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Sovereign Debt Maturity Structure Under Asymmetric Information

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Abstract

This papers studies the optimal choice of sovereign debt maturity when investors are unaware of the government's willingness to repay. Under a pooling equilibrium debt can be mispriced relative to the borrower's true fundamentals and the degree such mispricing can differ with the maturity of debt. Long-term debt becomes less attractive for safe borrowers since it pools more default risk that is not inherent to them. In response, safe borrowers issue low levels of debt with a shorter maturity profile -relative to the optimal choice under perfect information- and risky borrowers mimic the behavior of safe borrowers to preclude the market from identifying their type. In times of financial distress, the mispricing of long-term debt relative to short-term debt becomes stronger which makes borrowers reduce the amount of debt issuance and shorten its maturity profile, a fact that is observed among emerging economies. Under this framework, debt buybacks can be optimal when there is an overestimation of the country's risk perception in the market that was not present when long-term debt decisions were made.

JEL Classifications:H60,H63

Keywords: Debt maturity structure, asymmetric information.

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1 Introduction

Emerging economies have been characterized by experiencing frequent sovereign debt crises that have been associated with major output and consumption contractions.[1] The need to roll-over massive amounts of debt coming due may become so problematic for governments when interest rates spike and access to international credit markets is restricted, that it may end up precipitating sovereign default, debt restructuring -in terms less favorable for investors than originally agreed- or even worse, triggering self-fulfilling liquidity crisis, as originally stressed by Calvo (1988).[2] In this sense, a proper management of the maturity profile of sovereign debt helps restraining the likelihood of facing a financial crisis.[3]

It has also been stressed by the literature that given the inability of writing state-contingent contracts in the credit market for sovereigns, a rich maturity profile of debt can help complete markets by replicating state-contingent returns and thus allow countries to obtain a better hedge against idiosyncratic risks. It follows then that the management of the debt maturity structure is a key macro policy both for a precautionary motive -avoid facing debt crises- and a hedging motive.

A recent bulk of literature has studied the maturity composition of sovereign debt under the assumption of perfect information among all contracting parts. However, in the case of sovereigns debt contracts are non-enforceable and the repayment decisions depend on the subjective benefits and costs of default perceived by the government in office. While changes in governments are usually observable, changes in the balance of power, in the internal politics of the government or in the collective preferences are less likely to be observed by investors. Therefore, the market for sovereign debt in emerging economies may be characterized by asymmetric information about the borrower's willingness to pay. This asymmetry may play a relevant role in the agents' behavior by affecting, among other things, the optimal maturity structure of debt.

This paper provides a model of endogenous sovereign debt maturity choice that hinges in the presence of asymmetric information, departing from the ideas already explored by

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[1]See Reinhart and Rogoff (2008).

[2]Other studies that analyze the possibility of the emergence self-fulfilling crisis under heavy borrowing include Cole and Kehoe (1996) and Cole and Kehoe (2000).

[3]Existing empirical literature has shown that excessive reliance in short-term borrowing may increase the likelihood of facing a financial crisis. Rodrick and Velazco (1999) show, using panel data, that short-term debt is a robust predictor of financial crises and greater short-term exposure is associated with more severe crises.

recent literature. In the model investors are unaware of the repayment capacity of borrowers -that can exogenously choose to default on their debt- and extract information about it from the borrower's choice of debt allocations. The model thus features a signaling game in which debt is not only used to transfer consumption across time but also as a signal to reveal the type of the borrower. Bond prices -that compensate investors for the expected loss from default- are jointly determined in equilibrium with the maturity structure of debt.

The paper focuses on a pooling equilibrium in which both safe and risky borrowers choose the same levels of debt with the same maturity profile. Under this equilibrium safe borrowers issue lower levels of debt relative to the amount of debt they would issue if investors were aware of their type. They do so because their debt is mis-priced and contains an excessively high risk premium. Safe borrowers also choose a shorter maturity structure -relative to the optimal maturity structure they would choose if investors were aware of their type- since the debt mis-pricing is higher in long-term debt relative to short-term debt. Long-term debt is less attractive to safe borrowers since it pools more default risk that is not inherent to them. Risky borrowers, on the other hand, issue low levels of debt with a short maturity structure to mimic the behavior of safe borrowers and thus preclude the market from identifying their type. This way borrowers can gain a positive misinformation value by accessing debt at higher prices than those they should access if debt were priced according to their true fundamentals.

Times of financial distress in this model are characterized by periods where the *ex-ante* expected repayment capacity of borrowers deteriorates. In these periods, prices of long and short term debt fall and spreads increase. Additionally, consumption postponement becomes more valuable for safe borrowers. In fact, the longer the postponement the more valuable it becomes, since as time goes by, debt mis-pricing gradually reduces as it becomes more likely that risky borrowers reveal their type by defaulting on their debt. Therefore, it becomes optimal for safe borrowers, and also for risky borrowers that gain from pooling with safe borrowers, to reduce their overall level of debt issuance and shorten the maturity composition of debt.

The paper shows that this optimal behavior is consistent with sovereign debt issuance patterns in emerging economies. To analyze how the level of bond issuance and the choice of maturity structure of sovereign debt is related with movements in debt prices I construct and analyze a database of sovereign bond issuance and country spreads -defined as the interest rate premium that bonds from a particular country pay in excess of the US Treasury- for a representative sample of 34 financially integrated emerging economies.

The analysis of the data indicates that both the overall level of bond issuance and maturity of debt covary negatively with spreads. The overall level of issuance in times when spreads are above the country median is lower than the level of issuance in times when spreads are below the country median for the majority of the countries in the sample. Additionally, in times of high spreads the average bond maturity for the average emerging economy is longer than the average bond maturity in times high spreads. These findings argue that sovereign debt issuance decreases and its maturity structure shortens in times of financial distress relative to times of financial tranquility, generalizing what was already documented in previous studies for more reduced sets of countries.[4]

Recent studies have offered explanations for why we observe a negative relationship between debt maturity and spreads. Broner, Lorenzoni and Schmukler (2011) argue that governments in emerging markets issue shorter-term debt in a crisis because of changes in investors attitude towards risk. In particular, they claim that shocks to the investor's risk aversion increase the risk premium on long-term bonds more than on short-term bonds, inducing governments to shorten the maturity structure of their debt. Arellano and Ramanarayanan (2012) show that long-term debt provides a hedge against future fluctuations in interest rates, while short-term debt is more effective at providing incentives to repay. They argue that governments shift to short-term debt in periods of low spreads since in those times the incentive to repay provided by this type of debt is most valuable. Dovis (2013) finds that long-term debt prices can be more sensitive to shocks near default region and this can explain the shortening of maturities when spreads increase. This paper suggests an additional mechanism that may lay behind this observation that is associated with shocks to the ex-ante market risk perception in a context in which debt prices pool risk from different types of borrowers.

The model presented can also be used to analyze to what extent and under which circumstances may debt buybacks be optimal. During the 2010-12 European sovereign debt crisis the policy debate has raised concerns on whether certain European countries should buyback debt at such low prices. Under the analyzed pooling equilibrium, debt buybacks can be optimal when there is an overestimation of the country's risk perception in the market that was not present when long-term debt decisions were made. If, for example,

[4] Particularly, Broner, Lorenzoni and Schmukler (2011) show for a pool of 11 countries, along with other stylized facts, that emerging markets governments actively shift to shorter-maturity debt in a crisis and issue long-term debt in normal times. Arellano and Ramanarayanan (2012) find the same results analyzing four emerging countries.

markets unexpectedly assign borrowers a higher ex-ante probability of being risky then countries will find optimal to buyback debt previously issued since debt prices will be lower and hence debt will be less useful to transfer resources across time.

Related Literature

Several papers analyze the debt maturity structure of debt. The availability of both short and long-term debt is relevant in an economy with non-state-contingent debt. As shown in early work by Kreps (1982) and Duffie and Huang (1985), and more recently by Angeletos (2002) and Buera and Nicolini (2004), a rich maturity structure of bonds can help replicating allocations in an Arrow-Debreu economy with complete markets. In the model presented in this paper, the existence of both short and long-term debt is indeed essential for completing markets. If borrowers could only issue short term debt markets would be incomplete given the presence of uncertainty in risk-free interest rates.

Additionally, issuing debt with various maturities may be desirable for alternative reasons. Niepelt (2008) finds that in a model with lack of commitment paired with social losses in the wake of a default, issuing bonds with multiple maturities smooths the cost of issuing debt. On the other hand, the possibility of issuing debt with multiple maturities can give rise to debt dillution and an inefficient equilibrium. Hatchondo et al. (2010) show that the precense of debt dillution accounts for an important share of the default risk in emerging economies.

Long-term debt has been shown to be helpful for hedging motives. Lustig et al. (2006) and Arellano and Ramanarayanan (2012) argue that long-term debt helps hedge against future shocks. This paper shares this feature since long-term debt helps hedge against the need to roll-over debt with ex-ante uncertain risk-free interest rates.

On the other hand, recent literature has shown that short-term debt is good for providing incentives. Tirole (2003) shows that short-term debt enhances discipline and Jeanne (2009) argues that short-term debt gives incentives for sovereign governments to implement creditor-friendly policies, because creditors can discipline the government by rolling over the debt only after desired policies are implemented.[5] In this paper this effect is not present since the decision of defaulting is not modeled and comes exogenously. Broner, Lorenzoni and Schmukler (2011) argue that short-term debt may be more desirable to risk-averse creditors since they face more uncertainty when lending long-term. In this paper creditors

[5] This effect is also present in Arellano and Ramanarayanan (2012) and Dovis (2013).

will be assumed to be risk-neutral so there are no *a priori* preferences for them for any type of maturity.

The presence of asymmetric information in determining optimal maturity debt structure has been previously explored in the corporate finance literature. Flannery (1986) evaluates the extent to which a firm's choice of risky debt maturity can signal insiders' information about the firm's quality and finds conditions under which firms issue short-term debt as a result of a pooling equilibrium. Kale and Noe (1990) later show that this pooling equilibrium satisfies signaling equilibrium refinements. The model presented here, although set up in a different fashion -more adapted to a benevolent government that cares about risk averse households welfare, as opposed to modeling the firm's decision of how to finance a project with positive net present value-, will be driven by the same mechanisms and results. Diamond (1991) analyzes debt maturity choice as a trade-off between a borrower's preference for short-term debt due to private information about the future credit rating, and liquidity risk. In this paper, safe borrowers will face the trade-off between borrowing short-term to speculate with partial revelation of risky borrowers and consuming in the intermediate periods. The relevance of asymmetric information in explaining debt maturity structure has been tested empirically. Berger et al. (2005) find a strong quantitative role for asymmetric information in explaining debt maturity structure in the financial sector.

The remaining of the paper is organized as follows. Section 2 presents the empirical evidence on sovereign debt structure and spreads and documents the relationship between the level of debt issuance, the choice of maturity structure and bond spreads for emerging markets governments. Section 3 presents the theoretical model and analyzes equilibrium with asymmetric information and compares it to the equilibrium under the benchmark case of full information. Section 4 introduces the financial cycle in the model using a comparative statics analysis. Section 5 comments on the optimality of sovereign duybacks in the context of the model. Finally, section 6 concludes.

2 Sovereign Debt Structure and Spreads: Empirical Evidence

This section examines the empirical evidence on sovereign debt issuance, maturity structure and bond spreads. Specifically, it analyses two important relationships: *i.* how the levels of sovereign debt issuance covaries with bond spreads, and *ii.* how the maturity structure of sovereign debt covaries with bond spreads.

