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Abstract

This paper provides a framework for analyzing optimal government transfers of education when individuals are tempted to underinvest in education. The government may devise a transfer using a combination of free compulsory education, vouchers and price subsidies. I show that government intervention is needed if there is no deadweight loss associated with taxation. If there is a loss from taxation, government intervention is needed only if the level of temptation is sufficiently high. For high levels of temptation, free compulsory education or vouchers are optimal, whereas price subsidies may be optimal for intermediate levels of temptation.

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JEL classification : E21, H20, H4, H52, I28

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1 Introduction

Governments in many countries intervene heavily in the provision of education. Public spending on education is over three percent of the Gross Domestic Product (GDP) in most countries (Worldbank (2004)); in the U.S., it accounted for nearly six percent of the GDP in 2002. This expenditure may take a variety of forms: legislation requiring mandatory education, price modifications through tuition reductions, vouchers and tax deductions for education expenses.

The existing research neither suggests a rationale for government intervention strongly supported by empirical evidence, nor identifies the optimal instruments (Hanushek (2002)). For example, externalities, the inability to borrow against human capital, and redistributional purposes are suggested as justifications for government intervention. However, it is difficult to believe that marginal externalities are large enough to rationalize the current level of government spending (Hanushek (2003)). The inability to borrow against human capital may cause underinvestment in education (Becker (1993)). However, this hypothesis finds little empirical support (Cameron and Heckman (1999)). According to Hanushek (2003), a government may provide education for redistribution, but tuition support is generally dominated by wage subsidies if redistribution is the only goal.

This paper shows that temptation for immediate gratification can be a justification for government intervention in education. The benefits from education are realized in the future by raising future income while the opportunity costs are borne in the present. Hence, if individuals have temptation for immediate gratification, they will underconsume education. This paper also explicitly studies what type of instrument a government should use if government intervention is needed.

According to many experiments, the timing of a decision determines how an individual resolves the same intertemporal trade-off (see Frederick et al. (2002) for a survey). In the typical experiment, subjects choose between a smaller, period-1 reward and a larger, period-2 reward. If the choice is made at period 1, then the smaller-earlier reward is chosen. If the choice is made earlier (at period 0), the larger-later reward is chosen (Gul and Pesendorfer (2004)). Several studies argue that this behavior is important for understanding consumers' economic decisions (DellaVigna and Malmendier (2003), Laibson (1994, 1997, 1998)).

Two modelling approaches incorporating this different resolution of intertemporal decisions into preferences have emerged. One is time-inconsistent preferences, typically referred to as *hyperbolic discounting preferences* (Laibson (1994, 1997, 1998)). The other is preferences developed by Gul and Pesendorfer (2000, 2004), which are time-consistent (henceforth denoted as *Gul-Pesendorfer preferences*).

The first approach models an individual as a collection of independent decision makers (called *selves*). In each period, there is a single *self* whose preferences are allowed to be different from the preferences of *selves* in other periods. This approach typically assumes that a *self* discounts future utility geometrically but puts additional weight on current utility. Therefore, the period-0 *self* can prefer a larger, period-2 reward to a smaller, period-1 reward while period-1 *self* prefers the smaller period-1 reward because of the additional weight she places on period-1 utility. An individual with hyperbolic discounting preferences is called time-inconsistent, since she changes her preferences over time.

Gul and Pesendorfer, alternatively, developed an axiomatic framework in which preferences do not change over time (time-consistent). However, an individual suffers from *temptation* to increase current period consumption relative to standard preferences.[1] Therefore, the agent at period 0 does not suffer from temptation to increase period-1 consumption relative to period-2 consumption. Hence, she can

[1] Detailed information about Gul-Pesendorfer preferences can be found in Appendix A.

prefer a larger period-2 reward to a smaller period-1 reward, whereas she would prefer the smaller period-1 reward at period 1.

Under both approaches, an individual displays a demand for commitment. In hyperbolic discounting preferences, the period-0 *self* and the period-1 *self* are independent, and the choice by the period-1 *self* may not be optimal for the period-0 *self*. Therefore, the period-0 *self* would like to buy a commitment device to constrain period-1 *self*'s choice. In Gul-Pesendorfer preferences, the agent suffers more from temptation if she faces more alternatives. Therefore, she would prefer to reduce her choice set in advance. The demand for commitment allows for the possibility of welfare improvement through government provision of education. Education is a type of saving, which delays consumption for future periods, and it is difficult to undo. Therefore, the government's provision of education in exchange for taxation will serve as a commitment device.

Also note that neither approach dominates the other. In hyperbolic discounting preferences, the welfare criteria is ambiguous since an individual at each period is an independent decision maker. There exists, however, more experimental and empirical research to estimate parameters of these preferences. On the other hand, Gul-Pesendorfer preferences allow us to use standard welfare criteria, but there is little empirical research on them. Hence, I use both types of preferences to determine when government intervention is needed and what type of instrument is optimal when it is required.

I develop a model in which an agent faces a two-period decision problem of choosing allocations of a consumption good and education. The consumption good provides utility directly, while education increases her utility indirectly through an increase in second-period income. In this model, an agent has either hyperbolic discounting preferences or Gul-Pesendorfer preferences.

The government maximizes the agent's utility[2] using a combination of three instruments: free compulsory education, vouchers for education and price subsidies for education. Free compulsory education provides an agent with a certain level of education at no cost, but forces her to consume at least as much as that level of education. Vouchers give agents the right to receive education at no cost up to a certain level, but provide agents with the option of not using vouchers. Price subsidies reduce the price of education relative to the consumption good. Finally, the government must balance its budget, and there is a deadweight loss associated with taxation.

The difference between free compulsory education and vouchers is whether a transfer allows free-disposal. However, if an agent's utility is a strictly increasing function, her incentive to dispose of the vouchers is eliminated. Therefore, free compulsory education is equivalent to vouchers. Hence, we can limit our attention to the case in which the government uses no free compulsory education, which reduces the policy space to two dimension.

I obtain three main results. The first is that government intervention is optimal if temptation is high. Due to the cost of taxation, government intervention decreases the disposable income of an agent. However, government intervention imposes restrictions on the agent's budget set, thereby reducing her disutility from temptation compared to the case of no intervention. When the level of temptation is sufficiently high, the benefit from reducing temptation will dominate the cost associated with the decrease in disposable income, and government intervention is optimal.

