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ABSTRACT

This paper analyzes the requirements for a social welfare-optimized transition path toward a carbon-free economy, focusing particularly on the role of R&D and other technological measures to achieve timely supply-side transformations in the global production regime that will avert catastrophic climate instability. We construct a heuristic integrated model of macroeconomic growth constrained by geophysical system with climate feedbacks, including extreme weather damages from global warming driven by greenhouse gas emissions, and ‘tipping point’ for catastrophic runaway warming. A variety of options for technology development and implementation, and the dynamic relationships among them will be examined: interactions among the, each having a distinctive primary functionality CO₂ emissions control. The specifications recognize (i) the endogeneity and embodiment of technical innovations, and (ii) the irreversibility and long gestation periods of required intangible (R&D) and tangible capital formation. Efficient exercise of these options is shown to involve sequencing different investment and production activities in separate temporal “phases” that together form a transition path to a carbon free economy. To study the requirements of a timely (catastrophe-averting) transition, we formulate a sequence of optimal control sub-problems linked together by transversality conditions, the solution of which determines the optimum allocation of resources and sequencing of the several phases implied by the options under consideration. Ours is “planning-model” approach, which departs from conventional IAM exercises by eschewing assumptions about the behaviors of economic and political actors in response to market incentives and specific public policy measures. Solutions for each of several multi-phase models yields the optimal phase durations and rates of investment and production that characterize the transition path. Sensitivity experiments with parameters of economic and geophysical sub-systems provide insights into the robustness of the requirements analysis under variations in the technical and geophysical system parameters.

JEL codes: Q540, Q550, O310, O320, O330

Keywords: global warming, tipping points, catastrophic climate instability, technology fix options, R&D investments, capital-embodied innovations, optimal sequencing, IAM and DIRAM policy design approaches, multi-phase optimal control, sustainable endogenous growth

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1. Background and Motivation

Climate change is now convincingly linked to increasing atmospheric concentrations of GHG (greenhouse gases).¹ Among those who have examined the relevant scientific data there has been a growing consensus that this poses an inter-related host of worrisome problems. Moreover, rather than conveniently going away, or being gradually being accommodated by adaptations on the part of the world’s peoples, these problems are likely to grow worse. While on many specific points of climate science uncertainties, doubts and disagreement persist, the underlying physical and chemical processes responsible for anthropogenic “greenhouse effects” warming the Earth’ surface are firmly grounded, as is the accumulating mass of empirical observations attesting to the rise in mean global temperature that has taken place during the past two centuries.² Together, these findings firmly undergird the scientifically informed warnings about the consequences of continuing “business as usual” based on burning fossil fuels.

The rising levels of moisture that will accompany the warming of the Earth’s surface in the temperate latitudes can be expected to drive more frequent and more severe weather cycles, bringing heavier precipitation, stronger winds and weather-related damages to physical property and losses of human life. Moreover, it is more than conceivable that at some point in the not very distant future continuing the current rate of GHG emissions and unimpeded global warming will usher in an age of catastrophically abrupt climate changes, characterized by chaotic instabilities in Earth’s climates. This would drastically curtail the fraction of the world’s current population that would be able to achieve a state of “adaptive survival” preserving substantial resemblances to present state of civilization.

Mitigating environmental damages from CO₂, methane and dangerous particulate emissions by more extensively deploying known technologies, and replacing carbon-

¹ See IPCC (2007) and the Stern Review (Stern, 2006) for an extensive overview of the causes and consequences of climate change.

² See Solomon et al. (2007) on the physical basis and scientific evidence for IPCC conclusions regarding global warming and its sources in human activity. “Climate skepticism” and “global warming denial” should not be confused with the generally acknowledged scientific uncertainties that still surround the values some key parameters in quantitative models of the processes involved in anthropogenic global warming -- e.g., the magnitude of the “climate sensitivity” (linking changes in mean global surface temperature to relative gain in atmospheric GHG concentration), and the critical level of temperature gain that would trigger irreversible runaway warming and catastrophic climate instability. See further discussion below and details in footnotes 6-7.

based production systems with those that utilize new and economically efficient “carbon-free” technologies, are two obvious courses of action that may be able to avert these grim future prospects. Together with still more radical adaptations that possibly may be achieved by geo-engineering to capture and sequester existing atmospheric carbon and methane, these form the core of the technological options whose development and deployment are the focus of this exploratory examination of the dynamics of a feasible and timely transition to a sustainable low-carbon global economy.

A program of public and private R&D investments yielding directed technical changes of this kind should be viewed as a “supply-side strategy” in the campaign to stabilize global GHG concentrations at a viable level. Such a “technology fix” program would be complementary to carbon taxes, and the “cap and trade” schemes that have envisaged the development of a global market for carbon-emissions permits and derivative instruments. These latter mechanisms would work primarily by raising the current and expected future relative prices of carbon-intensive goods and services. Were they to have that effect, the volume demand for goods and services (e.g., electricity supply, transportation) that are especially carbon-intensive in their methods of production, would be reduced. Whereas the “carbon emissions-pricing” policy proposals to meet the challenges of climate change are directed toward achieving same ultimate goal as the “technology fix” strategy, they give at best secondary notice to the role that publicly supported R&D investments directed towards the generation of carbon-free technologies may play in the dynamics of a transition to a new, radically less carbon-intensive system of production for the world economy.

The “supply-side” approach that this paper explores, however, should not be understood merely as an optional supplement to “demand-management” through emissions-pricing policies that to date have received preponderant analytical and practical attention in the economics literature on climate change. Previous economic arguments for greater investment in R&D and diffusion of technologically enhanced production methods are relatively few and far between in the economics literature concerned with climate change, but it is notable that academic papers in that genre had begun to emerge even before the debacle of the attempt at the 2009 Copenhagen Conference strengthen and expand to Kyoto Treaty protocols.³ Neither these early

³ In the wake of the Stern Report (2007) and the Copenhagen Conference economic arguments for greater public and private investment in R&D, and subsidies for the diffusion of advanced technologies directed to lowering CO₂ emissions began appearing with greater more frequently – although these typically remained unspecific. Early statements proposing a “technology fix” approach include Arrow, Cohen, David et al. (2008), Nelson and Sarewitz (2008), Klemperer (2007/2009), David (2009), David, Huang, Soete and van Zon (2009), and Hendry (2010). None, to our knowledge, venture to examine the *required* behavior of directed R&D and capital formation embodying innovations that would lower the global rate of GHG emissions per unit of real output, which is the focus of the present work. Such theoretical and

forays into this area, nor other works to our knowledge, ventured to examine the required dynamic sequencing of *directed* R&D and other technology policies affecting public and private sector investments required for a timely transition to a low-GHG emitting production regime, the policy issue that is the focus of the research reported here.

It has been suggested that CO₂ emissions abatement achieved by the introduction of carbon-taxes or “cap-and trade” schemes for “pricing” transferable emissions permits, could indirectly achieve the what public research expenditures and tax subsidies for private investment in “green” R&D would otherwise have be directed to accomplish.⁴ The argument is that the effect of raising the cost of carbon-intensive processes and products to producers and consumers will be to stimulate producers’ demands for offsetting, carbon-free technologies and induce incremental private funding of the search of those innovative methods.

Although that is a possibility, so too is the opposite outcome. If raising carbon taxes does effectively curtail demand for highly carbon-intensive products, the result may well be a generally adverse expectations about market growth and a consequent weakening of incentives for R&D investment in all production techniques for in those lines of business.⁵ Moreover, the imposition of carbon taxation that is likely to be maintained and even rise in the future might well have the perverse first-order effect of depressing the present value of fossil fuel stocks in the ground, bringing forward in time the exploitation of those resource deposits and tending to depress prices in markets for

computational modelling that has been undertaken to date has largely following the pioneering work of Goulder and Schneider (1999), which is noted below.

⁴ Among the pioneering efforts to examine the proposition that raising the price of carbon would induce beneficial private investments in technological innovation, see the working papers by Nordhaus (1997), and Goulder and Mathai (1998). Goulder and Schneider (1999) carried this line of inquiry farther by using a computable multi-sector general equilibrium model to quantitatively assess the extent to which real GDP costs of imposing CO₂ abatement taxes would be altered by the effects of endogenous technological improvements resulting from R&D investment induced by the effect of the tax on the relative prices of carbon vs. non-carbon inputs. This modelling exercise did not, however, undertake an evaluation of the efficacy of the hypothesized induced technical changes in terms of the magnitude of the absolute abatement of the economy’s aggregate CO₂ emissions rate. More recent exercises in integrated climate policy assessment have undertaken to do so, as is noticed below (in section 2).

⁵ The question cannot be settled on theoretical grounds, but see Gans (2009) for analysis of the conditions under which carbon-taxes would have a perverse effect of discouraging carbon emissions-reducing technology research and innovation. Acemoglu, Aghion, Bursztynx, and Hemous (2010) explore the conditions for carbon taxes in a growth model with environmental (carbon emissions) constraints and endogenous directed technical change driven by private sector R&D investments. They find that with a sufficiently high elasticity of substitution between highly carbon-intensity and low carbon-intensity inputs in production, carbon taxes can switch directed R&D to “clean” (low carbon intensity) technological research, and that there is a complementary role for “temporary R&D subsidies”. But the complications of “Green Paradox” effects of carbon taxes (see fn. 6, below), and carbon-saving R&D subsidies are not addressed by their theoretical analysis.

carbon fuel.⁶ Beyond this possible drawback in reliance upon carbon taxes, there is still greater room for doubts regarding the efficacy of downstream cap-and-trade schemes, which price the emissions from burning carbon, but not the carbon material that is the energy source.

It is not at all obvious that putting a price on the emissions from the burning of hydrocarbons necessarily would suffice to raise the combined price of the material and its combustion, so the possibility of a perverse relative price change cannot be definitively ruled out.⁷ By contrast, the proposal of an “up-stream” implementation of the cap and trade mechanism (see Repetto 2011) is administratively attractive because effective restriction of the volume of fossil fuels available from “first sellers” it would necessarily raise the cost of carbon-based energy sources. Further monitoring and enforcement of the “caps” would be greatly simplified by the limited number of “first sellers” and the ease of identifying them – especially in comparison with the myriad downstream users that would require CO₂ emission permits. Theoretical analysts of the problematic “Green Paradox” effect of carbon taxes actually increasing the extraction and use of carbon fuels are agreed that this perverse outcome would not materialize if a high enough ad valorem tax rate could be imposed sooner rather than promised for the future. But, as efficient as administratively feasible as it might be for central government authorities in the largest economics to quickly impose coordinated tight regulatory

⁶ Discussion of the latter problem was broached first by Inn (2008, 2009: pp.10-13) who provided a label --“The Green Paradox” --under which a theoretical literature soon developed (see, e.g., van der Ploeg and Withagen (2010), Hoel (2010)). I, that the prospect of innovations that would provide more attractive substitutes for fossil fuel as an energy source would depress the value of those natural resource deposits, leading the owners to raise the current rate of extraction and use of carbon as an energy source --thereby offsetting the CO₂-mitigating intention of policies offering public subsidies for “green innovation.”

