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**Hyperbolic Discounting
and
Uniform Savings Floors**

By
Benjamin A. Malin
Stanford University

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Stanford Institute for Economic Policy Research
Stanford University
Stanford, CA 94305
(650) 725-1874

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Hyperbolic Discounting and Uniform Savings Floors

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Abstract

I develop a general equilibrium model populated by agents with varying degrees of hyperbolic discounting who vote for a uniform savings floor. Although partial equilibrium intuition suggests that all individuals will prefer to have some constraint on their consumption/savings decision, I find that even the smallest amount of heterogeneity in preferences leads to very large differences in preferred policies. In fact, policy preferences are extreme: each individual either prefers having no floor imposed on the population or having a floor so high that it eliminates all borrowing and lending. I demonstrate that both endogenously determined prices and dynamically inconsistent preferences are necessary for this result. Finally, I consider how the equilibrium savings floor depends on the average amount of self-control in the population.

Keywords: Hyperbolic Discounting, General Equilibrium, Commitment, Voting

JEL: E21, H0, H4, H55

*Department of Economics, Stanford University, 579 Serra Mall, Stanford, CA 94305 (email: bmalin@stanford.edu). I would like to thank Manuel Amador, Doug Bernheim, Bob Hall, Pete Klenow, Soo Lee, Antonio Rangel, Felix Reichling, Mark Wright, and especially Narayana Kocherlakota for helpful discussions throughout the course of this project. The financial support of the Kohlhausen Fellowship Fund at Stanford University is gratefully acknowledged.

1 Introduction

One common explanation for the public support of government involvement in providing retirement income is that people realize they will not save adequately on their own, and thus, they prefer to have a savings floor imposed on them. Modelling such preferences requires a deviation from the standard economic model; for example, assuming that individuals are hyperbolic rather than exponential discounters.

George Akerlof (1998) articulates this line of thought succinctly: "the hyperbolic model explains the uniform popularity of social security, which acts as a pre-commitment device to redistribute consumption from times when people would be tempted to overspend – during their working lives – to times when they would otherwise be spending too little – in retirement. ... The hyperbolic discounting model explains, as the standard model will not, why the young as well as the old should be particularly enthusiastic about social security" (p. 187).

This reasoning is based upon a hyperbolic discounting literature that focuses on an individual's decision problem given exogenously fixed prices (Laibson et al. (1998), Amador et al. (2003)) or on a representative agent¹ in a general equilibrium setting (Laibson (1997)). In these models, the agent will value commitment devices that restrict her ability to take certain actions: examples of commitment devices include minimum savings requirements (Amador et al.) and the restriction of financial market innovation (Laibson (1997)). Importantly, the use of the commitment device by a particular agent either does not affect the welfare of other agents (since prices are fixed in the decision-theoretic analysis) or affects all agents uniformly (by construction in the representative agent setting).

A state-mandated retirement savings program, however, is fundamentally different from the policies in the models just described in that it is a *public* commitment device. This device affects all members of society, not just those that suffer from self-control and realize they would benefit from constraint. Moreover, it may affect each member differently. Thus, analyzing its desirability in a general equilibrium framework with heterogeneous agents seems appropriate.

¹The representative agent is a stand-in for a continuum of identical agents.

Modern public retirement savings programs have many complex features: some are pay-as-you-go, others fully-funded, and most redistribute wealth across generations. A common feature, however, is that they redistribute resources across time for any particular individual. From an individual's perspective, a state-mandated retirement savings program reduces disposable income when she is young and ensures a minimum amount of consumption in old age; it is a savings floor. Thus, to focus on the role of a legislated retirement savings program as a commitment device, I will model it as a uniform savings floor. "Uniform" means that the savings floor applies to all individuals in the population in the same way. In my model, conditional on their income level, individuals face the same floor.

In this paper, I study the desirability of a uniform savings floor for mitigating self-control problems of individuals. I consider an endowment economy populated by individuals with varying degrees of hyperbolic discounting who vote for the savings floor.² My main finding is that even if all individuals have large biases for present consumption, a small amount of variation across individuals in the strength of the bias will cause vast differences in preferences for public commitment. The policy preferences are extreme: each individual either prefers having no savings floor imposed or having a floor so high that it eliminates all borrowing and lending.

The intuition behind this result is as follows. Consider an economy without a savings floor (i.e., no constraints on borrowing). *Ceteris paribus*, individuals who have relatively less present-bias will be net savers. The introduction of a savings floor will reduce the aggregate demand for loans, the interest rate, and thus, the wealth of savers. This negative wealth effect will counter any welfare gains from the mitigation of the time-inconsistency for these individuals. On the other hand, individuals who are originally borrowers will benefit from a positive wealth effect of the constraint and directly from the constraint itself.³

I demonstrate that the extreme-preferences result depends crucially on both endoge-

²The focus on an endowment economy in this paper is only for ease of exposition. It can be shown that all results of the model hold in a production economy as well.

³There is also a substitution effect that accompanies the changing interest rate. It will negatively affect both savers and (unconstrained) borrowers. This will be discussed in more detail in Section 3.

nously determined prices and dynamically inconsistent preferences. That is, the result does not hold in a setting in which individuals think prices are fixed; nor does it hold if individuals have perfect self-control. After characterizing individual preferences, I describe how the equilibrium savings floor is chosen and consider some comparative statics with respect to the average amount of self-control of the population. This paper will focus solely on analytical results as they most clearly show the economics at work.⁴

Adding heterogeneity in time-inconsistency seems like a natural way to enrich models of hyperbolic discounting preferences for a number of reasons. First, the experimental studies from the psychology literature that suggest discount functions are approximately hyperbolic also provide evidence of heterogeneity (Kirby and Herrnstein (1995)). In recent work, Ameriks and coauthors (2004) document differences across individuals in their level of self-control and find a link between self-control and wealth accumulation. Further evidence of heterogeneity of discount factors comes from the enormous dispersion in the accumulated wealth of families approaching retirement, even after conditioning on lifetime earnings, lifetime financial resources, and portfolio choice (Venti and Wise (2000)).⁵ Finally, adding heterogeneity is a simple way to allow for general equilibrium price effects while departing from the representative agent framework.

To my knowledge, the only other paper which analyzes the welfare implications of commitment devices in a general equilibrium environment that does not admit a representative agent is Ludmer (2004). He considers hyperbolic discounting agents who are ex-ante identical and trade a limited supply of illiquid assets. He shows that the availability of illiquid assets will generally not lead to welfare improvements and also describes asset prices and the portfolio holdings of agents. The welfare results in his model arise through changes in equilibrium asset prices in a manner similar to the interest-rate channel described above. My analysis differs from his in at least two ways.

⁴It can be shown that some assumptions necessary for closed-form expressions, namely Assumptions 1 and 2, can be relaxed and the qualitative results of interest still shown to hold by solving the model numerically.

⁵Venti and Wise (2000) do not test whether this heterogeneity is in the long-run discount factor or the time-inconsistency parameter, but the point to be made is that heterogeneity in preferences appears significant. In Section 6, I consider heterogeneity of both types in the model.