The second result is that if temptation is sufficiently high, vouchers are optimal. Since the agent cannot use vouchers to purchase the consumption good, vouchers guarantee the minimum level of education. However, price subsidies do

[2]For the case of hyperbolic discounting preferences, this paper evaluates welfare based on initial preferences, since this criterion is typically employed in the literature.

not ensure the minimum level of education. As temptation becomes severe, the agent chooses less education. Therefore, if temptation is sufficiently high (e.g., the agent at period 1 only values period-1 consumption), vouchers dominate price subsidies and no intervention.

The third result is that a convex combination of price subsidies and vouchers is not optimal if the agent has hyperbolic discounting preferences. The government uses vouchers only if it directly changes the allocation the agent chooses.[3] If the voucher does not directly change the allocation, then it is just a waste of resources due to the deadweight loss of taxation. Since the government uses vouchers only if the constraint binds, a small reduction of price subsidies does not change education choice but increases disposable income. Hence, the government uses only a pure voucher or a pure price subsidy.

The closest precursor of this paper is the contribution by Amador et al. (2004) that studied a rationale for forced minimum saving policies. An individual has preferences for commitment, but flexibility is valued because of taste shocks realized at period 1. The authors compared two policies: the minimum saving policy and the full separation policy, which makes an agent consume different allocations depending on her taste shocks. They showed a condition on the distribution of taste shocks that makes the minimum saving dominates the other. However, their paper neither explicitly consider when government intervention is needed nor other types of government instruments, such as tax deductions for savings.

The remainder of this paper is organized as follows: Section 2 presents the model. Section 3 characterizes the optimal government transfer, and Section 4 concludes.

[3] Vouchers affect the allocation the agent chooses through two channels: The first is the constraint to ensure the minimum level of education and the second is the income effect by deadweight loss from taxation. By "directly changes," I mean that vouchers change the allocation through the first channel, the constraint.

2 Model

The economy lasts for three periods ($t = 0, 1, 2$). Period 0 is a policy-setting date only. There exist two goods: a consumption good and education. The consumption good ($c_t, t = 1, 2$) directly increases an agent's utility at period t, while 1 unit of education purchased at period 1 increases utility indirectly by providing 1 unit of the consumption good at period 2.[4]

2.1 An Agent's Problem

There is a unit measure of agents, and an agent is endowed with y, at the beginning of period 1. An agent can suffer from temptation to overconsume the consumption good at period 1. Her preferences are represented either by hyperbolic discounting preferences, or by Gul-Pesendorfer preferences described in the following subsections.

2.1.1 Hyperbolic Discounting Preferences

Following Strotz (1956), Phelps and Pollack (1968), Laibson (1994, 1997, 1998), I assume that an agent has preferences which may change over time, as follows:

$$\text{Period 2} \qquad W_2 \equiv \underset{c_2 \in B_2}{Max}\ u(c_2) \tag{1}$$

$$\text{Period 1} \qquad W_1 \equiv \underset{(c_1, e) \in B_1}{Max}\ u(c_1) + \beta\delta u(c_2) \tag{2}$$

$$\text{Period 0} \qquad W_0 \equiv \underset{(c_1, e) \in B_1}{Max}\ u(c_1) + \delta u(c_2) \tag{3}$$

$$where\ 0 \leq \delta \leq 1, 0 \leq \beta \leq 1, B_t \text{ stands for period-t budget set}$$

[4] We can generalize this assumption that the return of education is r instead of 1. The optimal transfers with the return of r will be qualitatively the same as those when the return is 1, which will be characterized in Section 3. As r increases, government intervention dominates no intervention for a larger range of the level of temptation.

I assume $u', -u'' > 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. The period-0 *self* discounts the second period utility by δ, while the period-1 *self* discounts it more by $\beta\delta$. Because of this discrepancy between discount factors, the period-1 *self* underinvests in education from the period-0 *self*'s perspective. As β decreases, the period-1 *self* underinvests more in education. Hence, $1 - \beta$ represents the level of temptation, which is the same as $\gamma(\beta, \infty)$ with $\gamma(\beta, \lambda) \equiv \frac{\lambda(1-\beta)}{1+\lambda}$.[5]

2.1.2 Gul-Pesendorfer Preferences

Gul and Pesendorfer (2000, 2002, 2004) developed time-consistent preferences allowing the possibility that an agent can suffer from temptation to overconsume based on an axiomatic framework.[6]

If an agent has Gul-Pesendorfer preferences, her preferences are expressed as follows:

$$\text{Period 2} \qquad W_2 \equiv \underset{c_2 \in B_2}{Max}\{u(c_2) + \lambda V(c_2, 0)\} - \underset{c_2' \in B_2}{Max}\ \lambda V(c_2', 0) \tag{4}$$

$$\text{Period 1} \qquad W_1 \equiv \underset{(c_1, e) \in B_1}{Max}\{u(c_1) + \lambda V(c_1, e) + \delta W_2\} - \underset{(c_1', e') \in B_1}{Max}\ \lambda V(c_1', e') \tag{5}$$

$$where\ \lambda \geq 0, 0 < \delta \leq 1, B_t \text{ stands for period-t budget set}$$

I assume $u', -u'' > 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{c \to \infty} u'(c) = 0$. The function u represents a standard utility function. The function V, denoted as the temptation function, incorporates the agent's temptation to overconsume. If V puts more weight on period-1 utility than on period-2 utility, the agent will choose a lower level of education than the level of education she would choose under standard preferences.

[5] $\gamma(\beta, \lambda)$ is also used to denote the level of temptation in the case of Gul-Pesendorfer preferences.

[6] A brief review of Gul-Pesendorfer preferences is presented in Appendix.

The maximum temptation level, $\underset{(c_1', e') \in B_1}{Max} \ \lambda V(c_1', e')$, captures the possibility that the agent may prefer a more constrained budget set. Since this term increases weakly in the budget set, the agent with Gul-Pesendorfer preferences can prefer a strictly smaller set to a larger set, which contains the first set. ($A \succ B$ with $A \subsetneq B$).