⁷ This source of ambiguity regarding the effects of using the “cap and trade” mechanism to set a positive price on carbon emissions, it should be emphasized, is distinct from the concerns about the possible disincentive effects of carbon taxes’ negative impacts on demands for carbon-intensive goods and services – e.g., those relying heavily on production processes using energy sources such as coal, oil and natural gas. The theory of exhaustible natural resource pricing, following the classic work of Hotelling (1931) tells us that the resource valuation must rise at the rate of interest, and with marginal extraction costs constant, the price of the flow of materials traded in the market must rise *pari passus* with that of the resource deposit. Subsequent theoretical contributions on exhaustible resource economics, e.g. Heal (1976), Dasgupta, Heal and Majumdar (1978), Dasgupta and Heal (1980) -- have pointed out that downward pressure exerted by anticipations of advances in “backstop” technologies permitting substitution of carbon-free energy sources (and, more generally, more energy-efficient production process innovations) would lower the levels from which the market prices of coal, oil and natural gas would be trending upwards. This effect would work to slow, if not temporarily halt the diffusion of those innovations. Whether the magnitude of the first-order offset on prices in carbon markets would be large enough to perversely neutralize the intended impact of a global cap-and-trade scheme, or so large as to temporarily *lower* the after-tax unit cost of burning carbon fuels, is a distinct empirical question (of obvious policy-relevance) that deserves more attention than it has received. On first consideration, it seems that the answer must turn on the extent of the anticipated demand displacement in relation to the size of the resource reserves that could be accessed from the markets in question. We are grateful to Lawrence Goulder observation that the later consideration make the magnitude of the offset-effects greater in the case of high quality oil reserves than it would be for the world’s existing coal deposits.

caps on first vendors, such actions would be tantamount to directly restricting extraction and importation of fossil fuels. The political economy of domestic legislation and international treaty negotiations, however, presently and for some time to come would militate strongly against practical implementation of this approach to fixing the greenhouse gas externality, does not seem to give it any advantage vis-à-vis the more familiar “downstream” cap-and-trade mechanism.⁸

In these circumstances it should be remarked that the present and likely future social and political resistances to stringent and enforceable carbon-pricing policies might be weakened were there credible grounds for anticipating that the economic welfare costs entailed in switching to alternative energy sources would be significantly reduced by a concerted multi-national program of directed technological innovation. While the dynamics of this techno-politico nexus will need to be thought through carefully, the argument for systematic economic analysis of the potential role technology policies in effecting a timely, climate stabilizing transition to a viable “low carbon” economy transition therefore can, at least for the time being, rest in part on the possible temporal asymmetry of complementary (or super-modular) relationships between the two major policy approaches.

When viewed in that expanded and explicitly dynamic perspective, a program that starts with “tech fix” may make that policy initiative complementary to (and not a substitute for) fiscal and regulatory instruments designed to get market prices of carbon fuel to reflect their marginal social costs. This consideration provides another compelling reason for according greater attention to the requirements of a “technology push” strategy than it has received in previous and contemporary economic research on climate change policy.

2. Preliminaries for modelling a “Tech Fix” strategy’s dynamic requirements

All of the previously mentioned supply-side options, however, require paying explicit attention to major changes in production systems and/or changes in life-style and consumption patterns. Furthermore, to explore the dynamics of transitions towards a less carbon-intensive production regime and the role that research policy would have to play, it is necessary to consider the temporal duration of R&D activities directed to altering the performance characteristics of various classes of technology, as well as the duration of tangible capital formation processes required by technologies that must be “embodied” in new physical plant and equipment. Such questions can be most usefully

⁸ Indeed, it seems more than likely that government regulations banning unlicensed first sales of fossil carbon sources of energy is tantamount to state expropriation and control of those forms of natural resource wealth, and would meet with strong political opposition from powerful economic interests as “de facto socialization” of their privately owned mineral wealth.

explored in this paper within the framework of a heuristic model of endogenous global economic growth, which is the research approach pursued here. .

It is no less essential to begin by explicitly taking into account the dynamic behavior of the geophysical system within which the global economy is set. That larger physical context quite obviously imposes both natural resource and environmental constraints upon the production regime, as well as impinging directly upon human welfare – in this instance through the continuing anthropogenic emissions of greenhouse gases and their destabilizing impacts upon global climate and the accompanying alteration of weather patterns.

2.1 Geophysical system constraints

There are now firm scientific foundations for the growing concerns that to allow GHG emissions to continue at anything close to their present rate may soon set in motion irreversible runaway global warming, with potentially catastrophic consequences for global ecosystems and for human life as we know it. This conclusion has survived the persisting doubts voiced by a dwindling fringe of academic “climate change sceptics” and outright deniers of the seriousness of the problems that continued global warming will pose.⁹

The mass of evidence provided by chemical analysis of the gases in the deep ice cores that have been extracted from Arctic and mountain glaciers during the past decade-and-a-half has allowed climate scientists to document a history of abrupt alterations of the Earth’s climate(s) during pre-Holocene epoch. The record points to the possibility that even modest global warming driven by continued anthropogenic emissions of GHG could trigger the onset of irreversible global warming, but a form of climate instability that during the most recent transition between glacial and stadial epochs featured a

⁹ Despite the evidence presented by Solomon et al. (2007), doubts have been raised about conclusion that the warming trend observable during the second half of the 20th century resulted from radiative forcing *due to human activities*. These doubts are rooted in criticism of IPCC models of general climate circulation models (GCMs) on account of their poor performance in predicting the high frequency and high relative amplitude variations in global climate during the past 50-60 years. See, e.g., Scarfetta (2011) and references therein for studies showing that the latter variations can be more accurately predicted by statistically fitting harmonic (Fourier series) regression models that combine indicators of multiple astronomical cycles with differing frequencies. Such research, however, sheds scant light on the dynamics of the physical forcing processes that would account casually for the observed statistical correlation, nor does the existence of unexplained fluctuations around a trend vitiate the positive statistical significance of the trend itself. Recent theoretical efforts, such as that by Stockwell (2011), offer general physical basis for the skeptics’ contention that periodic radiative forcing from astronomical sources, not accounted for in the IPCC’s GCMs, could be responsible for the latter’s failures in predicting recent high frequency variations observed in the Earth’s temperature. The trouble with this line of speculative criticism seems obvious enough: variations in solar radiation are not thought to be a concurrently emerging feature in the history of our planet’s astronomical environment. Consequently it is hard to understand how that phenomenon could account for the “hockey-stick” up-turn observed in the last half-millennium’s worth of data indicating the long-term trend in the Earth’s temperature.

prolonged era of “climate flickering.”¹⁰ This latter term refers to a climate regime characterized by high frequency switching (with periodicities as short as 3 to 5 years) between markedly warmer and cooler average temperatures -- the latter differing by as much as half the 12° C. range between the lowest and highest extremes recorded during Earth’s glacial and stadial episodes. The possibility of catastrophic state change of that sort undermines the sanguine suppositions that the continued emissions of CO₂ would simply result in a smooth transition to a warmer global climate to which human societies would be able to successfully adapt.

The positive feedbacks that would drive runaway global warming dynamics --and its manifold entailed damages to human welfare – are those generated by the amplification of the linkage between elevated GHG concentration levels of anthropogenic origins and stronger radiative forcing that produces faster warming of Earth’s surface. The warming at various points triggers self-reinforcing alterations in the behaviour of the geophysical system. These feedback processes would operate to increase the strength of the radiative forcing resulting from a given atmospheric GHG concentration level; boost the surface absorption of heat, and hence the rate of warming produced by a given level of radiative forcing; supplement direct anthropogenic emissions of CO₂ by triggering releases of methane (a much more powerful GHG) – which, at lower temperatures, would have remained naturally sequestered in the permafrost beneath glacial ice and in the methane hydrates lying on the shallow seabed at the northern edge of the arctic ocean.

Some of the intervening steps of such sequences (depicted schematically in Figure 1) have a cascade-like sub-structure that serve to extend and further accelerate the positive feedback process.¹¹ For example, glacial retreat, by lowering *albedo* (the fraction of sunlight reflected by the earth’s surface) in the affected regions leads immediately to greater local heat absorption. This in turn promotes thawing of the exposed permafrost and the formation of “thaw lakes” from which (like other wetlands

¹⁰ Hall and Buhl (2006) review the findings from recent advances in climate science, and point out that its implications thoroughly undermine the supposition long maintained by mainstream contributors to the literature on energy and environmental economics, namely, that “climate change” driven by rising GHG concentration levels could be satisfactorily modelled as a smooth transition to a higher equilibrium level of the global mean surface temperature – as has been assumed by the integrated assessment models (IAMs) that have figured prominently and remain salient in the energy and environmental economics field. See Hall and Behl (2006: pp.461-462) for a detailed discussion of this and related features of the geophysical sub-system in Nordhaus and Boyer’s (2000) updating of the original (Nordhaus 1994) DICE model. The assumption that radiative forcing due to the accumulation of atmospheric CO₂ would drive a smooth transition to a higher equilibrium temperature of the Earth’s surface is retained by Nordhaus (2010) in the latest update of DICE (2010), as well as in the annualized version of the model that Cai, Judd and Lontzek (2012) create as a basis for DSICE, a stochastic version of the model.

¹¹ See Stern (2007), pp. 11-14, for a short overview of the positive feedbacks, including reduction of albedo through reduced ice-coverage of the arctic regions, thawing of permafrost and induced release of methane, as well as the release of methane from oceanic hydrate stores.

covering decaying organic material) methane will bubble to the surface. Similarly, the warming of the upper ocean, particularly in the shallower Arctic waters can destabilize the methane clathrate compounds that had formed there – and, more generally in the deeper ocean – during an earlier epoch. Similarly, warming of the large peat deposits in the arctic Siberian shelf would augment the release of methane from that natural source.

Methane in the atmosphere is over 20 times more potent than CO₂ in its greenhouse effects, and so clouds of the gas, bubbling up to the surface and flowing quickly to the atmosphere being will cause the CO₂-equivalent concentration level and the strength of radiant forcing to spike upwards. But, methane (CH₄) is an unstable molecule that is subject to immediate degradation by exposure to the water molecule called “OH radical,” which sets in motion a chemical chain reaction that within several days will have converted the newly added methane molecules to water and CO₂.¹² Consequently, the strong effect upon radiant forcing of a sudden jump in atmospheric methane concentration (unlike that of gains in CO₂ ppmv) is only transient and its persisting direct result upon the rate of global warming will be equivalent to that of the additional (unabated) CO₂ produced by the chemical reactions that degrade CH₄. A surge in warming resulting from a prolonged elevation of the rate of methane flux, nevertheless, may be sufficiently extended to induce further positive feedback effects that are self-reinforcing and contribute to raise the persisting atmospheric concentration of CO₂.¹³

Clouds of methane gas could be abruptly released by the warming of shallower ocean waters along the Arctic Ocean’s edges, since it is expected that a rise of only 2° C. in water temperature would be enough to destabilize the methane clathrate compounds that are holding CH₄ trapped in its cage-like molecular structure. This, the hypothesized “Clathrate Gun” effect – which would operate analogously to produce abrupt methane releases when the permafrost retreated, now thought to be a primary mechanism of the pre-historic episodes of pronounced high frequency temperature fluctuations that have left a record in the deep arctic ice-cores.¹⁴

¹² See Archer (2011:Ch5) for a lucid exposition of the methane cycle, drawing largely on Jacob (1999). The OH radical (denoted OH●) is produced naturally by the effect of sunlight on water, which causes the latter to lose a hydrogen atom, and to immediately steal one from an available methane molecule – yielding H₂O and the methyl radical (CH₃●), which a chain of simple chemical reactions ends up producing 2 stable molecules of water and one of carbon dioxide.