First, in my model *all* individuals can make use of the uniform savings floor, whereas only a fraction of agents in his model can utilize the illiquid assets. Second, the savings floor is endogenously determined, while the supply of illiquid assets is exogenously set.

The rest of the paper is organized as follows. Section 2 lays out the basic model and derives equilibrium conditions, while individual preferences regarding a uniform savings floor are characterized in Section 3. Section 4 illustrates the importance of general equilibrium, and Section 5 isolates the role of dynamically inconsistent preferences in the analysis. In Section 6, I generalize the extreme-preference result of Section 3 by considering richer forms of agent heterogeneity; specifically, agents differ in their patience (i.e., long-run discount factor) and self-control (i.e., present-bias). Section 7 analyzes how the equilibrium savings floor is affected by changes in the distribution of types and describes one counterintuitive finding. Section 8 concludes, and an Appendix collects some proofs.

2 Basic Model

In this section, I set up the model, describe the agents' decision problems, and derive conditions that must hold in equilibrium. In Section 3, these equilibrium conditions are used to characterize agents' preferences regarding a uniform savings floor.

2.1 Environment

There are three periods in the economy, $t = 0, 1, 2$. Period 0 is a policy-setting date only. In periods 1 and 2, agents get their endowments, trade with each other, and consume. There is a unit measure of agents, who differ only by their time-inconsistency parameter, β_i , and a type- β_i agent has preferences represented by the following utility functions:

$$\textit{Period 0} : u(c_1) + u(c_2)$$

$$\textit{Period 1} : u(c_1) + \beta_i u(c_2)$$

$$\textit{Period 2} : u(c_2)$$

where c_t is the agent's consumption in period t . I assume $u', -u'' > 0$, $\lim_{c \rightarrow 0} u'(c) = +\infty$, and $\lim_{c \rightarrow \infty} u'(c) = 0$. The distribution of agents is represented by a density $f(\beta_i)$ with c.d.f. $F(\beta_i)$ over the interval $B = [\underline{\beta}, \bar{\beta}]$, $\underline{\beta} > 0$, $\bar{\beta} \leq 1$, and all agents are endowed with $w_t > 0$ units of the consumption good in periods $t = 1, 2$. $(1 - \beta_i)$ is the *discrepancy* between period-0 and period-1 preferences for an agent of type β_i .

2.2 Decision Problems

Agents have two decisions: a period-0 vote and a period-1 consumption-savings decision. Let q denote the price in period 1 of a claim to consumption in period 2, and let ϕ be a savings floor (i.e., borrowing constraint). In period 1, given q and ϕ , each agent chooses savings, A_1 , to solve

$$\begin{aligned} \max_{A_1} & \left\{ u(w_1 - A_1) + \beta_i u \left(\frac{A_1}{q} + w_2 \right) \right\} \\ \text{s.t.} & \quad A_1 \geq \phi \end{aligned}$$

Define $A_1^*(\beta_i; \phi, q)$ to be the solution of this problem.

In period 0, agents vote (as-if-pivotal) over the period-1 savings floor. That is, each agent votes for the ϕ that solves

$$\begin{aligned} \max_{\phi} & \left\{ u(w_1 - A_1^*(\beta_i; \phi, q(\phi))) + u \left(\frac{A_1^*(\beta_i; \phi, q(\phi))}{q(\phi)} + w_2 \right) \right\} \\ \text{s.t.} & \quad \phi \in [\underline{\phi}, 0] \end{aligned}$$

where $\underline{\phi} \equiv -q(\underline{\phi})w_2$ is the natural borrowing limit.⁶ Define $\phi^*(\beta_i)$ to be the solution of this problem.

A few comments are in order concerning the period-0 voting problem. First, note that agents consider how the savings floor will impact the equilibrium price, $q(\phi)$. The equilibrium interest rate, $\frac{1}{q(\phi)}$, must adjust so that the supply of loans equals the quantity demanded by borrowers in period 1, and agents take this into account when they vote. Second, the savings floor cannot be greater than 0, because in an endowment economy with no storage technology, markets would not clear if it were. In a production economy, however, a positive savings floor is compatible with market clearing, and the main results of this paper still hold.

⁶Given Assumption 1 (to follow), there will exist a unique $\underline{\phi}$.

2.3 Equilibrium Conditions

Necessary conditions for an equilibrium include constrained optimality of individual choices and market clearing. Thus, given ϕ and q , equilibrium allocations must satisfy

$$u'(w_1 - A_1^*) \geq \frac{\beta_i}{q} u' \left(\frac{A_1^*}{q} + w_2 \right) \quad (1)$$

$$A_1^*(\beta_i; \phi, q) \geq \phi \quad (2)$$

where for constrained agents, (1) holds with strict inequality and (2) with equality, and for unconstrained agents, (1) holds with equality. At the equilibrium interest rate, markets must also clear. That is,

$$\int_{\underline{\beta}}^{\bar{\beta}} A_1^*(\beta_i; \phi) f(\beta_i) d\beta_i = F(\beta_C) \phi + \int_{\beta_C}^{\bar{\beta}} A_1^*(\beta_i; \phi) f(\beta_i) d\beta_i = 0 \quad (3)$$

where β_C solves

$$u'(w_1 - \phi) = \frac{\beta_C}{q(\phi)} u' \left(\frac{\phi}{q(\phi)} + w_2 \right) \quad (4)$$

Equations (1) – (4) jointly determine $A_1^*(\beta_i; \phi) \forall \beta_i$ and $q(\phi)$ for any ϕ . β_C is the cut-off between constrained and unconstrained individuals. Agents of type β_C optimally choose to save ϕ ; $\beta_i > \beta_C$ are unconstrained; and $\beta_i < \beta_C$ are constrained. As I am primarily interested in characterizing individual preferences over ϕ , it is not yet necessary to specify the procedure by which individual policy preferences are aggregated to determine the savings floor for the economy. In Section 6, the policy selection process will be made explicit.

To rule out multiple equilibria, I assume that the equilibrium interest rate, $\frac{1}{q(\phi)}$, is a function and that the following assumption holds:

Assumption 1 The interest rate, $\frac{1}{q(\phi)}$, is a (weakly) decreasing function of the savings floor. (i.e., $\frac{dq}{d\phi} \geq 0$)

The intuition for Assumption 1 is as follows: if the savings floor constrains some agents, an increase in the savings floor will shift the aggregate demand curve for loans to the left, and the interest rate will fall to equilibrate the loan market. Of course,

if the savings floor is not a binding constraint for any agent, the equilibrium interest rate will not change. Sufficient conditions⁷ for Assumption 1 to hold include

1. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma \leq 1$, or
2. $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, with $\sigma \leq 2$ and $w_1 = w_2$.

3 Preferences for Public Commitment

I now characterize the period-0 preferences of individuals for a public commitment device, namely ϕ , the period-1 savings floor. The main result is that all individuals prefer an extreme policy but not the same extreme. Given *any* nondegenerate distribution of types, some prefer having no floor imposed and others prefer a floor high enough to eliminate all borrowing and lending. This result stands in stark contrast to the conventional wisdom that ubiquitous lack of self-control leads to a consensus opinion on the desirability of a government-mandated savings floor.