If λ is sufficiently small, an agent with Gul-Pesendorfer preferences chooses an allocation different from the allocation to maximize her temptation ($i.e.$,$(c_1, e) \neq (c_1', e')$). If that is the case, the agent is called to exert *self-control.*[7]

While there is no theoretical constraint on the functional form of temptation functions, two forms are typically assumed. The first one is the hyperbolic utility ($i.e.$,$V(c_1, e) = u(c_1) + \beta\delta u(c_2)$), and the other is a strictly increasing function in c ($i.e.$,$V(c_1) = v(c_1)$ and $v' > 0$). The hyperbolic temptation function is often used to highlight differences between Gul-Pesendorfer preferences and hyperbolic discounting preferences, especially in a multiperiod setup. An increasing temptation function is assumed in an infinite-period model to guarantee a unique equilibrium (Gul and Pesendorfer (2004), Krusell et al. (2005)). Therefore, I limit my focus to these two temptation functions to study optimal government transfers. We can rewrite the agent's utility as follows:

$$
\begin{aligned}
W_2 &\equiv u(e) && (6)\\
W_1 &\equiv (1+\lambda)\ Max\ \{u(c_1) + \delta u(e) + \gamma(\beta,\lambda)(\frac{I(v(c_1) - u(c_1))}{(1-\beta)} - \delta u(e))\} \\
&- \ Max\ \lambda(u(c_1') + I(v(c_1') - u(c_1')) + \beta\delta u(e')) && (7)\\
&\quad where\ v' > 0, \gamma(\beta,\lambda) = \frac{\lambda(1-\beta)}{1+\lambda}, I = 1 \text{ if } \beta = 0 \text{ and otherwise } 0.
\end{aligned}
$$

[7] An agent with hyperbolic discounting preferences cannot exert *self-control*, since she does not consider period-0 self's utility and chooses the allocation which maximizes the current period *self*'s utility at each time. Therefore, the agent chooses the allocation to maximize temptation.

The third term in period-1 utility, $\gamma(\beta, \lambda)(\frac{I(v(c_1)-u(c_1))}{(1-\beta)} - \delta u(e))$, captures the discrepancy in allocations between no temptation and temptation. When the agent has hyperbolic temptation function, the agent chooses the allocation to maximize $\{u(c_1) + \delta(1 - \gamma(\beta, \lambda))u(e))\}$. As $\gamma(\beta, \lambda)$ increases (*i.e.,* β decreases or λ increases), the agent puts less weight on period-2 utility. When the agent has increasing temptation function, the agent chooses the allocation to maximize $\{u(c_1) + \delta u(e) + \frac{\gamma(1,\lambda)}{1-\gamma(1,\lambda)} v(c_1)\}$. As $\gamma(1, \lambda)$ increases (*i.e.,* λ increases),[8] the agent puts more weight on period-1 utility. Hence, $\gamma(\beta, \lambda)$ denotes the level of temptation.

2.2 The Government's Problem

At period 0, the government chooses a transfer mechanism that maximizes the representative agent's utility given the government budget balance constraints. In order to finance the expenditure of τ, the government needs to impose a tax amounting to $\tau + g(\tau)$ where $g(0) = 0$, $g'(\tau) \in [0, \infty)$. $g(\tau)$ represents the deadweight loss associated with taxation. The government can use a combination of free compulsory education, vouchers for education, and price subsidies for education, as defined below:

Definition 1 *A free compulsory education is a transfer that sets the minimum level of consumption of education. The free compulsory education, amounting to k, changes the budget set of an agent from $B_1 = \{(c_1, e)|0 \leq c_1, 0 \leq e, c_1 + e \leq y\}$ to $B_1 = \{(c_1, e)|0 \leq c_1 \leq y, k \leq e, c_1 + e \leq y + k\}$.*

[8] As $\gamma(1, \lambda)$ increases, does $\frac{\gamma(1,\lambda)}{1-\gamma(1,\lambda)}$.

Definition 2 *A voucher is a transfer of purchasing power that entitles an agent to obtain education free up to the amount of the voucher, without setting the minimum level of education consumption. The voucher transfer,amounting to* b*, changes the budget set of an agent to* $B_1 = \{(c_1, e)|0 \leq c_1 \leq y, 0 \leq e, c_1{+}e \leq y{+}b\}$.

Definition 3 *A price subsidy is a transfer that reduces the relative price of education to the consumption good. A price subsidy, with the rate of* s*, changes the budget set of an agent to* $B_1 = \{(c_1, e)|0 \leq c_1 \leq y, 0 \leq e, c_1 + \frac{1}{(1+s)}e \leq y\}$.

Therefore, a government transfer, (k, b, s), is a combination of free compulsory education, a voucher and a price subsidy. It changes the period-1 budget set of an agent to $B_1(k, b, s) = \{(c_1, e)|0 \leq c_1 \leq y, k \leq e, c_1 + \frac{1}{(1+s)}e \leq y + k + b\}$. Finally, the budget set of the agent with a government transfer (k, b, s) and taxation amounting to τ, is $\{(c_1, e)|0 \leq c_1 \leq y - \tau - g(\tau), k \leq e, c_1 + \frac{1}{(1+s)}e \leq y + k + b - \tau - g(\tau)\}$.

Definition 4 *A transfer* (k^*, b^*, s^*) *is optimal if*

(1) it maximizes an agent's utility, (i.e., it maximizes either equation (3) for the case of hyperbolic preferences or equation (7) for the case of Gul-Pesendorfer preferences)

(2) it satisfies the government's budget constraint $\tau = k + b + \frac{s}{1+s}e$*, where* e *is the agent's choice of education.*

3 Optimal Government Transfers

In this model, there is no market failure. Therefore, if there is no temptation (*i.e.,* $\gamma(\beta, \lambda) = 0$), no intervention is better than any government intervention.

Lemma 1 *(Standard Preferences) No government intervention is needed if there is no temptation (i.e., $\gamma(\beta, \lambda) = 0$).*

Proof. Government intervention decreases the agent's disposable income due to the cost of taxation, $g(\tau)$. However, when $\gamma(\beta, \lambda) = 0$ (*i.e.,* $\lambda = 0$ or $\beta = 1$), the government intervention does not increase the agent's utility by constraining her budget set. ■

However, if temptation exists (*i.e.,* $\gamma(\beta, \lambda) > 0$), government intervention can increase an agent's utility. In particular, if there is no deadweight loss associated with taxation (*i.e.,* $g(\tau) = 0$), government intervention achieves the *first-best*, which is the utility level that an agent with no temptation can obtain.

Proposition 1 *(First-best) There exists a transfer, (k,b,s), that achieves first-best if $g(\tau) = 0$ for all τ.*

Proof. If $g(\tau) = 0$ for all τ, a transfer $(0, b_1, 0)$, with $u'(y - b_1) = \delta u'(b_1)$, makes an agent choose the same allocation as in the first-best case. ■

If there is no deadweight loss associated with taxation, the government can implement the first-best allocation since there is no asymmetric information about either agent's preferences or endowments. However, when there exists a deadweight loss of taxation, not all government intervention is better than no intervention. The following propositions will help us to characterize optimal transfers when taxation induces a deadweight loss.

Proposition 2 reduces the policy space from three dimensional (*i.e.,* free compulsory education, voucher for education and price subsidies) to two dimensional. In this setting, free compulsory education and vouchers are equivalent.

