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Rising Wage Inequality: Does the Return to Management Tell the Whole Story?

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Abstract

This paper argues that the increased wage inequality observed in recent years is driven by changes in management compensation. The analysis is conducted within the framework of a two-sector search model with heterogeneous employees and heterogenous jobs i.e. employees with different educational levels who work in either management or the non-management sector of a firm. Individuals employed in the non-management sector search for management jobs while employed. This model characterizes the labor market flows, the firm's structure and the employee composition as well as the wage distribution in the firm. Using the personnel records from a large pharmaceutical company, the parameters of the model are estimated. This allows us to conclude that the increased wage inequality observed is due to amplified *within* and *between* group wage inequality which is driven by an increased gap between management and non-management wages.

Keywords: Wage inequality, Two-sector search model, Skill-biased technological change, Personnel data.

JEL codes: J3, J6, M5, O3

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1 Introduction

In recent decades, the United States and other developed countries have experienced increased wage inequality. This development is driven partly by a widening of the wage differentials *between* skilled and unskilled workers with a particular rise in the relative earnings of college graduates and partly by an increase in wage dispersion *within* narrowly defined education-experience cells.[1] The widening in the wage distribution is especially pronounced in the period after the mid-1970s and accelerated up through the 1980s. During the same period of time, management compensation is revolutionized. Over the period 1970 to 1996, CEO cash compensation doubled and realized pay nearly quadrupled, see Murphy (1999) and Hall and Murphy (2003). That the change in management compensation may have real and sizeable consequences for wage inequality is implicitly shown by Autor, Katz and Kearney (2005) who find that: *"fully 90 percent of the net increase in male 90-10 earnings inequality between 1979 and 2003 is accounted for by the rise in the 90-50 wage gap".* Thus the action in the wage distribution is clearly in the upper tail. Motivated by these observations, this paper argues that there is a close relation between the changes in management compensation and the development in wage inequality. In particular, we show that an increased wage gap between non-management and management employees can explain a significant part of the increased inequality both *between* and *within* educational groups.

The analysis of the wage distribution is conducted in a joint theoretical-empirical framework. The theoretical model is used to characterize the wage distribution of a representative firm. Furthermore, it provides a detailed decomposition of the wage distribution which can be used to identify the underlying driving forces leading to increased wage dispersion. In the empirical analysis, point estimates of the model's parameters are obtained. Given the structure imposed by the model, the empirical results reveal that the changes in the wage distribution is due to general and skill biased technological change. Finally, the identified technology shocks are used to predict employment responses.

The theoretical model which is presented in the next section is a two-sector search model with on-the-job search a la Pissarides (1994, 2000).[2] In the model, both jobs and workers are heterogenous. Workers are distinguished by their education level. Some workers have a basic education level and are referred to as "low-skilled". Other workers have a higher level of education and are referred to as "high-skilled". Both types of workers are employed in a firm with a non-management and a management sector. In our setup and analysis we follow an approach similar to McKenna (1996), Albrecht and Vroman (2002) and Gautier (2002). But contrary to these authors, we do not restrict the job search of low-skilled individuals.

The theoretical model provides a complete description of the employee flows related to a representative firm such as transitions from the external labor market and into the firm (hirings), reallocations within the firm (promotions) and separations. These flows fully identify the size, structure and em-

[1] These findings are documented by Katz and Murphy (1992), Murphy and Welch (1992), Juhn et al (1993), Levy and Murnane (1992), Bartel and Sicherman (1999), Katz and Autor (1999) and Autor, Katz and Kearney (2005).

[2] The job assignment model used by Bernhardt (1995), Gibbons and Waldman (1999) and Frederiksen and Takáts (2005) provides an alternative to the search model applied in the present analysis. In contrast to the job assignment model which is focused on the underlying selection and sorting processes and the individual's wage formation, the present analysis is more in line with the search literature which is focused on providing a complete treatment of worker flows and wage dispersion. See the discussions in Mortensen and Pissarides (1999) and Burdett and Mortensen (1998).

ployee composition in the firm which is the first component essential for the description of the wage distribution. The second component is the relative wages of the different employee subgroups. In the model these wages are determined by the employees' productivity and Nash bargaining.[3] Thus, the model provides a complete characterization of the wage distribution.

The model's parameters are estimated using seven years of monthly personnel records (1997 to 2003) from the main production site of an international pharmaceutical company. The advantage of these data compared to traditional labor market data sets is the information about the employees' allocation to jobs within the firm, i.e. whether the worker is employed in management or non-management.[4] This allows for estimation of the wage differentials across various employee subgroups and for studying how they develop over time.

Using the close link between wages and productivity established in the theoretical model, we can conclude that technology shocks increase the productivity level in the firm over the period leading to an overall real wage growth of 1.69 percent. Furthermore, the firm is hit by a series of skill-biased shocks that alter the relative wages. In particular, it is established that the relative wages of management employees have grown substantially over the period.

The main finding in the paper is that the increased wage inequality observed is due to amplified *within* and *between* group wage inequality which is driven by an increased gap between management and non-management wages. This follows from the observation that the overall return to education has increased by 2 percent over the period 1997 to 2003, but the return to education in non-management has increased by only 1.51 percent and the return to education in management decreased by 13.90 percent in the same period. This indicates that the within-rank developments in the relative wages between high- and low-skilled employees are unable to explain the overall increase in the return to education. Instead, it is driven by a divergence of relative wages between management and non-management employees.

Two key results follow from this observation. First, the higher *between* educational group wage differential (the returns to education) is driven by changes in the wage differentials between non-management and management. Second, a large part of the *within* educational group wage differential is due to job assignment. This shows that changes in the internal labor market of the firm such as relative wage dynamics and employee composition effects play a prominent role in explaining the recently observed changes in wage inequality.

Section 2 presents the theoretical model and provides a discussion of the effects of technological change on the wage structure. Section 3 presents the data and describes the changes in the firm's wage distribution. The empirical analysis is conducted in section 4. Section 5 examines the endogenous employment responses that will follow the technology shocks which are identified in the empirical analysis and study their effects on the wage structure. Finally, section 6 summarizes and concludes.

[3] The empirical relevance of the bargaining model is established by Lazear and Oyer (2004a, b) who show that firm-fixed-effects matter for wage determination.

[4] The use of employer-employee data has become widespread in recent years, see Abowd and Kramarz (1999). Few of these data sets, however, contain information about the hierarchical level of the employees. An exception is the Swedish data used by Lazear and Oyer (2004a, b).

2 The Model

Consider a (representative) firm facing a labor force of N workers. The workers employed by the firm are members of the internal labor market denoted by I, the remaining individuals constitute the external labor force, E. The size of the total labor force is normalized to unity, i.e. $N = I + E = 1$. All workers are distinguished by an observable education attainment where the proportion π (π is exogenously given) of the workers is low-skilled and the remaining $1 - \pi$ is high-skilled. High-skilled workers will be referred to as H-workers while low-skilled workers will be referred to as L-workers.

The firm has two sectors: Management (M) and non-management (NM). The jobs in management are heterogeneous as they have skill requirements. For instance, some management jobs in production require detailed knowledge about the production process which is only possessed by craftsmen (low-skilled). Supervisory administrative jobs, on the other hand, cannot be occupied by craftsmen as they do not have the required skills. Instead, these jobs are productive only if they are filled with high-skilled employees (accountants, lawyers etc.). For this reason, the model explicitly distinguishes between management jobs that require high-skilled and low-skilled workers.[5] The vacancies associated with these jobs are referred to as H-vacancies and L-vacancies. The vacancies in non-management are denoted as NM-vacancies and they can be occupied by a worker of any skill type as in Albrecht and Vroman (2002) and Gautier (2002).

The output generated by L- and H- workers in non-management and management are y_{Lj} and y_{Hj}, respectively, where $j = NM, M$. Two assumptions regarding the productivity of the different employee subgroups are imposed. First, high-skilled workers are more productive than low-skilled workers because schooling gives the workers a higher production capacity.[6] Thus it follows that $y_{Lj} < y_{Hj}$, $j = NM, M$. Second, we assume like Rosen (1982) that workers in non-management jobs have limited discretion over resources and hence are less productive than employees in management jobs who control resources. This implies that $y_{LNM} < y_{HNM} < y_{LM} < y_{HM}$.

When the firm opens up a vacancy, the type is determined ex-ante. Thus, the potential employees will take the restrictions on productivity and the skill requirements advertised with the job as given when looking for a new job. This allows for the following description of the job search behavior, see Figure (1). High-skilled individuals have the highest productivity in a management job, hence both H-external (e_H) and H-internal currently employed in a non-management job (i_{HNM}) will be looking for H-vacancies (v_H) in the management sector. L-external (e_L) and L-internal employed in the non-management sector (i_{LNM}) cannot get H-vacancies. So low-skilled workers are looking for vacancies matching their type. As for the high-skilled workers, the productivity of L-workers is highest in the management sector. This implies that both L-external and L-internal employed in the non-management sector are looking for L-vacancies in the management sector (v_L). The employees currently working in management have no incentives to search for a new job. Thus only individuals in the external labor market search for vacancies in the non-management sector (v_{NM}).

[5] This corresponds to assuming that high- and low-skilled employees are imperfect substitutes in management. For a detailed discussion of this assumption, see Freeman and Katz (1994).

[6] Gautier (2002) argues that high-skilled workers need not be more productive in all low-skilled jobs. However, he agrees that some high-skilled workers are more productive at some unskilled jobs than unskilled workers. We simply assume, in accordance with the data, that high-skilled workers are more productive than low-skilled workers in all types of jobs considered in this analysis.

Workers and vacancies meet each other randomly according to a matching function that is increasing in its argument, concave and homogenous of degree 1.[7] This implies that the matching process between jobs and workers can be expressed as follows

$$\begin{aligned} x_{HM} &= x_{HM}(v_H, \eta_H e_H + i_{HNM}) = \lambda(v_H)^{\alpha}(\eta_H e_H + i_{HNM})^{1-\alpha}, \\ x_{LM} &= x_{LM}(v_L, \eta_L e_L + i_{LNM}) = \lambda(v_L)^{\alpha}(\eta_L e_L + i_{LNM})^{1-\alpha}, \\ x_{NM} &= x_{NM}(v_{NM}, e_L + e_H) = \lambda(v_{NM})^{\alpha}(e_L + e_H)^{1-\alpha}. \end{aligned}$$

Where $0 \leq \eta_H, \eta_L \leq 1$. The parameters η_H and η_L characterize the firm's hiring strategy and reflects that the firm may have preferences for incumbent employees when filling management vacancies. Naturally, this reduces the external employee's (relative to the internal employee's) probability of getting a job in management.