Let $U(\beta_i; \phi)$ denote the period-0 indirect utility of an agent of type β_i when the savings floor is ϕ :

$$U(\beta_i; \phi) \equiv u[w_1 - A_1^*(\beta_i; \phi)] + u \left[\frac{A_1^*(\beta_i; \phi)}{q(\phi)} + w_2 \right]$$

Changes in ϕ can affect $U(\beta_i; \phi)$ through three channels. The first is the direct effect of constrained borrowing. The second and third are the substitution and wealth effects associated with a change in the equilibrium interest rate. I characterize $U(\beta_i, \phi)$ in a series of lemmas to pinpoint the channels through which changes in ϕ are operating. As ϕ changes, the entire equilibrium changes to be consistent with equations (1) - (4).

3.1 Regions of the Policy Space

For a given level of the savings floor, some individuals will be constrained and others unconstrained. The first lemma is that if an agent is constrained for some level of the

⁷I have not been able to establish necessary conditions for Assumption 1 to hold, although quantitative solutions of numerous parameterizations of the model have not revealed any cases in which the assumption is violated. Given the few restrictions imposed on the distribution of types and the functional form of utility, however, it is possible that a counterexample could be constructed.

savings floor, she will also be constrained by any higher savings floor.

Lemma 1 An increase in the savings floor, ϕ , must increase the cut-off between constrained and unconstrained agents, β_C .

Proof Assume ϕ increases. The left-hand side of (4) must strictly increase. On the right-hand side, note that $\frac{d\left(\frac{\phi}{q}\right)}{d\phi} = \frac{1}{q} - \frac{\phi}{q^2} \frac{dq}{d\phi} > 0$, so $u'\left(\frac{\phi}{q} + w_2\right)$ must strictly decrease. By Assumption 1, $\frac{1}{q}$ is also decreasing. Therefore, β_C must increase for (4) to hold.

QED

Another classification of individuals in this economy is into groups of savers and borrowers. The second lemma implies that if an individual is a borrower for some savings floor, then in an equilibrium with a higher savings floor, the individual will not be a saver. For a constrained agent, this follows immediately from Lemma 1 and $\phi < 0$.

Lemma 2 If an agent is an unconstrained borrower for some ϕ , she will not be a saver for any higher savings floor.

Proof I show that $\frac{\partial A_1^*}{\partial \phi}|_{A_1^* \leq 0} \leq 0$. $A_1^*(\beta)$ is implicitly defined by equation (1). Using the implicit function theorem yields

$$\frac{dA_1^*(\beta)}{dq} = -\frac{u'(c_1^*) + \beta u''(c_2^*) \frac{A_1^*}{q^2}}{-qu''(c_1^*) - \frac{\beta}{q} u''(c_2^*)}. \quad (5)$$

Then, note that $u'(c_1^*) = \frac{\beta}{q} u'(c_2^*)$ implies

$$\frac{dA_1^*(\beta)}{dq} < 0 \iff A_1^* < \frac{-qu'(c_2^*)}{u''(c_2^*)} \quad (6)$$

Finally, note that $\frac{-qu'(c_2^*)}{u''(c_2^*)} > 0$ and $\frac{dq}{d\phi} \geq 0$. Thus, $\frac{\partial A_1^*}{\partial \phi}|_{A_1^* \leq 0} \leq 0$. **QED**

The intuition for Lemma 2 is as follows: an unconstrained individual will not be affected by the constraint directly but will be affected by the decreased equilibrium interest rate. The substitution effect causes her to borrow more, and since she is

already a borrower, the wealth effect is positive and reinforces the desire to borrow more.

Figure 1 illustrates the implications of Lemmas 1 and 2 for a given agent. Start with the highest savings floor, $\phi = 0$. As the floor is lowered, the individual moves from being constrained to unconstrained, and as the interest rate rises, she possibly moves from being a net borrower to a net saver.

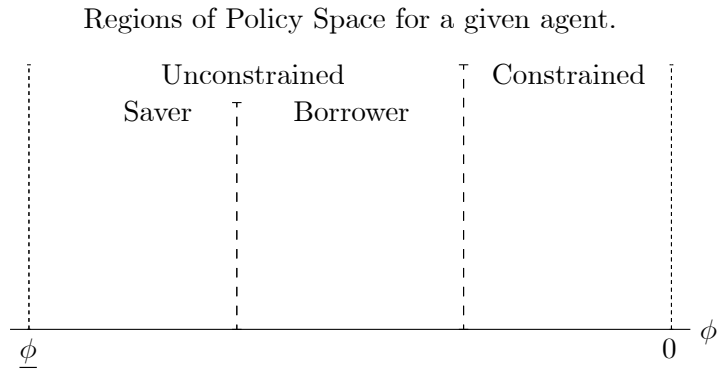


Figure 1

A few remarks about Figure 1 need to be made. First, some agents may never be borrowing constrained, even for the highest possible savings floor. For example, the individual with the most self-control in the economy will always have non-negative savings as she will have less desire to consume in period 1 than other individuals. Other individuals (consider $\underline{\beta}$) will never be savers, even for the lowest savings floor.

3.2 The Indirect Utility Function

I now proceed to characterize the shape of a given agent's indirect utility function over each region of the policy space. I move from the left to the right of Figure 1: first considering the unconstrained region and then the constrained region. Figure 2 will show the shape of the indirect utility function for any given agent.

For sufficiently low values of ϕ , no agents will be constrained (i.e., $F(\beta_C) = 0$). Thus, a small increase in the savings floor will have no impact on equilibrium quantities nor on the utility of the agents. This is shown by the flat portion of the indirect utility function in Figure 2. Figure 2 also graphically depicts Lemmas 3 and 4, which

characterize the preferences of unconstrained and constrained agents, over the savings floor when some agents are constrained (i.e., $F(\beta_C) \neq 0$).

Before stating and proving Lemma 3, one more assumption is needed.

Assumption 2 The price elasticity of savings is decreasing over a particular region of savings. Letting $\epsilon \equiv \frac{\frac{dA_1^*}{dq}}{\frac{A_1^*}{q}}$, I assume $\frac{d\epsilon}{dq} \leq 0$ for $\frac{u'(c_1^*)}{u''(c_1^*)} < A_1^* < 0$.

Remarks: This assumption is used only once in the proof of Lemma 3, and below, I will discuss its role in detail. I would prefer to make an assumption directly on agents' preferences but have not been able to map Assumption 2 into a more primitive form. A sufficient condition for the assumption to hold, however, is that $u(c) = \ln(c)$.

Lemma 3 $U(\beta; \phi)$ is quasiconvex in ϕ over the unconstrained region of the policy space.

Proof See the Appendix.