Data on sovereign bond issuance and spreads was collected for a comprehensive sample of emerging economies. The sample covers countries that are -or were once included- in J.P. Morgan's Emerging Markets Bond Index Global (EMBIG), subject to the constraint of having sufficient data availability. Being included in the EMBIG reflects both that the economy is emerging -and faces certain default risk- and that it is integrated to world capital markets. 34 countries met the sample criteria, namely, Argentina, Brazil, Bulgaria, China, Chile, Colombia, Croatia, Dominican Republic, Ecuador, Egypt, El Salvador, Hungary, Indonesia, Ivory Coast, Kazakhstan, Lebanon, Lithuania, Malaysia, Mexico, Morocco, Nigeria, Panama, Peru, Philippines, Poland, Russia, South Africa, South Korea, Thailand, Tunisia, Turkey, Ukraine, Uruguay and Venezuela. Further details about the description of the data can be found in Appendix 1.

Daily data for bond issuance was collected from Bloomberg and spreads data was obtained from Datastream. Bond issuance data covers bonds issued in all currencies. Given that the share of debt denominated in foreign currency varies greatly among countries in the sample it is important to not restrict the sample to dollar-denominated issuance.[6] The time period ranges from January 1994 -when EMBIG spreads are initially available- until May 2012. However, data on particular countries may start later or end earlier, depending on the availability of data on each country spreads.[7]

A bond spread is the excess return of the bond over a risk-free zero-coupon bond (i.e., a US Treasury) of the same maturity. A country's spread is a synthetic measure of the spreads of a representative basket of bonds issued by that country. It measures the implicit interest rate premium required by investors to be willing to invest in a defaultable bond of a particular country. A bond maturity is measured by the number of years until its maturity date.[8]

[6] For example, all debt issuance from Ecuador is dollar-denominated, whereas for the case of Brazil only 4% of the issuance in the sample was denominated in foreign currency. By focusing on bonds issued in all currency, currency risk is a relevant variable that can affect the decisions of issuance and maturity. The model presented in the paper will nevertheless abstract from any type currency risk.

[7] Issuance during periods of default were excluded from the sample. In particular, the default periods of Dec-01 to Jun-05 for Argentina and Aug-98 to Sep-00 for Russia were not considered. Additionally, debt issued under the Brady Plan or under a debt restructure were excluded from the analysis.

[8] An alternative, and perhaps more precise, measure of a bond's maturity is a bond's duration, defined in as the weighted average of the number of years until each of the bond's coupon payments. Due to poor data availability on bonds cash flow schedules, this measure cannot be computed for all bonds in all countries. However, as Arellano and Ramanarayanan (2012) document, the standard measure of maturity

Monthly average spreads, total issuance and average maturity were computed for each country in the sample. Maturities were weighted according to the volume of debt raised with each bond. Emerging economies pay an substantial positive premium over the US Treasury yield of an average of over 400 basis points. However, average spreads differ widely across countries, suggesting that the market poses different perceptions of default risk for different countries. Additionally, the average level of issuance and the weighted average bond maturity also differ across countries. Summary statistics of the data are reported in Table A2 in Appendix 1.

Average debt issuance levels and average maturity were computed for times of high (above the country sample median) and low (below the country sample median) spreads. The average levels of debt issuance during times of low spreads are higher than the levels of issuance during times of high spreads in 22 out of the 34 countries. Additionally, the weighted average maturity shortens in times of high spreads relative to times of low spreads in 19 countries.

As shown in Figure 1 panel a, debt issuance in the average emerging economy tends to be negatively correlated with the average spread. In periods of financial turmoil such as the Tequila crisis or the recent global financial crisis, debt issuance (measured as a percentage of GDP) dropped significantly while spreads more than doubled. Additionally, as depicted in Figure 1 panel b, the maturity structure of debt also tends to move in opposite directions with spreads. In periods of substantial increases in spreads like the Russian default in 1998, the Argentinean default in late 2001 and the 2007-09 global financial crisis, the maturity profile of debt shortened considerably.

To further illustrate the fact, two sets of panel data regressions were estimated. The first set of estimations regress the level of debt issuance on country spreads. An observation of debt issuance in the regression is the total amount of debt issued by a particular country in a particular month, measured as a percentage of GDP.[9] The level of debt issuance was regressed against the prevailing monthly average country spread. Country fixed effects and country-specific linear trends were included in the regression to account for possible heterogeneity in debt choices across countries. Two different specifications for this regression were estimated. The first one is a standard OLS specification and the second one is a Tobit model, estimated by maximum likelihood, that takes into account the fact that the level of debt issuance may be left censored at zero.

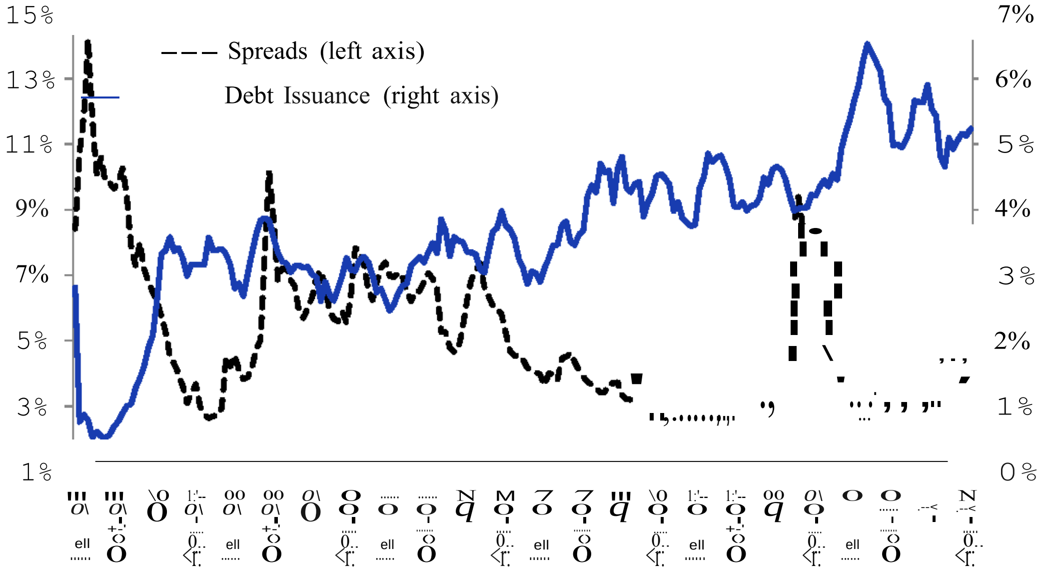
is a good substitute of the later.

[9] An observation thus includes all bonds issued in a particular month by one country.

Figure 1: Spreads, Debt Issuance and Debt Maturity in Emerging Economies

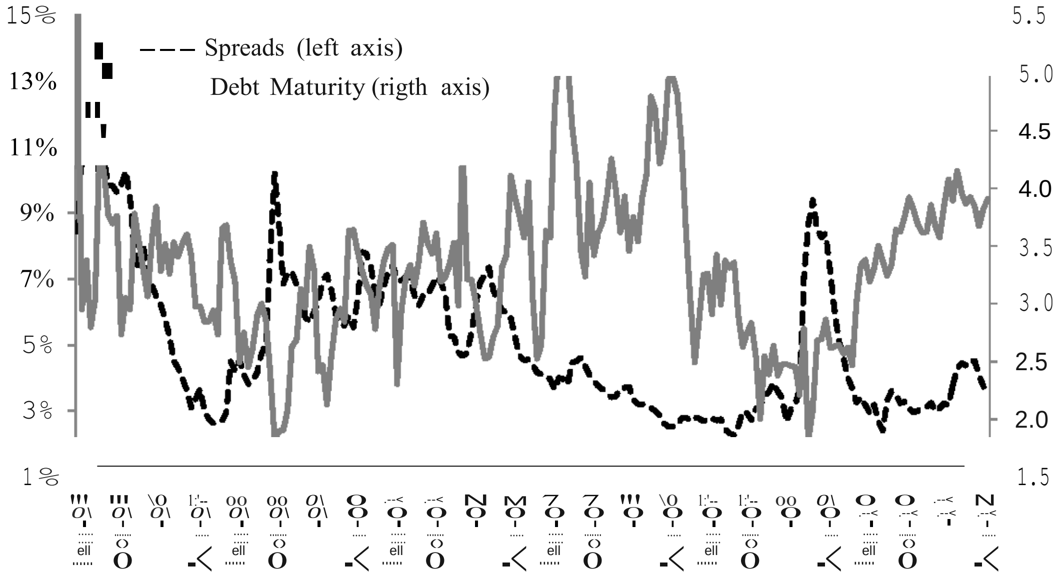
a. Spreads and Debt Issuance

(Spreads in percentage points; Debt Issuance, last 6 months, as a % of GDP)



b. Spreads and Debt Maturity

(Spreads in percentage points; Maturity in years, last 6 months weighted average)



Note: All variables are calculated as simple averages of the countries in the sample. Spreads are measured percentage points. Debt issuance is the issuance of the last 6 months measured as a percentage of GDP. Debt maturity is the average maturity in years weighted by the amount issued every month.

The estimations are reported in the second and third column of Table 1. Results indicate that the level of debt issuance is negatively related with country spreads. The coefficient on spreads is negative in both specifications and significantly different from zero at the 5% level. Standard errors are reported in parentheses. The coefficient on spreads under both specifications imply that an increase of 100 basis points in a country spread is associated with an decrease in the level of monthly debt issuance of the order of 0.1% of GDP.[10]

The second set of regressions estimate the relationship between debt maturity and country spreads. An observation of debt maturity is the weighted average maturity of all the bond issuance of a given country in a given month. Maturities were weighted by the volume of debt raised with each bond. This weighting accounts for the fact that even though countries may issue bonds with different maturities at the same time, the economic relevance of each bond is given by the amount of debt raised with each bond. The monthly average maturity was regressed against the prevailing monthly average country spread, country fixed effects and country-specific linear trends. Again, two different specifications were estimated. One specification is the standard OLS and the other is a Heckman specification that accounts for the incidental truncation of the data. Given that maturities are only available when countries decide to issue debt, the econometrician would tend to miss observations of maturities when countries decide not to issue debt which coincides with times of high spreads. The Heckman model takes into account this selection truncation by estimating a selection equation that estimates the decision of countries to issue debt and estimate the main equation of the maturity choice taking into account the selection model.[11]

The estimation results are reported in the last three columns of Table 1. The OLS specification is reported in the fourth column while the results of the maturity equation and the selection equation of the Heckman model are presented in the fifth and sixth column, respectively. Results indicate that the choice of sovereign debt maturity is affected by country spreads. The coefficient on spreads is negative and significant at the 1% level in both specifications. If we use the result of the Heckman specification, an increase in 100 basis points in a country's spread would lead to a reduction of 1 month in the average maturity structure of bonds.[12]

[10]As a benchmark to interpret that number, the median level of debt issuance in the entire sample is 0.3% of GDP.

[11]Given that the main equation and the selection equation have the same regressors under the Heckman specification, the source identification of the coefficients comes only from the nonlinearity of the functional form, i.e. the presence of the inverse Mills ratio.

[12]As a benchmark for fixing ideas with dimensions, spreads for emerging markets increased by more than

Table 1: Panel Regressions of Debt Issuance and Debt Maturity

	Dependent Var.: Issuance		Dependent Var.: Maturity		
	OLS	TOBIT	OLS	HECKMAN	
VARIABLES				Main Eq.	Selection Eq.
Spread	-0.00851**	-0.0134**	-0.0588***	-0.0639***	-0.00154
	(0.00373)	(0.00552)	(0.0223)	(0.0225)	(0.00683)
Constant	0.808	0.466	19.34***	16.60***	-2.650**
	(0.504)	(0.607)	(2.229)	(2.439)	(1.350)
Country Dummies	Yes	Yes	Yes	Yes	Yes
Time Trends	Yes	Yes	Yes	Yes	Yes
Observations	5,446	5,446	4,047	4,047	5,446
Censored Obs.		1,399			
R-squared	0.252		0.166		

Standard errors in parenthesis

*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Note: In all regressions the time trends are country specific. Tobit regression estimated by Maximul Likelihood. Censored observations are observations with zero issuance. Heckman regressions were estimated in twosteps. The selection equation estimates the decision to issue debt or not and the main equation estimates the choice of maturity.