Tournament games provide a motivation for choosing $\eta < 1$, see Lazear and Rosen (1981). If the number of vacancies in management are fixed ex-ante, it will create competition between the employees having a desire for the jobs which has a direct effect on the employee's productivity i.e. $y(\eta)$. McLaughlin (1988) showed that the incentives arising from the tournament are decreasing in the number of individuals competing for a particular vacancy. For this reason, the firm may want to reduce the probability that a management vacancy is given to an individual from the external labor market in order to sustain a sufficiently high level of productivity in the firm. This can be done by reducing η. The extreme case where $\eta = 0$ corresponds to a situation where the firm has ports of entry, see Doeringer and Piore (1971). This may not be optimal, however, because employees who know the competition may collude to shirk. Hence, to break any attempts of collusion the hiring from outside is potentially fruitful, see Chan (1996). These considerations determine the values of η_H and η_L.[8] In the following, we assume that η is chosen optimally ex-ante by the firm.[9]

The implications of the firm's hiring strategy on the flows in the labor market are illustrated in Figure 1. In the case where the firm does not have ports of entry, individuals in the external labor market are hired into both the non-management and management levels. If a person is hired into non-management, he or she may have the opportunity to move into management. All workers employed by the firm have an exogenous separation risk, hence there are flows from all hierarchal levels in the firm and back into the external labor market. In the case where the firm has ports of entry, all flows from the external labor market are into the non-management sector. This implies that the only way a person gets a job in management is by accepting a job in non-management and subsequently being promoted into management. As before separations take place from both sectors.

[7] The matching function gives the number of job matches that take place at each point in time. Thus it is the analogous of the production function but "inputs" are vacancies and job seekers. For more details on the matching function, the reader is referred to Mortensen and Pissarides (1999) and Pissarides (2000).

[8] An additional important point is that tournament games give incentives to sabotage and undermine team effort as shown in Lazear (1989). In industries where team work is important (and sabotage is potentially costly), tournament games can be counterproductive. Thus, when modeling incentives in such industries, competition among employees should be modeled explicitly.

[9] Since low and high skilled workers are imperfect substitutes in management there are two tournaments in the firm, i.e. one for the low skilled and one for the high skilled. This implies that the employees competing for a management vacancy have the same production capacity and hence they will exert the same level of effort in equilibrium. In other words, for given $\eta' s$ each employee subgroup can be associated with a unique level of production.

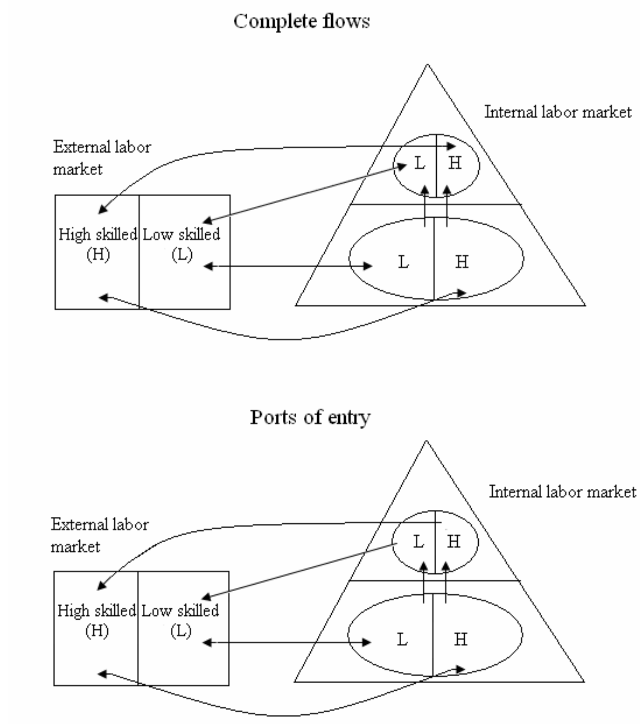

Figure 1: Flow diagrams.

To simplify notation, the following variables are defined

$$\begin{aligned}\theta_H &= \frac{v_H}{\eta_H e_H + i_{HNM}},\\ \theta_L &= \frac{v_L}{\eta_L e_L + i_{LNM}},\\ \theta_{NM} &= \frac{v_{NM}}{e_L + e_H}.\end{aligned}$$

Furthermore, let q_H be the probability that a H-job meets a H-worker in any time period. Similarly, if q_{NM} is the probability that a non-management job is matched to a worker and q_{LM} is the probability that a L-job in the management sector is matched to a worker, then under the CRS assumption it can be shown that[10]

$$\begin{aligned}q_{HM}(\theta_H) &= \frac{x_{HM}}{v_H} = \frac{x_{HM}(\theta_H, 1)}{\theta_H},\\ q_{LM}(\theta_L) &= \frac{x_{LM}}{v_L} = \frac{x_{LM}(\theta_L, 1)}{\theta_L},\\ q_{NM}(\theta_{NM}) &= \frac{x_{NM}}{v_{NM}} = \frac{x_{NM}(\theta_{NM}, 1)}{\theta_{NM}}.\end{aligned}$$

In addition, let p_H be the probability that a H-worker encounters a H-job in any time period, p_{NM} is the rate at which a worker meets a non-management job and p_{LM} is the rate at which a worker

[10] See for instance Pissarides (2000).

meets a L-job in management, then again under the CRS assumption

$$\begin{aligned} p_{HM}(\theta_H) &= \frac{x_{HM}}{\eta_H e_H + i_{HNM}} = x_{HM}(\theta_{HM}, 1), \\ p_{LM}(\theta_L) &= \frac{x_{LM}}{\eta_L e_L + i_{LNM}} = x_{LM}(\theta_{LM}, 1), \\ p_{NM}(\theta_{NM}) &= \frac{x_{NM}}{e_L + e_H} = x_{NM}(\theta_{NM}, 1). \end{aligned}$$

From this it follows that $p_{HM}(\theta_H) = \theta_H q_{HM}(\theta_H)$, $p_{NM}(\theta_{NM}) = \theta_{NM} q_{NM}(\theta_{NM})$ and $p_{LM}(\theta_L) = \theta_L q_{LM}(\theta_L)$.

2.1 Payoffs functions and wage determination

In the model, a match is created every time a firm and a worker meet and agree to an employment contract. In this process, the firm maximizes profits and the individual maximizes the present discounted value (PDV) of the expected income stream. For the firm to maximize profits it is faced with two questions. First, it has to make a decision on whether to open a vacancy or not. Second, it should maximize the PDV of the expected profit from recruiting a particular worker. For the individual the decision is to assess if the job offered is the most attractive given the alternative options such as other employment or continued job search. In addition, an important part of the job creation process is wages determination. A consequence of the job search process is that employment matches entail rent.[11] This rent is, following the convention, assumed to be split between the firm and the worker according to a Nash bargaining rule. Hence, a proportion of the value of the match is allocated to the firm and the rest is given to the worker in terms of wages. In the following, we are explicit about how these processes evolve.

2.1.1 The firm

The firm advertises three different types of jobs. It is as if the firm owns three different assets. It maximizes the profit from a vacant non-management job, a L-vacancy in the management sector and a H-vacancy in the management sector. Furthermore, the firm potentially recruits two types of workers. As outlined above, the productivity of the different workers matched with the different types of jobs can be ranked as follows: $y_{LNM} < y_{HNM} < y_{LM} < y_{HM}$. Hence, in practice the firm is employing four different subtypes of workers each generating a different profit since they have separate productivities and - as will be shown below - separate costs.

Let's denote the expected PDV of having a vacant non-management job by V_{NM}. The expected PDV of having a filled non-management job, however, depends on the productivity of the worker. For this reason, it is important to distinguish between expected PDV of having a non-management job filled by a low-skilled worker, J_{LNM}, and the expected PDV of having a non-management job filled by a high-skilled worker, J_{HNM}.

In a perfect capital market, the valuation of a vacant non-management job is such that the capital cost rV_{NM} (where r is the discount rate) equals the rate of return on the asset. Recalling that the job is filled by a high-skilled or a low-skilled worker with probability q_{NM}, then the rate of return on the

[11] The rent is a consequence of the friction imposed by the matching technology.

asset can be written as the difference between the cost of the vacant non-management job, c_{NM}, and the expected average return generated by having the job filled by a worker of any type. The average return of having a low-skilled or a high-skilled worker in the job is equal to the sum of the returns generated by low-skilled workers and the returns generated by high-skilled workers, weighted by the relative population size. From the discussion it follows that equation (1) will be satisfied in equilibrium

$$rV_{NM} = q_{NM}\left[\frac{e_L J_{LNM} + e_H J_{HNM}}{e_L + e_H} - V_{NM}\right] - c_{NM}. \tag{1}$$

Using the same intuition, the valuation of a non-management job filled by a low-skilled worker can be written such that the capital cost rJ_{LNM} equals the return on the asset. In this case, the net return on the asset is equal to the output produced by a low-skilled worker in a non-management job minus the cost of filling the job, w_{LNM}, which is the wage paid to the worker. In addition to this, the eventual loss of revenues that occur if the worker and the firm separate, which happens with probability s, and the potential loss if the worker finds a job in the management sector must be added. This amounts to[12]

$$rJ_{LNM} = y_{LNM} - w_{LNM} + s(V_{NM} - J_{LNM}) + p_{LM}(V_{NM} - J_{LNM}). \tag{2}$$

Similarly, the asset value of hiring a high-skilled worker to a non-management job, J_{HNM}, is

$$rJ_{HNM} = y_{HNM} - w_{HNM} + s(V_{NM} - J_{HNM}) + p_{HM}(V_{NM} - J_{HNM}), \tag{3}$$

where w_{HNM} is the wage earned by a high-skilled worker in non-management.

We can now derive the remaining expected PDVs of the firm's assets in the same way. The expected income streams of having a L- or H- vacancy, V_{LM}, V_{HM}, are

$$rV_{LM} = q_{LM}(J_{LM} - V_{LM}) - c_{LM}, \tag{4}$$

and

$$rV_{HM} = q_{HM}(J_{HM} - V_{HM}) - c_{HM}, \tag{5}$$

where c_{LM} and c_{HM} are the vacancy costs for L- and H-vacancies respectively.

Finally, the expected profit from recruiting a low-skilled worker to a management job, J_{LM} is given by

$$rJ_{LM} = y_{LM} - w_{LM} + s(V_{LM} - J_{LM}), \tag{6}$$

and the value to the firm of having a management job filled by a high-skilled worker, J_{HM} is

$$rJ_{HM} = y_{HM} - w_{HM} + s(V_{HM} - J_{HM}), \tag{7}$$

where w_{HM} and w_{LM} are the wages for high- and low-skilled workers in management.

[12] Note that in the special case where the firm has perfect ports of entry i.e. $\eta_L = \eta_H = 0$ then P_{LM} and P_{HM} are independent of e_L and e_H.

