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Firm Investment and the Term Structure of Uncertainty

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Firm Investment and the Term Structure of Uncertainty

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Why has firm activity been slow to recover from the Great Recession? I present theoretical and empirical evidence suggesting long-term uncertainty may be one reason. Specifically, I show the current level of uncertainty and expectations of future uncertainty – that is, the entire term structure of uncertainty – are negatively correlated with firm investment rates. I present a simple model generating these effects through real options channels. Using equity options to obtain forward-looking estimates of firm and aggregate uncertainty at different horizons, I then show that both the level and slope of the term structure of uncertainty have negative conditional correlations with capital investment rates, consistent with the model. Numerically, a one standard deviation increase in firm (aggregate) uncertainty over the next year relative to the next 30 days correlates with a decrease in firm capital investment equal to 3.1% (4.4%) of the mean firm investment rate over the next quarter. I also find the correlation between both short- and long-term uncertainty and R&D to be positive, supporting the theory that firms invest in growth options in the face of uncertainty. I discuss identification in this context and the particular relevance of my findings for government policy.

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1 Introduction

Why has firm investment and economic growth during the recovery from the Great Recession been low relative to previous recoveries? A survey of economists reveals how little consensus has been reached on the answer to this question. Sixty-two leading economists were asked what the single most important reason jobs have not returned more quickly in the United States has been. Only four responses were given by at least 8% of respondents. However, the most popular response was given by over 30% of respondents: uncertainty. This paper provides new theoretical and empirical evidence supporting this claim. Specifically, I examine the effects of short- and long-term uncertainty on firm investment and other economic outcomes.

The study of investment under uncertainty became popularized by McDonald and Siegel (1986), Dixit and Pindyck (1994) and Abel and Eberly (1996), among others, who were some of the first to explore this topic through the theory of real options. Since their work, both empiricists and theorists have flocked to this space, especially in the recent wake of the Great Recession. However, while economists have done much work striving to reach a consensus on the effect of uncertainty on investment, there is little understanding of what particular horizons of uncertainty are most relevant for firm investment. Indeed, the typical strategy for most work in this literature is to put a single measure of investment on the left-hand side of a linear regression or in a model framework, a single measure of uncertainty and control variables on the right-hand side or in a model framework, and end the analysis when the coefficient/effect of uncertainty is estimated statistically and/or economically significant in magnitude. Numerous authors have used this approach, with differing methods of identification. However, such work leaves one wondering what type of uncertainty matters most for firm decisions, and if firm investment responds differentially to uncertainties over different horizons.

In this paper I address this shortcoming by analyzing the differential relationships of different horizons of uncertainty with firm investment rates. That is, I investigate the relationship of firm investment with the entire term structure of uncertainty, rather than with just one uncertainty measure. First, I present a simple model of a firm making a one-time irreversible investment in a project that pays off for a finite number of periods, where there are two resolutions of uncertainty about project payoffs – one in the short-term and one in the longer-term. Investment is delayed only if uncertainty over project outcomes in the short-term or long-term is large relative to the expected project payoff, and if the effects of the uncertainty are long-lived.

Motivated by these results, I then analyze the empirical correlations of short- and long-term uncertainty with firm investment rates. I use firm and market option implied

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1 For example, the annual GDP growth rate in the two and a half years following the Great Recession was only 2.4%, compared with 5.9% during the period following the recession ending in 1982.

2 Survey details are can be found at [http://www.foreignpolicy.com/articles/2012/10/08/the_fp_survey_the_economy/](http://www.foreignpolicy.com/articles/2012/10/08/the_fp_survey_the_economy/). A list of the economists surveyed is contained in the appendix.

3 I note that these were not the first studies of investment under uncertainty. As outlined by Bloom (2014), other work such as Oi (1961), Hartman (1972), Abel (1983) and Bernanke (1983) had been done prior to these. These earlier works emphasized different channels by which uncertainty could affect investment. I list the three I do in particular since they appear to have been the studies that came out at the time focus began to turn more intensely to this area of research.