¹³ While it appears that there is little likelihood of this happening spontaneously through natural events (such as disruptions of the deep ocean seabed) that would take place on large enough scale to have catastrophic climate consequences, the size and instability of the Earth’s methane hydrate reserves and the uncertainties surround the present state of scientific knowledge lead a sober climate scientist like Archer (2011) to characterize those potentialities as “frightening”.

¹⁴ Even quite moderate warming of the ocean waters along the continental shelves is thought to be sufficient to destabilize the methane clathrate compounds that have formed on the seabed in northern

Yet another perverse feedback loop involves forest die-back. This worries scientists studying the boreal forest stretching across the northern latitudes of the north American continent and Russia -- a zone whose trees represent between 25 and 30 percent of the world's forest cover, and which is reported to have experienced the most pronounced temperature increases observed anywhere on the planet during the last quarter of twentieth century. In the northern latitude of Siberia the predominant needle-shedding larch forests are being replaced by evergreen conifers that grow more rapidly in the summer warmth, but the evergreen trees absorb more sunlight and their expansion is thus contributing to the global warming trend.¹⁵

The loss of predictability of local environmental changes accompanying profound ecological damage and losses of human life and welfare, and the socio-economic repercussions that would exacerbate the process thereby set in motion, can be left to the imagination at this point. Suffice it to say that the prospect, however remote, imparts considerable urgency to the task of developing a workable approach to identifying the requirements of an effective "technology fix" to averting its realization.

Our modelling approach in this paper builds upon the pioneering work of economists on climate policy analysis represented in so-called *integrated assessment models* (IAMs) that provide a simplified characterization of the essential features of the geo-physical system.¹⁶ These quantify the linkage between the flow of GHG emissions from economic activity and the augmented radiative forcing that drives higher mean global temperatures at the Earth's surface (the "green-house" effect). Modelling the carbon cycle takes account of the natural sequestration of CO₂ in the forests and oceans, and the consequent lagged effect in the adjustment of the accumulated atmospheric stock of GHG (equivalent CO₂ ppmv); the "climate sensitivity" parameter describes the equilibrium mean surface temperature's response to a doubling of the CO₂-e

latitudes. On the "Clathrate Gun" hypothesis, and its relevance as an explanation of abrupt climate change and the phenomenon of "climate flickering" at the end of the last ice age, see Kennett et al (2003), Maslin et al. (2004), Hall and Behl (2006), Reagan and Moridis (2007). The high frequency of the temperature fluctuations recorded in the ice-cores are held to militate against the earlier theory that this abrupt and catastrophic alteration of the climate system could have been produced by a "thermohaline collapse", which would result climate cycles of much lower frequencies.

¹⁵ Whether this is compensated by their greater capacity for absorbing CO₂ is not clear. If such is the case, it is being offset in the southern drier boreal zone, where water-stressed trees in the summer grow less rapidly and the forests are being replaced by grasslands and pasture.
http://en.wikipedia.org/wiki/Boreal_forest#Climate_change

¹⁶ For surveys and reviews of the evolving field of research on IAMs, after Nordhaus (1993), see Dowlatabadi (1995), Kelly and Kolstad (1999), Parker, Letcher, Jakeman et al (2003), Ackerman, DeCanio, Howarth and Sheeran (2009).

concentration in the atmosphere.¹⁷ Within that characterization of the physical framework, our model adds specifications of the production sector of an endogenous growth model in which directed R&D expenditures can raise the productivity of newly formed capital goods embodying novel “carbon-free” technologies, and hence enhance the economic efficiency of those additions to aggregate (sustainable) production capacity.¹⁸ This is the nexus through which public “research policy” interventions that boost investments in science and R&D can positively affect social welfare by lowering the costs of switching to production systems characterized by low, or in the limit “GHG emissions-free” technologies.¹⁹

But in this matter the timing as well as the magnitude of remedial action is a critical concern. The Intergovernmental Panel on Climate Change (IPCC) has concurred in the conclusion that there is a critical threshold at 2 degrees C. of global warming from the

¹⁷ For purposes of our model described below, atmospheric GHG (ppmv) concentration levels are given by an initial baseline level plus a scalar function of the integral of ($CO_2 - e$) emissions from the baseline date, the latter being proportional to the mean rate of CO_2 emitted per unit of output produced (proportional to utilized capacity) and not being absorbed by the forests and the upper and lower ocean. This simplification ignores the lag effects of emissions on changes in atmospheric CO_2 that result from the carbon-cycle diffusion of the gas between the atmosphere and the upper ocean, between the upper ocean and the lower ocean, and the upper ocean and the atmosphere. In addition it linearizes the relationship between the atmospheric concentration of carbon relative to its level at a base date (E_t/E_0 , in our notation) and the absolute gain in radiative forcing from the atmosphere, $F_t - F_0$. Widely used IAMs -- e.g., versions of DICE due to Nordhaus (2002, 2007) -- typically represent the change in atmospheric radiative forcing as taking the form $\Delta F_t = \eta[\log(E_t/E_0) - \log(2)]$ with adjustments for the direct and indirect effects of radiative forcing from the upper and lower oceans. Ignoring the latter, and the lags in the relationship between changes in the radiative forcing and those in Earth's surface temperature, the parameter η is the approximate “climate sensitivity”: the expected long-run gain in T relative to its base level resulting from doubling the atmospheric CO_2 concentration. Our simplified dynamics of the geophysical subsystem is more acceptable in modeling the transition paths to climate stabilization than it would be in simulating the course of carbon emissions and temperature changes over the 600-year long time span of optimally “moderated” CO_2 emissions envisaged by DICE (2007) and its precursor IAMs.

¹⁸ Presently, R&D outcomes are completely deterministic in our model, but we envisage the introduction of stochasticity in several relationships, including this one.

¹⁹ In proceeding in this way we have not ignored the controversial suggestion advanced by Weitzman (2009b) that the scientific uncertainty surrounding the behavior of the geo-physical system, such as the distribution of the “climate sensitivity” parameter, leaves open the possibility of catastrophic outcomes that may occur with vanishingly small “fat-tailed” probability. The dismal implication for economic policy design would seem to be the extreme form of the “precautionary principle” in which all other considerations are outweighed by this threat and the willingness to pay to avert it become infinite. To accept that view would militate utterly against any effort to pursue social welfare optimizing policy analysis within the framework of an equivalent deterministic system. But while the existence of a “fat-tailed” distribution follows from Bayesian statistics, as shown by Weitzman's mathematical result, the implications of his “dismal theory” concerning the economic policy implications have been found to come from the assumed constant relative risk aversion (CRRA) property of the traditional social utility function that has been posited by Weitzman, along with other economists working the welfare analysis of climate policy. Millner (2011), in a lucid and insightful review of the controversy ignited by Weitzman (2009b), shows that the “harmonic absolute risk aversion” (HARA) utility function -- a generalization the traditional (CRRA) utility function that is widely used in the finance literature -- not only yields finite welfare measures when risks are fat-tailed; under plausible parameter conditions it also makes policy evaluations relatively insensitive to the tails of the consumption distribution.

earth's preindustrial mean temperature beyond which the probability of being able to avert catastrophic runaway global warming begins to drop below 50-50. Due to the cumulative volume of persisting GHG emissions up until this point in time, however, the world already is committed to a future rise in mean global temperature (relative to the present) on the order of 0.5 to 1 degrees – even if carbon emissions were completely stopped. Even if the "tipping-point" were set at a 3 degree gain, this leaves relatively little room for further warming without catastrophic consequences, as the Stern Report pointed out (Stern, 2006: p.15). The implication is that under a "business as usual" rate of GHG emissions there may not be very much time left before the critical threshold will be crossed and the problem facing the world's population will be transformed to one of learning to adapt as best we can to the almost unimaginable mounting damage and societal disruptions entailed in coping with a destabilized climate system.

Against this worrisome background, it is a particularly harsh fact of present economic life that to make major changes in production systems also takes considerable time (and resources). This is because the transition from carbon-based production to carbon-free production involves first the (further) development of carbon-free alternatives and secondly the subsequent implementation of these alternatives through investment in tangible capital formation projects, including infrastructure modifications that will have long gestation periods. Furthermore, the development of the new technologies required will entail the commitment of resources to R&D projects of uncertain and possible extended durations. The claims of these programs of tangible and intangible investment therefore will press upon consumption levels for some period without yielding resource savings or significantly lowering the rate of GHG emissions.

Thus, the reality of transitioning towards a carbon-free economy in time to avoid crossing the critical temperature threshold into climate instability will be, at best, an uncomfortable, unremitting and perilous journey for humanity. The particular analytical challenge that is taken up in this paper is how best to schedule R&D expenditures and related tangible capital formation at the macro-level so as to maximise the societal welfare (or more realistically minimize the damage to social welfare) associated with the entire transition path towards a carbon-free production regime in a GHG-stabilized environment, taking into account the diversion of resources away from consumption that those investments entail. Towards this end we have constructed a multi-phase, multi-technology endogenous growth model that allows for the expenditure of R&D resources on the creation and improvement of "carbon-free technology" and acknowledges that this form of investment diverts real resources from consumption and hence adversely affects societal welfare in the short run (doing 'nothing' would have negative welfare effects in the long run, however).

Taken together, the economic and geo-physical sub-systems pose a timing problem, since old carbon-based technologies generate CO₂ emissions, and postponing the implementation of new, carbon-free technologies reduces the time available for building

up new capacity while cumulative emissions of CO₂ (and other GHG) remain below the associated critical threshold concentration (and associated mean global surface temperature) that can trigger the onset of climate instability. Starting soon enough to undertake the R&D (which takes both time and resources) can make available a better performing family of low-carbon and carbon-free technologies in time to embody that knowledge in new production facilities and avoid crossing that “tipping point”, but how soon is soon enough will depend upon the rate of R&D investments and their impact on the productivity of carbon-free production facilities, as well as the economy’s capacity to replace “dirty” carbon-using capacity with a carbon-free capital stock.

The following section of the paper demonstrates how this and related questions can be answered by formulating and solving a sequence of optimal control sub-problems for each distinctive phase in the transition from a “business-as-usual” carbon based economy to a sustainable carbon free economy, and then tying optimized phases together in “stacked Hamiltonians” by the use of transversality conditions that guarantee the optimality of the transition path as a whole. The sub-sections of 2 apply this approach in developing increasingly more complicated models, all constructed on the basis of the most elementary growth model. Section 3 comments on a number of insights that the three partial models provide about questions of timing in exercising various technology-fix options. It also reports preliminary results from the use of sensitivity analysis to assess the effects on optimal investment paths of variations in the assumed location of climate “tipping points”. The paper concludes in Section 4 with a brief review of what has been learned, what remains to be studied, and the proximate next steps in pursuing this line of research into the design and implementation requirement for a successful technology fix for global climate instability.