In summary, the results presented in this section argue that the volume of sovereign debt issuance and its maturity choice are affected by sovereign bond spreads. In particular, it is shown that: 1. levels of debt issuance covary negatively with spreads, and 2. debt maturity covaries negatively with spreads. In times of low spreads -or financial tranquility- governments issue higher levels of debt with longer maturities, whereas in times of high spreads -or financial distress- governments reduce their levels of debt issuance and shorten its maturities.

These findings are consistent with the empirical evidence presented in Broner, Lorenzoni and Schmukler (2011) and Arellano and Ramanarayanan (2012) that find the same results for a more reduced set of countries. Broner, Lorenzoni and Schmukler (2011) use a sample of eleven to show that debt levels decrease and debt maturity shortens when spreads are high.

500 basis points during the 2007-09 global financial crisis.

They find that high spreads are statistically significant determinants of lower debt issuance and shorter maturities by running instrumental variables regressions to identify the effects of idiosyncratic shocks to emerging economies. Arellano and Ramanarayanan (2012) use a measure of bond duration -as the weighted average of the number of years until each of the bond's coupon payments- and find that a bond's duration shortens when spreads increase. In this case the empirical evidence presented here generalizes the previously documented fact for larger sample of emerging countries and extends the sample for a longer time period that includes eight more years and the 2007-09 global financial crisis. The results obtained are still consistent with these previous studies.

3 A Model of Debt Maturity Choice

Consider a small open economy inhabited by an agent that lives for three periods, whose problem is to choose debt allocations to maximize lifetime expected utility of consumption. The agent (henceforth, the borrower) can be thought of as a government that cares about the representative consumer welfare or the welfare of a fraction of the population (e.g. the political party interests or a particular group of interest). The borrower has no commitment and can exogenously opt for defaulting on its debt. His ability to repay debt depends on his type θ.[13]

The borrower's preferences are time separable, concave and are represented by

$$U(\theta) = E[\log(c_0(\theta)) + \beta \log(c_1(s, \theta)) + \beta\beta_1 \log(c_2(s, \theta))] \tag{1}$$

where $\beta < 1$ represents the discount factor between time periods 0 and 1, β_1 is a stochastic discount factor between time periods 1 and 2 that will be realized in period 1 and s denotes the state of nature that will depend on whether the borrower defaults or repays and on the realization of the stochastic discount factor.

The borrower is endowed with a deterministic income stream (y_0, y_1, y_2). Let λ^θ be the probability that a borrower θ defaults on its debt at any time period. The borrower can be safe ($\theta = S$) or risky ($\theta = R$). It will be assumed without loss of generality that

[13] Several reasons can be thought for why governments can differ in their capacity for repaying debt. For example, Cole, Dow and English (1995) argue that repayment may be more likely in governments that have higher likelihood of remaining in power since this can be translated in higher discount factors. Another plausible reason is that governments may differ in their outside option of defaulting. Additionally, different preferences of political parties depending if they are in office or are opposition may also serve as a reasonable explanation. For examples of the later case see Alessina and Tabellini (1990).

safe borrowers never default, i.e., $\lambda^S = 0$, whereas risky borrowers default with probability $\lambda^R > 0$. Default occurs indiscriminately in both short and long-term outstanding debt. If the borrower defaults he can no longer access credit markets for all subsequent periods and consumes $c = y_{def} > 0$.

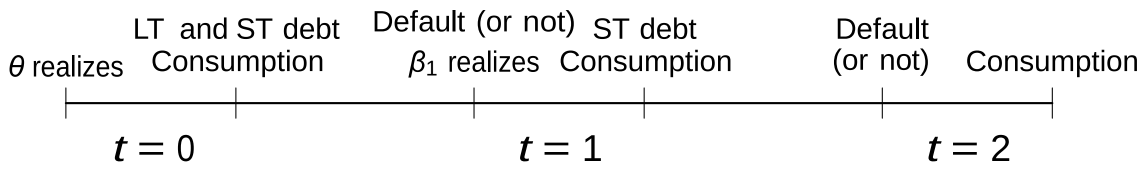
Borrowers also face an aggregate shock to the discount factor β_1 in $t = 1$. With probability p the borrower is patient and faces a discount factor of β^p, and with probability $1 - p$ the borrower is impatient and faces a discount factor of $\beta^i < \beta^p$.

There are infinitely many risk neutral investors that are perfectly competitive. They face the same discount factor as the borrower in $t = 0$ and the same aggregate shock to the discount factor in $t = 1$. Introducing stochastic intertemporal preferences in all agents in the economy will result in uncertainty in the risk free interest rate.[14]

The available debt instruments for borrower θ are short-term bonds $b_{t,1}(\theta)$ (zero coupon bonds issued at time t payable at date $t+1$) and long-term bonds $b_{t,2}(\theta)$ (zero coupon bonds issued at time t payable at date $t+2$). The prices of these bonds are denoted $q_{t,1}$ and $q_{t,2}$, respectively. I restrict the available set of securities to non-contingent debt contracts of different maturities since the optimal maturity structure is the focus of the paper and most of the debt in sovereign markets comes from non-contingent contracts (see Rogoff (2011)).

The timing of the decisions in the model is summarized in the following figure.

Figure 2: Timing of Decisions



Borrower θ consumption stream in repayment states will thus be given by

$$c_0(\theta) = y_0 + b_{0,1}(\theta)q_{0,1} + b_{0,2}(\theta)q_{0,2} \tag{2}$$

$$c_1(s,\theta) = y_1 - b_{0,1}(\theta) + b_{1,1}(s,\theta)q_{1,1}(s) \tag{3}$$

$$c_2(s,\theta) = y_2 - b_{0,2}(\theta) - b_{1,1}(s,\theta) \tag{4}$$

[14] The assumption that borrowers face the same shock to the discount factor as investors is without loss of generality. Main results will remain robust to imposing different discount factors for borrowers and investors.

where $s = \{p, \boldsymbol{i}\}$ depending on whether at $t = 1$ agents are patient or impatient. Note that default is not a relevant state since its probability of occurrence is exogenous and borrowers do not choose neither debt allocations nor consumption in those states. It is also worth stressing that there are complete markets for borrowers given that the number of debt instruments matches the number of states where there is repayment. In the absence of uncertainty in the discount factor of $t = 1$ and any other source of uncertainty, there would be asset redundancy in the economy as the payoffs of long-term debt can be exactly reproduced by issuing short-term debt and rolling it over in period 1.

The set of equilibria in this model will depend on the assumption about the information sets of the agents. In other words, equilibrium allocations and prices will depend on whether investors are aware or not of the borrower's type. The equilibrium with full information is now analyzed to serve as a benchmark for the equilibria with asymmetric information.

3.1 Equilibrium under Full Information Benchmark

In this specification investors have full information on the borrower's type. Hence, a bond issued by a borrower S will be a different security than a bond issued by borrower R, and thus will be priced differently. Equilibrium in this particular setting will be defined in the following way:

Definition 1. *An equilibrium in the full information setting is a set debt allocations* $\{b_{01}(\theta), b_{0,2}(\theta), b_{1,1}(s, \theta)\}$ *and prices* $\{q_{0,1}(\theta), q_{0,2}(\theta), q_{1,1}(s_1, \theta)\}$ *for* $s = p, \boldsymbol{i}$ *such that:*

(i) the borrower chooses debt allocations to maximize (1) subject to (2) - (4),

(ii) prices are determined by the discounted expected repayments to investors

Given the intertemporal preferences of investors and default probabilities of borrowers, the equilibrium prices of borrower's θ bonds are:

$$q_{0,1}(\theta) = \beta(1 - \lambda^{\theta}) \tag{5}$$

$$q_{0,2}(\theta) = \beta\beta_1^e(1 - \lambda^{\theta})^2 \tag{6}$$

$$q_{11}(s, \theta) = \beta^s(1 - \lambda^{\theta}) \tag{7}$$

for $s = \{p, \boldsymbol{i}\}$ and $\theta = S, R$, where $\beta_1^e = p\beta^p + (1-p)\beta^i$ is the expected value of the discount factor β_1 as of time $t = 0$.

Using the fact that long-term debt price is equal to the product of the expected prices of short-term debt, i.e., $q_{0,2} = q_{0,1}E_0[q_{1,1}]$, the solution to the borrower's problem is characterized by perfect consumption smoothing across time and states with

$$c_t^{F\prime}(s, \theta) = \frac{W(\theta)}{R(\theta)} \tag{8}$$

for all t, s and θ, where $R(\theta) = 1 + \beta(1 - \lambda^\theta) + \beta\beta_1^e(1 - \lambda^\theta)^2$ and $W(\theta) = y_0 + q_{0,1}(\theta)y_1 + q_{0,2}(\theta)y_2$ is the market value of the borrower's wealth. The presence of complete markets allows the borrower to attain perfect consumption smoothing which turns out to be optimal given that borrowers are risk averse and that investors and borrowers face the same intertemporal discount factor. There is a unique set of debt allocations that can attain the optimal level of consumption and is given by

$$b_{01}^{F\prime} = y_1 - \frac{W(\theta)}{R(\theta)} \tag{9}$$

$$b_{02}^{F\prime} = y_2 - \frac{W(\theta)}{R(\theta)} \tag{10}$$

$$b_{11}^{F\prime}(s) = 0 \tag{11}$$

The uniqueness of the equilibrium debt allocations comes from the existence of uncertainty in discount factors in $t = 1$. The existence of this uncertainty breaks down the possibility of replicating the payoffs of long-term debt by rolling-over short-term debt.

Due to the fact that borrowers and investors discount time at the same rate, the need for trading bonds comes only from the dispersion of endowments across time.[15] The borrower optimally chooses not to issue debt in $t = 1$ and issues short term debt to trade away period 1 endowment net of optimal consumption and long-term debt to trade away period 2 endowment net of the optimal level of consumption.

In order to ensure that the borrowers issue debt and do not save the following assumption is made.

Assumption 1.

$$y_1 \geq \frac{W(\theta)}{R(\theta)} \quad \text{and} \quad y_2 \geq \frac{W(\theta)}{R(\theta)} \tag{A1}$$

for $\theta = S, R$

[15] Note that when $y_0 = y_1 = y_2$ the borrower does not need to issue debt at all.

Low enough values of y_0 relative to y_1 and y_2 ensures that optimal debt allocations are non-negative. If the borrowers optimally chose to save with at least one security then that security would be mispriced since it should no longer reflect the default risk of the borrower.

The equilibrium in the full information specification will serve as a useful benchmark to the equilibrium analysis under the asymmetric information specification of the following subsection.

3.2 Equilibrium under Asymmetric Information

In this specification the borrower is still aware of his type. However, investors cannot distinguish the borrower's type and only know the *ex-ante* distribution of borrowers in the economy which is given by $\Pr(\theta = R) = \alpha_0 \in (0, 1)$. Under this setting the definition of equilibrium requires a specification of investors beliefs about the type of the agent; beliefs will determine the probability that a borrower is of type $\theta = R$ and will be a mapping between the cartesian product of the set of possible histories of states S_t and the set of possible histories of debt allocations in period t B_t to the [0,1] interval. Equilibrium is defined as follows.