4 See, for example, Leahy and Whited (1996), Stein and Stone (2013) and Gulen and Ion (2013).
volatilities at 30-day and 1-year horizons as uncertainty measures, and firm quarterly capital expenditures scaled by firm assets as my primary investment measure. Focusing on conditional correlations, I find that long-term uncertainty at both the firm and aggregate levels has statistically and economically significant negative correlations with firm investment rates. Numerically, a one standard deviation increase in firm uncertainty over the next year relative to the next 30 days correlates with a decrease in firm capital investment equal to 3.1% of the mean quarterly firm investment rate over the next quarter. A one standard deviation increase in long-term aggregate uncertainty over the next year (relative to the next 30 days) correlates with a decrease in firm capital investment equal to 4.4% of the mean quarterly firm investment rate over the next quarter. These results are robust to controlling for measures of firm profitability and investment opportunities such as Tobin’s Q, cash flow and sales growth. They are also robust to varying the horizons of uncertainty defined as “long-term” and “short-term,” and are not due to recession subsamples. Additionally, many endogeneity concerns are mitigated due to my specification. I will discuss identification and the remaining potential sources of endogeneity in the empirical results section.

With the continuation of slow growth following the Great Recession, analyzing what horizons of uncertainty are of greatest relevance for firm decisions is of particular importance for policy. The targets of economic and fiscal policy may be much different if long-term uncertainty is thought to be a cause of a persistent economic slowdown as opposed to short-term uncertainty. For example, the former may call for more stable long-term policies and tax codes with credible commitments, while the latter may call for faster response time and intervention in asset markets.

Some economists have argued specifically that long-term uncertainty about government policy has led firms to restrict investment (Greenspan (2012), Ohanian and Taylor (2012)). The idea that policy uncertainty is depressing investment has been supported by empirical results such as those in Figure ??, which plots gross private domestic investment (in structures, equipment and software) as a share of GDP and the policy uncertainty index of Baker, et al. (2013), which incorporates disagreement amongst economic forecasters, tax code expirations and policy uncertainty related news articles into a single index.

The negative relationship between uncertainty and investment in Figure 1 is stark. As Baker, et al. (2013) argue, one would think their policy uncertainty index captures uncertainty about long-term prospects and economic outcomes. However, no model and no empirical evidence has been presented investigating the effect of long-term uncertainty on investment differentially from short-term uncertainty. Empirical studies testing whether policy uncertainty appears to be depressing investment (such as Baker, et al. (2013) and Gulen and Ion (2013)) do not make any distinction between long- and short-term uncertainty. And while structural models are developed that incorporate multiple measures of uncertainty (e.g. firm and aggregate uncertainties as in Bloom (2009)), no such work incorporates uncertainties that vary by horizon.

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5Observing standard deviation changes in implied volatilities such as these is not unlikely. For example, market measures (which have less variation than firm measures due to diversification) moved significantly during and after the Great Recession. In my sample, a one standard deviation increase in my long-term market uncertainty measure is observed during the period following the Great Recession. At times the short-term market uncertainty measure increases by two standard deviations during the Great Recession.
Figure 1: Gross Private Domestic Investment as a share of GDP and Policy Uncertainty

Notes: Gross private domestic investment is investment in structures, equipment and software, courtesy of the Bureau of Economic Analysis via the FRED database. The U.S. policy uncertainty index is courtesy of Baker, et al. (2013). The data is quarterly from 1985 to 2013, with policy uncertainty observations being averages over the quarter.

Data on the term structure of uncertainty reveals that short- and long-term uncertainty do not always move together, particularly when long-term uncertainty is measured as expectations of future uncertainty relative to short-term uncertainty. Thus, each horizon of uncertainty may have differential effects on firm investment. Figure 2 plots the implied volatility on the S&P 500 index at different horizons both prior to (during the first half of 2007) and some time after (during the first half of 2013) the recent economic crisis.6

The data clearly illustrate that while short-term uncertainty (e.g. 30-day implied volatility) has been at levels similar to those of 2007, long-term uncertainty (e.g. 1-year implied volatility) has not.7 Indeed, 30-day implied volatility is less than 10% higher post-crisis than it was prior to the crisis, while 1-year implied volatility is more than 30%

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6As I will make clear in the data section, I obtain my implied volatility data from Optionmetrics. A kernel smoothing technique is applied to combine raw data on options with similar strikes, exercise styles and maturities to generate a series of “standardized options” with set maturities, for both the S&P 500 index and the firms in my sample. This allows me to have a data on a set of “constant maturity” options through time, and hence a set of “constant maturity” implied volatilities. An implied volatility estimates how volatile an underlying security price will be over a given horizon. The implied volatilities I use in this paper are obtained by inverting the Black-Scholes formula for the series of standardized options provided by Optionmetrics.