2. Modeling the phases of a timely transition to a “carbon-free” production system

2.0 An incremental model-building agenda

Rather than undertaking from the outset to specify the structure of an equivalent deterministic dynamical system that incorporates numerous features of each of the various possible technological strategies could be deployed to stabilize global climate, we proceed towards that goal in a step-by-step manner, taking three discrete partial model-building steps, and investigating what can be learned about the dynamics of an optimal transition path from each of them, considered separately.

We start from a basic model in which there are two available technological options, a mature carbon-intensive technology that is embodied in existing fixed production facilities and a carbon-free technology that has yet to be deployed by appropriate capital formation. The tableau in Figure 1 (below) provides a summary overview of activities that distinguish the three phases of the transition that transforms the

economy of our Basic Model from a carbon-burning “business as usual” regime to one in which the switch to producing exclusively with carbon-free capital facilities has stabilized the global climate. The tableau’s horizontal columns indicate the two different production technologies that can be used by the generic economy, whereas the different kind of tangible (and intangible investments) that may be undertaken are shown in the rows. The shading (in red, for the Basic Model) shows the combinations of concurrent investment and production activities that are compatible with efficient resource allocation in each phase of the transition path.

Next, we introduce into the Basic Model set-up the possibility of undertaking R&D expenditures during the “business as usual phase”. These investments in “directed technological change” are aimed to reduce the unit capital costs of production facilities embodying the carbon-free technology, raising the average productivity of K^B enough to match that of capacity based on the carbon-using technology. In this elementary model of a three-phase “endogenous R&D-driven transition”, only when that technological goal is attained does R&D expenditures (and further tangible investment in carbon-intensive production capacity) come to a halt, and tangible capital formation begins deploying the carbon-free technology. This is indicated in Figure 2 by the tableau’s blue shaded areas.

Figure 1: Basic Model (Red)

<i>Investment</i>	<i>Production (Capacity in Operation)</i>		
	Old Carbon High emission rate	Greened Carbon Lower emission rate	Carbon Free Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-free capital K^B	joint production		joint production
			carbon free production
R&D on carbon-free technology			
R&D on climate eng.			

Figure 2: Basic Model with R&D (Blue)

<i>Investment</i>	<i>Production (Capacity in Operation)</i>		
	Old Carbon High emission rate	Greened Carbon Lower emission rate	Carbon Free Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-free capital K^B	joint production		joint production
			carbon free production
R&D on carbon-free technology	production business as usual		
R&D on climate engineering			

In the third of the partial models to be examined, the structure of the basic model is extended by allowing for the exploitation of a third class of technological opportunities. These do not require significant R&D investments, since they make use of known “core” engineering techniques to provide an “intermediate”, less carbon-intensive set of production technologies that in effect serve to “green” existing carbon-using direct production facilities and infrastructure. Because they are characterized by lower rates of CO₂ emissions per unit of capital than the old carbon-based technologies (even if a higher capital cost per unit of output), they provide a reduced emissions-output ratio. Thus they can “buy time” to make the tangible investments needed to switch to a carbon-free production regime.

Figures 3.1 and 3.2 display in tableau form the pair of alternative 5-phase trajectories that arise in the case of the buy-time model. Each version can be solve for an optimal transition path, but to find the optimum optimum it is necessary to evaluate and compare the implied present value of the social welfare index associated with each of them.²⁰

²⁰ Note that the production constraints in the model, including those derived from the technological parameter values, may determine which, between alternative trajectories can satisfy the optimality requirements, so that answer to the question of which to choose become an empirical matter. See section 2.3 for further discussion.

Figure 3.1: Buy Time Model Trajectory 1 (Dark Green)

<i>Investment</i>	<i>Production (Capacity in Operation)</i>		
	Old Carbon High emission rate	Greened Carbon Lower emission rate	Carbon Free Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-economizing K^D (retrofitted K^A)	joint old & green carbon production	joint old & green carbon production	
		Greened carbon production	
Carbon-free capital K^B		Joint greened carbon and carbon-free production	Joint greened carbon and carbon-free production
			Carbon-free regime

Figure 3.2: Buy time model Trajectory 2 (Light Green)

<i>Investment</i>	<i>Production (Capacity in Operation)</i>		
	Old Carbon High emission rate	Greened Carbon Lower emission rate	Carbon Free Zero emission rate
Carbon-using capital K^A	production business as usual		
Carbon-economizing K^D (retrofitted K^A)	Joint old and green carbon production	Joint old and green carbon production	
Carbon-free capital K^B	Joint old and green carbon, carbon-free production	Joint old and green carbon, carbon-free production	Joint old and green carbon, carbon-free production
		Green carbon and carbon-free production	Green carbon and carbon-free production
			Carbon-free production

In the optimal control solutions each of the three foregoing “partial” models of a climate stabilizing transition, the internally optimized phases of the model are tied together by transversality conditions. That makes it possible to obtain the “requirements” of the entire optimal transition path, in terms of the optimum durations of its phases, and the optimized within-phase levels of activity for production, consumption, and investment rates (both the intangible R&D and the alternative tangible forms of technology-embodiment capital formation) – as well as the implied flows of CO₂ emissions up to the point at which the global economy’s production regime reaches the goal of a carbon-free state that averts the onset of irreversible climate instability. Even in the simplest of possible growth-model setting that has been specified, the solutions of each of these

successively more complicated models to find their respective overall optimal transition paths must be obtained by numerical analysis of the resulting systems of “stacked Hamiltonians” – the details of which following section discusses *ad seriatim*.

The presence of different phases of transition in which production capacity embodying new (comparatively “clean”) technologies is built up and that based on old (“dirtier”) techniques is run down and eventually discarded, is the feature that renders this modelling framework different from the standard AK growth model. A central task for our analysis of each of the models, therefore, must be to find the optimal configuration of specific investment and production activities, in combination with the optimality of their scheduling during the course of the transition to a viable and stabilized climate.

The questions addressed by this modelling exercise are not predictive. Rather than venturing to throw light on what will happen, they ask what has to be done, and when must it be done in order to get from a carbon-based production system to a carbon free production system that would avert runaway global warming, while maintaining the highest present value of social welfare that is consistent with attaining that goal.

We have begun by setting up a Hamiltonian system with a standard CIES inter-temporal utility function in which consumption (per capita) is its argument, since population and labor inputs) are suppressed and implicitly assumed to be constant.²¹ The latter is consistent with our making use of the simplest possible overall dynamic setting, namely, the AK-model of endogenous economic growth due to Rebelo (1991), and Barro and Sala-i-Martin (2004, ch. 4). We have extended the latter structure by replacing its specification of a single, linear capital-using production technology by the assumption that there are multiple discrete (linear) technologies that may be used either simultaneously or sequentially, including one that can be enhanced by directed investments in R&D activities.

The reduced unit cost of capital embodying the carbon-free technology as a result of directed R&D expenditures, however, is only realized through the technology’s subsequent deployment in gross tangible capital formation. To put a somewhat sharper point on this, it is not enough to think about R&D policies and programs. Mechanisms of diffusion into use, as distinct from the dissemination of information about technological innovations also must be figured explicitly among the requirements of a “technology fix.”

²¹ Readers interested in the details of the structural equations and solutions of the models discussed here may consult the “Technical Annex: Modeling Optimal Multiphase Transition Paths to Sustainable Growth” (July 2011), which is available at <http://siepr.stanford.edu/system/files/shared/OptimalMulti.pdf>, or can be downloaded as a supplement to an earlier version of this paper (presented on 12/14 at the SIEPR-GEEG Social Science and Technology Seminar) at: http://siepr.stanford.edu/programs/SST_Seminars/index.html.

Although the AK-setting is extremely simple, because we allow for different phases in the transition from carbon based to carbon free production, the resulting model is able to generate a set of time paths for the transition to a stabilized climate (and stationary level of atmospheric GHG concentration) that, in practice, can be determined only by the use of numerical methods.²² A future, more complicated extension of this set-up is envisaged, in which the cumulative increase in GHG concentration, or the mean global surface temperature gain associated with it, enters the utility function negatively.²³

The transition phases mentioned earlier are linked together through transversality conditions (cf. Leonard and Van Long (1992), ch. 7 in particular) that implicitly define the phase changes in terms of rather general optimality conditions that often give rise to conditions on the co-state variables that have a clear-cut economic interpretation. For example, it can be shown that it is never optimal to simultaneously invest in two existing technologies that are different with respect to their capital productivity and their emission characteristics.

Thus, in the current model setting, the existence of a cumulative emission tipping-point will generate notional emission costs associated with the use of carbon-based capital.²⁴ Investment in the latter type of capital will stop and that in carbon free capital will start the moment the shadow price of carbon based capital falls below that of carbon free

²² Given the fact that we distinguish explicitly between different sub-phases during the transition, we end up with a 'stack' of Hamiltonian problems that are linked together through a set of transversality conditions (TVC's), see further below in the text. The first order conditions, in combination with the TVC's and the given initial values of the state variables (and the given terminal value of cumulative emissions), give rise to a set of strongly non-linear constraints on the remaining initial values of the model's implied time paths for co-state and control variables. Using these initial values, the simplicity of the AK-setting allows us to obtain closed form analytical expressions by means of direct integration for the time paths of all variables—except for the cumulative stock of GHG emissions. The resulting paths themselves are in part highly non-linear and largely intractable (except for their underlying roots/structural equations) necessitating recourse to numerical exercises in order to investigate the properties of the model, some of which are not intuitively transparent *a priori*. Moreover, the path for cumulative emissions cannot be obtained by analytical means and requires numerical integration starting from some initial guess of the length of the BAU phase. The optimum duration of the BAU phase then is implied by the requirement that total cumulative emissions over the entire transition (all phases) should just match a pre-specified "threshold" value. Evidently a different approach will be required by a stochastic formulation of the transition process – of which the model discussed here can be viewed as the equivalent deterministic system.

²³ See Arrow (2009) and Weitzman (2009a, b) on the significance of specifying an additive temperature effect that is negative, interpreting this as a direct "environmental" effect upon social welfare. An alternative formulation is available -- in which the negative term in the social utility function is the inverse of the difference the critical "threshold," or "tipping point" temperature (beyond which the warming process is expected to become self-reinforcing) and Earth's prevailing global mean surface temperature. This formulation would have essentially the same consequences for inter-temporal resource allocation, but it would admit the adverse psychic effect of approaching the expected point at which humanity will, for all intents and purposes have lost its ability to stabilize the warming trend and avert the catastrophic onset of climate instability.

²⁴ We use the term "notional" here to denote the existence of a real cost, that is, however, generally not, or at least not fully, paid for in practice.

capital. As the cost of investment is represented by the welfare loss associated with consumption foregone, and as one unit of investment generates one unit of consumption foregone in both cases, this transversality condition implies the requirement that investment will take place in the technology that generates the highest marginal net welfare per unit of investment. Likewise, the discarding of existing capacity is typically an activity that signals the end of a particular phase and the beginning of the next one.