Definition 2. *In the asymmetric information setting a Perfect Bayesian Equilibrium (PBE) is a set of debt allocations* $\{b_{0,1}(\theta), b_{0,2}(\theta), b_{1,1}(s, \theta)\}$ *for* $s = p, i$, *prices* $q_{0,t} : B_0 \to \mathrm{R}$ *for* $t = 1, 2$ *and* $q_{1,1} : \{p, i\} \times B_1 \to \mathrm{R}$ *and beliefs* $\mu_t : S_t \times B_t \to [0, 1]$ *for* $t = 0, 1$, *such that:*

(i) The borrower chooses debt allocations to maximize (1) subject to (2) - (4),

(ii) prices are determined by the discounted expected repayments to investors given beliefs and

(iii) *where possible, beliefs are determined using Bayes rule, i.e.:*

$$\mu_t(s_t, b_t) = \frac{\Pr(\theta = R|s_t, b_t)}{\Pr(s = s_t, b = b_t)}$$

To make notation simpler any PBE will be referred to as the triplet $\{b(\theta), q(b), \mu(b)\}$. Two types of equilibrium may arise under this framework: a separating equilibrium, where different type of borrowers choose different allocations and thus investors can perfectly tell apart each borrower's type; and a pooling equilibrium, where both types of agents choose the same allocation and thus investors cannot distinguish their type.[16] A set of both pooling and separating equilibria exist in this game.

In order to restrict attention to a relevant notion of equilibrium I will focus in this paper on a particular equilibrium: the one that gives the highest utility to the safe borrower. This equilibrium selection seems a natural benchmark since it is precisely the safe borrower the one that is prone to suffer the most from the presence of asymmetric information either by being pooled with a riskier borrower or by engaging in distortionary allocations to separate from the risky borrower.[17] Equilibrium selection is captured in the following definition.

Definition 3. *In the asymmetric information setting a Best Perfect Bayesian Equilibrium for Safe Borrower (PBE-BS) is a triplet of debt allocations, prices and beliefs* $\{b(\theta), q(b), \mu(b)\}$ *such that:*

(i) $\{b(\theta), q(b), \mu(b)\}$ *is a PBE and*

(ii) $\{b(\theta), q(b), \mu(b)\}$ *yields the highest payoffs to the safe borrower, i.e.:*

$$U(b(S); S) \geq U(\tilde{b}; S)$$

for any other $\tilde{b}$ *sustained under a PBE.*

To analyze the PBE-BS I will first find the best PBE for the safe borrower among the set of pooling equilibria and then verify that this equilibrium will indeed be the best PBE-BS among all possible equilibria under specific parametric assumptions.

Beforehand, note that on any PBE, whether pooling or separating, prices will be determined by beliefs in the following way:

$$q^*_{01} = \beta(1 - \mu_0 + \mu_0(1 - \lambda^R)) \tag{12}$$

$$q^*_{02} = \beta\beta^e_1(1 - \mu_0 + \mu_0(1 - \lambda^R)^2) \tag{13}$$

$$q^*_{11}(s) = \beta^s(1 - \mu_1 + \mu_1(1 - \lambda^R)) \tag{14}$$

for $s = p, \boldsymbol{i}$.

[16] A semi-separating equilibrium is a third type of equilibrium that may also emerge in this game. In this equilibrium borrowers randomize the choice of debt allocations over intersecting sets. These type of equilibria will not be analyzed in this paper.

[17] The criterion of analyzing a specific equilibrium that is characterized by yielding the highest -or lowest- payoffs to a particular agent has already been used in the existing literature. See, for example, Aguiar and Amador (2011).

To find the pooling PBE-BS I will first compute the on-equilibrium prices, then solve for the optimal debt allocations for borrower S given those prices and finally construct beliefs that sustain those prices and allocations under a PBE.

Let $b^P = (b^P_{0,1}, b^P_{0,2}, b^P_{1,1}(s))$ be the equilibrium allocations under some pooling equilibrium. Then it must be true that on-equilibrium beliefs will be given by

$$\mu_0(b^P_{01}, b^P_{02}) = \alpha_0 \tag{15}$$

$$\mu_1(s, b^P_{11}) = \frac{\alpha_0(1-\lambda^R)}{1-\alpha_0\lambda^R} \tag{16}$$

for any s. These on-equilibrium beliefs are consistent with Bayes rule. Note that the belief of being a risky borrower in period one is lower than belief of being a risky borrower in period zero. The reason is that in $t = 1$ on-equilibrium beliefs are given by the probability of being a risky borrower conditional on not having defaulted in that period. Since, by definition, the risky borrower defaults at $t = 1$ with some positive probability and the safe borrower never defaults, it follows that the on-equilibrium beliefs should be lower in that period. On-equilibrium pooling prices will then be

$$q^*_{01}(b^P_{01}, b^P_{02}) = \beta(1-\alpha_0\lambda^R) \tag{17}$$

$$q^*_{02}(b^P_{01}, b^P_{02}) = \beta\beta^e_1(1-\alpha_0\lambda^R(2-\lambda^R)) \tag{18}$$

$$q^*_{11}(s, b^P_{11}) = \beta^s\frac{1-\alpha_0\lambda^R(2-\lambda^R)}{1-\alpha_0\lambda^R} \tag{19}$$

for $s = p, i$.

Now consider the following artificial problem of choosing debt allocations to:

$$\max_b U(b; S) \qquad s.t. \qquad U(b; R) \geq U^{F\prime}(R)$$

and also subject to (2) - (4) and prices given by (17) - (19). $U^{F\prime}(R)$ is the utility attained by borrower R under the full information allocation. This problem will yield the allocations that maximize borrower S utility such that borrower R finds it optimal to pool. Under certain parametric assumptions that will be discussed later, borrower R will find optimal to pool with the allocations that maximize the unrestricted problem for borrower S. It follows that the Lagrange multiplier associated to this restriction will be zero. The optimal consumption rule for borrower S

$$c^P_0 = \frac{W^P}{R(S)} \tag{20}$$

$$c^P_1(s) = \frac{\delta^P_1 W^P}{} \tag{21}$$

$R(S)$

$$c_2^P(s) = \frac{\delta_2^P W^P}{R(S)} \tag{22}$$

for $s = p, \boldsymbol{i}$, where $R(S)$ is defined as in the full information specification, $W^P = y_0 + q_{0,1}y_1 + q_{0,2}y_2$ is a measure of the borrower's total wealth valuated at the pooling prices and

$$\delta_1^P = \frac{1}{1 - \alpha_0\lambda^R} \quad \text{and} \quad \delta_2^P = \frac{1}{1 - \alpha_0\lambda^R(2 - \lambda^R)}$$

These parameters reflect a measure of the distortion at time t introduced by the presence of asymmetric information and the inability of investors to tell a borrower's type under a pooling equilibrium. The parameter δ_t^P is the ratio between the true probability of repayment of borrower S at time t -which was set to one at any time period without loss of generality- and the *ex-ante* probability of repayment that investors can infer at time t with the set of information available to them.

The unique set of debt allocations that are consistent with the optimal consumption rule are given by

$$b_{01}^P = y_1 - \frac{\delta_1^P W^P}{R(S)} \tag{23}$$

$$b_{02}^P = y_2 - \frac{\delta_2^P W^P}{R(S)} \tag{24}$$

$$b_{11}^P(s) = 0 \tag{25}$$

for $s = p, \boldsymbol{i}$.[18]

To ensure borrowers issue debt instead of saving and that borrower R prefers to pool as opposed to separate, the following parametric assumptions are made.

Assumption 2.

$$y_1 \geq \frac{\delta_1^P W^P}{R(S)} \quad \text{and} \quad y_2 \geq \frac{\delta_2^P W^P}{R(S)} \tag{A2.a}$$

$$\beta(1 - \lambda^R)\log(\delta_1^P) + \beta\beta_1^e(1 - \lambda^R)^2\log(\delta_2^P) > R(R)\left(\log\frac{W(R)}{R(R)} - \log\frac{W^P}{R(S)}\right) \tag{A2.b}$$

[18] The result that borrower's do not issue short term debt in period 1 and attain perfect consumption smoothing across states in $t = 1, 2$ is robust to the preferences specification. If instead of using log utility, we work with a general concave instantaneous utility function $u(c)$ then we would still have $b_{11}(s) = 0$ and the optimal consumption and debt allocation would be characterized by the following first order conditions

$$u^t(c_0) = \delta_1^P u^t(c_1)$$

$$u^t(c_0) = \bar{\delta}_2^P u^t(c_2)$$

Note that the solution to this maximization problem does not necessarily provide equilibrium debt allocations. The problem is just an artifact to find what would be the pooling allocations that yield the highest utility to borrower S, assuming these allocations configure a PBE. Now we need to prove that these allocations indeed constitute a PBE. For this purpose consider the following degenerate beliefs

$$\mu_0^P(b_{01}, b_{02}) = \begin{cases} \alpha_0 & \text{if } (b_{01}, b_{02}) = (b_{01}^P, b_{02}^P) \\ 1 & \text{if } (b_{01}, b_{02}) = (b_{01}^P, b_{02}^P) \end{cases}$$

$$\mu_1^P(b_{11}, s) = \begin{cases} \frac{\alpha_0(1-\lambda^R)}{1-\alpha_0\lambda^R} & \text{if } b_{11}(s) = b_{11}^P(s) \\ 1 & \text{if } b_{11}(s) = b_{11}^P(s) \end{cases}$$

for any s.[19]

Given these beliefs the pooling PBE-BS is characterized in the following Lemma.

Lemma 1. *Assume parameters satisfy (A2.a) and (A2.b), then debt allocations* $\{b_{0,1}^P,$ $b_{0,2}^P, b_{1,1}^P(s)\}$ *for* $\theta = S, R$ *and* $s = p, \boldsymbol{i}$*, prices* $\{q_{0,1}^*, q_{0,2}^*, q_{1,1}^*(s)\}$ *for* $s = p, \boldsymbol{i}$ *and beliefs* $\{\mu_0^P, \mu_1^P\}$ *configure a unique pooling PBE-BS.*

The proof can be found in Appendix 2.

It is worth comparing the optimal consumption rules in non-default states for both agents under the full information benchmark and the pooling equilibrium to assess the effects of the introduction of asymmetric information under this type of equilibrium. Recall that the full information benchmark was characterized by state and time consumption smoothing for safe and risky borrowers. Under this specification the maturity structure of debt is determined by the income dispersion (see Figure 2 panel a and b).

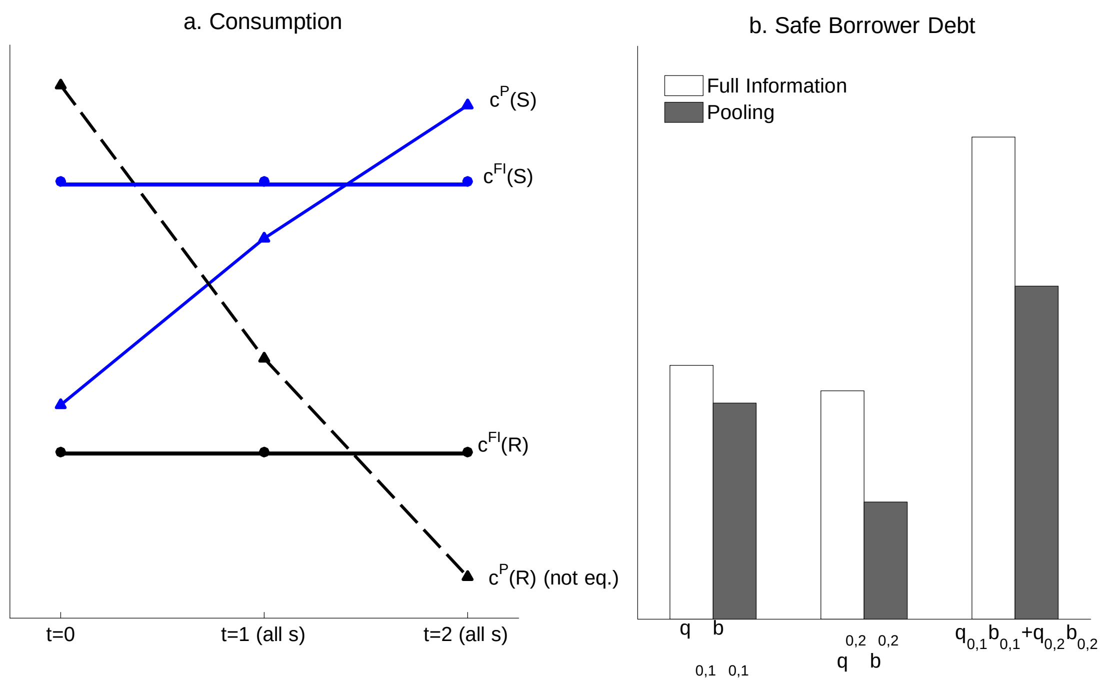
The presence of the pooling equilibrium in the asymmetric information setting leaves borrower S worse off respect to the full information setting: the utility obtained in the pooling PBE is strictly less than the utility obtained in the equilibrium with full information. The reason is that the prices that the borrower faces for issuing debt are now lower due to the fact that they reflect an average default risk that pools his true default risk with that of the other borrower who is riskier. However, the price effect for long and short term debt is asymmetric. Long-term debt prices decay more than short-term debt prices -relative to full information prices- since they pool default risk for two periods instead of one. In other words, long term debt is less attractive than short term debt for safe borrowers since it pools more default risk that is not inherent to them. The safe borrower optimally reacts to