7Unfortunately Optionmetrics makes available all data with a significant lag of more than a year. Hence I do not provide plots of more recent data here.
higher post-crisis than it was prior to the crisis. This supports the idea that specifically long-term uncertainty may be slowing the economic recovery. Put another way, Figure ?? plots the difference between the 30-day and 1-year implied volatilities on the S&P 500 index options, what I will call the “slope” of the term structure of uncertainty, over time. To see the same series cross-sectionally, Figure ?? plots the slope of the term structure of uncertainty for individual firms in different sectors of the economy over time.

Plotting the slope, I capture what expectations of future uncertainty are over the next year, relative to the next 30 days. When the slope is high, expectations of uncertainty over the next year are high, relative to expectations of uncertainty over the next 30 days. Both figures illustrate that firm and aggregate measures of long-term uncertainty have been consistently higher since the crisis than they have been at any other point in the past 20 years. This again illustrates that long-term uncertainty, specifically, has been high during the time the economy has been slow to recover.

\[ \frac{E^P[\sigma]}{E^Q[\sigma]} \] 

Given that the implied volatilities I present are Black-Scholes implied volatilities, they are risk neutral expectations of the volatility, \( E^Q[\sigma] \). However, I am really interested in what the effect of changes in expectations of volatility under the historical measure, \( E^P[\sigma] \), are on uncertainty. The difference between these has been termed the \textit{variance risk premium}. However, since the variance risk premium and the historical expectation of volatility tend to move together (see, for example, Bollerslev, Tauchen and Zhou (2009) and Han and Zhou (2011)), upward movements in \( E^P[\sigma] \) will, on average, be captured by upward movements in \( E^Q[\sigma] \), and vice versa.

\(^8\)Optionmetrics only makes data available starting in 1996, hence why I do not plot the time series back further.
Historically, two theoretical approaches have been taken to analyzing the effect of uncertainty on investment, as outlined in Bloom (2014): (1) that of a single firm real options framework where options are valued and then optimal exercise (i.e. investment timing) is determined, and (2) structural equilibrium models of an economy where firm investment policy functions are determined in conjunction with competitive dynamics. Most of these models predict a negative relationship between firm investment and uncertainty through various mechanisms including real options, precautionary savings and risk premium effects.\textsuperscript{10} In the theoretical portion of this paper I construct a general framework in the first class of models, building on the work of Dixit and Pindyck (1994).

Section ?? presents my simple model illustrating qualitatively that both short- and long-term uncertainty should be relevant for firm investment decisions. Section ?? presents the data and empirical strategy I use. Section ?? presents my empirical results and discusses identification. Section ?? discusses policy implications and outlines further research work that could be undertaken on this topic. Section ?? concludes.

\textsuperscript{10}Some mechanisms also predict a positive relationship between investment and uncertainty, such as growth options and the so-called Oi-Hartman-Abel effect (see Oi (1961), Hartman (1972) and Abel (1983)). A number of empirical studies have tested which of these theories are most at play, and in what scenarios. However, I will not attempt a complete review of the theoretical and empirical literature here but will instead proceed with my contribution. See Bloom (2014) for a survey on related literature.
2 A Simple Model

In this section I develop a simple model showing qualitatively that increases in both short- and long-term uncertainty are associated with firms delaying investment. I first present a general model that includes the numerical examples of Dixit and Pindyck (1994) as special cases, and then extend this to show the effects uncertainty at different horizons has on firm investment through real options channels.\footnote{Although models could be written down with multiple of the mechanisms at play, I focus on the real options channel in this model. I do this because my objective is not to demonstrate which of the mechanisms have stronger effects, but rather to show that different horizons of uncertainty can have differential effects on investment. Real options is a standard mechanism used to study the effect of uncertainty on (partially) irreversible investment, such as capital expenditure.}