The intuition for Lemma 3 comes from the expression for the slope of the indirect utility function⁸:

$$\frac{dU(\beta; \phi)}{d\phi} = \left[u'(c_2^*) \left(-\frac{A_1^*}{q^2} \right) + \left(\frac{1-\beta}{\beta} \right) u'(c_1^*) \frac{dA_1^*}{dq} \right] \frac{dq}{d\phi} \quad (7)$$

First, recall Assumption 1: an increase in the savings floor causes the interest rate to decrease, i.e., $\frac{dq}{d\phi} \geq 0$. Next, the second additive term within the brackets is negative because an unconstrained agent will save less as the interest rate decreases, i.e., $\frac{dA_1^*}{dq} < 0$. This is exactly the time-inconsistency problem. The larger the discrepancy between period-0 and period-1 preferences – that is, the larger is $(1-\beta)$ – the greater the utility cost. Note that in the usual case of time-consistent preferences, $\beta = 1$, the envelope condition applies, and the substitution effect associated with $\frac{dA_1^*}{dq}$ is negligible.

The first additive term in equation (7) can be positive or negative depending on the sign of A_1^* . If the agent is a lender, the decrease in the interest rate decreases wealth and utility.

⁸This intuition is for $\frac{u'(c_1^*)}{u''(c_1^*)} \leq A_1^* \leq \frac{-qu'(c_2^*)}{u''(c_2^*)}$. If A_1^* lies below this interval, utility increases with a decreasing interest rate, and if A_1^* lies above this interval, utility decreases with a decreasing interest rate.

If the agent is a borrower, however, utility can increase due to the positive wealth effect. Thus, a borrower views the decreased interest rate as a mixed blessing. On the one hand, the agent becomes wealthier because she can borrow the same amount at a lower price, but on the other hand, this reduced price exacerbates the agent's commitment problem.

The magnitude of the price elasticity of savings determines which force is more important for the borrower. If the price elasticity of savings is large, $\epsilon > \frac{1}{1-\beta}$, the borrower's time-inconsistency will dominate the wealth effect, and she will suffer from the decrease in the equilibrium interest rate. Assumption 2 ensures that the price elasticity of savings decreases with a decrease in the interest rate, so that for tighter borrowing constraints, indirect utility may be increasing. This delivers the quasiconvexity of the indirect utility function over the unconstrained region of the policy space.

Having characterized the policy preferences of an unconstrained individual, Lemma 4 describes how the indirect utility of a constrained agent changes with an increase in the savings floor.

Lemma 4 Period-0 utility is increasing over the constrained region of the policy space.

Proof Recall $\beta_{con} < \beta_C \leq \beta_{unc}$.

$$\frac{dU(\beta_{con}; \phi)}{d\phi} = u'(w_1 - \phi)(-1) + u' \left(\frac{\phi}{q} + w_2 \right) \left(\frac{1}{q} - \frac{\phi}{q^2} \frac{dq}{d\phi} \right) \quad (8)$$

For an unconstrained agent, $u'(w_1 - A_1^*) = \frac{\beta_{unc}}{q} u' \left(\frac{A_1^*}{q} + w_2 \right)$ and $A_1^* > \phi$. Thus, $u'(w_1 - \phi) < \frac{\beta_{unc}}{q} u' \left(\frac{\phi}{q} + w_2 \right) \leq \frac{1}{q} u' \left(\frac{\phi}{q} + w_2 \right)$. Rewriting (8), it is easy to see that

$$\frac{dU(\beta_{con}; \phi)}{d\phi} = \left[\frac{1}{q} u' \left(\frac{\phi}{q} + w_2 \right) - u'(w_1 - \phi) \right] - \frac{\phi}{q^2} \frac{dq}{d\phi} u' \left(\frac{\phi}{q} + w_2 \right) > 0 \quad (9)$$

QED⁹

⁹Note that the proof relies on at least one type of agent being unconstrained, but this is kosher. If all agents are constrained, $\phi = 0$ (by market clearing). In other words, in all cases where $\phi < 0$, some agent is unconstrained and the proof goes through.

The interpretation of Lemma 4 is as follows: an increase in the savings floor directly benefits agents who are borrowing constrained through giving them more commitment. This is reflected by the term in square brackets in (9). The term outside of the brackets shows that a decrease in the interest rate also makes these agents better off because they are borrowers, i.e., $\phi < 0$.

One might at first be surprised by Lemma 4. As ϕ increases, the equilibrium interest rate is decreasing. It may seem that for low enough values of the interest rate, a tighter constraint cannot possibly be welfare improving. In equilibrium, however, the interest rate will never get too low. It is bounded below by the marginal rate of substitution of agent $\bar{\beta}$ in an autarkic equilibrium.

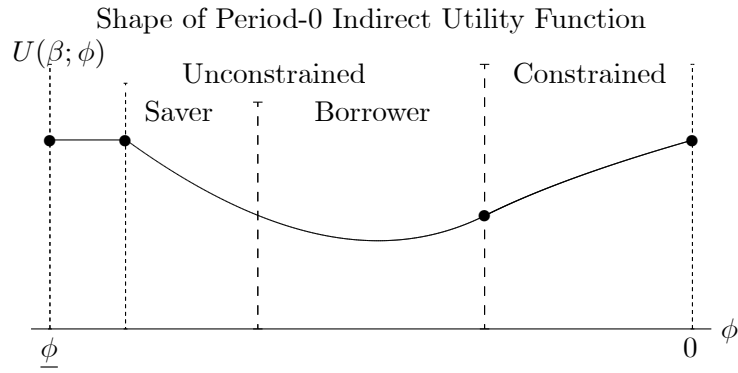


Figure 2

3.3 Extreme Preferences

The lemmas of the previous two sections described the policy preferences of any particular agent. For the entire distribution of agents, a two-part "Extreme Preferences" result emerges. All individuals prefer an extreme policy but not the same extreme: some individuals prefer having no savings floor imposed while others prefer a floor so high that it eliminates all borrowing and lending. First, the "all" part is shown by Proposition 1 and Figure 3.

Proposition 1 Each individual's preferred policy is either (effectively) no savings floor, $\underline{\phi}$, or the maximum savings floor, $\bar{\phi} = 0$.

Proof It follows directly from Lemmas 1-4 that $U(\beta_i; \phi)$ is quasiconvex in ϕ , $\forall \beta_i$.
QED

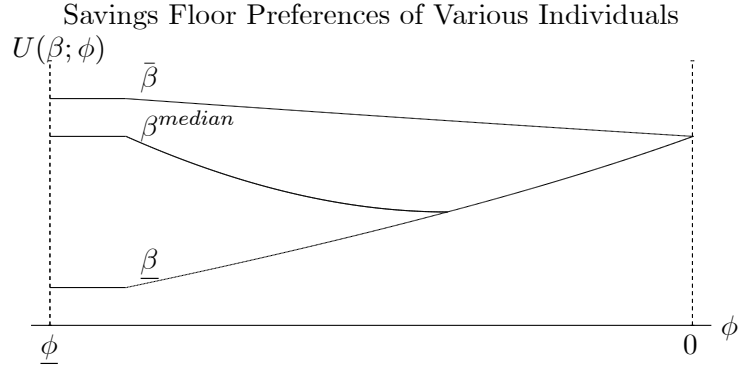


Figure 3

The "not the same" part of the result is shown by Proposition 2 and Figure 4. Recall that $\phi^*(\beta)$ denotes the preferred savings floor of a type β agent.