[19]These are not the unique beliefs that can sustain these debt allocations as a PBE.

this negative asymmetric effect on prices by lowering the amount of debt and shortening its maturity profile. These debt allocations entail an increasing consumption path across time (see Figure 2 panel a and b).[20]

On the other hand, borrower R sees himself benefited from the presence of asymmetric information.[21] Given the on-equilibrium pooling prices borrower R would prefer to attain a decreasing path of consumption across time (see Figure 2 panel a). However, by choosing that allocation he would be revealing his type and this would not constitute an equilibrium. In order to preclude the market from identifying his type borrower R mimics the behavior of borrower S. This way he can gain a positive misinformation value by accessing to cheaper debt than that he should access if debt were priced according to his true fundamentals.

Figure 3: The Effect of Asymmetric Information



It is worth noting that the asymmetric price effect on debt is non-monotonic in the default probability of borrower R. The price effect on long and short-term debt will differ

[20] Interestingly, as in the full information benchmark, under the pooling equilibrium agents also attain state consumption smoothing. Given the presence of complete markets, the fact that $q_{0,2} = q_{0,1}E_0[q_{1,1}]$ and the fact that borrowers and investors share the same discount factors, the optimal consumption rule is characterized by perfect consumption smoothing across states in $t = 1, 2$.

[21] If this were not true, it would not be a pooling equilibrium, as borrower R would prefer to separate and choose the full information optimal allocation.

more as λ^R moves away from 0 and 1. If borrower R defaults with a small probability (i.e., $\lambda^R \sim 0$) then the effect of asymmetric information is negligible on both long and short-term debt since both type of borrowers are alike. If borrower R defaults for sure (i.e., $\lambda^R = 1$) then all the default risk from the investors perspective is accumulated in $t = 1$ and there is no additional default risk at $t = 2$. Therefore, in this case the price effect of asymmetric information will be the same for long and short-term debt.

The existence of this pooling equilibrium relies on Assumption 2.b, that ensures borrower R prefers pooling and mimicking borrower S by choosing the same debt allocations, to separating and choosing the full information optimal allocation. Borrower R will only prefer to pool if the utility obtained from the price gain is higher than the disutility from issuing distorted debt allocations.

Figure 3 depicts the combinations of ex-ante probabilities of a borrower being type R (α_0) and default probabilities for type R (λ^R) such that the described pooling equilibrium exists. Note that this pooling equilibrium exists for all levels of α_0 less or equal than a certain threshold. This is due to the fact that the lower the ex-ante probability of being a risky borrower, the higher the positive wealth effects that borrower R obtains from pooling. The threshold level of α_0 is decreasing in the levels of default probabilities of the risky borrower. This is because higher default probabilities of the risky borrower imply higher differences in the intertemporal trade-offs between the safe and risky borrower and thus larger distortions for borrower R from choosing the pooling allocations.[22]

Now that the pooling PBE-BS has been fully characterized the next part shows that this equilibrium is indeed the PBE-BS among all the set of equilibria. For this it will be necessary to characterize the PBE-BS among the set of separating equilibria which is done in the next lemma.

Let $\{b^S_{0,1}, b^S_{0,2}, b^S_{1,1}(s)\}$ be the debt allocations that solve the following problem

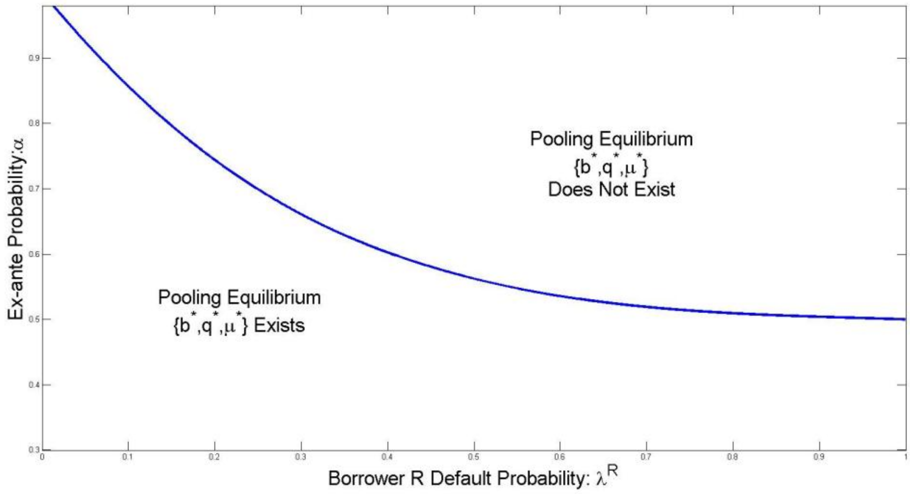
$$\max_b U(b; S) \qquad s.t. \qquad U(b; R) \leq U^{F\prime}(R)$$

and also subject to (2) - (4) and prices given by (5) - (7) for $\theta = S$. This problem will yield the allocations that maximize borrower S expected utility such that borrower R finds it optimal to separate and choose his full information allocations regardless of the fact that he will reveal his type by doing so.

Lemma 2. *Debt allocations* $\{b^S_{0,1}, b^S_{0,2}, b^S_{1,1}(s)\}$ *for borrower* S*, debt allocations* $\{b^{F\prime}_{0,1}, b^{F\prime}_{0,2}, b^{F\prime}_{1,1}(s)\}$

[22] For very low values of y_0 -relative to y_1 and y_2- and high values of λ^R close to one, the threshold level of α_0 is increasing as the wealth effect due to higher values of λ^R gains relevance.

Figure 4: Existence of Pooling Equilibrium



Note: This figure assumes $(y_0, y_1, y_2) = (0.5, 1, 1)$, $\beta = \beta_1^e = 0.9$, an illustrative example.

for borrower R, prices $\{q^*_{0,1}, q^*_{0,2}, q^*_{1,1}(s)\}$ *for* $s = p, i$ *configure a unique separating PBE-BS sustained by degenerate beliefs*

$$\mu_t^S(b_t, s) = \begin{cases} 0 & \text{if } b_t = b_t^S \\ 1 & \text{otherwise} \end{cases}$$

for any t, s.

The proof can be found in Appendix 2.

The presence of the separating equilibrium in the asymmetric information setting leaves borrower S worse off respect to the full information setting given that in order to separate from borrower R, borrower S engages in some distortionary consumption path. He does so by consuming more in states in which his valuation of consumption is highest relative that of borrower R. Given that borrower R defaults with positive probability in periods $t = 1, 2$ the highest consumption valuation of borrower S relative to borrower R occurs in late repayment states. Although for different reasons, the optimal consumption rule features an increasing consumption path across time, as in the previously analyzed pooling equilibrium. In order to attain this increasing consumption path the safe borrower optimally chooses to issue low levels of debt with shorter maturities, relative to the optimal issuance under the full information benchmark.

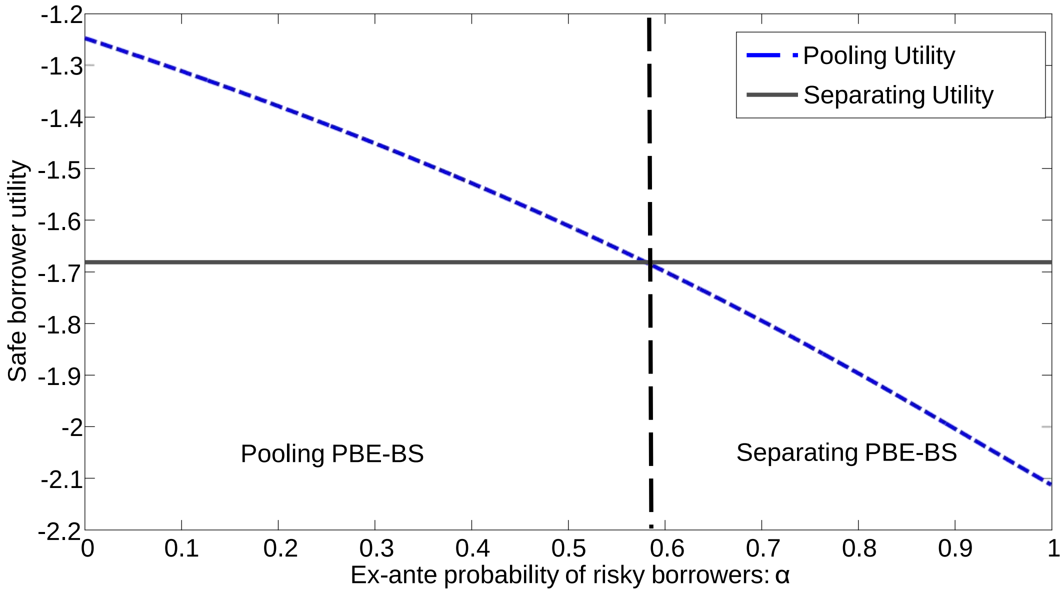
Borrower R is indifferent between separating and pooling and decides to separate by choosing the debt allocations that maximize his utility in the full information benchmark. In other words, he prefers to choose debt in order to optimally transfer resources across time regardless of the fact that he reveals his type by doing so. A more detailed analysis of the separating PBE-BS can be found in Appendix 3.

Given the characterization of the separating PBE-BS we can now analyze when is the case that the PBE-BS is pooling which is done in the following proposition.

Proposition 3. *There exist some threshold ex-ante probability of being a risky borrower* $\tilde{\alpha}_0 \in (0, 1]$ *such that for* $\alpha_0 \leq \tilde{\alpha}_0$ *the pooling PBE* $\{b^P(\theta), q^*(b), \mu^P(b)\}$ *for* $\theta = S, R$ *is the PBE-BS.*

The proof can be found in Appendix 2.

Figure 5: Best PBE for Safe Borrower: Characterization



Note: This figure assumes $(y_0, y_1, y_2) = (0, 1, 1), \beta = 0.9, \beta^p = 1, \beta^i = 0.8, p = 0.5$ and $\lambda^R = 0.5$, an illustrative example.

The intuition for the result is straightforward. The utility attained by borrower S in the separating equilibrium does not depend on the ex-ante probability of being a risky borrower since types are fully revealed in this type of equilibrium. On the other hand, the utility attained by borrower S in the pooling equilibrium is decreasing in the ex-ante probability of being a risky borrower. The reason is that the higher the ex-ante probability of being a risky borrower, the more negative wealth effect borrower S gets from being pooled with a risky borrower. It follows that for low values of α_0 borrower S benefits more from the pooling equilibrium.