2.1.2 The worker

In the following, the returns to the worker will be derived. A low-skilled worker who is not employed by the firm will search for a job in both the management and non-management sectors. In contrast, if the low-skilled worker is already employed in the non-management sector, he or she will only search for jobs in management. When employed in non-management, the worker will earn w_{LNM} and given employment in management the wage is w_{LM}. For simplicity we assume that the income the worker would get when unemployed, i.e. unemployment insurance benefits, is set to zero.

Let E_L be the present discounted value of the expected income stream of a low-skilled worker not employed by the firm. In unit time, the individual may move either to a job in the non-management sector or to a L-job in the management sector. The first event occurs with probability p_{NM}. The worker would then earn the present discounted value of the expected income stream W_{LNM} until he finds a job in management or separates from the firm. If the individual instead gets a L-job in the management sector (which occurs with probability p_{NM}), he will earn the present discounted value of the expected income stream W_{LM} until a separation occurs. In summary, E_L is equal to the expected capital gain from a change of state or,

$$rE_L = p_{NM}(W_{LNM} - E_L) + p_{LM}(W_{LM} - E_L). \tag{8}$$

In a similar way, the asset value of being in the external labor market for a high-skilled worker, E_H can be determined as

$$rE_H = p_{NM}(W_{HNM} - E_H) + p_{HM}(W_{HM} - E_H). \tag{9}$$

The permanent income of a low-skilled worker employed in non-management W_{LNM} is given by the wage, w_{LNM}, the risk premium against unemployment and the option value of getting a job in management

$$rW_{LNM} = w_{LNM} + s(E_L - W_{LNM}) + p_{LM}(W_{LM} - W_{LNM}), \tag{10}$$

where W_{LM} is the expected income stream earned by a low-skilled employee in a management job.

Employees already working in management do not search for a new job, but they face the separation probability s. Thus, the permanent income of a low-skilled individual employed in management is

$$rW_{LM} = w_{LM} + s(E_L - W_{LM}). \tag{11}$$

In a similar way, we can derive the permanent income of high-skilled individuals employed in a non-management job, W_{HNM}, and a management job, W_{HM}, as follows

$$\begin{aligned} rW_{HNM} &= w_{HNM} + s(E_H - W_{HNM}) + p_{HM}(W_{HM} - W_{HNM}), && (12) \\ rW_{HM} &= w_{HM} + s(E_H - W_{HM}). && (13) \end{aligned}$$

2.1.3 Wages determination

The final step is to derive an expression for wages. Following the convention in the literature, the firm and the worker negotiate about the wage using a Nash bargaining rule. Furthermore, it is assumed that a worker negotiating with a potential employer cannot simultaneously bargain for another job. In addition, the employer observes the skill of the worker when they start the bargaining.[13]

[13] Here we assume that bargaining fails at the same rate as jobs are destroyed, Gautier (2002).

The disagreement payoff, D_{LNM}, for low-skilled workers in the external labor force bargaining for the wage in non-management satisfies the asset equation[14]

$$rD_{LNM} = s(E_L - D_{LNM}) + p_{LM}(W_{LM} - D_{LNM}). \tag{14}$$

In words, the present discounted value of the expected disagreement payoff of a low-skilled worker who is not employed by the firm is equal to the PDV the individual would expect to get from bargaining with the firm and simultaneously continue searching for a job in the management sector. If the negociation fails (with probability s) he goes back to the external labor market and earns E_L. If the firm and the worker do not disagree or if the worker does not find a job in management (with probability p_{LM}), the bargaining goes on and the worker earns the payoffs of a non-management job. The remaining disagreement payoffs are derived in the same way. D_{HNM} is the disagreement payoff for external high-skilled workers bargaining for non-management wages, and D_{LM} and D_{HM} are the payoffs for low- and high-skilled workers employed in a non-management job. They satisfy

$$\begin{aligned} rD_{LM} &= s(E_L - D_{LM}), & (15)\\ rD_{HNM} &= s(E_H - D_{HNM}) + p_{HM}(W_{HM} - D_{HNM}), & (16)\\ rD_{HM} &= s(E_H - D_{HM}). & (17) \end{aligned}$$

Finally, the value of the match $(W - D) + (J - V)$ is shared between the worker and the firm using the Nash bargaining rule, see Bimore et al. (1986).[15] This requires that the proportion β of the match value goes to the firm and the remaining part is given to the worker. Hence, the following expressions will be satisfied

$$\begin{aligned} (1-\beta)(W_{LNM} - D_{LNM}) &= \beta(J_{LNM} - V_{NM}), & (18)\\ (1-\beta)(W_{HNM} - D_{HNM}) &= \beta(J_{HNM} - V_{NM}), & (19)\\ (1-\beta)(W_{LM} - D_{LM}) &= \beta(J_{LM} - V_{LM}), & (20)\\ (1-\beta)(W_{HM} - D_{HM}) &= \beta(J_{HM} - V_{HM}). & (21) \end{aligned}$$

2.2 Equilibrium

The steady state equilibrium is characterized by two restrictions. First, the labor market flows are stable. Second, all profit opportunities in the market are exhausted. An immediate consequence of focusing on the steady state is that all the flows related to the firm such as transitions from the external labor market and into the firm, reallocations within the firm, and separations can be described. Given these flows are stable, the structure of the firm and the composition of the workforce can be determined. This involves characterizing the proportion of the employees working in management and non-management as well as identifying the skill composition in both sectors. Finally, the wages for each employee subgroup are derived. Combining this information with knowledge about the workforce composition, the wage structure in the firm can be determined.

[14]Employees leave their current job when bargaining for a new job.

[15]Previously, this apporach has been used by Diamond (1982), Mortensen (1982) and Pissarides (1987).

2.2.1 Steady state flows and the employment composition

The first steady state condition equates the flow of low-skilled employees into non-management to the flows out of that state

$$p_{NM}e_L = (s + p_{LM})i_{LNM}. \tag{22}$$

That is, the number of low-skilled in the external labor market who are successful in finding non-management jobs equals the number of individuals leaving that state either for a job in management or due to a separation from the firm. A similar equation can be derived for high-skilled workers in non-management

$$p_{NM}e_H = (s + p_{HM})i_{HNM}. \tag{23}$$

An additional steady state condition which has to be satisfied is that the flows of low-skilled workers into and out of management are equal

$$p_{LM}(\eta_L e_L + i_{LNM}) = si_{LM}. \tag{24}$$

Finally, the fourth steady state condition equates the flows into and out of management for high-skilled workers

$$p_{HM}(\eta_H e_H + i_{HNM}) = si_{HM}. \tag{25}$$

Important to note is that in a firm with perfect ports of entry ($\eta_L = \eta_H = 0$) individuals in the external labor market are restrained from getting management jobs, hence equations (24) and (25) are reduced to[16]

$$\tilde{p}_{LM}i_{LNM} = si_{LM}$$

and

$$\tilde{p}_{HM}i_{HNM} = si_{HM}.$$

Thus, as expected the firm's hiring policy has implications for the steady state flows and as will be seen below also for the equilibrium composition of the employees.

Recalling that the proportion of low-skilled in the labor market is denoted by π, i.e.

$$\begin{aligned} i_{LNM} + i_{LM} + e_L &= \pi, \\ i_{HNM} + i_{HM} + e_H &= 1 - \pi, \end{aligned}$$

[16] The $\tilde{}$'s are used to denote that the probabilities are altered together with η.

the steady state employee composition can be derived from the equations (22) to (25)

$$i_{LNM} = \frac{s p_{NM} \pi}{(p_{LM}+s)(p_{NM}+\eta_L p_{LM}+s)}, \tag{26}$$

$$i_{LM} = \frac{p_{LM}\pi\left[p_{NM}+\eta_L(p_{LM}+s)\right]}{(p_{LM}+s)(p_{NM}+\eta_L p_{LM}+s)}, \tag{27}$$

$$e_L = \frac{s\pi}{p_{NM}+\eta_L p_{LM}+s}, \tag{28}$$

$$i_{HNM} = \frac{s p_{NM}(1-\pi)}{(p_{HM}+s)(p_{NM}+s+\eta_H p_{HM})}, \tag{29}$$

$$i_{HM} = \frac{p_{HM}(1-\pi)\left[p_{NM}+\eta_H(p_{HM}+s)\right]}{(p_{HM}+s)(p_{NM}+s+\eta_H p_{HM})}, \tag{30}$$

$$e_H = \frac{s(1-\pi)}{p_{NM}+s+\eta_H p_{HM}}. \tag{31}$$

From the steady state proportions it follows that the composition of low- and high-skilled employees in the firm reflects the composition in the labor market, i.e. the proportion of low-skilled employees in both non-management and management increase in π.

2.2.2 The wage structure

Wages are determined by the firm's incentives to maximize profits and the individual's desire to increase the present discounted value of the expected income stream that can be obtained from working. Furthermore, in equilibrium all profit opportunities from new jobs are exploited and driven to zero by free entry. In other words, we have $V_{NM} = V_{LNM} + V_{HNM} = 0$ and $V_{LM} = V_{HM} = 0$. From this, the following equilibrium condition can be derived

$$J_{LNM} = \frac{y_{LNM}-w_{LNM}}{r+s+p_{LM}}, \tag{32}$$

$$J_{HNM} = \frac{y_{HNM}-w_{HNM}}{r+s+p_{HM}}, \tag{33}$$

$$J_{LM} = \frac{y_{LM}-w_{LM}}{r+s}, \tag{34}$$

$$J_{HM} = \frac{y_{HM}-w_{HM}}{r+s}. \tag{35}$$

These equations state that in equilibrium the profits from an occupied job are equal to the difference between the worker's productivity and his wage divided by the firm's implicit discount rate[17]. Furthermore, we have that

$$W_{LNM} - D_{LNM} = \frac{w_{LNM}}{r+s+p_{LM}},$$

$$W_{HNM} - D_{HNM} = \frac{w_{HNM}}{r+s+p_{HM}},$$

$$W_{LM} - D_{LM} = \frac{w_{LM}}{r+s},$$

$$W_{HM} - D_{HM} = \frac{w_{HM}}{r+s}.$$

[17] The firm's implicit discount rate when the worker is employed in a management job is equal to the sum of the discount rate plus the separation rate. When the worker is employed in a non-management job, the firm also takes into account the fact that the worker is searching for a management job.