Proposition 2 *Consider two transfers,* (k, b, s) *and* (k', b', s)*. If* $k + b$ *is equal to* $k' + b'$*, the utility of an agent with* (k, b, s) *is the same as that with* (k', b', s)*.*

Proof. The budget constraint of an agent under (k, b, s) and that under (k', b', s) can be expressed as follows:

$$B_1(k,b,s)=\{(c_1,e)|0 \le c_1 \le y - \tau - g(\tau), k \le e, c_1 + \frac{1}{(1+s)}e \le y + k + b - \tau - g(\tau)\}$$

$$B_1(k',b',s)=\{(c_1,e)|0 \le c_1 \le y - \tau' - g(\tau'), k' \le e, c_1 + \frac{1}{(1+s)}e \le y + k' + b' - \tau' - g(\tau')\}$$

Since the agent's utility increases in education, the agent's choice of the education level is greater than the sum of free compulsory education and voucher (*i.e.,* $e \ge k + b$ and $e' \ge k' + b'$). Therefore, the budget set $B_1(k, b, s)$ is equivalent to $B_1(k', b', s)$ in the agent's perspective. ■

This property comes from the assumption that the agent's utility is strictly increasing in education, conditioning on the level of the consumption good. The difference between vouchers and free compulsory education is that vouchers allow for disposal of the transfer, while free compulsory education does not. However, strictly increasing utility in education nullifies this difference because she will use all vouchers. If the agent's utility is not strictly increasing in education,[9] she can be better off with vouchers. From now on, I limit my attention to transfers with no free compulsory education, which is denoted as $(b, s) \equiv (0, b, s)$.

Next I show that government intervention is optimal if temptation is high. Government intervention decreases the disposable income of an agent due to the cost of taxation. However, government intervention imposes restrictions on the agent's budget set, reducing her disutility from temptation compared to the case of no intervention. When the level of temptation is sufficiently high, the benefit from reducing temptation will dominate the cost associated with the decrease in disposable income, and government intervention is optimal.

[9] For example, if education requires the agent to spend time studying and if the agent values leisure, then the agent's utility may not strictly increase in education.

Lemma 2 *There exists a* $\gamma(\beta^*, \lambda^*)$ *s.t. for all* $\gamma(\beta, \lambda) \geq \gamma(\beta^*, \lambda^*)$ *no intervention is not optimal.*

Proof. See Appendix B. ■

The utility with no intervention decreases as γ increases while that with sufficiently large amount of vouchers is constant. Therefore, there exists a γ^* such that for all $\gamma(\beta, \lambda) \geq \gamma(\beta^*, \lambda^*)$, a voucher transfer dominates no intervention.

In the following two subsections, I study the optimal government transfers when an agent has either hyperbolic preferences or Gul-Pesendorfer preferences, and a deadweight loss is strictly increasing in the amount of taxation (*i.e.*, $g' > 0$)

3.1 Hyperbolic Preferences

The following two propositions tell us that we only need to consider a pure voucher transfer, $(b, 0)$, or a pure price subsidy transfer, $(0, s)$.

Proposition 3 *If a government uses a transfer program* (b, s) *with* $b > 0$, *and under* (b, s) *an agent chooses education strictly greater than b (non-binding case),*[10] *then there exists a transfer program* (b', s) *with* $b' < b$ *that dominates* (b, s).

Proof. Since the constraint associated with vouchers does not bind, the first order condition determines the choice of (c_1, e) by period-1 *self* (*i.e.,* (c_1, e) is determined by $u'(y - e(b, s) - g(b + \frac{s}{1+s}e(b, s)) = (1 + s)\beta\delta u'(e))$.

Using the implicit function theorem, we find that

$$\frac{\partial e}{\partial b} = \frac{-g'(\tau)}{\{(1+s)\beta\delta\frac{u''(e)}{u''(c(b,s))} + 1 + \frac{s}{1+s}g'(\tau)\}} \tag{8}$$

$$\frac{\partial c}{\partial b} = -\frac{\partial e}{\partial b} - g'(\tau)(1 + \frac{s}{1+s}\frac{\partial e}{\partial b}) \tag{9}$$

[10] A non-binding case refers to a case in which the constraint of $\{c_1 \leq y - \tau - g(\tau)\}$ does not bind, or equivalently, the constraint of $\{b \leq e\}$ does not bind

Given the assumptions we made for u and g, $\frac{\partial e}{\partial b}$ and $\frac{\partial c}{\partial b}$ are negative. Therefore, we can construct a transfer $(b-\epsilon, s)$ with $\epsilon > 0$ and $\epsilon \simeq 0$ such that both education and consumption good chosen by period-1 *self* under $(b-\epsilon, s)$ are larger than that under (b, s). This result comes from the fact that disposable income ($y^d \equiv y + b - \tau - g(\tau)$) increases as the government changes its transfer from (b, s) to $(b-\epsilon, s)$. Because of this income effect, the agent can consume more of both the consumption good and education under $(b-\epsilon, s)$ and, therefore, achieve higher utility. Hence, (b, s) cannot be optimal. ■

When an agent chooses education higher than the amount of vouchers, the decrease of vouchers indirectly affects the consumption of education through the changes of disposable income. Because of concavity of the utility function, the total amount of tax decreases as vouchers decrease. Hence, a government will use vouchers only if it constrains an agent's choice.

Proposition 4 *Suppose a government uses a transfer program (b, s) with $s > 0$ and under this program an agent chooses education level b (i.e., binding case). Then there exists a transfer (b, s'), with $s' < s$ that dominates (b, s).*

Proof. Let's denote $(c_1^N(b, s), e^N(b, s))$ as the agent's choice if there is no restriction on the usage of vouchers. Since the vouchers constrain the agent's choice, the agent's choice of education, $e(b, s)$, is larger than $e^N(b, s)$ (*i.e.*, $e(b, s) > e^N(b, s) \Rightarrow e(b, s) = b$).

Using the implicit function theorem, we find that

$$\frac{\partial e^N}{\partial s} = \frac{-\{u''(c^N)g^{'}(\tau)\frac{1}{(1+s)^2}e^N + \beta\delta u'(e^N)\}}{\{u''(c^N)(1+\frac{s}{1+s}g'(\tau)) + (1+s)\beta\delta u''(e^N)\}} \tag{10}$$

Since $\frac{\partial e^N}{\partial s}$ is finite, we can construct another transfer, $(b, s-\epsilon)$ with $\epsilon > 0$ such that $e(b, s-\epsilon) > e^N(b, s-\epsilon)$ (*i.e.*, $e(b, s-\epsilon) = b$). Since the level of consumption of education remains the same but the subsidy rate declines, the amount of tax, τ, decreases. Hence, the agent's utility with $(b, s - \epsilon)$ is higher than that with (b, s) because $c(b, s - \epsilon) = y - \tau' - g(\tau')$ is larger than $c(b, s)$. ■

The decrease of the price subsidy rate has two effects: the first is the price effect that reduces the agent's choice of education given the same disposable income; the second is the income effect that is caused by the change in taxation. Given the price subsidy rate, the income effect will increase the agent's choice of education. Since under (b, s) the constraint associated with vouchers binds, we can find a sufficiently small ϵ where the constraint still binds under $(b, s - \epsilon)$. In this case, there is no price effect but disposable income increases because of decrease of tax.[11] Hence, (b, s) with $s > 0$ is not optimal.