Proposition 2 There exists an individual $\tilde{\beta} \in (\underline{\beta}, \bar{\beta})$ who is indifferent between $\underline{\phi}$ and $\bar{\phi}$. For $\beta < \tilde{\beta}$, $\phi^* = \bar{\phi}$. For $\beta > \tilde{\beta}$, $\phi^* = \underline{\phi}$.

Proof See the Appendix.

Figure 4 shows that individuals with more of a time-inconsistency problem (lower β) prefer $\phi = 0$, while those whose preferences change relatively less over time prefer $\phi = \underline{\phi}$. In this particular setting, individuals who are net savers in the no-savings-floor equilibrium are the ones who would have a decrease in utility by moving to an equilibrium with a high savings floor.

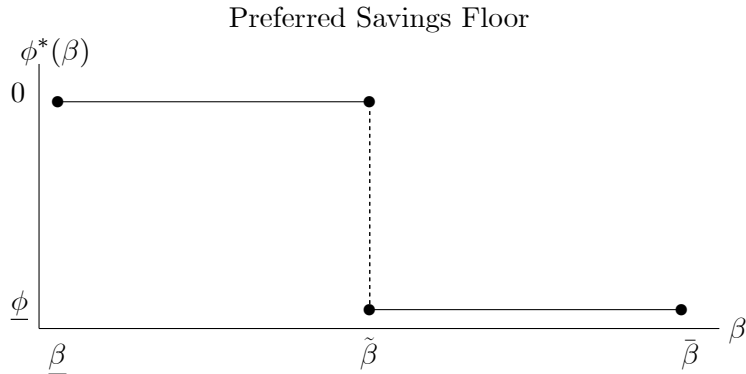


Figure 4

4 Equilibrium Prices vs. Fixed Prices

Allowing for the interest rate to change with movements in the savings floor is crucial for the extreme-preferences result of the previous section. To demonstrate this, I consider a fixed price version of the model in which the interest rate is no longer pinned down by market clearing. Instead, it is a constant that does not depend on the equilibrium savings floor. Equations (1) and (2) still characterize the consumption/savings decision of period-1 individuals.

Let $U^{PE}(\beta_i; \phi)$ denote the period-0 indirect utility of an agent of type β_i when the savings floor is ϕ :

$$U^{PE}(\beta_i; \phi) \equiv u[w_1 - A_1^*(\beta_i; \phi)] + u\left[\frac{A_1^*(\beta_i; \phi)}{q} + w_2\right]. \quad (10)$$

As ϕ changes, it only impacts an agent's welfare through her savings, $A_1^*(\beta_i; \phi)$, and has no effect on the interest rate, $\frac{1}{q}$. Furthermore, it is apparent from equations (1) and (2) that savings only change if the savings floor is a binding constraint.

In this setting, a two-part result emerges that is the opposite of the previous section's two-part extreme-preferences result: all individuals prefer the same savings floor, and the preferred policy may not be an "extreme". The use of quotes reflects the fact that the extremes of the policy space are no longer defined by equilibrium conditions, and thus, they can be arbitrarily chosen. For comparison to the previous section, I will assume $\phi \in [-qw_2, 0]$.

Proposition 3 All individuals prefer the same savings floor.

Proof See the Appendix.

Because individuals are identical except for their time-inconsistency parameter and because the interest rate does not vary with the policy, it is easy to see from (10) that the optimal savings from the period-0 perspective is independent of β_i . Denote the optimal savings as A_1^0 . All individuals will prefer the savings floor in the policy space that is closest to A_1^0 . If this savings floor is a binding constraint on their period-1 savings decision, the individual strictly prefers it to other floors; if this savings floor does not bind, the individual is indifferent between this floor and others.

Proposition 4 The preferred policy may not be an extreme policy.

Proof See the Appendix.

Proposition 4 is most easily demonstrated by considering the case in which $w_1 = w_2 = w$. The optimal (from a period-0 perspective) savings satisfies

$$u'(w - A_1^0) = \frac{1}{q} u' \left(\frac{A_1^0}{q} + w \right).$$

If $\frac{1}{q} > 1$, then $A_1^0 > 0$ and the agents' preferred savings floor is an extreme, $\phi^* = 0$. If $\frac{1}{q} < 1$, then $A_1^0 < 0$ and the agents' preferred savings floor is an interior policy, $\phi^* = A_1^0$. Figures 5 and 6 depict individuals' policy preferences for these two cases.

Savings Floor Preferences: $A_1^0 > 0$

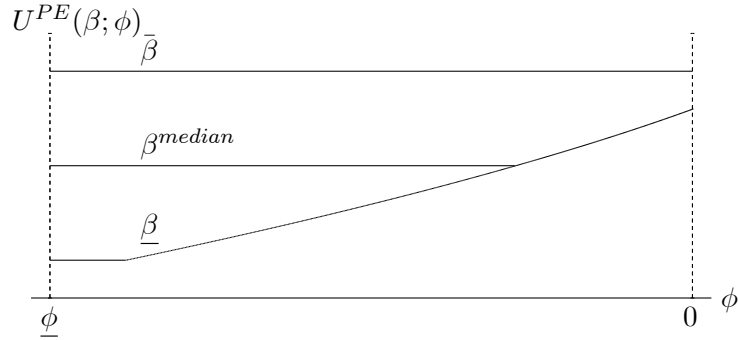


Figure 5

Savings Floor Preferences: $A_1^0 < 0$

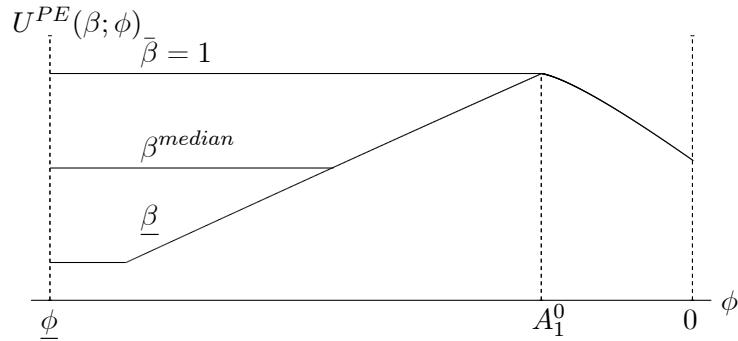


Figure 6

The partial equilibrium intuition alluded to in the introduction is easily seen in Figures 5 and 6. If prices are fixed, all individuals will reach a consensus regarding the optimal savings floor. In general equilibrium, however, a small bit of heterogeneity in individuals' need for commitment leads to big differences in policy preferences. Those agents with less self-control problems (high β) prefer to get their commitment from the high interest rate associated with a low savings floor rather than having a high floor.

5 Time-Inconsistency vs. Impatience

The extreme preferences result above differs from results in the time-inconsistency (i.e., hyperbolic discounting) literature partly because the interest rate changes with the equilibrium savings floor. The general equilibrium effects of a policy change, however, will occur even with standard, time-consistent preferences. To isolate the role of time-inconsistency for the result, I consider an environment identical in all ways to the one in Section 2 except that individuals now have time-consistent preferences.