2.1 Model with Only Short-Term Uncertainty

There are $T + 1$ periods. There is no time discounting (i.e. the interest rate, $r$, equals $0$). A risk neutral firm\footnote{Alternatively, one can interpret the risk of the project as being fully diversifiable, which is standard in the real options literature to justify this assumption.} is deciding whether or not to undertake a one-time irreversible investment in a project or not, at some time $t = 0, 1, ..., T + 1$. The firm can pay $I$ at any time $t$ and collect payoffs from the project at all times $t + 1, t + 2, t + 3, ..., T + 1$.
The project pays 0 at time \( t = 0 \).

The firm has uncertainty about the project’s payoff stream. Specifically, a resolution of uncertainty takes place at time \( t = 1 \), and is characterized in the following way:

- With probability \( p \) the project will pay off \( V_H \) each period \( t = 1, 2, \ldots, T + 1 \), and with probability \( 1 - p \) the project will pay off \( V_L \) at each period \( t = 1, 2, \ldots, T + 1 \), where \( V \equiv V_H - V_L > 0 \). Let \( \tilde{V} \) denote the random variable that is the payoff of the project from time \( t = 1 \) to time \( t = T + 1 \).

This framework results in a project payoff structure given by the tree in Figure 5. I assume the project is not valuable to have undertaken if the bad state (\( L \)) of the world results, i.e. \( (T + 1)V_L < I \). However, I assume it is valuable to have undertaken the project if the good state (\( H \)) of world results, i.e. \( (T + 1)V_H > I \).

Figure 5: Model Decision Tree Illustrating Short-Term Uncertainty

Since the firm is risk neutral, it will implement a strategy based on net present values (NPVs) of the different strategies at time \( t = 0 \). Thus, to determine the optimal strategy of the firm I compute the NPVs associated with each potential strategy. Note that the strategy space is quite large. However, this can be reduced significantly by noting that the firm will only consider investing in periods \( t = 0 \) and \( t = 1 \). By investing at some time \( t^* > t = 1 \) the firm forgos additional periods of payoff that it could have had by investing earlier. Since the cost of investing is static (\( I \)) and since there is no time discounting and no information revelation except at time \( t = 1 \), the only difference between investing at \( t = 1 \) and \( t = t^* \) is the additional payoff from investing earlier. Hence, it cannot be optimal to invest at some time \( t^* > t = 1 \). Thus, the following strategies are the only ones remaining that can potentially be optimal for the firm:
1. Never invest
2. Invest at time $t = 0$
3. Invest at time $t = 1$ but only if the state is $H$
4. Invest at time $t = 1$ but only if the state is $L$
5. Invest at time $t = 1$ regardless of the state of world

I can, in fact, reduce the strategy space even further by noting that the strategy to invest “regardless” of the state of the world cannot be optimal, and that waiting to invest and then investing if the bad state of the world is realized can never be optimal. Investing regardless of the state of the world at time $t = 1$ cannot be optimal because if it were then the firm should have just invested at time $t = 0$ and collected the additional payoff. Investing if the bad state of the world is realized cannot be optimal for similar reasoning. At any given time $t$, if it is optimal to invest in the bad state of the world then it must also be optimal to invest in the good state of the world since good state payoffs dominate bad state payoffs. This implies it is optimal to invest at time $t$ regardless of the state of the world, which I just showed cannot be optimal. Thus, investing at a time $t$ but only in the bad state of the world cannot be an optimal strategy.