The optimum timing of such a phase change, and hence the phases' durations, will be governed by a general transversality condition, namely that the shadow price of an incremental unit of production capacity embodying the particular type of technology should be zero at the exact moment that it is taken out of production; otherwise, if it had a positive productive use, why discard it? This transversality condition turns out to be equivalent to the requirement that a unit of existing carbon-based capital should be discarded at the moment that the marginal welfare benefits of using that unit of carbon-based capacity drop below the corresponding marginal emission costs.²⁵

The features of the Basic Model of "technology switching" that have been presented above may be formally summarized in the equations set out below, which describe the objective (social welfare function, U , system of intertemporal production constraints and the transversality conditions that determine the optimal solution for the multi-phase transition path (see Table 0) . This, the simplest of the models, is discussed in further detail, and then extended in the sections that follow.

2.1 The Basic Model

The first and foundational version of our suite of multi-phase transition models is called (appropriately enough) the Basic Model. In this there are two already available production technologies, one that is carbon-based and already in use, and a carbon-free technology that at the outset is yet to be deployed.²⁶ This set-up creates the possibility of there being three distinct phases.

²⁵ This condition closely resembles the negative quasi-rent condition that governs the economic lifetime of clay-clay vintages in a perfect competition setting (see Malcomson (1975), for example).

²⁶ For purposes of this analysis it is supposed that the period under consideration commences when a carbon-free (B) technology becomes available, although its average productivity is lower than that of the capital (K^A) embodying the carbon-using technology; or, equivalently, that the cost of the production capacity (K^B) per unit of output exceeds that of K^A . ²⁷ As the BAU technology has higher capital productivity than the CFR technology by assumption, welfare would not be maximised if the flow of CO2 emissions would stop before the cumulative emission threshold would be reached.

Table 0

Phase-Specific Equations of the Basic 3-Phase Transition Model	
$Y^A = AK^A$	(production, BAU, JPR)
$\dot{K}^A = Y^A - C - \delta^A K^A$	(net investment, BAU)
$\dot{K}^A = -\delta^A K^A$	(net investment, JPR)
$\dot{E}^A = \varepsilon^A Y^A$	(emissions, BAU, JPR)
$Y^B = BK^B$	(production JPR, CFR)
$\dot{K}^B = Y^B - C - \delta^B \cdot K^B$	(net investment CFR)
$\dot{K}^B = Y^A + Y^B - C - \delta^B \cdot K^B$	(net investment JPR)
$\bar{E} = E_0 + \int_0^{TJPR} \dot{E}^A dt + \int_{TJPR}^{TCFR} \dot{E}^A dt$	(Cum. Emissions)
$U = \int_0^\infty \{e^{-\rho \cdot t}\} \{C^{1-\theta} / (1 - \theta)\} dt$	(intertemp. social welfare, ALL phases)
$H_{TJPR}^{BAU} = H_{TJPR}^{JPR} \Leftrightarrow \lambda^A = \lambda^B$	(TVC defining TJPR (equality Hamiltonians))
$H_{TCFR}^{JPR} = H_{TCFR}^{CFR} \Leftrightarrow \lambda^B A = -\varepsilon \lambda^S$	(TVC defining TCFR: deactivation YA)
$\lambda_{TCFR}^A = 0$	(TVC value of deactivated capital = 0)
$\lim_{t \rightarrow \infty} \lambda^B \cdot K^B = 0$	(TVC, standard AK TVC CFR phase)

In first phase, which can conveniently be labelled BAU (for 'Business as Usual') the carbon-free technology remains un-deployed, and the only positive tangible capital formation taking place adds to the stock of carbon-using production facilities, which are the current source of the system's real gross output. We have assumed that the available carbon-free production technology cannot match the carbon-based alternative in the average productivity of the capital facilities that embody it. To start deploying and using it immediately therefore would entail an immediate sacrifice of consumption that could be enjoyed by continuing to invest and rely upon carbon-based capacity. Better, therefore, to plan to sacrifice future consumption, the present utility value will be discounted – until the future negative shadow price of the associated CO2 emissions begins to approximate the incremental utility of the extra consumption flow. This is the economic logic of prolonging the BAU phase, up until the a point when it ceases to be

optimal to go further along that path and gross capital formation switches from carbon-using to carbon-free production facilities.

A second phase opens with the start of positive investment in facilities embodying the CFR technology, and the cessation of BAU capital formation, therefore marks the opening of the Basic model's second phase. From this point forward there is negative net investment in carbon-using facilities, since the physical depreciation of the existing stock is no longer being offset by positive gross investment in capital of that type. This phase of the Basic model is referred to as 'the joint production' (JPR) phase, as capital goods embodying both classes of technology being used to generate output. The transition to a carbon-free production regime and climate stabilization is complete with the shut-down of the remaining carbon-using facilities and, hence, stoppage of the flow of CO₂ emissions just as the cumulative stock of emissions (and the temperature) approach their respective critical threshold levels. That marks the beginning of the third and final phase of system's transition to sustainable, carbon-free growth.²⁷

The intensity of the flows during each phase, as well as the moments in time at which the various phase changes are scheduled, all follow from the first-order conditions (FOCS), the transversality conditions and from the given initial and terminal values for the state-variables that are part of the optimum control problem. Events and phase-

²⁷ As the BAU technology has higher capital productivity than the CFR technology by assumption, welfare would not be maximised if the flow of CO₂ emissions would stop before the cumulative emission threshold would be reached.

changes in the context of the Basic Model are summarised in Figure 4 (below), which

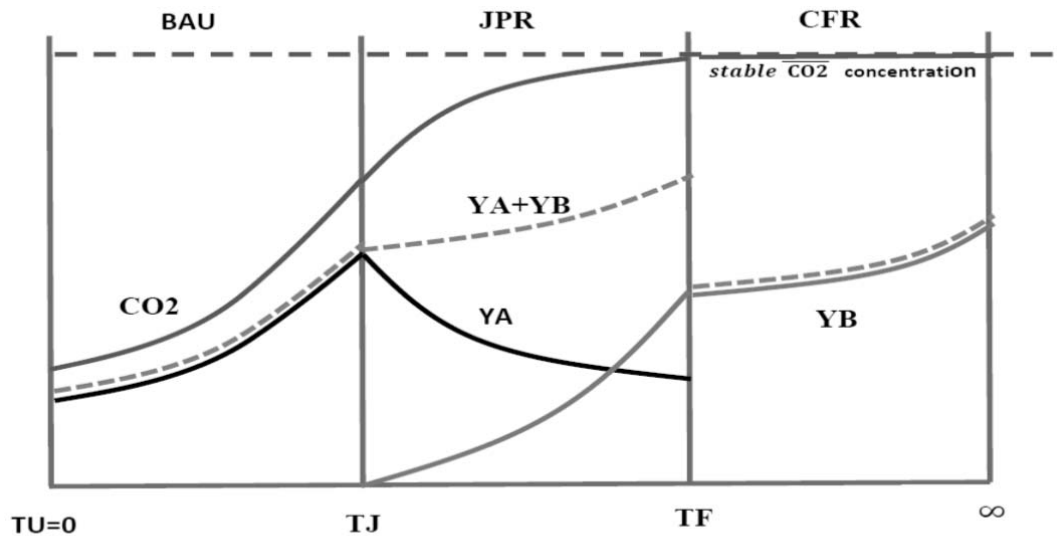


Figure 4. The 3 - Phase “Basic” (Technology Switching) Model:

This transition path to a Carbon Free Economy requires an eventual cut in production just before the “trigger-point” atmospheric concentration of CO₂-equivalent ppmv is reached, but output growth (YB) can resume thereafter.

shows the development over time of the stock of carbon-based capital K^A , the stock of carbon-free capital K^B and cumulative emissions, ΣCO_2

The points marked on the time-axis of the Figure (TU, TJ, TF) indicate the optimal moments at which the respective phases (BAU, JPR, CFR) begin. It may be noted that although the cumulative stock of emissions continues to rise during the JPR phase, it does so at a decreasing rate as the stock of carbon-based capital K^A is run down and replaced by its carbon-free substitute. The final, carbon-free production phase starts when cumulative emissions atmospheric concentration (ΣCO_2) almost reaches the expected climate “tipping-point, and the curve marked CO_2 becomes flat to the right of TF.

Evidently, the various phases in the model are qualitatively different. The first phase, i.e. the BAU phase, uses a high growth technology which unfortunately quickly raises the stock of cumulative CO₂ emissions that is bounded from above by the cumulative threshold. Before that level is reached, investment in carbon free technology must have taken place during phase JPR to bring carbon free production capacity up to a level where the switch from using carbon-based capacity to using carbon free capacity would not force changes in consumption levels that are too disruptive, since consumers dislike consumption shocks, as is implied by our use of a social welfare function that depends

upon the present (discounted) discounted levels of per capita consumption, and is of the (CIES) form allowing for relative risk aversion.

For $t > T_F$ the world is “green”, and, given the more expensive carbon-free production facilities that are required to stabilize the atmospheric CO₂-equivalent concentration level, the output (per capita) will have to grow for some while at the relatively slow pace at which the K^B is accumulating with the gross carbon-free capital formation rate constrained not to cut too heavily into consumption. Eventually, however, the build-up of the carbon-free capital stock is sufficient to support both rising consumption and a higher rate of investment in K^B , which thereafter accumulates at a quickened pace.²⁸

2.2 A Basic Model with Endogenous R&D

In addition to the Basic Model, we have specified a version in which R&D can be undertaken to improve the productivity of the CFR technology before its actual implementation through gross investment in the CFR technology. R&D requires resources to be invested now in return for a future intangible asset in the form of knowledge of how to embody CFR production techniques in production facilities whose capital cost per unit of output will be lower than is the case at present. Surprisingly, this view of the role to be played by R&D in controlling future GHG emissions is something of a novelty in the small economic literature that has employed integrated assessment modelling of climate policy options, because insofar as the contribution of investments in R&D to “directed” technology change and innovation has been considered at all, the “direction” has been taken to be the lowering of CO₂ emissions per unit total output in the economy.²⁹ To sharpen the contrast with the latter approach, Section 2.3 (below)

²⁸ The structure of the Basic Model has some commonalities with the analysis in Valente (2003) of a two-phase endogenous growth model in which production based on essential (energy) inputs obtained from exhaustible resources switches to a “backstop technology” that provides a constant supply of sustained (e.g., solar) energy. R&D investments permit growth through productivity improvements that occur at the same rate with either technologies (although the levels of productivity can differ), and the optimal timing of technology switching is determined by welfare maximization in which utility depends upon discounted per capita consumption. Valente similarly obtains optimal control solutions for each stage and ties these paths together with transversality conditions. But, unlike in the present analysis, the point at which the renewable technology can be efficiently embodied in tangible capital used in production is not endogenously determined by directed R&D expenditures and the diversion of output to building the “renewables-base” capital stock. Nor is a positive shadow price explicitly attached to using the exhaustible resource in Valente’s analysis, because it abstracts from the geophysical climate-system constraints that affect the optimal transition path. As will be seen, the research approach here examines much more complicated, multi-stage transitions.