Specifically, all individuals have perfect self-control, but some are more impatient than others. Preferences are represented by

$$\textit{Period} - 0 : u(c_1) + \delta_i u(c_2)$$

$$\textit{Period} - 1 : u(c_1) + \delta_i u(c_2)$$

$$\textit{Period} - 2 : u(c_2)$$

where the distribution of types is represented by a density $f(\delta)$ over the interval $D = [\underline{\delta}, \bar{\delta}]$, $\underline{\delta} > 0$, $\bar{\delta} \leq 1$.

5.1 Qualitative Features of $\phi^*(\delta)$

When agents had time-inconsistent preferences, I showed that an agent's most preferred uniform savings floor was an extreme. The analysis that follows shows that time-inconsistency of preferences is necessary for this extreme result.

Let $U(\delta; \phi) \equiv u[w_1 - A_1^*(\delta; \phi)] + \delta u \left[\frac{A_1^*(\delta; \phi)}{q(\phi)} + w_2 \right]$ be the time-0 indirect utility of an agent of type δ when the savings floor is ϕ . Thus,

$$\frac{dU(\delta; \phi)}{d\phi} = \left(\frac{\delta}{q} u'(c_2^*) - u'(c_1^*) \right) \frac{dA_1^*}{d\phi} - \delta u'(c_2^*) \frac{A_1^*}{q^2} \frac{dq}{d\phi} \quad (11)$$

Lemmas analogous to Lemmas 1 and 2 will hold in this environment as well, so that Figure 1 will continue to describe the regions of the policy space. Therefore, I characterize preferences first for an unconstrained individual (Lemma 5) and then for one who is constrained (Lemmas 6 and 7).

Lemma 5 $U(\delta; \phi)$ is quasiconvex in ϕ over the unconstrained region of the policy space.

Proof By the envelope theorem, the first term in (11) is equal to 0. Thus, $\frac{dU(\delta; \phi)}{d\phi}$ has the opposite sign of $A_1^*(\delta; \phi)$. Finally, by Lemma 2, as the interest rate decreases, a borrower will never become a saver. **QED**

For constrained individuals, Lemma 6 characterizes the slope of the indirect utility function at the point in the policy space when the individual becomes constrained, and Lemma 7 considers the slope as the constraint approaches its upper bound.

Lemma 6 For a "just"-constrained agent, $\frac{dU(\delta; \phi)}{d\phi}$ is positive.

Proof This follows very closely from the proof of Lemma 5. Note that for a "just-constrained" individual, the first term is to a first-order approximation 0, while the second term is strictly positive. **QED**

Lemma 7 For a constrained agent, as $\phi \rightarrow 0$, $\frac{dU(\delta; \phi)}{d\phi}$ is negative.

Proof As $\phi \rightarrow 0$, the second term in (11) goes to 0, while the first term is strictly negative. **QED**

In contrast to the extreme preferences result obtained with time-inconsistent preferences, Lemmas 5-7 establish that an individual with time-consistent preferences can have a moderate policy as his preferred policy. When individuals had time-inconsistent preferences, it was not possible for them to be overly constrained, but with standard preferences, the direct effect of a constraint is to decrease utility. There is a wealth effect that allows for a small amount of constraint to be welfare improving, but this effect is overwhelmed by the distortion of optimal choice that becomes more severe as the constraint increases. The extreme-preferences result, therefore, crucially depends on the presence of time-inconsistency in preferences.

6 Heterogeneity in Present-Bias and Patience

In Section 2, consumers differ only by the strength of their present-bias, β , whereas in Section 5, they differ only in their level of patience, δ . This section characterizes conditions under which the extreme preferences result holds for the more general case in which individuals are heterogenous in both dimensions. Unless otherwise noted, the model is identical to the one developed in earlier sections.

6.1 Environment

There is a unit measure of people, indexed by i , who are heterogeneous in their present-bias factor, β_i , and long-run discount factor, δ_i . They have preferences represented by the following utility functions

$$\textit{Period} - 0 \quad : \quad u(c_1) + \delta_i u(c_2)$$

$$\textit{Period} - 1 \quad : \quad u(c_1) + \beta_i \delta_i u(c_2)$$

$$\textit{Period} - 2 \quad : \quad u(c_2)$$

where $(\beta_i, \delta_i) \in [\underline{\beta}, \bar{\beta}] \times [\underline{\delta}, \bar{\delta}]$, $\underline{\beta}, \underline{\delta} > 0$, $\bar{\beta}, \bar{\delta} \leq 1$. Assume that β_i and δ_i are stochastically independent.

6.2 Policy Preferences

If there is variation in the present-bias factor across individuals but no heterogeneity in the long-run discount factor, Proposition 1 states that individuals most preferred policy will be an extreme. On the other hand, with heterogeneity only in the long-run discount factor, Lemmas 6 and 7 establish that an interior policy may be most preferred. The following proposition gives sufficient conditions for an extreme policy to be the most preferred policy of an individual.

Proposition 5 Individual i will prefer an extreme policy if $\frac{\delta_i}{\delta} \geq \bar{\beta}$.

Proof See the Appendix.

Individuals make a trade-off between gains from trade and gains from commitment when choosing the level of the uniform savings floor. Smaller gains from trade because individuals have similar preferences (i.e., less variation in δ) and a greater need for commitment (i.e., lower levels of $\bar{\beta}$) are associated with individuals preferring an extreme constraint.

For the special case of heterogeneity only in β , all individuals prefer an extreme policy. If, on the other hand, δ is the only form of variation, an extreme policy is guaranteed to be the equilibrium policy only if $\bar{\beta}$ is low enough and there is not much variation in δ .

7 Political Economic Comparative Statics

Having characterized individual preferences for a uniform savings floor, I now consider some comparative statics of the model. I return to the baseline model from Section 2 and consider how a shift in the distribution of types affects the savings floor selected in equilibrium. To do so, it becomes necessary to specify how individual preferences are aggregated to select the equilibrium policy.

Fortunately, a median voter theorem applies in this setting because, by Proposition 2, an individual's preferred savings floor is a monotonic function of her type. Since $\phi^*(\beta)$ is monotonic, I assume that $\phi^* = \phi^*(\beta_{median})$ ¹⁰ and evaluate how changes in the distribution of types, $F(\beta)$, affect ϕ^* . Restricting the class of utility functions to those that satisfy the first sufficient condition for Assumption 1, the following proposition shows that a decrease in the average amount of self-control in the economy can actually lead to less commitment in this environment.

Proposition 6 Any downward (in FOSD sense) shift in $F(\beta)$ that leaves β_{median} unchanged leads to a (weak) decrease in ϕ^* .

Proof A downward shift in the distribution leads to a decrease in q . To see this, note that if q stays constant, the left-hand side of equation (3) decreases because $\frac{dA_1^*}{d\beta} > 0$. An increase in q would decrease $A_1^*(\beta)$, $\forall \beta$ ¹¹. Thus, q must decrease in order for (3) to hold.