This results in the space of potentially optimal strategies for the firm being further reduced to:

1. Never invest
2. Invest at time $t = 0$
3. Invest at time $t = 1$ but only if the state is $H$

I now compute net present values (from the perspective of time $t = 0$, of course) of the different possibly optimal strategies, and then compare these in order to make statements about the effect of uncertainty on a firm’s decision to invest, not invest, or delay investment. The NPVs of the corresponding three strategies above are found directly to be:

1. $0$
2. $p(T + 1)V_H + (1 - p)(T + 1)V_L - I$
3. $p(TV_H - I)$

The optimal strategy will, of course, depend on parameters. Comparing NPVs reveals how.
2.1.1 Delay Investment Until After the Resolution of Uncertainty

When will investment occur but be delayed until after the resolution of uncertainty? This will happen if (A) the net present value of waiting (strategy 3 above) is higher than the net present value of investing at time \( t = 0 \) (strategy 2 above), and (B) the net present value of waiting (strategy 3 above) and actually investing is higher than the net present value of not investing (strategy 1 above). Requirement (B) is important since I want to study the timing of investment and, hence, cases where investment actually occurs but is just delayed. \(^{13}\) Recall that, by assumption, I am in the good state, \( H \), if investment is being undertaken at \( t = 1 \).

Mathematically, (A) being satisfied is equivalent to:

\[
p(TV_H - I) > p(T + 1)V_H + (1 - p)(T + 1)V_L - I
\]

Cancellation of common components and rearranging results in:

\[
(1 - p)I > pV_H + (1 - p)V_L + (1 - p)TV_L
\]

\[\iff\]

\[
\frac{I}{T} > \frac{1}{T} \left[ \frac{p}{1 - p} V_H + V_L \right] + V_L \tag{1}
\]

\[\iff\]

\[
\frac{I}{T} > \frac{E(\tilde{V}_1)}{T(1 - p)} + V_L \tag{2}
\]

Equation (2) simply amounts to saying that if the project cost is sufficiently large relative to the expected project payoff, the payoff is too small in the bad state relative to the project cost, or the probability of the bad state of the world is large enough, then the firm will delay the investment decision rather than invest at \( t = 0 \) since the project is only worth having if the good state is realized and costly to have undertaken if the bad state is realized.

(B) being satisfied is equivalent mathematically to:

\[
p(TV_H - I) > 0
\]

\[\iff\]

\[
V_L + V > \frac{I}{T} \tag{3}
\]

Equation (3) simply amounts to saying that the per period project payoff in the good state of the world must be sufficiently large relative to the per period cost of the project for the project to be undertaken after delay has occurred.

Combining (2) and (3) yields

\[
V_L + V > \frac{I}{T} > \frac{E(\tilde{V}_1)}{T(1 - p)} + V_L
\]

\(^{13}\)If requirement (B) is not imposed then a firm might not invest simply because it does not have profitable investment projects.

\(^{14}\)This is the simply the bad news principle of Bernanke (1983).
$$TV(1-p) > E(\tilde{V}_1)$$

$$\iff T\frac{\sqrt{1-p}}{\sqrt{p}}\sigma(\tilde{V}_1) > E(\tilde{V}_1)$$

(4)

where $\sigma(\tilde{V}_1) = p(1-p)(V_H - V_L)^2$ as computed in the appendix. This result is summarized in the following theorem.

**Theorem 1.** The decision to invest will be delayed until after the resolution of uncertainty only if (??) holds.

Theorem ?? yields a number of insights. First, the investment decision is delayed only if the expected payoff of the project satisfies (??). If the expected payoff is sufficiently large then the firm will invest at $t = 0$ to collect one more period of payoff, rather than wait to decide whether to invest or not. Second, the investment decision will be delayed only if the standard deviation of the state payoffs is sufficiently high relative to the expected value. In essence, the theorem states that if the decision to invest is not delayed, then uncertainty must be sufficiently low. Third, the investment decision will be put off only if the probability of the bad state, $1 - p$, is sufficiently large relative to the probability of the good state and the expected payoff. Fourth, the investment decision will be delayed only if the period of time after the resolution of uncertainty is sufficiently large. If the firm is putting off its investment decision, it must be that it is at risk of getting stuck with a poor project payoff stream for a long period of time. It is useful to note that (??) can be rearranged as

$$T\frac{\sqrt{1-p}}{\sqrt{p}} > \frac{E(\tilde{V}_1)}{\sigma(\tilde{V}_1)}$$

(5)

I point out that the right-hand side term, $\frac{E(\tilde{V}_1)}{\sigma(\tilde{V}_1)}$, is a Sharpe ratio. If Sharpe ratios of projects are sufficiently high, then the decision to invest cannot have been delayed.