²⁹ To the best of our knowledge, the earliest previous investigation of the effects of allowing for *endogenous* technological change in a computational climate policy assessment model appears in the related papers by Goulder and Mathai (1998), and Goulder and Schneider (1999), which specify the effect of R&D expenditures on changes in an economy-wide total factor productivity coefficient. In these pioneering investigations of the impact of induced technological change on the attractiveness of CO₂ abatement policies, absolute changes in productivity (or real marginal cost) levels, rather than the proportionate rate of change in productivity was specified as a positive increasing (decreasing) function of R&D expenditures. In Goulder and Schneider (1999: pp.216, 223) this functional relationship not

examines the option of achieving greater efficiencies in the use of carbon-based energy can be achieved *without R&D expenditures*, but by investments in the engineering implementation of existing technological knowledge that would result in higher unit (tangible) capital costs of carbon-based production facilities.³⁰

Furthermore, in specifying the impact of such R&D investments on the unit cost of CFR capital goods (K_B) we deliberately depart from the “R&D production function” formulation that is familiar in endogenous growth models following Lucas (1988), Romer (1990) and Aghion and Howitt (1991). Rather than taking the instantaneous rate of improvement in the productivity of CFB-embodying capital, \dot{B} / B , to be proportional to the current absolute flow of R&D resource inputs, R , as in

$$\dot{B} = \zeta \cdot R \cdot B \quad 0 < \zeta < 1 ,$$

an alternative specification is proposed here:

$$\dot{B} = \zeta \cdot R^\beta \cdot (\bar{B} - B), \quad 0 < \beta < 1 ,$$

where ζ is a constant productivity parameter, and \bar{B} represents the maximum value that B can attain.³¹

There are several features of this specification that argue for its’ use in the present applications context. Firstly, it has the advantage of introducing decreasing returns to

further restricted in their mathematical analysis of a 2-period model; whereas the linear specification, $\dot{B} = \zeta \cdot R$ was used in their dynamic multi-period general equilibrium simulation studies. Nordhaus (2002) takes a different approach, in effect assuming that induced research effort focuses solely on reducing the CO₂ emissions intensity of production. Nordhaus posits a separate production function for carbon-based energy inputs which are used in fixed proportion to the non-energy inputs in an economy-wide production function. He then specifies that the rate of change of the carbon-intensity coefficient in energy production is given by $(d\sigma/\sigma) = -[\Psi R^\beta - \Omega]$, where $\Psi > 0$ is a constant and $\Omega > 0$ is a (constant) rate of obsolescence of the technical knowledge gained through R&D and reflected in σ . The empirical findings of Griliches (1973), Hall (1995) and others on the link between commercial R&D and industrial productivity growth are cited by Nordhaus in support of this specification, although in general the time-spans to which the data on industry-level and firm-level data relate are far shorter, and hardly of the global scope typically contemplated in IAM models.

³⁰ Of course, this distinction is not necessary because one can model the situation in which R&D expenditures are allocated between the two “directions” of technological change, lowering raising the ratio of gross output per unit of CO₂ emissions, and raising the productivity of carbon-free tangible capital used in production. But because the present investigation is confined to modeling the paths to successful climate stabilization, and the time scale for that is far shorter than the hundreds of years contemplated in the existing variants of the Nordhaus (1994, 1999, 2002, 2007) DICE model, it is not unreasonable to suppose that the presently existing stock of implementable techniques for enhancing the productivity of carbon-energy inputs could suffice for a considerable number of decades without requiring “refreshment” by focused investment of R&D efforts. See further discussion of the “buy time” option in section 2.3, below.

³¹ With \bar{B}, ζ, β as constant positive parameters, B and R therefore become additional state- and control variables in our dynamic system.

R&D in a setting that, unlike the conventional endogenous growth models, excludes the possibility of a specific technology being rendered infinitely productive (and so resulting in infinitely rapid growth) merely by the application of more and more massive R&D expenditures at any particular moment in time.³² Allowing decreasing marginal returns in R&D recognizes that at a given stage in the advance of knowledge the state of fundamental scientific understanding of the physical processes involved may still be inadequate to permit the effective application of more and more resources to the solution of a particular practical problem -- such as the further improvement of the productivity of a particular class of technology-embodied capital facilities.

Secondly, this formulation of the effects of investment in R&D activities may be thought to reflect a Platonic world in which a finite number of solution possibilities for technical transformations are present from the start of time, but these as a rule will not reveal themselves spontaneously. They can be uncovered, however, and formulated for practical application through costly research and development procedures based upon the existing state of fundamental scientific knowledge, rather than being created *de novo* and without limit by the expenditure of resources in the performance of R&D activities.³³ That more restrictive view of the transformative power of investment in R&D and is appropriate not only for the foregoing general reasons, but also because the concern in this context is not with the undirected global expansion of the technological opportunity set typically envisaged in theoretical growth models. Rather, the aim of the “directed R&D” in the present model is to enhance the economic properties of particular kinds of process inventions, with new product inventions only insofar as alterations in product characteristics are consequential for the raising the efficiency of capital inputs into carbon-free production processes.³⁴

Within the framework created by introducing this (or any other) R&D production function into the Basic model, there are again three distinct phases of the transition to a stabilized climate. Table 1 (below) compares the phases in the original basic model with the version introducing R&D. It should be noted that while the R&D model resembles

³² The equilibrium (steady-state) growth rate in a standard AK-model rises linearly with the rate growth in the productivity of capital (cf. Barro and Sala-i-Martin, 2004), and therefore with the flow rate of R&D expenditures. The existence of technology-specific intrinsic productivity limits set – in the limit -- by the physical properties of the materials, chemical and electrical processes entailed in production makes this so implausible that even its assertion as a metaphor is of dubious usefulness.

³³ It also has the advantage of being jointly concave in B and R, which is a necessary condition for the welfare maximization problem to have a solution.

³⁴ Following this interpretation, adding endogenous technological change to the Basic Model allows us to characterize the optimal path of *global R&D that is directed to increasing the productivity of CFR capital*. Correspondingly the impact of R&D investment on economic welfare is modeled as being felt indirectly, rather than directly in the form of pure product quality enhancements. In other words, the welfare gains come through reduction of the sacrifice of consumption utility required in the transition, and for the subsequent sustained growth of (per capita) consumption under stabilized climatic conditions.

the Basic Model, because it has three phases and two optimum switching moments, it differs in having three state variables and two control variables.

Table 1. Comparison of the Basic and Endogenous R&D Models

<i>Basic model</i>			
Phase	BAU1 → Business as usual	JPR1 → Joint production	CFR1 Carbon-free
Investment in	Carbon-based capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	Carbon-free capital only
<i>Endogenous R&D Model</i>			
Phase	BAU2 → Business as usual	JPR2 → Joint production	CFR2 Carbon-free
Investment in	Carbon-based capital	Carbon-free capital	Carbon-free capital
R&D investment in	Carbon-free technology	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and carbon-free capital	Carbon-free capital only

The reason is that it turns out to be optimal to start doing R&D to improve a particular technology from the very first moment that information about a workable CFR-technology becomes available. On the assumption that the CFR technology is discovered at time zero but is less productive than the mature carbon-using technology when embodied in equally costly production facilities, R&D activity then be undertaken (only) during the phase when tangible capital formation and production adhere to the carbon-dependent features of the Basic model's BAU phase. Following that, a JPR phase will commence with gross investment directed to deploying the improved CFR technology in KB-type production facilities.

2.3 A “Buy-Time Model” – Greening Carbon-based Capital in the Basic Model

We now turn to the third of the partial models, in which the structure of the basic model is enriched by introducing a third category of technologies, namely a known “intermediate” class of carbon-using production techniques that are characterized by lower rates of CO₂ emissions per unit of capital but capital productivity per unit of output that is lower than of the mature carbon-based technologies, but not as much lower than that of the initial (pre-R&D) versions of the carbon-free techniques of

production.³⁵ This conceptualization corresponds broadly to the variety of well-grounded engineering methods may be used to upgrade carbon-using production facilities, whether by improving the energy efficiency of residential, business and government office buildings by better insulation, heating and cooling systems, and reducing the passive consumption electricity by electrical and electronic appliances by configuring them to switch off completely when not in use, and so forth. But it also subsumes proposals for the expanded exploitation of natural gas as a reduced GHG-emissions source of energy, including the acknowledgement that incremental costs per BTU would be required to curtail the environmental damage currently associated with “fracking” methods.

Break-through research and novel engineering principles, however, are not required to exploit this class of carbon-based energy sources and production technologies here, but specific applications of core engineering knowledge adaptation of existing designs to local contexts will raise the average unit capital costs carbon-using plant equipment that has undergone this kind of “green-upgrading,” as well as that of a “less dirty” energy source such as natural gas, and safer nuclear power-plants with provisions for long-term sequestration of their toxic waste. The gain to be had by seizing this “low-hanging fruit” comes in the reduced rate of CO₂, which means undertaking the entailed incremental capital expenditures constitutes a way to “buying time” early in the transition in order to be able to subsequently proceed more slowly in replacing the whole carbon-based production regime with one based on new, carbon-free production facilities.

³⁵ On the latter, see the recent noticed example of the annual consumption of over \$3 billion work of electric power in the U.S. by HD TV “set-top” boxes supplied by cable companies, due to the choice of low cost designs that do not actually stop draining power when they are switched off (see, “Atop TV Sets, a Power Drain that Runs Nonstop,” *The New York Times*, June 26, 2011: p.1.). The McKinsey Global Institute devoted considerable attention in the years before the financial crisis to studies of current and near-term options for energy efficiency routes to CO₂ emissions-reductions and private costs saving through upgrading of production and distribution systems. See, e.g., Farrel and Rennes (2008); Grove and R. Burgelman (2008). In the U.S., numerous proposals for this kind of “retrofitting” have had difficulty gaining policy-traction due to the prevailing policy bias towards research subsidizes to support “innovation” in renewable energy technologies.” More recently, the case for expanded exploitation of natural gas and greater investment in other opportunities to improve the productivity of carbon-based technologies that (similarly) offer higher output per volume of GHG emitted has been cogently and vigorously advanced by Burton Richter (2010). During the crisis and recession years the attention of the European Commission shifted away from the expensive and longer-term research envisaged by its ambitious Strategic Energy Technologies Plan [SET, COM (2007) 723]; it focused instead on a variety of shorter-term tactics aimed at stimulating aggregate demand in ways that would implement already available technologies for “green” purposes, notably : retro-fitting buildings for greater energy efficiency, supporting the automotive industry to increase production of low-CO₂ vehicles using electric batteries and second generation bio-fuels (see discussion in David, *op.cit.* (2009). In the U.S., numerous proposals for this kind of “retrofitting” have had difficulty gaining policy-traction due to the prevailing policy bias towards research subsidizes to support “innovation” in renewable energy technologies.

Not surprisingly, therefore, the extent to which the “buy time option” will be attractive to exploit in the context of our Basic model, by building up production capacity (K^D) in this intermediate “greened” form rather than capital that embodies the mature, more carbon-intensive technology, depends not only on the associated capital productivity but also on the reduction gained in emissions per unit of output. It turns out that introducing the possibility of utilizing this third class of technologies gives rise to a model in which there are 5 distinct phases that can be arranged in either of two alternative transition trajectories, as indicated by Figures 3.1 and 3.2 and detailed in Table 2.³⁶

Moreover, which trajectory will be the one that is optimal for this simple economy to follow depends upon the parameter configuration that sets the BTM technology’s output/emission ratio. Given these ‘technical data’, the choice between the alternative trajectories that will be followed in exploiting the “buy time option” is prescribed by comparing the welfare valuations of the two solutions of the two optimal control programs.³⁷ Trajectory 1, as described by the following Table, is found to be the welfare dominant member of the pair when the that ratio is high, whereas when the ratio is low it is better to follow Trajectory 2, which calls for a shorter period of investment in building stocks of K^D and an earlier switch (in the third phase rather than the fourth) to gross capital formation embodying the carbon-free technologies.