The criterion for equilibrium selection in this paper is based on analyzing such equilibrium that yields the highest utility to the agent that has the most to loose from the presence of asymmetric information. One may also focus on equilibria that survive certain refinements on off-equilibrium beliefs. For example, Cho and Kreps (1987) propose the intuitive criterion that rules out equilibria in which there exists an allocation such that one agent would be willing to deviate if he is believed to be his true type and all other agents would not prefer to do so regardless of what type they are believed to be. In our particular
setting, if we do not allow savings as part of the signaling set, i.e., $(b_{0,1}, b_{0,2}) \in \mathbb{R}^+_2$, it can be shown that under certain parametric conditions the analyzed pooling PBE-BS satisfies the intuitive criterion.

4 Introducing the Financial Cycle

The model presented so far describes how the presence of a pooling equilibrium can preclude borrowers from revealing their true repayment capacity to investors and distort the relative prices of consumption at different time periods. However, it does not explain why governments choose to issue lower levels of debt with shorter maturities in times of financial distress. The most important challenge ahead is use the model to give some insight on the empirical facts documented in section 2. This section provides an explanation for this fact using a comparative statics approach.

I will restrict the attention to the set of parameters that are such that the PBE-BS is pooling. For this equilibrium I consider how variations in the ex-ante probability of being a risky borrower affect the debt allocations and prices of the pooling equilibrium that yields the highest utility to the safe borrower. The results are summarized in the following proposition.

Proposition 4. *Under the pooling PBE-BS the prices of both long and short-term debt decrease and the optimal level of debt issuance decreases when the ex-ante probability probability of being a risky borrower increases. Additionally, there exists a small enough y_0 such that the optimal maturity composition shortens when the the ex-ante probability probability of being a risky borrower increases.*

The proof can be found in Appendix 2.

The intuition for the first result is straightforward. When a given borrower is more likely to be risky the prices at which investors are willing to buy his debt are lower because the ex-ante default risk is higher. Lower debt prices come hand in hand with higher spreads. Additionally, given lower debt prices borrowers optimally choose to issue lower levels of debt since debt is now a less effective instrument to transfer consumption across time.

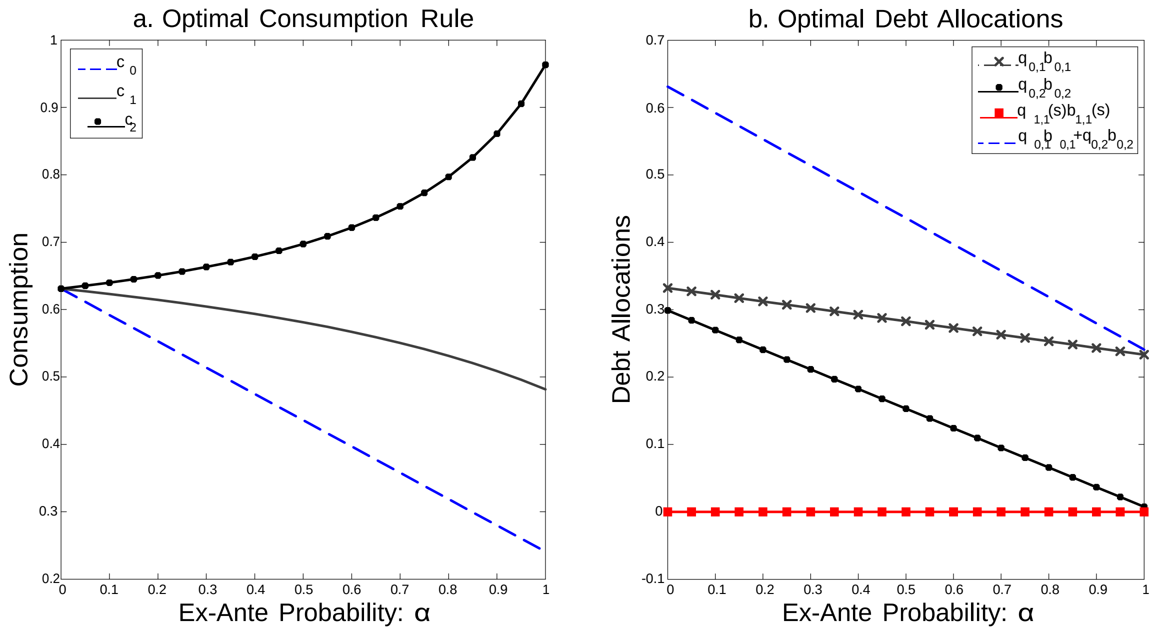
The intuition for the second result is a little more intricate. First note that prices of short and long-term debt do not react in the same way to increases in the ex-ante probability of being a risky borrower. Long-term debt prices drop more than short-term prices. The reason is that long-term debt prices are affected by the probability of the borrower defaulting in either period 1 or 2 and the ex-ante distribution of borrowers affects the probabilities of both events. On the other hand, short-term debt prices are affected by the probability of default in the period that immediately follows its issuance and thus the ex-ante distribution of borrowers affects the price only through the likelihood of that single event. Given this asymmetric effect on prices, an increase in the ex-ante probability of being a risky borrower leads to consumption in period 2 becoming cheaper relative to consumption in period 0 and 1, and consumption in period 1 becoming cheaper relative to consumption in period 0. The safe borrower optimally responds to this change in relative prices by consuming more in period 2 relative to period 1 and consuming more in period 1 relative to period 0 (see Figure 2, panel a, for a graphical illustration of consumption behavior as a function of the ex-ante probability of being a risky borrower).

In other words, in periods of financial distress consumption postponement becomes desirable for safe borrowers and the longer the postponement the more desirable it becomes, since risky borrowers will reveal their type by defaulting on its debt. Therefore, it becomes optimal for safe borrowers to reduce their overall level of debt issuance and shorten the maturity composition of debt (see Figure 2, panel b).

Risky borrowers still find optimal to mimic the behavior of safe borrower and preclude investors from identifying their type and thus face a positive wealth effect from being pooled with safe borrowers.

All in all, the model predicts a decrease in overall levels of debt and a shortening of its maturity structure when spreads increase, two facts that are consistent with the empirical observations documented in section 2.

Figure 6: Optimal Consumption and Debt Allocation in Pooling Equilibrium $\{b^*, q^*, \mu^*\}$



Note: This figure assumes $(y_0, y_1, y_2) = (0, 1, 1)$, $\beta = 0.9$, $\beta^p = 1$, $\beta^i = 0.8$, $p = 0.5$ and $\lambda^R = 0.5$, an illustrative example.

5 Debt Buybacks under Asymmetric Information: A Comment

During the 2010-12 European sovereign debt crisis, the interest rate spreads of certain European countries reached historical peaks as the market perception of an eventual default increased substantially.[23] In this context, the policy debate has focused, among other topics, on sovereign debt buybacks. In particular, concerns have been raised on whether countries like Greece or Spain should buyback debt at such low prices.

This section uses the model presented in this paper to assess to what extent and under what circumstances may debt buybacks be optimal under a context of asymmetric information. Debt buybacks are characterized by situations in which previously issued long-term debt is bought in the secondary market at prevailing market prices. In the model's frame-

[23] The highest peak in spreads was registered in Greece where spreads reached a maximum of 3500 basis points in March 2012. In other countries like Ireland, Spain and Portugal spreads reached levels higher than 500 basis points.

work, a debt buyback is a situation where

$$b_{0,2} > 0 \qquad \text{and} \qquad 0 > b_{1,1}(s) > -b_{0,2}$$

To analyze debt buybacks in the model, as in the previous section, I will use a comparative statics approach. Under the analyzed pooling PBE-BS consider an *unexpected* in the ex-ante probability of being a risky borrower in $t = 1$: $\alpha_1 = \Pr(\theta = R | t = 1)$ increases.

It can be shown that the optimal response of safe borrowers (and also of risky borrowers that gain from mimicking safe borrowers) is

$$b_{11} = \frac{W^P}{\delta_1^P R(S)} \quad \frac{1}{1 - \alpha_1 \lambda^R} - \frac{\delta_2^P}{\delta_1^P} \quad < 0$$

which implies a debt buyback in $t = 1$.

This behavior is optimal given that the price of debt is now lower as a response of investors assigning a higher probability of borrowers being risky. Given the new debt prices it is now less attractive for borrowers to frontload consumption. They prefer to postpone consumption by buying back long-term debt at low prices.

More generally, debt buybacks can be optimal when there is an overestimation of the borrower's risk perception in the market that was not present when long-term debt decisions were made. The overestimation of the borrower's risk perception can come from an increase in the ex-ante probability of being a risky borrower (as in the current comparative statics exercise), or by an increase in the borrower's willingness to repay that is not priced in the market. This overestimation creates a wedge between the the market price of debt and the borrower's true intertemporal valuation of consumption that makes the borrower postpone consumption with debt buybacks.

It is important to stress that this analysis of debt buybacks is carried out with the strong assumption that the likelihood of default does not depend on the amount of debt issued. This implies that potential buybacks would not have any effect on debt prices. Bulow and Rogoff (1988) show that the price effect of buybacks may be harmful for borrowers and thus buybacks need not be beneficial for them. It is also worth stressing that the optimality of buybacks under the pooling equilibrium comes from a wedge between the borrower's intertemporal valuation of consumption and the market price of debt that exists given the presence of asymmetric information. The analysis here does not focus on the role of buybacks as signals of the borrowers type that can be another potential reason for why buybacks may be optimal.[24]

[24] For an analysis of buybacks as signaling devices see Acharya and Diwan (1993).

6 Conclusion

The contribution of this paper is the analysis of the optimal choice of sovereign debt maturity structure under the assumption that investors are anaware of the governments willigness to repay debt. Under a pooling equilibrium debt can be mispriced relative to the borrower's true fundamentals and the degree such mispricing can differ with the maturity of debt. Long-term debt becomes less attractive for safe borrowers since it pools more default risk that is not inherent to them. Safe borrowers optimally reacts to this negative asymmetric effect on prices by lowering the amount of debt and shortening its maturity profile. Risky borrowers mimic the behavior of safe borrowers in order to preclude the market from identifying their type, thus gaining a positive misinformation value by accessing to cheaper debt than that they should access if debt were priced according to their true fundamentals.

The relationship between maturity choice of sovereign debt and bond prices is introduced using a comparative statics approach. Times of financial distress in this model are characterized by periods where the *ex-ante* expected repayment capacity of borrowers deteriorates. In these periods, prices of long and short term debt fall and spreads increase. Additionally, consumption postponement becomes more valuable for safe borrowers. In fact, the longer the postponement the more valuable it becomes, since as time goes by, it becomes more likely that risky borrowers reveal their type by defaulting on their debt. Therefore, it becomes optimal for safe borrowers, and also for risky borrowers that gain from pooling with safe borrowers, to reduce their overall level of debt issuance and shorten the maturity composition of debt.

The predictions of the model are shown to be consistent with the observed co-movement of spreads and debt levels and maturities. Using data for a comprehensive sample of 34 emerging economies, this paper analyzed the relationship between the level of sovereign debt issuance, its maturity structure and country spreads. Results of panel data regressions indicate that: 1. the level of debt issuance covaries negatively with country spreads and 2. the maturity of debt covaries negatively with country spreads. In times of financial distress spreads increase, countries reduce their overall level of debt and shorten its maturity structure.

The paper finally uses the model to analyze policies of debt buybacks and argues that buybacks can be optimal when there is an overestimation of the country's risk perception in the market that was not present when long-term debt decisions were made.

The relevance of asymmetric information in explaining the choice of sovereign debt maturity structure follows naturally from the findings of this paper. Empirically testing the presence of imperfect information in the market for sovereign debt can shed light into whether the explanation offered in this paper plays a significant role in the governments choice of maturity structure of their debt. There has been studies that address this question for the case of the banking sector. Berger et al. (2005) find a strong quantitative role for asymmetric information in explaining debt maturity structure in the financial sector.