A decrease in q increases $U(\beta; \phi)$, $\forall \beta$. To see this note (following the proof of Lemma 3) that $\frac{dU(\beta; \phi)}{dq}$ has the same sign as $\frac{-A_1^*}{q} + (1 - \beta)\frac{dA_1^*}{dq}$. The second term is always negative, and the sign of the first term is negative if the agent is a saver. If the agent is a borrower, we need $\frac{\frac{dA_1^*}{dq}}{\frac{A_1^*}{q}} > \frac{1}{(1-\beta)}$, and this is true in the class of utility functions under consideration.

Therefore, $\tilde{\beta}$ decreases. If $\tilde{\beta}^{new} < \beta_{median} \leq \tilde{\beta}^{old}$,¹² ϕ^* decreases. Otherwise, ϕ^* remains the same. **QED**

Thus, a shift in the distribution of agents that lowers the average amount of personal commitment ability in the economy may also lead to policies that provide less commitment ability, not more as is suggested in many models of a representative agent with time-inconsistent preferences. If the average self-control of agents relative to the median voter is smaller, the median voter will benefit more from others' lack of constraint and may vote against public commitment.

¹⁰My results will still hold for any super-majority voting rule.

¹¹Either of the two sufficient conditions for Assumption 1 would have been sufficient for this step.

¹²I assume that agent of type $\tilde{\beta}$ votes for $\phi^* = 0$.

On the other hand, if the shift also changes the identity of the median agent to a lower type, ϕ^* may increase. The result will depend on the sign of $\tilde{\beta}^{new} - \beta_{median}^{new}$.

8 Conclusion

I extend the analysis of the economics of commitment to a general equilibrium setting in which individuals vote over a public commitment mechanism, namely a uniform savings floor. The model demonstrates that even if individuals have very small differences in their levels of self-control, they will have large differences in their preferred policies. Each individual either prefers no savings requirement or a savings requirement that forces everyone to save the same amount. This extreme result is driven by the endogenously determined interest rate and would not hold in models of private commitment devices or models that admit a representative agent.

It should be emphasized that the simplicity of the model (endowment economy with heterogeneity in preferences) was primarily for expositional purposes, but the extreme result will hold for a wider class of models. Specifically, it continues to hold in a production economy where individuals can transfer resources across periods in the form of productive capital. The main complications include redefining the policy space to allow for a positive minimum savings floor and considering the effect that changes in the capital stock have on wages. Another extension is to allow for alternative forms of heterogeneity, such as differences in the slope of the agents' endowment profiles. This is formally equivalent to the case of heterogeneity in the long-run discount factors discussed in sections 5 and 6.

Throughout the paper, I modelled the public commitment device as a uniform savings floor and suggested that government-mandated retirement savings programs essentially have this feature. This is, of course, a rough approximation. Although all U.S. workers have 12.4% of their earnings automatically saved for them through Social Security, it is well known that the payroll tax is regressive (because of the cap on taxable earnings) and the benefit schedule is progressive. This paper shows that even if we abstracted from the redistribution aspects of social security and focused

solely on its role as a commitment device, we still would not expect to see a consensus opinion on its desirability.

9 Appendix

9.1 Proof of Lemma 3

First, I need to establish some properties of $\frac{dA_1^*}{dq}$. Equations (5) and (6) imply that a decrease in the interest rate leads to an increase in period-1 consumption if and only if period-1 savings are not too high. Second, note by the implicit function theorem that

$$\frac{d\left(\frac{A_1^*(\beta)}{q}\right)}{dq} = \frac{\frac{dA_1^*}{dq}}{q} - \frac{A_1^*}{q^2} \quad (12)$$

$$= \frac{u'(c_1^*) - A_1^* u''(c_1^*)}{q^2 u''(c_1^*) + \beta u''(c_2^*)} \quad (13)$$

Thus,

$$\frac{d\left(\frac{A_1^*(\beta)}{q}\right)}{dq} < 0 \iff A_1^* > \frac{u'(c_1^*)}{u''(c_1^*)} \quad (14)$$

In words, a decrease in the interest rate leads to a decrease in period-2 consumption if and only if period-1 savings are not too low.

Now, I prove the lemma.

$$\frac{dU(\beta; \phi)}{d\phi} = \left[u'(w_1 - A_1^*)(-1) \frac{dA_1^*}{dq} + u'\left(\frac{A_1^*}{q} + w_2\right) \frac{1}{q} \left(\frac{dA_1^*}{dq} - \frac{A_1^*}{q} \right) \right] \frac{dq}{d\phi} \quad (15)$$

$$= \left[u'(c_2^*) \left(-\frac{A_1^*}{q^2}\right) + \left(\frac{1}{\beta} - 1\right) u'(c_1^*) \frac{dA_1^*}{dq} \right] \frac{dq}{d\phi}. \quad (16)$$

The second equality comes by substituting the first-order condition of the agent's period-1 decision problem into the expression and collecting like terms. Since we are interested in the sign of this expression, divide through by $u'(c_2^*) \frac{dq}{d\phi} > 0$ to get

$$\frac{dU(\beta; \phi)}{d\phi} = -\frac{A_1^*}{q^2} + \frac{u'(c_1^*)}{u'(c_2^*)} \left(\frac{1}{\beta} - 1\right) \frac{dA_1^*}{dq} \quad (17)$$

$$= -\frac{A_1^*}{q^2} + \frac{\beta}{q} \left(\frac{1}{\beta} - 1\right) \frac{dA_1^*}{dq}. \quad (18)$$

Again, the second equality comes from substituting in the first-order condition. (18) has the same sign as

$$\frac{-A_1^*}{q} + (1 - \beta) \frac{dA_1^*}{dq}. \quad (19)$$

If $A_1^*(\beta; \phi) > \frac{-qu'(c_2^*)}{u''(c_2^*)}$, $\frac{dU(\beta; \phi)}{d\phi}$ is negative as consumption in both periods decreases with an increase in q .

If $0 \leq A_1^*(\beta; \phi) \leq \frac{-qu'(c_2^*)}{u''(c_2^*)}$, $\frac{dU(\beta; \phi)}{d\phi}$ is negative as both terms in (19) are negative.

If $\frac{u'(c_1^*)}{u''(c_1^*)} < A_1^*(\beta; \phi) < 0$, the first term in (19) is positive. Thus, (19) is negative if and only if

$$\frac{\frac{dA_1^*}{dq}}{\frac{A_1^*}{q}} > \frac{1}{1 - \beta}. \quad (20)$$

Since $\frac{dA_1^*}{dq} = \frac{q^2 u'(c_1^*) + \beta u''(c_2^*)}{q^2 u''(c_1^*) + \beta u''(c_2^*)}$ is decreasing in q for $\frac{u'(c_1^*)}{u''(c_1^*)} < A_1^*(q) < 0$, there is a cutoff level, \tilde{A}_1^* , below which $\frac{dU(\beta; \phi)}{d\phi}$ is positive. Since $\frac{dA_1^*}{dq} < 0$ on this region, once $\frac{dU(\beta; \phi)}{d\phi}$ is positive for some ϕ , it will be positive for any greater ϕ .