When the voucher constraint binds, the marginal change in the price subsidy rate does not affect the consumption of education. However, as the price subsidy rate decreases, the total amount of tax decreases, which in turn increases disposable income. Hence, if the government uses vouchers, it will not combine them with a positive price subsidy.

According to Propositions 3 and 4, a government will use vouchers only if the constraint of vouchers binds and if there is no price subsidy. Therefore, we can limit our attention to studying which type of transfer is optimal among a pure voucher transfer,$(b, 0)$, a pure price subsidy transfer,$(0, s)$, and no intervention,$(0, 0)$.

From Lemma 1, we know when $\beta = 1$ ($i.e.$,$\gamma(\beta, \infty) = 0$), no intervention is optimal, and the following proposition shows that a voucher transfer is the best when $\beta = 0$ ($i.e.$,$\gamma(\beta, \infty) = 1$).

[11] The tax under $(b, s - \epsilon)$ is $(1 + s - \epsilon)b$, which is smaller than the original level, $(1 + s)b$.

Proposition 5 *When* $\gamma(\beta, \infty) = 1$, *the voucher transfer amounting to* b_2 *is optimal, where* b_2 *is defined by* $u'(y - b_2 - g(b_2))(1 + g'(b_2)) = \delta u'(b_2)$.

Proof. When $\gamma(\beta, \infty) = 1$ ($i.e., \beta = 0$), any level of voucher constrains an agent choice. Therefore, the vouchers amounting to b_2 is optimal among voucher transfers, where b_2 is defined by $u'(y - b_2 - g(b_2))(1 + g'(b_2)) = \delta u'(b_2)$. The agent's utility with b_2 is $u(y - b_2 - g(b_2)) + \delta u(b_2)$. In contrast, if there is no government intervention or any price subsidy, the *period-1 self* will consume all endowment to purchase consumption goods when $\beta = 0$. Therefore, her utility is $u(y)$. Since $u'(0) = \infty$, the voucher transfer dominates both no intervention and price subsidies. ■

No intervention and price subsidies cannot eliminate the case that period-1 *self* would consume no education. However, vouchers constrain the agent to consume education equivalent to at least as much as the amount of vouchers. Because of *Inada* condition, the voucher transfer dominates no intervention and price subsidies.

To characterize optimal government transfer for less extreme values of γ ($0 < \gamma < 1$), we need further assumptions. Therefore, I deliver the optimal transfers for a specific example, rather than imposing additional assumptions:

Example 1 *(Log Utility and Linear Cost Function) Suppose an agent has log utility and a government's loss function is linear* ($g(\tau) = \tau$). *Then, the government's problem can be written as below:*

$$\max_{(c,e) \in B_1(b,s)} \quad \{\log(c_1) + \delta \log(e)\}$$

$$\text{where } (c_1, e) \in \arg\max_{(c_1,e) \in B_1(b,s)} \log(c_1) + \beta\delta \log(e)$$

$$\tau = k + b + \frac{s}{1+s} * e \text{ and } \tau \geq 0$$

In this example, when $\beta = 0$, the voucher is optimal. For $\beta \in (0, \frac{1}{2+\delta})$, the price subsidy is optimal. For $\beta > \frac{1}{2+\delta}$,no intervention is optimal.[12]

As β decreases, period-1 *self* will consume less education so that the period-0 *self*'s utility level under no intervention decreases. However, the price subsidy can alleviate this problem by putting more weight on the marginal utility of education, while it does not constrain the agent's choice. Therefore, if β becomes small enough, the price subsidy can be better than no intervention. However, since it does not reduce the agent's budget set to a singleton, the price subsidy becomes dominated by the voucher program when β is sufficiently small. The assumption of a log utility and linear cost function delivers three intervals of β in which one of three instruments is optimal.

3.2 Gul-Pesendorfer Preferences

I now ask what the optimal transfer would be for an agent with Gul-Pesendorfer preferences. According to Lemma 1, when $\gamma(\beta, \lambda) = 0$, no intervention is optimal. Similar to the hyperbolic discounting preferences case, the voucher transfer is optimal if $\gamma(\beta, \lambda) = 1$.

Proposition 6 *When $\gamma(\beta, \lambda) = 1$, the voucher is optimal.*

Proof. Case 1 ($v = u$ and $\beta \geq 0$) : Since the agent's preferences converge to the hyperbolic discounting preferences as $\gamma(\beta, \lambda)$ goes to 1, the voucher transfer defined in Proposition 5 is optimal.

Case 2 ($\beta = 0$) : As λ goes to infinity, the agent will purchase the consumption good as much as possible. Hence, her utility is $u(y)$ if there is no government

[12] The proof can be found in Appendix B.

intervention, and it is less than but close to $u(y)$ if there is a strictly positive price subsidy. However, if the government provides vouchers, the agent's budget constraint for the consumption good binds. Therefore, she does not suffer the disutility from self-control and her utility is $u(y - b_2 - g(b_2)) + \delta u(b_2)$ with b_2 defined by $u'(y - b_2 - g(b_2))(1 + g'(b_2)) = \delta u'(b_2)$. Since b_2 is not zero, the utility with voucher transfer is higher than that with no intervention or that with a price subsidy. ■

In the case of the hyperbolic-discounting temptation function, the same intuition in Proposition 5 applies. In the case of a strictly-increasing temptation function, as $\gamma(0, \lambda)$ increases, the utility level with no intervention decreases. A price subsidy cannot completely eliminate the disutility from temptation. However, the optimal voucher provides a singleton choice set to the agent and fully eliminates disutility from temptation. Therefore, when $\gamma(0, \lambda)$ is sufficiently large, the benefit from eliminating disutility dominates the disutility from constraining choice, and the voucher transfer dominates no intervention and a price subsidy.

Unless the agent has the hyperbolic-discounting temptation function and can exert *self-control,* the optimal transfer for an agent with Gul-Pesendorfer preferences will be either a pure voucher transfer or a pure price subsidy.