### 2.2 Model with Both Short- and Long-term Uncertainty

I now extend the model to the more general case of having two resolutions of uncertainty about projects payoffs – one in the short-term and one in the long-term – rather than just one. Since the intuition and mechanisms leading to the key equations of interest are similar in this model and the previous model, I put the derivations in the appendix (section ??) rather than here.

There are $T$ periods. As before, there is no time discounting ($r = 0$), the firm is risk neutral and is deciding whether or not to undertake a one-time irreversible investment in a project or not, at some time $t = 0, 1, ..., T$. The firm can pay $I$ at any time $t$ and collect payoffs from the project at all times $t + 1, t + 2, t + 3, ..., T$. The project pays $0$ at time $t = 0$.

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15 More formally, a mean preserving spread between $V_H$ and $V_L$ decreases the incentive to invest at time $t = 0$, since if the spread is not mean preserving it could also change the right-hand side $E(\tilde{V}_1)$. 

10
This model is new relative to the previous one in that the decision of the firm is complicated by two resolutions of uncertainty as opposed to one, which gives some insight as to how the timing and term structure of uncertainty affects firm decisions. One resolution of uncertainty takes place at time $t = 1$, while another resolution of uncertainty takes place at time $t = t_1 + 1$ where $t_1 > 0$ and $t_1 < T$. These resolutions of uncertainty are characterized in the following way:

- At $t = 1$ uncertainty is resolved in that with probability $p_1$ the project will pay off $V_H$ each period $t = 1, 2, ..., t_1$, and with probability $1 - p_1$ the project will pay off $V_L$ at each period $t = 1, 2, ..., t_1$, where $V \equiv V_H - V_L > 0$. Let $V_1$ denote the random variable that is the payoff of the project from time $t = 1$ to time $t = t_1$.

- At $t = t_1 + 1$ uncertainty is resolved in that with probability $p_2$ the project will pay off $V_{HH} > V_H$ each period $t = t_1 + 1, t_1 + 2, ..., T$ if the first resolution was $H$, and with probability $1 - p_2$ the project will pay off $V_{HL} < V_H$ at each period $t = t_1 + 1, t_1 + 2, ..., T$ if the first resolution was $H$, where $V_H \equiv V_{HH} - V_{HL} > 0$. Similarly, uncertainty is resolved in that with probability $p_2$ the project will pay off $V_{LL} > V_L$ each period $t = t_1 + 1, t_1 + 2, ..., T$ if the first resolution was $L$, and with probability $1 - p_2$ the project will pay off $V_{HL} < V_L$ at each period $t = t_1 + 1, t_1 + 2, ..., T$ if the first resolution was $L$, where $V_L \equiv V_{LL} - V_{HL} > 0$. I also assume $V_{HL} \geq V_{LL}$, although this could be relaxed as it is not essential for the results I study. Let $V_2$ denote the random variable that is the payoff of the project from time $t = t_1 + 1$ to time $t = T$.

This framework results in a payoff structure given by the tree in Figure ??.

For simplicity, I assume that in no state where a bad state has ever been realized ($L, LL, LH, HL$) is the investment valuable to have. This effectively wipes out the entire bottom branches of the tree from consideration as being optimal states for investment. Conversely, it is profitable to have the investment when both good states ($H, HH$) of the world have resulted. Mathematically, $(T - t_1)V_{LL} + t_1V_H < (T - t_1)V_{HL} + t_1V_L \leq (T - t_1)V_{HH} + t_1V_H < I$ and $(T - t_1)V_{HH} + t_1V_H > I$.