Finally, if $A_1^*(\beta; \phi) \leq \frac{u'(c_1^*)}{u''(c_1^*)}$, $\frac{dU(\beta; \phi)}{d\phi}$ is positive as consumption in both periods increases with an increase in q . **QED**

9.2 Proof of Proposition 2

By Proposition 1, $\phi^*(\beta) \in \{\underline{\phi}, 0\}, \forall \beta$.

If $\phi = 0$, all agents have the same utility since they have identical endowments and period-0 preferences. Simply put,

$$U(\beta; 0) = u(w_1) + u(w_2) \equiv U(0), \forall \beta$$

If $\phi = \underline{\phi} \equiv -q(\underline{\phi})w_2$, all agents are unconstrained, and period-0 utility is denoted by $U(\beta; \underline{\phi})$. Furthermore, $U(\beta; \underline{\phi}) < U(0)$ since an agent of type $\underline{\beta}$ is the first to become

constrained, and hence, his utility is nondecreasing over the policy space. At the other extreme, $U(\bar{\beta}; \underline{\phi}) > U(0)$ since a type $\bar{\beta}$ agent will always be an unconstrained saver, and thus, her utility is nonincreasing over the policy space. Finally, $U(\beta; \underline{\phi})$ is strictly increasing in β .

$$\begin{aligned} \frac{dU(\beta; \underline{\phi})}{d\beta} &= (-1)u'(w_1 - A_1^*(\beta)) \left(\frac{dA_1^*(\beta)}{d\beta} \right) + u' \left(\frac{A_1^*(\beta)}{q} + w_2 \right) \frac{dA_1^*(\beta)}{d\beta} \\ &= (1 - \beta) \frac{u'(c_2^*)}{q} \frac{dA_1^*(\beta)}{d\beta} > 0 \end{aligned}$$

Thus, there exists some $\tilde{\beta} \in (\underline{\beta}, \bar{\beta})$ such that

$$\begin{aligned} U(\beta; \underline{\phi}) &< U(0), \text{ if } \beta < \tilde{\beta} \\ U(\beta; \underline{\phi}) &= U(0), \text{ if } \beta = \tilde{\beta} \\ U(\beta; \underline{\phi}) &> U(0), \text{ if } \beta > \tilde{\beta} \end{aligned}$$

QED

9.3 Proof of Proposition 3

Let $\tilde{\phi}(\beta_i)$ denote the period-1 savings choice of an unconstrained agent of type β_i .

That is, $\tilde{\phi}(\beta_i)$ is implicitly defined by

$$u'[w_1 - \tilde{\phi}] = \frac{\beta_i}{q} u' \left[\frac{\tilde{\phi}}{q} + w_2 \right].$$

Let $\phi^*(\beta_i) = \phi^*$ denote the optimal (from a period-0 perspective) savings choice of a type- β_i agent. ϕ^* is implicitly defined by

$$u'[w_1 - \phi^*] = \frac{1}{q} u' \left[\frac{\phi^*}{q} + w_2 \right].$$

Note that $\tilde{\phi}(\beta_i) \leq \phi^*$ for $\beta_i \leq 1$, with strict inequality unless $\beta_i = 1$. More generally, note that $\phi^* - \tilde{\phi}(\beta)$ has the same sign as $(1 - \beta)$ for $\beta > 0$.

I proceed by characterizing the shape of $U^{PE}(\beta_i; \phi)$ over the policy space. For $\phi < \tilde{\phi}(\beta_i)$, the agent is unconstrained and $U^{PE}(\beta_i; \phi)$ is flat. For $\phi \geq \tilde{\phi}(\beta_i)$, the agent

is constrained by the savings floor. For a constrained agent,

$$\begin{aligned}\frac{dU^{PE}(\phi)}{d\phi} &= \frac{1}{q}u' \left[\frac{\phi}{q} + w_2 \right] - u'(w_1 - \phi) \\ &= (1 - \beta) \frac{u' \left[\frac{\tilde{\phi}}{q} + w_2 \right]}{q}, \text{ where } \beta \text{ is chosen so that } \tilde{\phi}(\beta) = \phi.\end{aligned}$$

Thus, $U^{PE}(\beta_i; \phi)$ is increasing over the region $\tilde{\phi}(\beta_i) \leq \phi \leq \phi^*$ and decreasing over the region $\phi \geq \phi^*$.

If ϕ^* is in the policy space, it is the preferred policy of all agents. Otherwise, all agents prefer the point in the policy space closest to ϕ^* . **QED**

9.4 Proof of Proposition 4

Let $w_1 = w_2 = w$ and $\frac{1}{q} < 1$. An individual's optimal (from a period-0 perspective) savings A_1^0 satisfies

$$u'(w - A_1^0) = \frac{1}{q}u' \left(\frac{A_1^0}{q} + w \right).$$

Thus, $A_1^0 < 0$ and the agents' preferred savings floor is an interior policy, $\phi^* = A_1^0$. **QED**

9.5 Proof of Proposition 5

Let $U(\beta_i, \delta_i; \phi) \equiv u[w_1 - A_1^*(\beta_i, \delta_i; \phi)] + \delta_i u \left[\frac{A_1^*(\beta_i, \delta_i; \phi)}{q(\phi)} + w_2 \right]$ denote the time-0 indirect utility of agent i when the savings floor is ϕ .

It is straight-forward to extend Lemmas 1, 2, and 3 to this more general case. Thus, once an agent becomes constrained, she will also be constrained in an equilibrium with a higher savings floor. Furthermore, $U(\beta_i, \delta_i; \phi)$ is quasiconvex in ϕ for any unconstrained agent.

For a constrained agent,

$$\frac{dU(\beta_i, \delta_i; \phi)}{d\phi} = \underbrace{\left[\frac{\delta_i}{q}u' \left(\frac{\phi}{q} + w_2 \right) - u'(w_1 - \phi) \right]}_{\text{substitution effect}} + \underbrace{\left[-\delta_i u' \left(\frac{\phi}{q} + w_2 \right) \frac{\phi}{q^2} \frac{dq}{d\phi} \right]}_{\text{wealth effect}}$$

The wealth effect is always positive but approaches 0 as $\phi \rightarrow 0$. The substitution effect is non-negative for a ‘just-constrained’ individual and becomes smaller as ϕ increases. If the substitution effect is always non-negative, then the individual will have an extreme policy as her most preferred policy. Thus, we need to check the sign of $\frac{\delta_i}{q} u' \left(w_2 + \frac{\phi}{q} \right) - u'(w_1 - \phi)$ as $\phi \rightarrow 0$.

The equilibrium price when $\phi = 0$ must satisfy

$$u'(w_1) \geq \frac{\beta_i \delta_i}{q(0)} u'(w_2) \quad \forall i.$$

This implies

$$q(0) = \bar{\beta} \bar{\delta} \frac{u'(w_2)}{u'(w_1)}.$$

Thus,

$$\frac{\delta_i}{q(0)} u'(w_2) - u'(w_1) = \left(\frac{\delta_i}{\bar{\beta} \bar{\delta}} - 1 \right) u'(w_1) \geq 0 \iff \frac{\delta_i}{\bar{\delta}} \geq \bar{\beta}. \quad \mathbf{QED}$$

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