Using the Nash bargaining rule, the notation

$$y_{LNM} = y,\ y_{HNM} = \mu_1 y,\ y_{LM} = \mu_2 y \text{ and } y_{HM} = \mu_3 y \qquad \text{where } \mu_j > 1\ ,\ j = 1,2,3 \tag{36}$$

and

$$\begin{aligned} \kappa_0 &= i_{LM}/(i_{LNM} + i_{HNM} + i_{LM} + i_{HM}), \kappa_1 = i_{HM}/(i_{LNM} + i_{HNM} + i_{LM} + i_{HM}), \\ \kappa_2 &= i_{LNM}/(i_{LNM} + i_{HNM} + i_{LM} + i_{HM}), \text{ and } \kappa_3 = i_{HNM}/(i_{LNM} + i_{HNM} + i_{LM} + i_{HM}), \end{aligned}$$

the following proposition regarding the wage structure in the firm can be stated

Proposition 1 *Under the equilibrium conditions:*

a) *Wages are proportional to productivity i.e.* $w_j = \beta y_j\ ,\ j \in \{LNM, HNM, LM, HM\}$

b) *The distribution of wages in the firm is*

$$w = \begin{cases} w_{LNM} & \text{with Probability } \kappa_0 \\ w_{HNM} & \text{with Probability } \kappa_1 \\ w_{LM} & \text{with Probability } \kappa_2 \\ w_{HM} & \text{with Probability } \kappa_3 \end{cases}$$

with mean

$$\begin{aligned} E(w) &= \kappa_0 w_{LNM} + \kappa_1 w_{HNM} + \kappa_2 w_{LM} + \kappa_3 w_{HM} \\ &= y\beta(\kappa_0 + \kappa_1\mu_1 + \kappa_2\mu_2 + \kappa_3\mu_3), \end{aligned}$$

and variance

$$\begin{aligned} Var(w) &= E(w^2) - E(w)^2 \\ &= Var(\kappa_0, \kappa_1, \kappa_2, \kappa_3, \mu_1, \mu_2, \mu_3, y, \beta). \end{aligned}$$

The parameters of Proposition (1) will be estimated in the empirical section of the paper.

2.3 Discussion

The wage distribution has previously been analyzed by Juhn et al. (1993) and Katz and Autor (1999). It has been convention in these studies to decompose the wage dispersion into a part reflecting *between* (educational) group wage differentials and a residual part capturing the *within* (educational) group wage differentials.[18] The decomposition proposed above is more detailed than convention because it

[18] The breakdown of the wage dispersion may be even more detailed in terms of the observable demographic characteristics, see Katz and Autor (1999) for a survey.

treats the wage differentials between management and non-management employees explicitly. However, in order to make our discussion of the wage structure comparable with the previous research in the area the following definitions are made. First, the wage differential of employees with different levels of education who are working in the same type of job will be denoted as the *between* group wage inequality. Second, the wage differential of employees with the same level of education who are working in different jobs is referred to as the *within* (educational) group wage inequality. These definitions are summarized in Lemma (1)

Lemma 1 *Under the equilibrium conditions:*

a) *The* ***between*** *groups wage differentials are*

i) Non-management

$$\frac{w_{HNM}}{w_{LNM}} = \frac{\beta y_{HNM}}{\beta y_{LNM}} = \mu_1$$

ii) Management

$$\frac{w_{HM}}{w_{LM}} = \frac{\beta y_{HM}}{\beta y_{LM}} = \frac{\mu_3}{\mu_2}$$

b) *The* ***within*** *group wage differentials are*

i) Low-skilled

$$\frac{w_{LM}}{w_{LNM}} = \frac{\beta y_{LM}}{\beta y_{LNM}} = \mu_2$$

ii) High-skilled

$$\frac{w_{HM}}{w_{HNM}} = \frac{\beta y_{HM}}{\beta y_{HNM}} = \frac{\mu_3}{\mu_1}$$

Furthermore, as information on the employee allocation to jobs within the firm is unavailable in conventional labor market studies, it is interesting also to provide a decomposition of wage dispersion which does not require this information. Lets define *"the return to education"* as the wage differential between high- and low-skilled employees in the firm and for the sake of completeness lets define *"the return to management"* as the wage differential between management and non-management employees. These definitions are summarized in Lemma (2)

Lemma 2 *Under the equilibrium conditions:*

i) The return to education is

$$\frac{w_H}{w_L} = \frac{\psi_3 w_{HM} + \psi_1 w_{HNM}}{\psi_2 w_{LM} + \psi_0 w_{LNM}} = \frac{\psi_3\mu_3 + \psi_1\mu_1}{\psi_2\mu_2 + \psi_0}$$

where $\psi_0 = i_{LNM}/(i_{LNM} + i_{HNM})$, $\psi_1 = i_{HNM}/(i_{LNM} + i_{HNM})$, $\psi_2 = i_{LM}/(i_{LM} + i_{HM})$ *and* $\psi_3 = i_{HM}/(i_{LM} + i_{HM})$, *ii) The return to management is*

$$\frac{w_M}{w_{NM}} = \frac{\psi_2 w_{LM} + \psi_3 w_{HM}}{\psi_0 w_{LNM} + \psi_1 w_{HNM}} = \frac{\psi_2 \mu_2 + \psi_3 \mu_3}{\psi_0 + \psi_1 \mu_1}$$

Lemma (2) combines the parameters $\{\mu_1, \mu_2, \mu_3\}$ with the steady state proportions derived in equations (26) to (31). Thus, the returns to education and the returns to management are shown to be functions of the underlying *within* and *between* wage inequality measures. This observation carries an important message because it shows that the measures used to decompose the wage distribution in conventional labor market studies are affected by the structure of the firm, i.e. the relative size of the management sector and wage differentials between management and non-management employees. This suggests that information on within firm job assignment is important in order to understand the structure of wages. Furthermore, Lemma (1) can be used to identify the underlying driving forces behind the changes in the more aggregate measures (such as the returns to education) that are applied in conventional studies of wage inequality.

The results obtained in Proposition (1), Lemma (1) and Lemma (2) are discussed further below.

2.3.1 Technological progress

Proposition (1) establishes a close relation between productivity and wages. This implies that technology shocks changing the productivity of the workers will have consequences for wages and potentially for wage inequality. First, technological progress increasing the common productivity level, y, will increase the average wage level in the firm, i.e. $\partial E(w)/\partial y > 0$. Furthermore, it will increase wage dispersion. The intuition is that the wage for the group of low-skilled non-management employees is increased by $\beta \Delta y$ whereas the wages for the remaining employee subgroups are increased by $\beta \mu_j \Delta y$, $j = 1, 2, 3$ where $\mu_j > 1$. Hence, simultaneously with the right shift in the wage distribution it becomes more right-skewed. Second, positive shocks to the relative productivity (such as skill-biased technological change) of the different employee subgroups increases the wage level, i.e. $\partial E(w)/\partial \mu_j > 0$, $j = 1, 2, 3$. A change in the relative productivity of the employees will in general have ambiguous effects on wage dispersion. Simulations around the observed equilibrium for the firm studied in this paper show, however, that an increase in any of the parameters $\{\mu_1, \mu_2, \mu_3\}$ will increase the wage dispersion. These results are summarized in Lemma (3)

Lemma 3 ***Technological progress*** *increases the wage level and the wage dispersion in the firm.* ***Skill-biased technological progress*** *increases the wage level in the firm, but in general it has an ambiguous effect on wage dispersion.*

Lemma (1) describes the *within* and *between* groups wage differentials. These measures are not altered by skill-neutral technology shocks as the common productivity component nets out. In contrast, they are affected by changes in the relative productivity. Hence, they respond to skill-biased technological change.

Finally, as Lemma (2) is constructed from the parameters $\{\mu_1, \mu_2, \mu_3\}$ and the steady state employment compositions, it is unaffected by skill neutral technological change as long as the structure of employment does not respond to changes in technology.[19] Skill-biased technological change, however, affects the measures. In particular, an increase in the relative productivity of high-skilled management employees ($\mu_3 \nearrow$) has a positive effect on the returns to education.

The empirical literature has documented that the recent increase in wage inequality is driven partly by a widening of the wage differentials *between* high- and low-skilled workers and partly by an increase in wage dispersion *within* narrowly defined educational groups, see the survey in Katz and Autor (1999). These findings provide a test for the model as they can be used to evaluate its ability to replicate the empirically observed dynamics.

Lets focus on an increase in the relative productivity of high-skilled management employees caused by a skill-biased technological shock ($\mu_3 \nearrow$). From Lemma (1) the shock can be seen to increase the relative wage gap between high- and low-skilled employees in management and to increase the wage dispersion within the group of high-skilled employees. Furthermore, as argued above, the change in relative productivity will increase the returns to education. Thus, a skill-biased shock to the productivity of high-skilled employees in management is capable of producing the wage dynamics observed in the labor market. Below, we show that this line of reasoning has empirical relevance.

2.3.2 Estimation procedure

The measures of wage inequality defined above are crucial for the subsequent analysis because they form the basis for obtaining empirical estimates of the model's parameters. In particular, there is a close link between the Mincer wage equation and the measures defined in Lemma (1).

The Mincer wage equation takes the following form

$$\ln(w_t) = \alpha + \mu_1^* \xi_{(w_t = w_{HNM},t)} + \mu_2^* \xi_{(w_t = w_{LM},t)} + \mu_3^* \xi_{(w_t = w_{HM},t)} + X_t' \delta + \epsilon_t$$

Where $\xi_{(.)}$ is an indicator function, X is a set of explanatory variables, ϵ is an error term and δ is a set of parameters.[20]

The point estimates obtained from the regression can be used to predict the expected log-wages for the different employee subgroups. For instance, $\ln \hat{w}_{LNM} = \hat{\alpha} + X_t' \hat{\delta}$ and $\ln \hat{w}_{HNM} = \hat{\alpha} + \hat{\mu}_1^* + X_t' \hat{\delta}$. Thus,

$$\ln \hat{w}_{HNM} - \ln \hat{w}_{LNM} = \hat{\alpha} + \hat{\mu}_1^* + X_t' \hat{\delta} - \hat{\alpha} + X_t' \hat{\delta} = \hat{\mu}_1^* \tag{37}$$

Recall from Lemma (1) that,

$$\frac{\hat{w}_{HNM}}{\hat{w}_{LNM}} = \mu_1 \Leftrightarrow \ln\left(\frac{\hat{w}_{HNM}}{\hat{w}_{LNM}}\right) = \ln \mu_1 \Leftrightarrow \ln \hat{w}_{HNM} - \ln \hat{w}_{LNM} = \ln \mu_1 \tag{38}$$

hence, combining equations (37) and (38) we get

$$\hat{\mu}_1^* = \ln \mu_1 \Leftrightarrow \mu_1 = \exp(\hat{\mu}_1^*)$$

Similar links between the parameters of the theoretical model and the parameters estimated by the Mincer equation can be made.

[19] We elaborate on this issue below.

[20] The regression is constructed such that the low-skilled employees in non-management constitute the reference group, meaning that the wage differentials captured by the parameters $\{\mu_1^*, \mu_2^*, \mu_3^*\}$ are with respect to this group only. Wage differentials between other employee subgroups not involving the low-skilled non-management employees can be obtained by combining the parameters appropriately, see Lemma (1).