In Table 2 (below), the header lines contain the labels for the sub-phases in each trajectory.³⁸ The entries below each phase label belonging to a trajectory contain a shorthand description of the activities taking place during or at the beginning of the sub-phases. Notice that the main difference between both trajectories is the time at which the carbon-using A-technology is de-activated. As a consequence, output during sub-phase BTM22 of trajectory 2 comes from three different sources, so the equation describing the accumulation of BTM capital is more complicated than that for trajectory 1.³⁹

³⁶ Note that the act of ‘buying time’ involves using a technology with a relatively high output/emission ratio. Hence, logically speaking, buying time is associated with production using that technology rather than with investment in that technology. Therefore, in this case, the BTM phase is subdivided into three sub-phases, in which we have positive output from the BTM technology (here indexed ‘D’), and the CFR phase starts when the BTM-technology is deactivated. Typically, the j-th BTM sub-phase of trajectory k is labeled BTM.

³⁷ That the model itself directs attention in this way to the relevance of empirical information about certain parameters is worth notice, because it may help in prioritizing areas warranting more concrete and detailed empirical research in engineering and applied economics.

³⁸ Typically, the j-th BTM sub-phase of trajectory k is labeled BTM_{j,k}.

³⁹ Although this makes the entire model considerably more difficult to solve, it was solvable for *Mathematica*.

Table 2. Alternative trajectories involving the buy time (BTM) technology

<i>Trajectory 1</i>					
Phase	BAU1 → Business as usual	BTM11 → Buying time	BTM21 → Buying time	BTM31 → Buying time	CFR1 Carbon-free
Investment in	Carbon-based capital	Reduced emission capital	Reduced emission capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	Reduced emission capital only	Carbon-free capital and reduced emission capital	Carbon-free capital only
<i>Trajectory 2</i>					
Phase	BAU2 → Business as usual	BTM12 → Buying time	BTM22 → Buying time	BTM32 → Buying time	CFR2 Carbon-free
Investment in	Carbon-based capital	Low emissions capital	Carbon-free capital	Carbon-free capital	Carbon-free capital
Output using	Carbon-based capital	Both carbon-based and reduced emission capital	All three types of capital	Carbon-free capital and reduced emission capital only	Carbon-free capital only

By solving the stacked optimum control problems associated with the model versions outlined above, we arrive at a complete and intertemporally consistent description of the nature and timing of the various phases, as well as the shape of the time-paths of tangible and intangible capital investment, levels of production and consumption, as well as cumulative emissions as a function of the structural parameters of the model. These parameters include the probable location of the cumulative emissions threshold, the productivity of the R&D process as well as the productivity differences between carbon-based and carbon-free technologies, and the ‘standard’ preference parameters for the consumption component of the social utility function, i.e. the rate of time discount and the intertemporal elasticity of substitution.

3. Preliminary results, experiments with physical system interactions

To this point in the present research program, the models described in the foregoing pages have not been calibrated on actual data for the global economy. Nor have we completed our work on the integration of the BTM model with the endogenous R&D

model. The latter is a matter of particular interest because being able to extend the length of time during which R&D expenditures are maintained is (as far as we presently can see) likely to be the most productive use of the added time “bought” by CO₂ emission-rate reductions effected by exercising the BTM option. Using known techniques to upgrading carbon-using infrastructures and directly productive capital facilities certainly would be a socially more productive purpose than hastening the inevitable descent into climate instability just for the sake of prolonging current enjoyment of the higher consumption levels associated with “business as usual.”

Comparisons of the results obtained with the different partial models, however, yields a number of useful insights about the role of directed R&D investment in the transition to climate stabilization. One striking and intuitively understandable finding that emerges from the optimized solution of the endogenous R&D model is highlighted by its juxtaposition with the features of the transition path found for the basic model. In the latter case, business-as-usual investment in carbon-using capital (K^A) comes to a halt when carbon-free technology can be embodied in production facilities, regardless of their greater unit capital cost; whereas in the former model the BAU phase ends only when R&D succeeds in rendering the unit costs of production facilities embodying CFR techniques competitive with that of K^A .⁴⁰

Allowing for the possibility of investment in R&D directed towards lowering the unit capital costs of carbon-free production processes has the effect of raising the tangible investment in carbon-using production facilities, shortening the absolute and relative duration of the BAU phase -- leaving the length of the subsequent joint production phase essentially the same. In other words, being able to invest in the research required for a “technical fix” has the expected result of speeding the beginning of CFR production, the cessation of investment in carbon-using production facilities, and correspondingly, the retirement of the old carbon-based capital stock.

The underlying economic logic here is that the anticipation of being able to build more productive CFR capacity in the future generates a heightened *derived* demand for BAU capacity, and hence a higher rate of tangible capital formation in carbon-based facilities -- since the desired stock of CFR capital is a produced means of production. The

⁴⁰ Strictly speaking, the last part of this statement is not quite true in the model on which this discussion is based, because the upper limit on the capital productivity of the CFR technology, \bar{B} is set below the that for the carbon-using technology. R&D removes as much as possible of the productivity deficit of K^B relative to K^A before implementing the CFR technology tangible capital. B. But this is in no way essential, and there are no a priori reasons for not entertaining the possibility that CRF technologies can (eventually) have lower unit capital costs than carbon-based technologies, so that if the exponent and constant in the productivity change-R&D input function are quite small, the model is the model is not inconsistent with the observation that the world we live in is (still) a dependent upon a carbon-using regime of production.

resulting higher volume of K^A generates a larger output flow, providing a greater pool of resources that can be spent on R&D investment during the BAU phase, and subsequently for CFR-embodied capital formation in the JPR phase. It should be noted, however, that this would be a dangerous plan to have pursued were the technical improvements in carbon-free production systems that had been expected to result from R&D expenditure to fall short of expectations. In such circumstances – hardly unrealistic in view of the uncertainties surrounding the performance of R&D -- it would be necessary to eventually make the required switch to much more costly CFR production capacity, attended by consequently greater losses of consumption and welfare.

This suggests the practical relevance of combining the endogenous R&D model and the BTM model, because, by adding the “buy time option” of reducing the CO₂ emissions rates on the carbon-using production capacity, there would be more time left to build-up the necessary carbon-free capital stock. Since that capital formation process would weigh all the more heavily of consumption levels, exercising the “buy time option” can serve as a partial insurance against the adverse welfare consequences of disappointed expectations regarding the effectiveness of R&D investment in enhancing the productivity production facilities embodying carbon-free technologies. Although the two options often seem to be discussed as alternative, substitute “climate policies” they are more properly seen to be complements in an expanded “technology fix” portfolio.

It is a fairly straightforward matter to conduct sensitivity experiments with these partial models, in order to get a qualitative impression of the impact of parameter variations that, given the deterministic form the model can begin to suggest the range of distributions in the dynamics the system exhibit were the model component reformulated more realistically in stochastic terms. Moreover, such sensitivity experiments also shed light on the way in which the requirements for a successful climate-stabilizing transition will be affected by -- and therefore need to make ex ante allow for -- alterations in the perceived and actual dynamic feedbacks arising from interactions between the economic and the geo-physical subsystems.⁴¹

Since at this stage of our work the results of the solution of the model considering both the buy-time option and R&D directed to innovations in carbon free production technology are still being analysed, we report the results of using sensitivity experiments with the “external” physical specifications of our partial economic models as a preliminary means of exploring the impacts of their interactions. Among the variety of computational experiments that can be readily executed, the following pair has yielded results that are of principal interest: (i) lowering the cumulative GHG emissions tipping-point, and (ii) increasing the annual rate of physical depreciation of carbon-based production capacity.

⁴¹ For example, a more cautionary stance towards the dangers of surpassing the cumulative emissions threshold may involve a lowering of the ‘model’- threshold below its ‘real world’ expected level.

The first of these offers a simple way to assess the gross effects of making precautionary allowances for the ambiguity surrounding the exact level of the atmospheric GHG concentration level that will be a “tipping point” into the domain of irreversible climate instability.⁴² The robust result we find for all variants of lowering the critical threshold for temperature gain (or equivalently, the cumulative emissions tipping point) is that the required length of the transition to a stabilized climate would be shortened by abridgement of the BAU phase, leaving the duration of the joint production phase essentially unaltered. Initially, output is reduced but consumption is raised, so that investment in tangible capital formation is postponed; although consumption subsequently will be reduced, there also is less investment in the new CFR technology, and, indeed and lower levels of activity across the board – compared to those found for the (basic) reference model when the temperature-gain threshold is set at a higher, less constraining level.

The general finding, then, is that with less scope for CO₂ emissions before the expected critical threshold will be reached, production with carbon-using capacity must be curtailed and the length of the transition abridged; with less time to build up carbon-free capital goods, the eventual growth of output and consumption has to be deferred until climate stabilization with a carbon-free production regime has been achieved. A further, consistent result applies in the specific cases of the endogenous R&D and BMT models: because a lower critical threshold leaves less time to do R&D or to buy time, order to counter that effect to some extent, the distribution of R&D activity over time must be shifted to in favour of R&D now and against R&D later.

Thus, the planning messages to be taken from this is that setting a lower GHG ppm target --in keeping with greater aversion to risk ambiguity and consequent adherence to the maximin strategy dictated by the “precautionary principle,” calls for rapidly ramping up the rate of R&D expenditures. Similarly, it warrants boosting at the outset the rate of capital formation in production facilities with upgraded output/emissions performance. Later, both types of investment will have to decline, first relative to output and then absolutely, once the build-up of carbon free production capacity gets under way.

⁴² The implications of allowing for the ambiguity in the location of the “tipping point” beyond which warming becomes a self-reinforcing process even where GHG emissions of immediate anthropogenic origins have ceased completely will need to be assessed within the context of a fully integrated stochastic version of our model. Lemoine and Traeger (2011) introduce probabilistic functions for crossing a climate regime tipping point into a recursive formulation of the DICE integrated assessment model for establishing optimal climate policies. The tipping point's probability and timing thus become endogenously determined by the chosen emission policy. Recognizing that the probability distribution for the “temperature Threshold” corresponding to the “tipping point” is not known with confidence, policy decisions can be made that reflect “ambiguity aversion” by selecting strategies that reduce the downside variance around the expected “threshold temperature. Lemoine and Traeger’s simulations show that under reasonable parameterization of the DICE model, allowance for tipping points in this way can raise the near-term social cost of carbon emissions by as much as 50%.

Our second set of experiments provide a simplified way of emulating the higher frequency of damages that can be expected to result from extreme weather events associated with changing climate(s), and to assess the impact on the optimized transition plan of anticipating the consequences of warming that already is “in the pipeline,” due to the past history of GHG emissions. An increased rate of unscheduled capital losses (damages from increased climate instability) has the effect of postponing the arrival of the CFR phase, primarily because production using the carbon based technology is negatively affected, leading to a lower flow of CO₂ emissions in the process. This is “the good news”, but, unfortunately it is not the whole story.