When an agent has the hyperbolic-discounting temptation function and cannot exert *self-control,* her preferences are equivalent to the hyperbolic discounting preferences. Hence, a pure voucher or a pure price subsidy will be used if there is government intervention, according to Propositions 3 and 4. The following Propositions show that when an agent has a concave temptation function and the net disutility from temptation decreases in disposable income,[13] a government will

[13] The disutility from temptation means $v(c_1) - v(y^M)$ where c_1 is the level of consumption good which maximizes $u(c_1) + \lambda v(c_1) + \delta u(e)$, and y^M is the maximum amount of endowment within the agent's budget set.

use either a pure voucher or a pure price subsidy if it is needed.

Proposition 7 *Suppose* $\beta = 0$, v *is concave and* $\frac{\partial\{v(c_1)-v(y^M)\}}{\partial y^D} \leq 0$ *where* c_1 *is the agent's choice for the consumption good,* $y^D = y+b-\tau-g(\tau)$ *and* $y^M = y^D - b$. *Then, if a government uses a transfer program* (b,s) *with* $b > 0$ *and under* (b,s) *an agent chooses education strictly greater than* b *at period 1 (i.e., a non binding case), there exists a transfer program* (b',s) *with* $b' < b$ *that dominates* (b,s).

Proof. See the Appendix B. ■

Proposition 8 *Suppose* $\beta = 0$, v *is concave and* $\frac{\partial\{v(c_1)-v(y^M)\}}{\partial y^D} \leq 0$ *where* c_1 *is the agent's choice for the consumption good,* $y^D = y + b - \tau - g(\tau)$ *and* $y^M = y^D - b$. *If a government use a transfer program* (b,s) *with* $s > 0$ *and under this program an agent chooses education level* b*(i.e., binding case), then there exists a transfer,*(b,s')*, with* $s' < s$ *that dominates* (b,s).

Proof. See the Appendix B. ■

The intuition of Propositions 7 and 8 is as follows: as shown in Propositions 3 and 4, the discounted utility, $u(c) + \delta u(e)$ increases as a government changes its transfer from (b,s) to (b',s) or from (b,s) to (b,s'). By assumption, the net disutility from temptation decreases as disposable income increases. Therefore, Propositions 7 and 8 hold.

I will conclude this subsection by deriving the optimal transfers for a specific example:

The net distutility decreases in the disposable income ($y^d \equiv y+b-\tau-g(\tau)$) if the temptation function v is log.

Example 2 *(Log Temptation Function) Suppose* $u(c) = v(c) = \log(c), g(\tau) = \tau$ *and* $\beta = 0$*. Then, the government's problem is to choose (b,s), which solves the following problem:*

$$\max_{(c,e)\in B_1(b,s)} \quad \{(1+\lambda)\log(c_1) + \delta\log(e)\} - \lambda \max_{(c)\in B_1(b,s)} \log(c_1)$$
$$where\ \tau = k + b + \frac{s}{1+s} * e \geq 0$$

By Proposition 7 and 8, the optimal policy is either no intervention, pure voucher transfer, or pure price subsidy. In this example,There exists a λ_1 *with* $1 < \lambda_1 \leq 1+\delta$ *s.t. for all* $\lambda \geq \lambda_1$*, the voucher is optimal and for* $1 \leq \lambda \leq \lambda_1$*, no intervention is optimal. When* $\lambda \in (0,1)$*, either voucher or no intervention is optimal depending on parameter values and when* $\lambda = 0$*, no intervention is optimal.*[14]

4 Conclusion

This paper has provided a framework for analyzing optimal government transfers of education when private agents are tempted to underinvest in education and the government devises a transfer using free compulsory education, vouchers and price subsidies. I have shown that government intervention is optimal if there is no deadweight loss induced by taxation. If there is a deadweight loss, then government intervention is optimal if the level of temptation is sufficiently high. For high levels of temptation, free compulsory education or vouchers are optimal, whereas for intermediate levels of temptation, price subsidies may be optimal.

This paper has several possible extensions. First, it would be interesting to study a multiperiod decision problem. Suppose that an agent lives T periods, and

[14] The proof can be found in Appendix B.

her temptation to underinvest in education decreases as she becomes older. We would expect such a model to predict that free compulsory education or vouchers are optimal for primary education because the level of temptation is comparatively high and that price subsidies are optimal for higher education. Moreover, job retraining can be viewed as a form of education for workers who are beyond the age of normal education. This model is consisted with the observation that governmental intervention in job retraining is less common than in primary or higher education.

Second, we can use this model to study a government's provision of other goods in addition to education that increase future income, such as social security and durable goods. Suppose there are three investment goods, $i \in \{1, 2, 3\}$ with return r_i, where $r_1 \geq r_2 \geq r_3$. In such a setting, the government is more likely to provide a good that has a high return. This property may possibly explain why the government intervenes in education and social security but not in durable goods.

5 Appendix A

This section illustrates Gul-Pesendorfer preferences (Gul and Pesendorfer (2000, 2004)) in a two-period deterministic environment. A consumption problem (CP) is defined as a set of choices, each of which yields a consumption $c \in C$ for the current period and a consumption problem starting the next period. Consider an agent who faces a consumption problem in each period. The standard approach to this problem is to define preferences for the agent over sequences of consumption realizations. This approach excludes preferences that depend on what could have been chosen in addition to the consumption realization. However, direct dependence on the opportunity set can be used to capture temptation and preference for self-control. Therefore, Gul and Pesendorfer model preferences over the opportunity set (*i.e.,* the set of CPs, say Z), and the following axioms characterize the preferences:

We use x, y,or z to denote elements of Z.

Axiom 1 *(complete and transitive)* $\succeq$ *is a complete and transitive binary relation.*

Axiom 2 *(Strong Continuity) The sets* $\{x : x \succeq z\}$ *and* $\{x : z \succeq x\}$ *are closed.*

Axiom 3 *(Stationarity)* $\{(c, x)\} \succeq \{(c, y)\}$ $\; iff \;$ $x \succeq y$

Axiom 4 *(Set Betweenness)* $x \succeq y$ *implies* $x \succeq x \cup y \succeq y$.

Axiom 5 *(Temptation by Immediate Consumption)* $y^1 = z^1, \{x\} \succ \{x, y\} \succ \{y\}$ *and* $\{x\} \succ \{x, z\} \succ \{z\}$ *implies* $\{x, y\} \;\tilde{}\{x, z\}$

Axioms 1-2 imply that $\succeq$ may be represented by a continuous function. Axiom 3 characterizes a preference that is stationary. The main difference between Gul-Pesendorfer preferences and standard preferences comes from Axiom 4 (Set

Betweenness). Under standard preferences, the availability of a less preferred set does not decrease utility. However, Set Betweeness allows for the possibility that the utility of an agent can be decreased by the availability of a less preferred set.