In the empirical analysis conducted below various versions of the Mincer wage regression will be used.

3 The Data

Seven years of monthly personnel records from the main production size of an international pharmaceutical company are used in the empirical analysis. The average employment in the plant over the period 1997 to 2003 is 6175 persons, and the share of management workers in the firm is close to 4.5 percent in all years.[21] Empirically, the firm is close to having ports of entry since 99 percent of all workers are hired into non-management.[22] For the individual worker, the promotion probability is almost 2 percent a year and the separation rate from the firm is 3.52 percent on average.

The full data set consists of 519,016 observations. These data contain information about the employee's age, gender, tenure, educational level, job assignment and wages. The observations with missing information (mainly caused by lacking information on education) are deleted which leaves us with a sample of 487,514 observations. This is the sample used in the analysis below.

	Mean (standard deviation)
Age	39.532
	(8.946)
Tenure	7.796
	(7.266)
Gender (woman = 1)	0.550
Education (high-skilled = 1)	0.348
# observations	487,514

Table 1: Descriptive statistics, 1997-2003.

The descriptive statistics for the monthly employee-based observations used in the analysis are presented in Table 1. The firm employs 55.01 percent women and the average employee is 39.53 years old. These individuals have 7.80 years of tenure on average. The firm is operating in a segment where product development is crucial for survival. This implies that a large proportion of the employees are engaged directly in research. In addition, the production process is highly automated, and in order to meet the strict requirements of the FDA (Food and Drug Administration) product testing is an

[21] Only permanent full-time employment is considered in this analysis which corresponds to 91.76 percent of the individuals employed at the workplace. In addition, the executive management (CEO, executive vice presidents and senior vice presidents) is excluded due to lack of information on the compensation package for these individuals. For recent surveys on executive compensation, see Murphy (1999) and Hall and Murphy (2003).

[22] Doeringer and Piore (1971) argued that the presence of ports of entry and exit are crucial for the existence and sustainability of an internal labor market. This result, however, was questioned in the seminal work by Baker et al. (1994a,b) who found no evidence for ports of entry and exit in the firm they studied. Nevertheless, they argue that there was clear evidence that an internal labor market was at work. Our firm seems to be more in line with the firms studied by Doeringer and Piore.

essential part of production. This structure explains the high education level in the firm where 34.80 percent of the firm's employees have a bachelor degree or above. In the sequel, we term individuals with education levels below a bachelor degree as low-skilled and the remaining part as high-skilled.

A preliminary look at the real wage structure in the firm shows some interesting features. In Table 2, the yearly average wages for the four groups: low-skilled non-management, high-skilled non-management, low-skilled management and high-skilled management are presented. As expected, the wage level increases with education level and rank. Furthermore, it can be seen that all four groups experience real wage growth over the 7-year period, however, the growth rates differ substantially across groups. Most pronounced is the wage growth for low-skilled employees in management who have a wage increase of 25.39 percent. In contrast, the real wage increase for low-skilled in non-management is only 3.92 percent. Similar tendencies can be seen for high-skilled individuals. Thus, the wage gap between management and non-management employees increased significantly over the period.

	Employee subgroup			
	Low-skilled Non-management	High-skilled Non-management	Low-skilled Management	High-skilled Management
1997	25,950	36,520	40,425	55,104
	(3,801)	(9,738)	(7,375)	(11,869)
1998	26,300	37,021	44,768	57,912
	(3,834)	(9,781)	(8,151)	(12,735)
1999	26,478	37,666	44,446	57,055
	(3,793)	(10,212)	(8,870)	(11,329)
2000	26,572	38,017	45,836	58,060
	(3,857)	(10,388)	(10,887)	(11,757)
2001	26,858	38,446	49,712	61,097
	(4,414)	(11,061)	(11,659)	(16,071)
2002	26,864	37,529	51,470	60,688
	(4,583)	(10,371)	(17,136)	(16,150)
2003	26,967	37,847	50,689	60,636
	(4,156)	(9,302)	(12,169)	(13,358)
Real wage growth 1997-2003	3.92%	3.63%	25.39%	10.04%
# observations	316,614	149,554	1,227	20,119

Tabel 2: Real wages, 1997-2003.

4 Developments in Wage Inequality

In this section, the wage inequality in the firm is analyzed empirically and the parameters of Proposition (1) are estimated. A first step in the analysis is to determine that the observed general and relative changes in productivity and hence in wages have significant (sizeable) consequences for the wage structure in the firm. Secondly, an econometric model that accommodates the changes over time is estimated and point estimates of $\{\mu_1^*, \mu_2^*, \mu_3^*, y\}$ are obtained. This exercise pines down the magnitude by which the overall and relative productivity and wages change over the period 1997 to 2003. Hence the underlying mechanisms driving the changes in the wage structure are identified. Finally, Lemma

(1) and Lemma (2) are used to relate the findings to the existing empirical literature. We observe patterns in the data similar to those observed in other recent studies of wage inequality and conclude that the increase in wage dispersion is explained by amplified *within* and *between* educational group wage inequality which is driven by a widening in the gap between management and non-management wages.

Proposition (1) proposed a decomposition of the overall wage differential into four education-rank categories: low-skilled non-management, high-skilled non-management, low-skilled management and high-skilled management. Using Mincer wage equations, the wage differentials between these employee subgroups can be estimated and point estimates of the parameters of Proposition (1) are obtained from this regression. The results are presented in Table 3. In the first model where only the skill-rank categories are included the wage differential between low- and high-skilled non-management workers (μ_1^*) is estimated to 32.1 percent. The wage gaps between low-skilled non-management workers and low- and high-skilled management workers (μ_2^* and μ_3^*) are estimated to 55.6 percent and 78.8 percent, respectively. Adding further information to the model, i.e. including time dummies and demographic and tenure variables, reduces the wage gains from management slightly reflecting that these workers in general are older and have longer tenure than non-management workers. Overall the results have changed only marginally and the highly significant wage differentials across the four groups persist.[23]

	(1)	**(2)**	**(3)**
Low-skilled in Non-management	-	-	-
High-skilled in Non-management	0.321 (0.006)	0.322 (0.006)	0.327 (0.006)
Low-skilled in Management	0.556 (0.046)	0.557 (0.045)	0.477 (0.047)
High-skilled in Management	0.788 (0.009)	0.788 (0.009)	0.716 (0.009)
Constant	10.173 (0.003)	10.150 (0.003)	9.089 (0.089)
Demographic and tenure variables	NO	NO	YES
Time dummies	NO	YES	YES
R-squared	0.326	0.327	0.398
# observations	487,514	487,514	487,514

Note: Standard errors are clustered with respect to individuals. The demographic and tenure variables used in the regression are: gender, age, age squared, tenure and tenure squared.

Table 3: Mincer wage equations.

The models presented in Table 3 estimate the average wage differentials between the different

[23]The explanatory power of the model increases from 32.6 percent to 39.8 percent when information on time, demographics and tenure is included in the model. The explanatory power is high compared to standard Mincer wage equations estimated on traditional labor market data sets. However, compared to the study by Baker et al.(1994a,b) who explain 70 percent of the cross-sectional wage variation in the firm using only the hierarchical level of the firm our explanatory power is moderate.

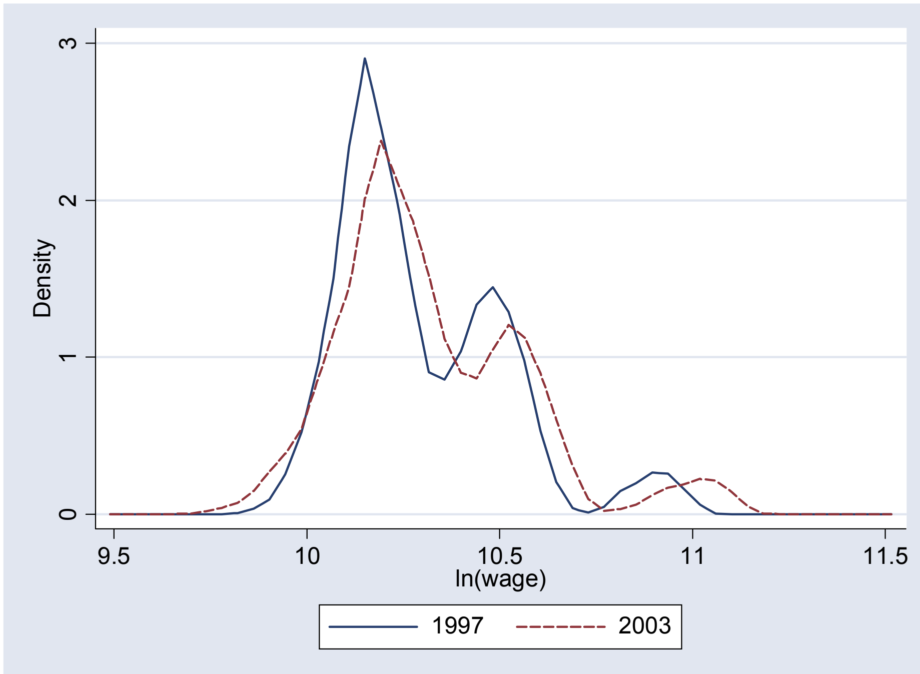

Figure 2: Wage distributions for 1997 and 2003.

employee subgroups for the period 1997 to 2003. The regressions accommodate that the wage level may be time-varying (time dummies are included), but they restrict the relative wages to be time-invariant. This restriction, however, is rejected by the data given the evidence presented in Table 2 where the relative wages are documented to vary substantially over time.

In order to assess the importance of the changes in the relative wages for the wage structure, a model similar to model 3 in Table 3 is estimated for the two years 1997 and 2003. The point estimates from these regressions are used together with the characteristics of the workforce in 1997 to predict the wages for the two years.[24] This reveals that the standard deviation of the predicted log-wages increases from 0.217 to 0.245 over the period. A result that is expressed visually in Figure 2 where kernels for the two predicted distributions are presented. This documents that the wage structure is affected significantly by the changes in relative wages.

To fully accommodate the shifts in the parameters, we estimate a model that allows for overall wage change and shifts in the relative wages over time. In practice, a model where dummies for belonging to a particular employee subgroup, time dummies and interaction terms between employee subgroup and the time dummies are estimated. The results of this estimation are presented in Figures 3 and 4.[25]

Figure 3 shows the changes in the real wage level from 1997 to 2003. The real wages increase for all employee subgroups from 1997 and up to 2001. In the context of the model, this shows that the overall productivity captured by the variable y increases in this period. The period of growth is followed by two years of decline such that the general wage level in 2003 is at the same level as in 2000.[26] Over

[24] The 1997 workforce characteristics are used in both predictions to avoid employee composition effects.

