuations. Moreover, all available household data suggests the volatility of individual household income and consumption is very high. We would conjecture that it is likely that the increased stability of the aggregates have not been matched by an ex-post increased stability of household consumption. This may be analogous to the observations of Campbell, Lettau, Malkiel and Xu (2001) that the volatility of individual stock returns increased vs. the increased stability of the aggregate market. But then, if individual ex-ante cost of volatility is high, what is the role of stabilization policy and how can it contribute to improved social welfare?

We argue that aggregate volatility is a public “bad” and that stabilization policy can mitigated it by offering important insurance services as public goods. This complements the traditional view of stabilization policy as one designed to bolster the economy’s output to attain its potential level by providing adequate demand. The insurance perspective of stabilization, offered here, suggests that without public policy each agent selects optimal private resources to be used for self insurance against aggregate volatility. Agents with substantial liquid assets can protect themselves better than those without such holdings. Effective stabilization policy can reduces the private cost of self insurance and hence it is an efficient way to save private resources. An important channel for stabilization is the reduction in the volatility of prices as seen in this paper. Such policy is then a public good from which all benefit, particularly agents who do not have the assets needed for self insurance. Due to limited individual liquid asset holdings, public stabilization policy can attain higher level of benefits than attainable in a free market with the existing, unequal, distribution of wealth.

References


**APPENDIX**

Construction of the Random Variables $\eta_{t+1}^j(g_t^j)$

The variables $\eta_{t+1}^j(g_t^j)$ are our tools to enable agents to exhibit subjective beliefs with “fat” tails reflecting over confidence. We define $\eta_{t+1}^j(g_t^j)$ by specifying its density, conditional on $g_t^j$:

\[
p(\eta_{t+1}^j|g_t^j) = \begin{cases} 
\psi_1(g_t^j)\Phi(\eta_{t+1}^j) & \text{if } \eta_{t+1}^j \geq 0 \\
\psi_2(g_t^j)\Phi(\eta_{t+1}^j) & \text{if } \eta_{t+1}^j < 0 
\end{cases}
\]

where $\eta_{t+1}^j$ and $\tilde{g}_{t+1}^j$ (in (22)) are independent and

\[
\Phi(\eta) = \frac{1}{\sqrt{2\pi}}e^{-\frac{\eta^2}{2}}.
\]
(ψ₁(g^j), ψ₂(g^j)) are specified later. To show the agent can hold such a belief and be rational see Appendix A of Kurz, Jin and Motolese (2003b). A sufficient condition requires that if G(g^j) is the empirical density of g^j then we need to ensure that

\[ \int p(\eta^j | g^j)G(g^j)dg^j = \Phi(\eta^j), \] a Normal density of \( \eta^j \).

(A3) follows from (A1) and by the two conditions

\[ \int \psi_1(g^j)G(g^j)dg^j = 1 \text{ and } \int \psi_2(g^j)G(g^j)dg^j = 1. \]

To explain, since the empirical distribution of g^j is normal, averaging over g^j generates a random variable \( \bar{\eta}_{t+1}^j \) such that \( \bar{\eta}_{t+1}^j \sim N(0, 1) \). Averaged components of \( \Psi_{t+1}(g_t^j) \) (i.e. \( \lambda_t(\cdot)\bar{\eta}_{t+1}^j + \bar{u}_{t+1}^j \)) are then normally distributed with mean 0 and variance determined by the rationality conditions.

As for interpreting \( \eta_{t+1}^j(g_t^j) \), the functions \( \psi_i(g^j) \) i = 1, 2 allow agents to construct different probabilities of \( \eta_{t+1}^j \) being positive or negative, conditional upon \( u_t^j \): \( \psi_1(g^j) \) is rising with \( g^j \) towards 2 and \( \psi_2(g^j) \) is declining with \( g^j \) towards 0. When they converge rapidly to their asymptotic values the densities take the form

(i) For large positive \( g^j \) the mean value of the density becomes positive since

\[ p(\eta_{t+1}^j | g^j \text{ very large}) \approx \begin{cases} 2\Phi(\eta_{t+1}^j) & \text{for } \eta_{t+1}^j > 0 \\ 0 & \text{for } \eta_{t+1}^j < 0 \end{cases} \]

(ii) For large negative \( g^j \) the mean value of the density becomes negative since

\[ p(\eta_{t+1}^j | g^j \text{ very small}) \approx \begin{cases} 0 & \text{for } \eta_{t+1}^j > 0 \\ 2\Phi(\eta_{t+1}^j) & \text{for } \eta_{t+1}^j < 0 \end{cases} \]

As a practical approximation we have selected the function

\[ \psi(g^j) = \frac{1}{1 + e^{bg^j}}, \quad B = \int \psi(g^j)G(g^j)dg^j = \frac{1}{2}, \quad \psi_1(g) = 2\psi(g). \]

Direct calculations reveal the three empirical moments of the random variable \( \eta^j \) under discussion

\[ E[\eta^j] = 0, \quad E[(\eta^j)^2] = 1, \quad E[\eta^j g^j] = \frac{4}{\sqrt{2\pi}} E[\psi(g^j)g^j]. \]

The parameter \( b \) is the over confidence parameter as it specifies the degree of fat tails in the unconditional distribution of \( \eta_{t+1}^j(g_t^j) \). It actually regulates the speed at which the random variable \( \eta_{t+1}^j \) fluctuates, over time, from positive to negative values.
FIGURE 1: $\Pi_K(P)$