Consider the following case: Suppose an agent named Mike has high blood pressure. In order to maintain his health, he should abstain from saturated fat. However, Mike prefers bacon, which has saturated fat, to vegetables. If his wife provides him only with a vegetarian dish for dinner, he has no choice but to eat the vegetarian dish. Thus he does not suffer from temptation. However, if his wife provides both bacon and vegetables and he chooses not to eat the bacon, he would still have decreased utility due to the cost of self control.

If an agent's preferences satisfies Axioms 1 - 4 and are non-degenerate,[15] they are uniquely represented by the following function:

$$W(z) \equiv \max_{(c,z')\in C\times Z'} (u(c) + V(c,z') + \delta W(z')) - \max_{(c'',z'')\in C\times Z'} V(c'',z'') \qquad (11)$$

$$\textit{where } \delta \in (0,1)$$

Axiom 5 requires that the correlation between the current consumption and the continuation problem does not affect preferences. This axiom simplifies temptation preferences such that they only depend on the current consumption (*i.e.,* $V(c,z') = v(c)$). Axiom 5 guarantees a unique equilibrium when an agent with Gul-Pesendorfer preferences lives infinitely.

[15] Gul and Pesendorfer define the preference $\succeq$ as non-degenerate if there exists x, y such that $y \subset x$ and $x \succ y$.

6 Appendix B

6.1 Lemma 2

Proof. When an agent has hyperbolic preferences, her utility without government intervention, $W_1^H(0,0)$, is $u(c) + \delta u(e)$ where (c, e) is defined by $u'(c) = \beta\delta u'(e)$ and $c + e = y$. $\frac{\partial W_1^H(0,0)}{\partial \gamma}$ is negative since $\frac{\partial W_1^H(0,0)}{\partial \beta} > 0$ and $\frac{\partial \beta}{\partial \gamma} < 0$.

When an agent has Gul-Pesendorfer preferences with concave temptation function (*i.e.,* $V(c,e) = v(c)$ and $v', -v'' > 0$),her utility without government intervention, $W_1^{GP1}(0,0)$, is $u(c) + \lambda v(c) + \delta u(e) - \lambda v(y)$ where (c, e) is defined by $u'(c) + \lambda v'(c) = \delta u'(e)$ and $c + e = y$. $\frac{\partial W_1^{GP1}(0,0)}{\partial \gamma}$ is negative since $\frac{\partial W_1^{GP1}(0,0)}{\partial \lambda} < 0$ and $\frac{\partial \lambda}{\partial \gamma} > 0$.

Finally, when an agent has Gul-Pesendorfer preferences with hyperbolic temptation function (*i.e.,* $V(c,e) = u(c) + \beta\delta u(e)$), her utility without government intervention, $W_1^{GP2}(0,0)$, is $(1+\lambda)\{u(c) + \delta\gamma(\beta,\lambda)u(e)\} - \lambda\{u(c') + \delta u(e')\}$ where (c, e) is defined by $u'(c) = \delta\gamma(\beta,\lambda)u'(e)$ and $c + e = y$ and (c', e') is defined by $u'(c) = \delta\beta u'(e)$ and $c'+e'= y$. $\frac{\partial W_1^{GP2}(0,0)}{\partial \gamma}$ is negative since $\frac{\partial W_1^{GP2}(0,0)}{\partial \lambda} < 0$, $\frac{\partial \lambda}{\partial \gamma} > 0$, $\frac{\partial W_1^{GP2}(0,0))}{\partial \beta} > 0$ and $\frac{\partial \beta}{\partial \gamma} < 0$.

Suppose a government provides vouchers large enough to constrain the agent's choice. The optimal level of vouchers among those which constrain the agent's choice is $b = \max\{b^*, b^c\}$ where b is defined by $u'(y - b - g(b))(1 + g'(b)) = \delta u'(b)$ and b^c is the minimum level of vouchers which constrains the agent's choice. The agent's utility with vouchers amounting to b is $W_1^H(b,0) = W_1^{GP1}(b,0) = W_1^{GP2}(b,0) = u(y - b - g(b)) + \delta u(b)$.

Since $\frac{\partial b^*}{\partial \gamma} = 0$ and $\frac{\partial b^c}{\partial \gamma} < 0$, $\frac{\partial b}{\partial \gamma} = 0$ and hence the agent's utility is constant (*i.e.,* $\frac{\partial W_1^H(b,0)}{\partial \gamma} = \frac{\partial W_1^{GP1}(b,0)}{\partial \gamma} = \frac{\partial W_1^{GP2}(b,0)}{\partial \gamma} = 0$) if γ is sufficiently large.

While the agent's utility without government intervention decreases in γ, the agent's utility with vouchers is constant if γ is sufficiently large. Therefore, there exists a γ^* s.t. for all $\gamma \geq \gamma^*$ no intervention is not optimal. ■

6.2 Proposition 7

Proof. Since the constraint associated with vouchers does not bind, the first order condition determines the choice of (c_1, e) by period-1 *self* (*i.e.,* (c_1, e) is determined by $u'(c) + \lambda v'(c) = (1+s)\delta u'(e)$ and $c = y - e(b,s) - g(b + \frac{s}{1+s}e(b,s))$.

Using the implicit function theorem, we find that

$$\frac{\partial e}{\partial b} = \frac{-g'(\tau)}{\{(1+s)\delta\frac{u''(e)}{\{u''(c)+\lambda v''(c)\}} + 1 + \frac{s}{1+s}g'(\tau)\}} < 0 \tag{12}$$

$$\frac{\partial c}{\partial b} = -\frac{\partial e}{\partial b} - g'(\tau)(1 + \frac{s}{1+s}\frac{\partial e}{\partial b}) < 0 \tag{13}$$

Given the assumptions we made for u and g, $\frac{\partial e}{\partial b}$ and $\frac{\partial c}{\partial b}$ are negative. Therefore, we can construct a transfer $(b-\epsilon, s)$ with $\epsilon > 0$ and $\epsilon \simeq 0$, such that both education and the consumption good chosen by period-1 *self* under $(b-\epsilon, s)$ are larger than that under (b, s). This result arises from the fact that disposable income $(y^d \equiv y + b - \tau - g(\tau))$ increases as the government changes its transfer from (b, s) to $(b-\epsilon, s)$. The agent's utility, W_1, is $\{u(c_1) + \delta u(e)\} + \lambda\{v(c_1) - v(y^M)\}$ with $y^M = y - \tau - g(\tau)$ and $\tau = b + \frac{s}{1+s}e$. Since the agent consumes more goods with $(b-\epsilon, s)$ and $\frac{\partial b}{\partial y^M} < 0$, $W_1(b-\epsilon, s)$ is larger than $W_1(b, s)$ if $\frac{\partial v(c_1) - v(y^M)}{\partial y^D} < 0$. ■

6.3 Proposition 8

Proof. Let's denote $(c_1^N(b,s), e^N(b,s))$ as the agent's choice if there is no restriction on the use of vouchers. Since the vouchers constrain the agent's choice, the agent's choice of education, $e(b,s)$, is larger than $e^N(b,s)$ (*i.e.*, $e(b,s) > e^N(b,s) \Rightarrow e(b,s) = b$).