Since it seems reasonable for purposes of this analysis to assume that these negative climate effects will impact old carbon-based facilities most severely, if not exclusively,⁴³ the rate of return on investment in carbon-using facilities will be lower than is the case in the absence of allowance for this feedback of climate destabilization (i.e., in the base model). The “bad news” is that this results in lower initial rates of carbon-using capital formation (gross investment in KA and KD) and higher rates of consumption in an extended BAU phase. Consequently the constraint place on the growth of productive capacity during this early phase of the transition curtails the rate of R&D investment, and the global economy’s subsequent ability to rapidly accumulate the necessary stock of capital embodying carbon-free technologies.

One implication of the foregoing findings would seem to be that expenditures aimed at averting “unscheduled losses” in production capacity due to climate instability-related damages, may be quite a “good investment”. It should be noted that under the assumption of increased damages, the level of R&D activity also is affected negatively. But, considering that in a fully endogenous integrated model, climate-related capital-losses will be driven by rising GHG concentration levels and higher mean surface temperatures, they are likely to take a heavier toll on existing production capacity at dates farther in the future, rather than from the outset of the transition.

That particular form of non-stationarity can be emulated (albeit roughly) by a further simulation experiment in which the physical depreciation rate on carbon-using capital jumps to the higher level only after the tangible investment in that type of capital has stopped. We may suppose that the anticipation of those future capital losses, as before, will exert a depressing effect (albeit less heavily) upon the expected net rate of return to investment in KA during the BAU phase. Consequently, although the induced near-term reduction in tangible (carbon-technology using) capital facilities, and the rise of

⁴³ The consideration underlying this assumption is the a large portion of the carbon-based infrastructure and directly productivity capital stock will be legacies from the BAU regime and the historically antecedent state, it will be located in temperate latitudes and built in fashions that will leave it more vulnerable to the effects of coastal and riverine flooding, and serve wind-damage from hurricanes and monsoons, than the recently constructed carbon-free facilities. The latter, ideally, will have been designed and sited with those risks in mind.

consumption per capita will occur during the BAU phase, that would not happen from the phase's outset. Instead, it would be concentrated in the period just before the carbon-using stock attains its maximum, when (under the assumed timing of the increased rate of unscheduled capital losses) the weather-created damages start to occur with greater frequency.⁴⁴ The overall effect of these alterations in the timing of intangible (R&D) investments upon the terminal value of the productivity of capital embodying CFR technologies is bounded, however, because the BAU phase during which R&D is taking place will have been stretched out. That change compensates for the cumulative effect of a decrease in the flow rate of R&D inputs, and so ameliorates the latter's negative impact upon the extent of productivity improvements in K^B .

A major message that emerges from the foregoing modelling exercise and parameterization exercises is one that might well have been anticipated at its outset. What we learn from heuristic model building of the present kind is how to think about the problem, and not necessarily what to conclude about the best course of action in meeting the challenge it poses. Even the highly simplified dynamical system that we are studying has sufficient interconnected and mutually interdependent "moving parts" to tax one's unassisted intuitions as to the ways that variations in the parameters of the geophysical and economic subsystems will alter the optimized transition paths to a stabilized global climate.

Moreover, while the simplicity of the heuristic growth model makes more transparent the logic of changes in the directions of investment and production activity in their various forms, the sequenced phase structure of the transition is not so immediately obvious. Nor can the impacts of parametric variations on the optimal phases' relative durations be ascertained without undertaking explicitly quantitative analyses. The latter can serve to prioritize areas for empirical research, by identifying technical parameters (and in models with richer specifications of agents' behaviors, critical behavioural parameters)--the magnitudes of which are found to strongly impact the welfare properties and shape the resource allocation requirements in the early stages of the optimal transition path. Building such models, and investigating the implications of more sophisticated, integrated systems of economic-climate interactions, should not be seen as a pursuit intended to produce a substitute for the exercise of intuitive judgements about matters of economic policy design. In the end, the latter will have to weigh many important practical considerations of human behavior, culture and politics that will resist accurate capture in tractable quantitative models. Instead, we regard

⁴⁴ This difference in timing has the effect of releasing resources that allow for a faster build-up of carbon-free capital (by assumption, designed so as to be not susceptible to the elevated severity of the weather). In addition, due to the specification adopted for the R&D production function in our model, the release of those resources at a later point in the future, after considerable R&D expenditures have taken place, is less costly (in terms of CFR productivity gains foregone) than is the case when the volume of R&D is lowered from the outset.

the exercise of experience-based intuitions and quantitative analyses to be complementary ingredients in the policy design process; and believe they will work more effectively and reliably when each is allowed to inform, sharpen and qualify the conclusions to which one is led by their joint employment.

4. What is to be learned from the next stage in this research program?

The present paper focuses on understanding the deterministic system of the modelling framework we have constructed, and has indicated the directions in which we can proceed to complete the integration of the partial models within a complete endogenous economic and geo-physical system.

In the next major stage of the program, it will be important to move to a stochastic integrated model, by specifying the probability distributions governing the “climate sensitivity” of the physical system, which is the principal nexus between the altered production system and the welfare consequences of the climate changes brought about by global warming.⁴⁵ Similarly, we also must recognize the stochastic nature of the output of R&D expenditure inputs and the payoffs of technological advances permitting higher productivity in the available stock of capital embodying carbon-free technologies. But for the present there are many interesting things to learn, and questions to answer by exploring the equivalent deterministic version of our model.

For different parameter constellations we are able to show how the timing of the phases will change, and how the evolution over time of welfare per capita will be affected, but also how the required R&D expenditures and the volume of gross capital formation will change, both in terms of their distribution and intensity over time. We also show how uncertainty regarding the position of the climate tipping point in combination with different degrees of precautionary behaviour would affect the optimum timing and intensities of the various activities distinguished in the model.

The flip side of the latter computational analysis is our ability to explore the welfare damage of “policy implementation failures”. Starting from the optimal program for the transition to a viable and stable GHG concentration level, it is feasible to show the effects of budgetary or political constraints that result in specifically timed postponements of public investment in R&D, or uncompensated private cuts in the upgrading of carbon-based production facilities, or in the roll-out of the carbon-free

⁴⁵ The nature of the distribution of the feedback parameter describing the physical system’s response (in terms of global mean surface temperature to a doubling of the atmospheric GHG concentration level) is surrounded by very substantial uncertainties arising from the complexities of the interactions among the physical and chemical processes that underlie the feedback sequences indicated in the introduction. In the present context there is an important issue as to whether it is reasonable to assume that the effects on the distribution around the mean “climate sensitivity” of changes in the mean GHG concentration level will be variance-preserving.

technology when, under normal business conditions it would be advantageous to undertake installation of the required capital investments. How much the timing of action matters in this system, and the penalties for deviation from the optimal path can be shown under the assumption that no effort is made to re-optimize from the position that has resulted in the aftermath of such “shocks”, and also under the assumption of trying to “catch up” after a particular range of delays by means of re-optimizing the transition path. Thus it is possible computationally within this heuristic modelling framework to describe the relationship between the lengths of implementation delays during specific phases of a successful transition to climate stability, and the consequent costs in terms of net-welfare foregone.

5. Getting “Tech Fix” into the climate policy mix: A beginning, not a conclusion:

This initial stage in our projected explorations of the “requirements” for a socially optimal transition to sustainable global economic growth in a viable stabilized climate has been motivated primarily by the belief that it is vitally important to give “technology fix” options a central place in structured analysis of policies that can respond effectively to the challenge of climate destabilization. Environmental and energy economists have been quick to emphasize the allocational efficiency of various fiscal and regulatory means of raising the market prices of carbon fuels, and suggested that the efficacy of that policy approach would be enhanced by their effects in inducing “carbon-saving” technical innovation from private sector.

As has been pointed out, however, the latter proposition presupposes that raising the cost of fossil fuel sources of energy would not have negative income effects that dampened private R&D expenditures by energy-intensive sectors of the economy. Furthermore, the integrated assessment models that economists have developed as quantitative tools to guide policies aiming to raise the price of carbon typically fail to show the conditions under which they would elicit a sufficient flow of innovations that were directed specifically to curtailing and eventually displacing fossil fuels as a dominant global source of energy. To address this “policy assessment gap” would entail giving greater attention to specifying the relevant characteristics of the array of energy-using technologies, and the corresponding “endogenous technological progress” sub-system, and to connect them with the other components of the aggregate economic growth models that will govern the pace and extent of deployment of improved production techniques.

The dynamic integrated requirements analysis modeling (DIRAM) approach that has been introduced here may be viewed as a preliminary step towards a more illuminating representation within the context of familiar IAMs of the potential role of technological measures in the transition from the present dominance of carbon-based energy to a low carbon global regime of production. It surely will occur to some readers to question

according priority to elaborating the technological specifics of existing IAMs, on the ground that many other features of these highly stylized models also warrant further elaboration, and greater policy relevance calls for a spatially disaggregated approach that would take account of geographical and ecological variations affecting both the global economy and the climate system. Indeed, there already have been many efforts along just such lines, impelled by a policy interest in assessing the differential regional and national incidence of the costs of curtailing global GHG emissions. Others might argue for the importance of recognizing the endogeneity of changes in the size, age structure and spatial distribution of global population, and more sophisticated welfare framework to account for demographic as well as economic changes in the transition to a viable stabilized climate.

even were the latter qualification to be waved away, there should be room to consider that arriving at domestic and international agreements to impose future increasing prices for fossil fuels would be politically less arduous where there good prospects for significant mitigation of such commitments' adverse impacts on future real profit rates and consumption levels. Just that context would be created by a prior commitment on the part of the already developed and scientifically and technically advance countries to major coordinated technological programs -- such as those aimed at significantly increasing the efficiency of energy distribution and use, and at lowering the real unit costs of non-fossil fuel sources of energy.

Rising prices for carbon-based energy would encourage any form cost-saving innovation activities and, by that token, it would raise the perceived marginal social payoff from expanding public support for science and technology research – if only to create a knowledge infrastructure that would lower the costs of directed innovation in the affected sectors and lines of business. Nevertheless, the augmented public R&D funding would have to be forthcoming, and additional, differential subsidy measures would need to be introduced to direct private R&D (and subsequent deploy the results) toward lowering the unit costs of carbon-based and alternative energy sources. Alternatively, a well thought out supply-side climate policy that started with the goal of expanding a variety of applications-oriented R&D programs would tend to create credible expectations of substantial future resource savings with carbon-free production facilities, and a concomitantly smaller sacrifice of material welfare entailed in restricting GHG emissions. That could contribute to weakening economically motivated resistance to a schedule of gradually rising carbon taxes and therefore impart wider and stronger commitments to national and international agreements on coordinated and variable actions that would curtail GHG emissions. Bundling proposed commitments from lower income developing economies with reciprocal loan subsidies and cost concessions in the transfer to new “greener” and carbon-free technologies would constitute a credible package for negotiations that would grow in its attractiveness as the R&D programs matured.

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