[25] The full regression results are presented in the appendix.

[26] There has been significant real wage growth over the period even thought the precision of the estimates in the latter

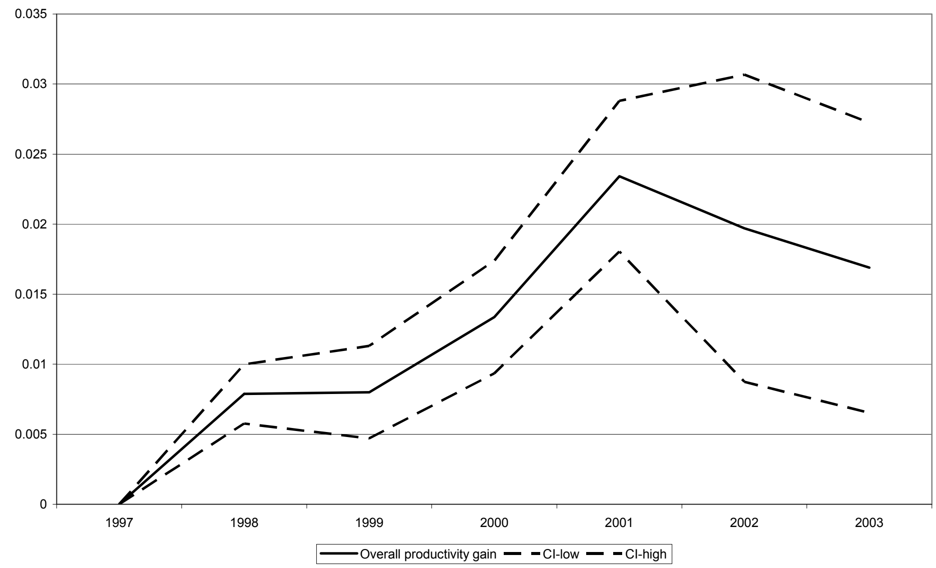

Figure 3: General wage increase, 1997-2003.

the full period, the general real wage level increases by 1.69 percent.

In Figure 4, the development in relative wages for the period 1997 to 2003 is presented. The relative wages between high- and low-skilled employees in non-management have increased by 1.51 percent. This change is small, however, compared to the increased wage gap between non-management and management workers. The wage differential between high-skilled in management and low-skilled in non-management is altered by 8.70 percent and the increase in the wage gap between low-skilled in management and non-management is as high as 22.54 percent. These observations clearly indicate that the technology shocks hitting the firm in the period enhance the productivity of management employees relatively more than the productivity of non-management employees.

Thus, skill-neutral technology shocks changing y and skill-biased technological shocks altering the parameters $\{\mu_1, \mu_2, \mu_3\}$ increase the overall and relative wage levels resulting in a significant increase in wage dispersion in the firm over the period 1997 to 2003. These findings can be related to previous empirical findings through Lemma (1) and Lemma (2) where the focus is on the *within* and *between* educational group measures of wage inequality, see Juhn et al. (1993) and Katz and Author (1999).

years is moderate.

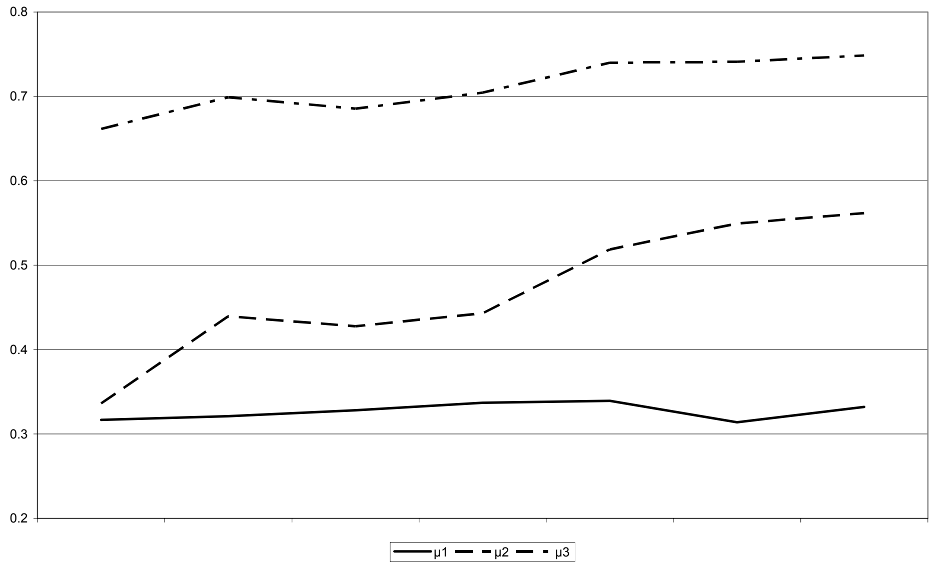

Figure 4: Development in relative wages, 1997-2003.

	Model (1)	Model (2)	Model (3)	Model (4)
Constant	9.023 (0.090)	9.029 (0.089)	9.090 (0.089)	9.096 (0.088)
Low skilled	-	-	-	-
High skilled (HS)	0.372 (0.006)	0.358 (0.007)	0.326 (0.006)	0.317 (0.006)
Non-management			-	-
Management (M)			0.395 (0.010)	0.344 (0.013)
HS*1998		0.009 (0.003)		0.003 (0.003)
HS*1999		0.013 (0.004)		0.010 (0.005)
HS*2000		0.023 (0.005)		0.019 (0.005)
HS*2001		0.025 (0.006)		0.021 (0.006)
HS*2002		0.002 (0.013)		-0.004 (0.014)
HS*2003		0.020 (0.009)		0.013 (0.009)
M*1998				0.038 (0.007)
M*1999				0.018 (0.011)
M*2000				0.028 (0.013)
M*2001				0.064 (0.015)
M*2002				0.091 (0.019)
M*2003				0.080 (0.016)
Demographic and tenure variables	YES	YES	YES	YES
Year dummies	YES	YES	YES	YES
R-squared	0.351	0.351	0.398	0.399
# observations	487,514	487,514	487,514	487,514

Note: Standard errors are clustered with respect to individuals.

Table 4: Mincer wage equations.

The returns to education measure presented in Lemma (2) are constructed such that it reflects the measure of *between* group wage inequality, used in conventional labor market studies of wage inequality i.e. it ignores information on job assignment within the firm. Using the Mincer wage equation, the return to education, captured by an indicator for high skills, is estimated to 37.2 percent, see model 1 of Table 4. This estimate represents the average return to education over the period 1997 to 2003. Model 2 goes into more detail by studying the change over time. This is done by adding interaction terms between being high-skilled and the time dummies. The results show that the return to education has increased from 35.8 percent in 1997 to 37.8 in 2003 - an increase of 2 percent. Hence the *between* group wage inequality clearly increased in this period.

The 2 percent increase in the return to education is remarkable given the results obtained above. The reason is that the return to education in non-management increases by only 1.51 percent and the return to education in management is reduced by 13.90 percent over the same period.[27] Hence, the overall increase in the returns to education cannot be explained by the development in the wage differentials within the hierarchal levels. Instead, the focus should be on the divergences in wages between hierarchal levels.

In model 3, the return to education is estimated simultaneously with the returns to management. Compared with model 1, there is a reduction in the returns to education of 4.6 percent which indicates that a substantial part of the wage differential contributed to differences in the level of education in model 1 reflects wage differentials between non-management and management employees. Furthermore, the explanatory power of the model is increased by 4.7 percentage points emphasizing the importance of including information about the hierarchal level into the wage regression.

The interesting result is presented in model 4 where the time development in both the return to education and the return to management are estimated. In this regression, the increase in the return to education is 1.3 percent (and insignificant) as opposed to the estimated increase in model 2 of 2 percent. In contrast to these low numbers, the increase in the return to management is as high as 8 percent over the period. This clearly shows that part of the estimated increase in the returns to education in model 2 is driven by a divergence of wages between management and non-management.

To sum up, the results presented above show that both the *between* and *within* group wage inequality have increased over the period, i.e. the wage gap between high- and low-skilled employees has widened and the wage gaps between management and non-management for similar educational groups have increased. Furthermore, it is established that these changes are driven by a divergence in the wage levels of management and non-management employees. This allows us to conclude that our findings are in accordance with previous studies of wage inequality and that we have identified the substantial changes in management compensation to be the main driving force behind the observed increase in wage inequality.

5 Employment Responses and Wage Inequality

In the empirical section, it is documented that the firm experiences an increase in productivity ($y \nearrow$) that leads to a wage growth of 1.69 percent. Furthermore, the different employee subgroups experienced specific productivity shocks resulting in differences in the relative wage growth, i.e. μ_1, μ_2 and μ_3 increased with different rates in the period. The effects of these shocks are not isolated to wage

[27] This result is obtained by realizing that the wage gap between high- and low-skilled workers in management is μ_3/μ_2 (see Lemma 1).

responses. They also affect the employment composition.[28] This can be seen most clearly from the model's transition probabilities in steady state which can be shown to depend on the productivity level[29]

$$P_{HM}(\theta_{HM}) = \lambda^{\frac{1}{1-\alpha}} \left[\frac{(1-\beta)\,\mu_3 y}{r+s} \right]^{\frac{\alpha}{1-\alpha}}, \tag{39}$$

$$P_{LM}(\theta_{LM}) = \lambda^{\frac{1}{1-\alpha}} \left[\frac{(1-\beta)\,\mu_2 y}{r+s} \right]^{\frac{\alpha}{1-\alpha}}, \tag{40}$$

$$P_{NM}(\theta_{NM}) = g(y, \mu_1, \mu_2, \mu_3, \Omega). \tag{41}$$

where Ω is a set of parameters.

5.1 Technological change

Studying equations (39), (40) and (41), it is apparent that a technology shock (an increase in y) has a positive effect on P_{HM} and P_{LM}, i.e. $\partial P_{HM}/\partial y > 0$ and $\partial P_{LM}/\partial y > 0$. This effect is coming from a positive relation between the value of the employment match and the productivity of the employee. Thus, the probability of getting a management job gets higher when the general productivity level increases. The effect of y on the probability of getting a non-management job is more involved. First, the higher productivity increases the value of the match in non-management. Second, it becomes easier for both the low- and high-skilled employees in non-management to be promoted into a management job (due to the higher P_{HM} and P_{LM}). This has a negative effect on the match value as the non-management jobs will be dissolved at a faster rate. The net effect is derived analytically in the appendix, and it is established that the positive technology shock reduces the match value of non-management jobs. Ultimately, this reduces the probability of getting jobs in non-management, $\partial P_{NM}/\partial y < 0$.