Using the implicit function theorem, we find that

$$\frac{\partial e^N}{\partial s} = \frac{-\{(u''(c_1^N) + \lambda v''(c_1^N))\frac{g'(\tau)}{(1+s)^2}e^N + \delta u^{'}(e^N)\}}{(1+s)\delta u''(e^N) + (u''(c_1^N) + \lambda v''(c_1^N))(1 + \frac{s}{1+s}g'(\tau))} \tag{14}$$

Since $\frac{\partial e^N}{\partial s}$ is finite, we can construct another transfer, $(b, s-\epsilon)$ with $\epsilon > 0$ such that $e(b, s-\epsilon) > e^N(b, s-\epsilon)$ $(i.e., e(b, s-\epsilon) = b)$.Since the level of education consumption remains the same but the subsidy rate declines, the amount of tax, τ, decreases. Hence, the agent's utility with $(b, s-\epsilon)$ is higher than that with (b, s) because $c(b, s-\epsilon) = y - \tau' - g(\tau')$ is larger than $c(b, s)$. The agent's utility, W_1, is $\{u(c_1) + \delta u(e)\} + \lambda\{v(c_1) - v(y^M)\}$ with $y^M = y - \tau - g(\tau)$ and $\tau = b + \frac{s}{1+s}e$. Since the agent consumes more goods with $(b, s-\epsilon)$ and $\frac{\partial s}{\partial y^M} < 0$, $W_1(b, s-\epsilon)$ is larger than $W_1(b, s)$ if $\frac{\partial v(c_1) - v(y^M)}{\partial y^D} < 0$. ■

6.4 Example 1

Suppose an agent has log utility and a government's loss function is linear. Then, the government's problem can be written as follows:

$$\begin{aligned} \max_{(c,e)\in B_1(b,s)} \quad & \{\log(c_1) + \delta \log(e)\} \\ & \text{where } (c_1, e) \in \operatorname*{arg\,max}_{(c_1,e)\in B_1(b,s)} \log(c_1) + \beta\delta \log(e) \\ & \tau = k + b + \frac{s}{1+s}e, \; g(\tau) = \tau \geq 0 \end{aligned}$$

The agent's utility with no intervention is $(1+\delta)\log(y) + \delta\log(\beta\delta) - (1+\delta)\log(1+\beta\delta)$. The optimal price subsidy rate, s, is $\frac{1-2\beta-\delta\beta}{2\beta}$ if $\beta \in (0, \frac{1}{2+\delta})$ and zero otherwise. The agent's utility with the price subsidy is $\log\frac{y}{(1+\delta)(1-\beta\delta)} + \delta\log\frac{\delta y}{2(1+\delta)}$ if $\beta \in (0, \frac{1}{2+\delta})$. The optimal voucher,$b$, is $\frac{\delta y}{2(1+\delta)}$ if $\beta < \frac{1}{2}$ and otherwise $\frac{\beta\delta}{1+2\beta\delta}y$.

The agent's utility under optimal voucher is $\log \frac{y}{(1+\delta)} + \delta \log \frac{\delta y}{2(1+\delta)}$ if $\beta < \frac{1}{2}$, and $(1+\delta)\log(y) - (1+\delta)\log(1+2\beta\delta) + \delta \log(\beta\delta)$ if $\beta > \frac{1}{2}$.

According to Proposition 5, when $\beta = 0$, the voucher is optimal. For $\beta \in (0, \frac{1}{2+\delta})$, the price subsidy is optimal. For $\beta > \frac{1}{2+\delta}$,no intervention is optimal.

6.5 Example 2

Suppose $u(c) = v(c) = \log(c), g(\tau) = \tau$ and $\beta = 0$. Then, the government's problem is to choose (b,s), which solves the following problem:

$$\max_{(c,e)\in B_1(b,s)} \quad \{(1+\lambda)\log(c) + \delta \log(e)\} - \lambda \max_{(c')\in B_1(b,s)} \log(c') \tag{15}$$
$$\text{where } \tau = k + b + \frac{s}{1+s} * e$$

The agent's utility with no intervention,$u(0,0)$, is $(1+\delta)\log(y) + (1+\lambda)\log(\frac{1+\lambda}{1+\lambda+\delta}) + \delta \log(\frac{\delta}{1+\lambda+\delta})$. The optimal price subsidy rate,s, is $\frac{\lambda-(1+\delta)}{2}$ if $\lambda \geq (1+\delta)$ and zero otherwise. Then, the agent's utility is $u(0,0) + (1+\delta)\log(\frac{1+\lambda+\delta}{1+\lambda+\delta(\lambda-\delta)}) + \delta \log(\frac{\lambda+1-\delta}{2})$ if $s = \frac{\lambda-(1+\delta)}{2}$. The optimal voucher,$b$, is $\frac{\delta}{2(1+\delta)}y$ if $\lambda \geq 1$ and $\frac{\delta}{(1+\lambda+2\delta)}y$ if $0 \leq \lambda \leq 1$. The agent's utility under the optimal voucher is $(1+\delta)\log(y) + \log(\frac{1}{(1+\delta)}) + \delta \log(\frac{\delta}{2(1+\delta)})$ if $\lambda \geq 1$ and $(1+\delta)\log(y) + \log(\frac{(1+\lambda+\delta)}{(1+\lambda+2\delta)}) + \delta \log(\frac{\delta}{(1+\lambda+2\delta)})$ if $0 \leq \lambda \leq 1$.

There exists a λ_1 with $1 < \lambda_1 \leq 1+\delta$ s.t. for all $\lambda \geq \lambda_1$, the voucher is optimal and for $1 \leq \lambda \leq \lambda_1$, no intervention is optimal. When $\lambda \in (0,1)$, either voucher or no intervention is optimal, depending on parameter values and when $\lambda = 0$, no intervention is optimal.

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