Overall, the model presented here has provided useful insight in how the presence of asymmetric information interacts with the choice of optimal debt maturity structure for governments in emerging markets. It has also offered an explanation for the negative comovement we observe in the data between average maturity and bond spreads.

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Appendix 1

This appendix discusses in further detail the data collected for the empirical analysis conducted in section 2 and presents additional robustness calculations.

A country was included in the sample if it is included -or was once included- in J.P. Morgan's EMBI Global (EMBIG), subject to the constraint of having sufficient data availability. Being included in the EMBIG reflects that both that the economy is emerging -and faces certain default risk- and that it is integrated to world capital markets. To be included in the EMBIG index, countries have to satisfy one of the following criteria:

(i) Be classified as low or middle per capita income by the World Bank;

(ii) Have restructured external or local debt in the past 10 years;

(iii) Have restructured external or local debt outstanding.

For a given particular bond to be included in the index, it must have a face value of over 500 million dollars, maturity of more than two and a half years and verifiable daily prices and cash flows.

For all countries in the sample data on bond issuance and country spreads was collected. Daily data for bond issuance was collected from Bloomberg and spreads data was obtained from Datastream. Table A1.1 reports the 34 countries that were included in the sample with the number of bond issuance observations and the sample period for which there is data availability on spreads. The resulting sample of countries turned out to be balanced across regions: 34% of the countries in the sample are from Latin America, 21% from Emerging Asia, 24% from Emerging Europe and 21% from Middle East and Africa.

Table A.1: Data and Sample Description

Country	Obs.	Sample Period	Country	Obs.	Sample Period
Argentina	879	Jan/94 - May/12	Lithuania	811	Dec/09 - May/12
Brazil	1464	May/94 - May/12	Malaysia	2014	Nov/96 - May/12
Bulgaria	236	Aug/94 - May/12	Mexico	1315	Jan/94 - May/12
Chile	5464	Jun/99 - May/12	Morocco	923	Jan/94 - Nov/06
China	1626	Jan/94 - May/12	Nigeria	695	Jan/94 - May/12
Colombia	720	Mar/97 - May/12	Panama	149	Aug/96 - May/12
Croatia	1651	Sep/96 - May/12	Peru	740	Apr/97 - May/12
Dominican Rep.	409	Dec/01 - May/12	Philippines	2186	Jan/98 - May/12
Ecuador	604	Mar/95 - May/12	Poland	1253	Nov/04 - May/12
Egypt	1626	Aug/01 - May/12	Russia	360	Sep/97 - May/12
El Salvador	205	May/02 - May/12	South Africa	26	Jan/95 - May/12
Hungary	1529	Feb/99 - May/12	Thailand	3262	Jun/97 - Mar/06
Indonesia	1201	Jun/04 - May/12	Tunisia	74	Jun/02 - Mar/11
Ivory Coast	16	May/98 - May/12	Turkey	813	Jul/96 - May/12
Kazakhstan	1062	Jul/07 - May/12	Ukraine	1271	Jun/00 - May/12
Korea	7361	Jan-94 - Apr/04	Uruguay	6195	Jul/01 - May/12
Lebanon	1934	May/98 - May/12	Venezuela	2585	Jan/94 - May/12

Note: The default periods of Dec/01 - Jun/05 for Argentina and Aug/98 - Sep/00 for Russia were excluded from the sample. The period of Jul/04 - Nov/09 for Croatia was also excluded due to lack of data on spreads. Issuance under the Brady Plan or under a debt restructure were excluded.

Bond issuance data covers bonds issued in all currencies. On average, 17% of a country's issuance is denominated in foreign currency. Nevertheless, there is great variability of the share of foreign currency-denominated debt across countries. For example, all debt issuance from Ecuador is dollar-denominated, whereas for the case of Brazil only 4% of the issuance in the sample was denominated in foreign currency. It is thus relevant not to restrict the analysis by analyzing only foreign-currency denominated debt.

Table A.2 displays s series of summary statistics on the data. Monthly average spreads, total issuance and average maturity were computed for each country in the sample. Spreads are measured in percentage points, monthly issuance is measured as a percentage of annual GDP and maturities are measured in years and are monthly weighted country averages, weighted according to the volume of debt raised with each bond.

Emerging economies pay an substantial positive premium over the US Treasury yield of an average of over 400 basis points. However, average spreads differ widely across countries, suggesting that the market poses different perceptions of default risk for different countries. The average level of monthly debt issuance is 0.6% of GDP and the average maturity is 3.3 years. Additionally, the average level of issuance and the weighted average bond maturity also differ across countries.

Average debt issuance levels and average maturity for times of high (above the country sample median) and low (below the country sample median) spreads are also reported in Table A.2. The average levels of debt issuance during times of low spreads are higher than the levels of issuance during times of high spreads in 22 out of the 34 countries. Additionally, the weighted average maturity shortens in times of high spreads relative to times of low spreads in 19 countries.

Table A.2: Bond Spreads, Issuance and Maturities in Emerging Economies

Country	Spread (Median)	Average Issuance Uncond.	Average Issuance Low Spreads	Average Issuance High Spreads	Average Maturity Uncond.	Average Maturity Low Spreads	Average Maturity High Spreads
Argentina	6.81	0.51	0.56	0.47	3.68	3.8	3.56
Brazil	5.22	1.15	1.09	1.21	3.83	4.97	2.85
Bulgaria	3.58	0.27	0.05	0.48	2.89	2.77	3.08
Chile	1.4	0.75	1.09	0.57	1.52	0.45	2.18
China	1.05	0.42	0.56	0.29	5.25	3.82	7.02
Colombia	3.32	0.19	0.17	0.2	2.79	2.53	2.99
Croatia	2.75	0.73	0.87	0.67	1.64	1.68	1.61
Dominican Rep.	4.63	0.53	0.43	0.63	4	3.34	5.1
Ecuador	9.21	0.09	0.1	0.07	3.74	3.06	4.31
Egypt	1.51	0.9	1.06	0.81	0.94	1.25	0.77
El Salvador	2.95	0.38	0.27	0.45	2.68	4.24	2.08
Hungary	0.87	1.18	1.59	0.93	2.11	2.34	1.97
Indonesia	2.63	1.61	2.1	1.41	1.35	2.16	0.97
Ivory Coast	23.14	0.13	0.07	0.19	5.6	3.21	8.94
Kazakhstan	3.88	0.46	1.02	0.35	1.99	1.61	2.07
Korea	1.18	1.13	0.44	1.4	2.61	4.05	2.05
Lebanon	3.62	2.05	2.32	1.83	3.2	3.06	3.32
Lithuania	3.16	0.45	0.55	0.44	2.01	2.35	1.98
Malaysia	1.56	0.88	1.02	0.74	2.19	2.15	2.23
Mexico	3.02	0.6	0.68	0.53	2.27	2.25	2.29
Morocco	4.37	0.45	0.5	0.42	5.37	5.91	5.06
Nigeria	9.6	0.23	0.25	0.21	1.83	1.32	2.32
Panama	3.18	0.46	0.51	0.41	4.14	4.52	3.86
Peru	3.49	0.33	0.44	0.22	4.07	4.71	3.39
Phillipines	3.81	0.65	0.67	0.63	3.37	4.34	2.73
Poland	1.77	0.61	0.68	0.54	3.65	4.44	2.85
Russia	2.63	0.21	0.14	0.27	4.6	6.12	3.78
South Africa	2.03	0.03	0.02	0.03	11.26	12.87	9.95
Thailand	1.19	1.75	0.61	2.18	1.79	1.75	1.8
Tunisia	1.39	0.12	0.11	0.13	3.22	3.12	3.37
Turkey	3.29	1.05	1.19	0.93	2.64	2.87	2.43
Ukraine	4.83	0.17	0.12	0.21	2.54	3.69	2.04
Uruguay	2.97	0.74	0.98	0.63	3.13	2.95	3.21
Venezuela	9.07	0.39	0.4	0.38	3.18	3.97	2.39
Average	4.09	0.63	0.67	0.61	3.27	3.46	3.25

Appendix 2

This appendix provides the proofs of lemmas 1 and 2 and propositions 3 and 4.

Proof of Lemma 1. It suffices to show that $\{b^P, q^*, \mu^P\}$ configures a pooling PBE. The fact that it will be the pooling PBE-BS follows by construction given that the debt allocations b^P attain the maximum utility for borrower S given on-equilibrium pooling prices.

Since it was already shown that beliefs μ^P are consistent with Bayes rule on equilibrium and that prices q^* are determined by the discounted expected repayment given beliefs, it only remains to be proven that both borrowers find it optimal to choose allocations b^P to show that $\{b^P, q^*, \mu^P\}$ indeed configures a pooling PBE.

Note that borrower S prefers to chose allocation b^P since, by definition it yields higher utility than any other allocation with pooling prices, and thus also yields higher utility than any allocation with prices set by beliefs $\mu_t = 1$ -since these are lower than pooling prices-.

Borrower R prefers allocation b^P if the maximum utility attained with prices set by beliefs $\mu_t = 1$ for all t is lower than the utility obtained by choosing b^P. The debt allocations that yield the highest utility to borrower R are given by (9)-(11) for $\theta = R$. The utility obtained from choosing these allocations is

$$U(b^{F\prime}, R) = R(R) \log \left(\frac{W(R)}{R(R)}\right) + \left(\beta\lambda^R + \beta\beta_1^e(1-(1-\lambda^R)^2)\right)\log(y_{def})$$

On the other hand, the utility attained from choosing b^* is

$$\begin{aligned} U(b^P, R) &= \log\left(\frac{W^P}{R(S)}\right) + \beta(1-\lambda^R)\log\left(\frac{\delta_1 W^P}{R(S)}\right) + \beta\beta_1^e(1-\lambda^R)^2\log\left(\frac{\delta_2 W^P}{R(S)}\right) \\ &\quad + \left(\beta\lambda^R + \beta\beta_1^e(1-(1-\lambda^R)^2)\right)\log(y_{def}) \end{aligned}$$

Borrower R will prefer to choose b^P as long as $U(b^P, R) > U(b^{F\prime}, R)$ which is precisely what is stated in Assumption 2.b.

Finally, uniqueness of the pooling PBE-BS comes from the fact that allocations b^P are the unique maximizers of the utility of borrower S given on-equilibrium pooling prices. ∎

Proof of Lemma 2. Again it suffices to show that $\{b^S(S), b^{F\prime}(R), q^*, \mu^S\}$ configures a PBE. The fact that it will be the separating PBE-BS follows by definition of $b^S(S)$.

Note that beliefs $\mu_t^S(b_t, s)$ are consistent with Bayes rule on equilibrium since $\mu_t^S(b^S(S), s) = 0$ and $\mu_t^S(b^{F\prime}(R), s) = 1$ for all t. Additionally, prices q^* are determined by the discounted expected repayment given beliefs. It only remains to be shown that borrower R finds optimal to choose $b^{F\prime}(R)$ and borrower S prefers to choose allocations $b^S(S)$ given prices $q^*(b)$.

Recall that $b^S(S)$ solves the following problem

$$\max_b U(b;S) \qquad s.t. \qquad U(b;R) \leq U^{F\prime}(R)$$

and also subject to (2) - (4) and prices given by (5) - (7) for $\theta = S$. In Appendix 3 I show that this is a well defined problem that has a unique solution.

By definition of $b^S(S)$, borrower R is indifferent between $b^{F\prime}(R)$ and $b^S(S)$. Also by definition of $b^{F\prime}(R)$, borrower R prefers it to any other allocation $b = b^S(S)$. It follows that borrower R finds it optimal to choose $b^{F\prime}(R)$.

In Appendix 3 it is shown that borrower S will choose the allocations $b^S(S)$ given prices $q^*(b)$.