The change in the transition probabilities has implications for the steady state stocks of employees in the internal and external labor markets. The probability of getting a job in management has increased, hence the stock of managerial employees is higher in the new equilibrium. In the non-management sector there are two effects. First, the promotion probability has increased meaning that it has become easier for the non-management employees to move into a management job. Second, it has become more difficult to get non-management jobs. These two effects result in a reduction in the steady state stock of non-management employees. Finally, the net effect on the flows into the firm is positive which results in a reduction in the number of individuals remaining in the external labor market. The discussion is summarized in Proposition (2).

Proposition 2 *Technological progress increases employment in management, reduces employment in non-management and lowers the stock of employees in the external labor market:*

$\partial i_{LNM}/\partial y < 0$, $\partial i_{LHM}/\partial y < 0$, $\partial i_{LM}/\partial y > 0$, $\partial i_{HM}/\partial y > 0$, $\partial e_L/\partial y < 0$, $\partial e_H/\partial y < 0$.

[28] A third effect not considered in this paper is how changes in labor market prices (such as relative wage changes driven by technology shocks) affect educational choice. In the context of the model this leads to changes in π. This issue has recently been studied by Galor and Moav (2000) and Lee (2005)

[29] See the derivation in the appendix.

Proposition (2) has some interesting implications. First, a firm that is hit by a positive productivity shock will grow. Hence, more productive firms become relatively bigger in size. Second, a more productive and hence relatively bigger firm will allocate relatively more employees to management jobs. This is due to the higher probability of getting a management job for both internal and external individuals and the reduced probability for the individuals in the external labor market to get a job in non-management.

Corollary 1 *Technological progress increases employment and makes the firm allocate relatively more employees to management sector jobs.*

Proposition (2) shows that technology progress increases the relative size of the management sector. This result combined with the observation that management wages are higher than non-management wages has the implication that the technology-driven employment responses will add to an increase in wage dispersion. Thus, from Lemma (3) and Proposition (2) we can conclude that there are two channels through which technological progress leads to increased wage dispersion. First, there is a direct effect on wages. Second, there is an effect coming from the employment responses to the shock.

Corollary 2 *Technological progress increases wage dispersion through a wage effect and an employment effect.*

The comparative statistics presented above show that technological progress increases employment and makes it optimal for the firm to allocate relatively more employees to management jobs. Since these changes in the employment composition affect the wage structure, they have to be accommodated when analyzing changes in wage inequality.

6 Conclusion

Wage inequality has been rising since the mid-1970s. In this paper, we study the driving forces behind the increased wage dispersion using a general equilibrium model with heterogeneous employees who work in a firm with a management sector and a non-management sector. The parameters of the model are estimated using seven years of personnel records from a large pharmaceutical company. The results show that the firm experienced increased wage inequality over the period which is driven by amplified *between* and *within* educational group wage inequality, i.e. the wage gaps between high- and low-skilled employees has widened and the wage dispersion within educational groups has increased. Furthermore, it is established that these changes are driven by a divergence in the wage levels of management and non-management employees.

The results carry an important message as they show how information on within firm dynamics is important for understanding changes in the wage distribution. In particular, we show an example of how the development in wage inequality *between* educational groups can differ across organizational levels as inequality is documented to grow at the non-management level and to decline at the management level. Another important observation is that a large part of the *within* educational group wage differential, which is normally treated as a regression residual, can be explained by employee assignments to management and non-management jobs. These observations emphasize the importance of integrating information about the firm structure into the analysis of wage inequality.

The theoretical model presented in the paper has several innovations. First, it explicitly treats all the flows related to a firm i.e. transitions from the external labor market and into the firm (hirings), reallocations within the firm (promotions) and separations. Second, the wage distribution is constructed from a complete specification of the employee composition in the firm and the different employee subgroup's relative wages. Finally the model allows for studying the consequences of skill neutral and skill biased technological change on the wage structure.

The main contribution of the paper, however, is the joint theoretical-empirical approach to analyzing the driving forces behind the increased wage inequality. In this respect our paper is intended to motivate future research using the same approach in order to advance our understanding of labor market dynamics.

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7 Appendix

7.1 Steady state conditions

The first steady state condition equates the flow of low-skilled workers into a non-management job to the flows of low-skilled workers out of that state

$$e_L = \frac{s + p_{LM}}{p_{NM}} i_{LNM}. \tag{42}$$

Also, the flows in and out of management for low-skilled workers have to be equal in the steady state. In other words,

$$i_{LM} = \frac{p_{LM}(s + \eta p_{LM} + p_{NM})}{s p_{NM}} i_{LNM}, \tag{43}$$

where s is the separation rate. The third steady state equation equates the flows into and out of management for high-skilled workers.

$$\frac{p_{HM}(\eta e_H + i_{HNM})}{s} = i_{HM}. \tag{44}$$

Hence, the number of high-skilled workers in the external labor market who are successful in finding non-management jobs will equal the number of individuals leaving that state either for a job in management or due to a seperation from the firm. A similar equation can be stated for high-skilled workers in non-management

$$e_H = \frac{(s + p_{HM})}{p_{NM}} i_{HNM}. \tag{45}$$

Since

$$i_{LNM} + i_{LM} + e_L = \pi$$

the following equations can be derived from (22) and (25)

$$\begin{aligned} \pi &= i_{LNM} + i_{LM} + e_L, \\ sp_{NM}\pi &= sp_{NM} i_{LNM} + p_{LM}(s + \eta p_{LM} + p_{NM}) i_{LNM} + s(s + p_{LM}) i_{LNM}, \end{aligned}$$

$$i_{LNM} = \frac{sp_{NM}\pi}{(s + \eta p_{LM} + p_{NM})(s + p_{LM})}, \tag{46}$$

$$i_{LM} = \frac{sp_{LM}\pi[\eta(s + p_{LM}) + p_{NM}]}{(s + \eta p_{LM} + p_{NM})(s + p_{LM})}, \tag{47}$$

$$e_L = \frac{s\pi}{(s + \eta p_{LM} + p_{NM})}. \tag{48}$$

And in a similar fashion

$$e_H = \frac{s(1 - \pi)}{(s + \eta p_{HM} + p_{NM})},$$

$$\begin{aligned} i_{HM} &= \frac{p_{HM}(1 - \pi)}{(s + \eta p_{HM} + p_{NM})(s + p_{HM})}, \\ i_{HNM} &= \frac{sp_{LNM}(1 - \pi)}{(s + \eta p_{HM} + p_{NM})(s + p_{HM})}. \end{aligned}$$

7.2 Closing the model

Applying the free entry condition $rV_{HM} = rV_{LM} = rV_{NM} = 0$ and substituting (35) into (7), the following equation is obtained

$$J_{HM} = \frac{(1 - \beta) y_{HM}}{r + s}.$$

So, using (5) it follows that

$$q_{HM} = \frac{c(r + s)}{(1 - \beta) y_{HM}}.$$

Assuming that the matching technology is Cobb-Douglas, i.e

$$x_{HM} = \lambda (v_{HM})^{\alpha} (e_H + i_H)^{1 - \alpha},$$

then

$$q_{HM} = \lambda \theta_{HM}^{\alpha - 1}, \tag{49}$$

$$\theta_{HM} = \left(\frac{\lambda(1 - \beta) y_{HM}}{c(r + s)} \right)^{1/1 - \alpha}, \tag{50}$$

$$p_{HM} = \lambda \theta_{HM}^{\alpha}. \tag{51}$$

Upon substituting (34) into (6), it follows that

$$J_{LM} = \frac{(1-\beta)y_{LM}}{r+s}.$$

Furthermore, if

$$x_{LM} = \lambda(v_{LM})^{\alpha}(e_L + i_L)^{1-\alpha},$$

we obtain

$$\begin{aligned} q_{LM} &= \frac{c(r+s)}{(1-\beta)y_{LM}}, && (52)\\ \theta_{LM} &= \left(\frac{\lambda(1-\beta)y_{LM}}{c(r+s)}\right)^{1/1-\alpha}, && (53)\\ p_{LM} &= \lambda\theta_{LM}^{\alpha}. && (54)\end{aligned}$$

From (32), (33), (2) and (3) we have that

$$\begin{aligned} J_{LNM} &= \frac{(1-\beta)y_{LNM}}{r+s+p_{LM}},\\ J_{HNM} &= \frac{(1-\beta)y_{HL1}}{r+s+p_{HM}}.\end{aligned}$$

So using (1) and setting $\theta_{LNM} = \theta$, one can derive

$$c = \lambda(\theta)^{\alpha-1}(1-\beta)y\left[\frac{\pi(s+\eta p_{HM}+p_{NM})(s+r+p_{HM})+(1-\pi)(s+\eta p_{LM}+p_{NM})(s+r+p_{LM})}{(s+r+p_{HM})(s+r+p_{LM})\left[\pi\eta p_{HM}+(1-\pi)\eta p_{LM}+p_{NM}\right]}\right]. \quad (55)$$

By setting $\alpha = 1/2$ and using the value of p_{LM} and p_{HM} obtained in (54) and (51), we get

$$c = G(\mu_{1,}\mu_{2,},\mu_{3,}\alpha,\pi,c,\lambda,\eta,\beta,b,r,s,\theta,y,) \quad (56)$$

where $G(\mu_{1,}\mu_{2,},\mu_{3,}\alpha,\pi,c,\lambda,\eta,\beta,b,r,s,\theta,y,) =$

$$\frac{\lambda\sqrt{\theta}(1-\beta)y\tilde{f}(\mu_3)}{c(r+s)d(\pi\eta\lambda^2((1-\beta)\mu_{3,}y+(1-\pi)(\eta\lambda^2((1-\beta)\mu_{2,}y)+\lambda\sqrt{\theta}c(r+s))}$$
$$+\frac{\lambda\sqrt{\theta}(1-\beta)y\breve{f}(\mu_2)}{c(r+s)d(\pi\eta\lambda^2((1-\beta)\mu_{3,}y+(1-\pi)(\eta\lambda^2((1-\beta)\mu_{2,}y)+\lambda\sqrt{\theta}c(r+s))}$$

and

$$\begin{aligned} d &= (c(r+s)^2+\lambda^2((1-\beta)\mu_{3,}y))(c(r+s)^2+\lambda^2((1-\beta)\mu_{2,}y))\\ \tilde{f}(\mu_{3,}) &= \left[(\pi(\lambda c(r+s)\sqrt{\theta}+\eta\lambda^2((1-\beta)\mu_{3,}y)+sc(r+s))(c(r+s)^2+\lambda^2((1-\beta)\mu_{3,}y))\right]\\ \breve{f}(\mu_2) &= \left[((1-\pi)(\lambda c(r+s)\sqrt{\theta}+\eta\lambda^2((1-\beta)\mu_{2,}y)+sc(r+s))(c(r+s)^2+\lambda^2((1-\beta)\mu_{2,}y))\right]\end{aligned}$$

Setting $\beta = \lambda = \pi = 1/2$, one can compute the following derivatives

$$\begin{aligned}\frac{\partial G}{\partial\theta} &= \frac{c(r+s)y\mu_1(\mu_2+\mu_3)\tilde{h}(\mu_2)}{(64\sqrt{\theta}(c(r+s)^2+1/8\eta\mu_{2,}y)(c(r+s)^2+1/8\eta\mu_{3,}y)(8c(r+s)\sqrt{\theta}+(1-\beta)y\eta(\mu_2+\pi\mu_3))^2}\\ &+\frac{c(r+s)y(\mu_2+\mu_3)\breve{h}(\mu_3)}{(64\sqrt{\theta}(c(r+s)^2+1/8\eta\mu_{2,}y)(c(r+s)^2+1/8\eta\mu_{3,}y)(8c(r+s)\sqrt{\theta}+(1-\beta)y\eta(\mu_2+\pi\mu_3))^2}\end{aligned}$$

where

$$\begin{aligned}\tilde{h}(\mu_2) &= 8c(r+s)^2 + y\eta\mu_2)(32c^2(r+s)^2\theta + y\eta(8c(r+s)(s+\sqrt{\theta}) + \eta\mu_{2,}y \\ \breve{h}(\mu_3) &= 8c(r+s)^2 + \eta\mu_{3,}y)(32c^2(r+s)^2\theta + y\eta(8c(r+s)(s+\sqrt{\theta}) + \eta\mu_{3,}y).\end{aligned}$$

Thus,

$$\frac{\partial G}{\partial \theta} > 0.$$

We can also compute

$$\begin{aligned}\frac{\partial G}{\partial y} &= \frac{8c^2(r+s)^2\sqrt{\theta}(8c(r+s)^2 + y\eta\mu_3)^2\tilde{\rho}(\mu_3)}{(8c(r+s)^2 + y\eta\mu_3)^2(8c(r+s)^2 + y\eta\mu_2)^2(8c(r+s)\sqrt{\theta} + y\eta(\mu_3+\mu_2))^2} \\ &\quad + \frac{8c^2(r+s)^2\sqrt{\theta}\mu_1(8c(r+s)^2 + y\eta\mu_2)^2\breve{\rho}(\mu_2)}{(8c(r+s)^2 + y\eta\mu_3)^2(8c(r+s)^2 + y\eta\mu_2)^2(8c(r+s)\sqrt{\theta} + y\eta(\mu_3+\mu_2))^2}\end{aligned}$$

where $\tilde{\rho}(\mu_3) =$

$$((2s+\sqrt{\theta})(64c^2(r+s)^3\sqrt{\theta} - y^2\eta^2\mu_2^2) + (y\eta(32c(r+s)^2\sqrt{\theta} + \eta y(2r+\sqrt{\theta})\mu_2)\mu_3 + 2(r+s)y^2\eta^2\mu_3^2,$$

and $\breve{\rho}(\mu_2) =$

$$((2s+\sqrt{\theta})(64c^2(r+s)^3\sqrt{\theta} - y^2\eta^2\mu_3^2) + (y\eta(32c(r+s)^2\sqrt{\theta} + \eta y(2r+\sqrt{\theta}))\mu_3)\mu_2 + 2(r+s)y^2\eta^2\mu_2^2.$$

Hence, we see that

$$\begin{aligned}\frac{\partial G}{\partial y} &> 0 \text{ if for all } \mu_i \in (1, +\infty\ ),\ i = 2,3 \\ 64c^2(r+s)^3\sqrt{\theta} - y^2\eta^2\mu_i^2 &> (<)0 \\ \rho(\mu_i) &> (<)0.\end{aligned}$$

Some computations show that $\tilde{\rho}(\mu_3) = 0$ has two roots

$$\begin{aligned}\mu_3^1 &= \frac{-32c(r+s)^2\sqrt{\theta}y\eta - \eta^2y^2(2r+\sqrt{\theta})\mu_2) - \sqrt{\Delta(\mu_2)}}{4(r+s)y^2\eta^2}, \\ \mu_3^2 &= \frac{-32c(r+s)^2\sqrt{\theta}y\eta - \eta^2y^2(2r+\sqrt{\theta})\mu_2) + \sqrt{\Delta(\mu_2)}}{4(r+s)y^2\eta^2},\end{aligned}$$

where

$$\Delta(\mu_2) = (32c(r+s)^2\sqrt{\theta}y\eta + \eta^2y^2(2r+\sqrt{\theta})\mu_2)^2 - 8(r+s)y^2\eta^2(64c^2(r+s)^3\sqrt{\theta} - y^2\eta^2\mu_2^2)(2s+\sqrt{\theta}).$$

By symmetry $P(\mu_2) = 0$ has two roots

$$\begin{aligned}\mu_2^1 &= \frac{-32c(r+s)^2\sqrt{\theta}y\eta - \eta^2y^2(2r+\sqrt{\theta})\mu_3) - \sqrt{\Delta(\mu_3)}}{4(r+s)y^2\eta^2}, \\ \mu_2^2 &= \frac{-32c(r+s)^2\sqrt{\theta}y\eta - \eta^2y^2(2r+\sqrt{\theta})\mu_3) + \sqrt{\Delta(\mu_3)}}{4(r+s)y^2\eta^2},\end{aligned}$$

where

$$\Delta(\mu_3) = (32c(r+s)^2\sqrt{\theta}y\eta + \eta^2y^2(2r+\sqrt{\theta})\mu_3)^2 - 8(r+s)y^2\eta^2(64c^2(r+s)^3\sqrt{\theta} - y^2\eta^2\mu_3^2)(2s+\sqrt{\theta}).$$

If $64c^2(r+s)^3\sqrt{\theta}-y^2\eta^2\mu_i^2>0$ then the four roots are negative. It follows that

$$\frac{\partial G}{\partial y}>0 \text{ fo rall } \mu_i\in(1,+\infty), i=2,3.$$

If $64c^2(r+s)^3\sqrt{\theta}-y^2\eta^2\mu_i^2<0$ then μ_3^2 and μ_2^2 are positive.

$$\frac{\partial G}{\partial y}>0 \text{ fo rall } \mu_i\in(1,\mu_i^2), i=2,3.$$

In other words, using the implicit function theorem, we can solve (56) for θ_{LNM} and obtain

$$\theta_{LNM}=f(\alpha,\pi,c,\lambda,\eta,\beta,b,r,s,y).$$

7.3 Comparative statics

Using (46) the following derivative can computed

$$\frac{\partial i_{LNM}}{\partial y}=s\pi\frac{\partial p_{LNM}}{\partial\theta}\frac{\partial\theta}{\partial y}(\eta p_{LM}+s)-\frac{\partial p_{LM}}{\partial y}s\pi p_{LNM}(2\eta p_{LM}+2\eta s+s).$$

From (54) it is known that

$$\frac{\partial p_{LM}}{\partial y}>0 \quad ,\frac{\partial p_{LNM}}{\partial\theta_{LNM}}>0.$$

Thus, using the implicit function theorem, we have that

$$\frac{\partial\theta}{\partial y}=-\frac{\partial G}{\partial y}/\frac{\partial G}{\partial\theta_{LNM}}>0 \text{ for all } \mu_i\in(1,+\infty),\ i=2,3$$

and

$$\begin{aligned}\frac{\partial\theta}{\partial y} &< 0 \text{ if } 64c^2(r+s)^3\sqrt{\theta}-y^2\eta^2\mu_i^2>0 \text{ and } \mu_i\in(1,+\infty), i=2,3 \text{ or}\\ \frac{\partial\theta}{\partial y} &< 0 \text{ if } 64c^2(r+s)^3\sqrt{\theta}-y^2\eta^2\mu_i^2<0 \text{ and } \mu_i\in\mu_i\in(1,\mu_i^2), i=2,3 \text{ or}\end{aligned}$$

the sign of $\partial i_{LNM}/\partial y$ follows. Using the steady state population state derived above we can compute

$$\frac{\partial i_{HNM}}{\partial y}=s(1-\pi)(\eta p_{HM}+s)\frac{\partial p_{LNM}}{\partial\theta}\frac{\partial\theta}{\partial y}-\frac{\partial p_{LM}}{\partial y}s\pi p_{LNM}(2\eta p_{LM}+2\eta s+s),$$

$\partial i_{LM}/\partial y=$

$$\pi(p_{LM}+s)(1-\eta)\left(\frac{\partial p_{LNM}}{\partial\theta}\frac{\partial\theta}{\partial y}+\eta\frac{\partial p_{LM}}{\partial y}\right)+\pi(p_{LNM}+\eta(p_{LM}+s)(p_{LNM}+\eta p_{LM}+s)\frac{\partial p_{LM}}{\partial y},$$

and $\partial i_{HM}/\partial y=$

$$(1-\pi)(p_{HM}+s)(1-\eta)\left(\frac{\partial p_{LNM}}{\partial\theta}\frac{\partial\theta}{\partial y}+\eta\frac{\partial p_{HM}}{\partial y}\right)+\pi(p_{LNM}+\eta(p_{HM}+s)(p_{LNM}+\eta p_{HM}+s)\frac{\partial p_{HM}}{\partial y}.$$

The last two derivatives are ambiguous. However if

$$\begin{aligned}-\frac{\partial p_{LNM}}{\partial\theta}\frac{\partial\theta}{\partial y} &< \eta\frac{\partial p_{LM}}{\partial y},\\ -\frac{\partial p_{HNM}}{\partial\theta}\frac{\partial\theta}{\partial y} &< \eta\frac{\partial p_{HM}}{\partial y},\end{aligned}$$

then we get the signs given in Proposition (2).

7.4 Additional regressions

	Linear regression
Constant	9.096 (0.088)
Low-skilled in non-management (LNM)	-
High-skilled in non-management (HNM)	0.317 (0.006)
Low-skilled in management (LM)	0.336 (0.060)
High-skilled in management (HM)	0.661 (0.013)
HNM*1998	0.004 (0.003)
HNM*1999	0.011 (0.005)
HNM*2000	0.020 (0.006)
HNM*2001	0.022 (0.006)
HNM*2002	-0.003 (0.014)
HNM*2003	0.015 (0.009)
LM*1998	0.103 (0.024)
LM*1999	0.091 (0.030)
LM*2000	0.107 (0.053)
LM*2001	0.182 (0.077)
LM*2002	0.213 (0.083)
LM*2003	0.225 (0.077)
HM*1998	0.038 (0.007)
HM*1999	0.024 (0.011)
HM*2000	0.043 (0.013)
HM*2001	0.079 (0.014)
HM*2002	0.087 (0.015)
HM*2003	0.102 (0.006)
Demographic and tenure variables	YES
Year dummies	YES
R-squared	0.400
# observations	487,514

Mincer wage regression.