$\blacksquare$

Proof of Proposition 3. Note first that the utility attained by borrower S under the pooling PBE-BS is continuous and strictly decreasing in α_0, whereas the utility attained by borrower S under the separating PBE-BS does not depend on α_0.

In the extreme case when $\alpha_0 = 0$ then the pooling PBE-BS coincides with the full information equilibrium for borrower S which yields borrower S a higher utility than the separating PBE-BS.[25] Therefore, for $\alpha_0 = 0$ the PBE-BS is the best pooling PBE.

For the other extreme case of $\alpha_0 = 1$ we may have that the PBE-BS is either pooling or separating. In the first case given the monotonicity of $U(\cdot;S,\alpha_0)$ under the pooling PBE-BS it follows that the threshold $\tilde{\alpha}_0 = 1$. In the second case again by monotonicity of $U(\cdot;S,\alpha_0)$ under the pooling PBE-BS it follows from Bolzano's Theorem that there exists some threshold $\tilde{\alpha}_0 \in (0,1)$ such that for $\alpha_0 \leq \tilde{\alpha}_0$ the pooling PBE-BS is the PBE-BS.

$\blacksquare$

Proof of Proposition 4. The first result of the proposition follows directly from the on-equilibrium price expressions.

To show the second result I need to show that

$$\frac{\partial q_{0,1}b^P_{0,1} + q_{0,2}b^P_{0,2}}{\partial \alpha_0} < 0$$

[25] It will yield the same utility as the separating PBE-BS when the restriction $U\left(b^S(S);R\right) \leq U^{F\prime}(R)$ is not binding since in this case the separating PBE-BS will also coincide with the full information equilibrium.

Note that $q_{0,1}b^P_{0,1} + q_{0,2}b^P_{0,2} = c^P_0 - y_0$ so proving the second result is equivalent to proving that

$$\frac{\partial c^P_0}{\partial \alpha_0} < 0$$

but this follows immediately from the fact that $\frac{\partial W^P}{\partial \alpha_0} < 0$.

Finally, to show the last result I need to show that

$$\frac{\partial q_{0,1}b^P_{01}/q_{0,2}b^P_{02}}{\partial \alpha_0} > 0 \text{ for small } y_0$$

It suffices to show:

$$\frac{\partial q_{0,1}/q_{0,2}}{\partial \alpha_0} > 0, \qquad \frac{\partial b^P_{01}}{\partial \alpha_0} > 0 \qquad \text{and} \qquad \frac{\partial b^P_{02}}{\partial \alpha_0} < 0$$

Proving the first inequality holds is equivalent to showing:

$$\frac{\partial \log(q_{0,1})}{\partial \alpha_0} > \frac{\partial \log(q_{0,2})}{\partial \alpha_0}$$

We can calculate these two derivatives:

$$\frac{\partial \log(q_{0,1})}{\partial \alpha_0} = -\frac{\lambda^R}{1-\alpha_0\lambda^R} > -\frac{\lambda^R(2-\lambda^R)}{1-\alpha_0\lambda^R(2-\lambda^R)} = \frac{\partial \log(q_{0,2})}{\partial \alpha_0}$$

Similarly, proving the second inequality holds is equivalent to showing:

$$\frac{\partial \log(\delta_1)}{\partial \alpha_0} + \frac{\partial \log(W^P)}{\partial \alpha_0} < 0$$

The previous derivative can be calculated:

$$\begin{aligned}
\frac{\partial \log(\delta_1)}{\partial \alpha_0} + \frac{\partial \log(W^P)}{\partial \alpha_0} &= \frac{\lambda^R}{1-\alpha_0\lambda^R} - \frac{\lambda^R(\beta y_1 + \beta\beta^e_1(2-\lambda^R)y_2)}{q_{01}y_1 + q_{02}y_2} \\
&= \frac{\lambda^R}{1-\alpha_0\lambda^R} - \frac{\lambda^R}{1-\alpha_0\lambda^R}\frac{(q_{01}y_1 + (2-\lambda^R)\beta\beta^e_1(1-\alpha_0\lambda^R)y_2)}{q_{01}y_1 + q_{02}y_2} \\
&< \frac{\lambda^R}{1-\alpha_0\lambda^R} - \frac{\lambda^R}{1-\alpha_0\lambda^R}\frac{W^P}{W^P} \\
&= 0
\end{aligned}$$

The first equality uses the fact that $y_0 = 0$, and the inequality uses the fact that $q_{01} > q_{02}$ and $\lambda^R < 1$.

It remains to show the third inequality. Again proving the third inequality holds is equivalent to showing:

$$\frac{\partial \log(\delta_2)}{\partial \alpha_0} + \frac{\partial \log(W^P)}{\partial \alpha_0} > 0$$

$$
\begin{aligned}
\frac{\partial \log(\delta_2)}{\partial \alpha_0} + \frac{\partial \log(W^P)}{\partial \alpha_0} &= \frac{\lambda^W(2-\lambda^R)}{1-\alpha_0\lambda^R(2-\lambda^R)} - \frac{\lambda^R(2-\lambda^R)(\beta y_1(2-\lambda^R)^{-1} + \beta\beta_1^e y_2)}{q_{01}y_1 + q_{02}y_2} \\
&= \frac{\lambda^R(2-\lambda^R)}{1-\alpha_0\lambda^R(2-\lambda^R)} \quad 1 - \frac{\beta\frac{(1-\alpha_0\lambda^R(2-\lambda^R))}{2-\lambda^R}y_1 + q_{02}y_2}{q_{01}y_1 + q_{02}y_2} \\
&> \frac{\lambda^R(2-\lambda^R)}{1-\alpha_0\lambda^R(2-\lambda^R)} \quad 1 - \frac{W^P}{W^P} \\
&= 0
\end{aligned}
$$

Here, the first equality uses the fact that $y_0 = 0$, and the inequality uses the fact that $(1-\alpha_0\lambda^R(2-\lambda^R))(2-\lambda^R)^{-1} < 1-\alpha_0\lambda^R$.

Appendix 3

This appendix characterizes the best separating equilibrium for borrower S.

To solve for the separating PBE-BS I will follow the same steps as in the pooling equilibrium: first solve for the optimal debt allocations for borrower S given separating prices subject to the constraint of borrower R preferring not to pool; and secondly construct beliefs that sustain those prices and allocations under a PBE.

Consider the following artificial problem that chooses debt allocations to:

$$\max_b U(b;S) \qquad s.t. \qquad U(b;R) \leq U^{F\prime}(R)$$

and also subject to (2) - (4) and prices given by (5) - (7) for $\theta = S$. This problem will yield the allocations that maximize borrower S expected utility such that borrower R finds it optimal to separate and choose his full information allocations regardless of the fact that he will reveal his type by doing so.

Let φ be the Lagrange multiplier associated to the separating restriction. Then the first order conditions associated to this problem are

$$\frac{\beta}{c_0} - \beta \left(\frac{p}{c_1(p)} + \frac{1-p}{c_1(\boldsymbol{i})} \right) = \varphi \left(\frac{\beta}{c_0} - \beta(1-\lambda_R) \left(\frac{p}{c_1(p)} + \frac{1-p}{c_1(\boldsymbol{i})} \right) \right) \tag{26}$$

$$\frac{\beta\beta_1^e}{c_0} - \beta \left(\frac{p\beta^p}{c_2(p)} + \frac{(1-p)\beta^i}{c_2(\boldsymbol{i})} \right) = \varphi \left(\frac{\beta\beta_1^e}{c_0} - \beta(1-\lambda^R)^2 \left(\frac{p\beta^p}{c_2(p)} + \frac{(1-p)\beta^i}{c_2(\boldsymbol{i})} \right) \right) \tag{27}$$

$$\frac{\beta^s}{c_1(s)} - \frac{\beta^s}{c_2(s)} = \varphi \left(\frac{\beta^s(1-\lambda^R)}{c_1(s)} - \frac{\beta^s(1-\lambda^R)^2}{c_2(s)} \right) \tag{28}$$

The optimal consumption stream that is implied by these first order conditions is

$$c_0^S = \frac{W(S)}{D^S}$$

$$c_1^S(s) = \frac{\delta_1^S W(S)}{D^S}$$

$$c_2^S(s) = \frac{\delta_2^S W(S)}{D^S}$$

for $s = p, \boldsymbol{i}$, where $D^S = 1 + \beta\delta_1^S + \beta\beta_1^e\delta_2^S$, $W(S)$ is the market value of wealth valued at safe borrower's debt prices and

$$\delta_1^S = \frac{1-\varphi(1-\lambda^R)}{1-\varphi} > 1 \qquad \text{and} \qquad \delta_2^S = \frac{1-\varphi(1-\lambda^R)^2}{1-\varphi} > \delta_1^S$$

It is interesting to note the parallelism between to the solution to this problem and the optimal consumption stream in the pooling PBE-BS. As in the pooling PBE-BS the optimal consumption stream is to consume a distorted fraction of wealth, now valued at prices that reflect the true repayment capacity of the safe borrower. In this case the distortion is introduced via the parameters δ_1^S and δ_2^S that reflect the intertemporal valuations of consumption of borrower S relative to borrower R.

The value of the Lagrange multiplier is obtained by setting the separating restriction to bind, i.e. φ will be such that

$$\log\left(\frac{W(S)}{D^S}\right) + \beta(1-\lambda^R)\log\left(\delta_1^S\frac{W(S)}{D^S}\right) + \beta\beta_1^e\log\left(\delta_2^S\frac{W(S)}{D^S}\right) = U^{F\prime}(R)$$

It can be shown that $\varphi < 1$. This result ensures that consumption will be non-negative and increasing across time.

The unique optimal debt allocations associated with this consumption stream are given by

$$b_{01}^S = y_1 - \frac{\delta_1^S W(S)}{D^S} \tag{29}$$

$$b_{02}^S = y_2 - \frac{\delta_2^S W(S)}{D^S} \tag{30}$$

$$b_{11}^S(s) = 0 \tag{31}$$

Finally, to complete the proof of Lemma 2 I now show that borrower S will find optimal to choose $b^S(S)$ given prices q^*. This will be true when the utility of choosing $b^S(S)$ and revealing his type is higher than the utility of choosing the best allocation conditional on being believed to be a risky borrower. The allocations that yield the highest utility to borrower S given debt priced with borrower R default risk are given by:

$$b_{0,1} = y_1 - \frac{W(R)}{(1-\lambda^R)R(S)} \qquad b_{0,2} = y_2 - \frac{W(R)}{(1-\lambda^R)^2R(S)} \qquad b_{1,1}(s) = 0$$

It then follows that borrower S will prefer allocations $b^S(S)$ if

$$\log\left(\frac{W(S)}{D^S}\right)(1+\beta+\beta\beta_1^e) + \beta\log\left(\delta_1^S\right) + \beta\beta_1^e\log\left(\delta_2^S\right) >$$
$$\log\left(\frac{W(R)}{R(S)}\right)(1+\beta+\beta\beta_1^e) - (\beta+2\beta\beta_1^e)\log\left(1-\lambda^R\right)$$

The presence of the separating equilibrium in the asymmetric information setting leaves borrower S worse off respect to the full information setting given that in order to separate from borrower R, borrower S engages in some distortionary consumption path. He does so by consuming more in states in which his valuation of consumption is highest relative that of borrower R. Given that borrower R defaults with positive probability in periods $t = 1, 2$

the highest consumption valuation of borrower S relative to borrower R occurs in late repayment states. Although for different reasons, the optimal consumption rule features an increasing consumption path across time, as in the previously analyzed pooling equilibrium. In order to attain this increasing consumption path the safe borrower optimally chooses to issue low levels of debt with shorter maturities, relative to the optimal issuance under the full information benchmark.

Borrower R is indifferent between separating and pooling and decides to separate by choosing the debt allocations that maximize his utility in the full information benchmark. In other words, he prefers to choose debt in order to optimally transfer resources across time regardless of the fact that he reveals his type by doing so.