As before, to determine the optimal strategy of the firm I compute the NPVs associated with each potential strategy. After reducing the strategy space as outlined in the appendix, the only potentially optimal investment strategies are:

1.* Never invest

2.* Invest at time $t = 0$

3.* Invest at time $t = 1$ but only if the state is $H$

4.* Invest at time $t = t_1 + 1$ but only if the state is $HH$

The NPVs of these strategies above are computed directly as:

1.* $0$

2.* $t_1[p_1V_H + (1 - p_1)V_L] + (T - t_1)[p_1(p_2V_{HH} + (1 - p_2)V_{HL}) + (1 - p_1)(p_2V_{LL} + (1 - p_2)V_{HL})] - I$
To determine how the optimal strategy will depend on parameters I compare NPVs. Recall that I am investigating specifically how short- and long-term uncertainty affect the timing of investment. Thus, although I could investigate the circumstances under which each of the four strategies are optimal, in the subsections below I only investigate cases where delayed investment is optimal relative to earlier investment, and when extended delay (until after both resolutions of uncertainty take place) of the investment decision occurs.

2.2.1 Delay Investment Until After the First Resolution of Uncertainty

When will the decision to invest be delayed until just after the first resolution of uncertainty? This will happen if (A*) the net present value of waiting (strategy 3* above) is higher than the net present value of investing at time \( t = 0 \) (strategy 2* above), and (B*) the net present value of waiting (strategy 3* above) and investing is higher than the net present value of not investing (strategy 1* above). Requirement (B*) is important since, as stated before, I want to study cases where investment actually occurs but is delayed

\[ 3.* \quad p_1[(t_1 - 1)V_H + (T - t_1)(p_2V_{HH} + (1 - p_2)V_{HL}) - I] \]

\[ 4.* \quad p_1p_2[(T - t_1 - 1)V_{HH} - I] \]
due to uncertainty, rather than cases of investment not occurring due to there being no profitable investment opportunities for the firm. Recall that, by assumption, I will be in the high state, $H$, if investment is being undertaken at $t = 1$.

As shown in the appendix, ($A^*$) and ($B^*$) being satisfied is mathematically equivalent to:

$$1 + \sqrt{p_1(1-p_1)} \frac{T - t_1}{t_1 - 1} \frac{(E(\tilde{V}_2|H) - E(\tilde{V}_2|L))}{\sigma(\tilde{V}_1)} > \sqrt{p_1} \frac{E(\tilde{V}_1)}{(t_1 - 1)\sqrt{1-p_1} \sigma(\tilde{V}_1)} \quad (6)$$

$$\iff \frac{\sqrt{1-p_1}}{\sqrt{p_1}} [(t_1 - 1)\sigma(\tilde{V}_1) + (T - t_1)\sqrt{p_1(1-p_1)}(E(\tilde{V}_2|H) - E(\tilde{V}_2|L)) > E(\tilde{V}_1)] \quad (7)$$

This result is summarized in the following theorem.

**Theorem 2.** The decision to invest will be delayed until after the first resolution of uncertainty only if ($??$) holds.

Theorem ?? yields a number of insights. The decision to invest will be delayed until after the first resolution of uncertainty only if, simultaneously, the expected payoff at time $t = 1$, $E(\tilde{V}_1)$, is sufficiently small, the variance of the first resolution state payoff is sufficiently large, the spread between the expected payoffs after the second resolution of uncertainty conditional on the first period state resolution is sufficiently large, the probability of the bad state, $1 - p_1$, is sufficiently large, or the period of time after the first resolution of uncertainty, $T - t_1 = (T_t - t_1) + (t_1 - 1)$, is sufficiently large. These results are similar to and carry the same intuition as the results in the previous model, except that in addition I now find that the larger the second resolution of uncertainty is, the less incentive there is for the firm to choose to invest at $t = 0$.

In addition to these insights, looking at ($??$) rearranged in the form of ($??$) leads to further insights, although it is difficult to make comparative static statements since various expressions are on both sides of the inequality. Examining the second term on the left-hand side of ($??$), I see that investment will be delayed only if one or more of the following are true: the period after the first resolution of uncertainty, $T - t_1$, is sufficiently long relative to the period of time between the resolutions of uncertainty, $t_1 - 1$; the spread in second resolution outcomes relative to the standard deviation of first period outcomes, $\frac{E(\tilde{V}_2|H) - E(\tilde{V}_2|L)}{\sigma(\tilde{V}_1)}$, is sufficiently large, the probability of the bad state, $1 - p_1$, is sufficiently large, or the Sharpe ratio, $\frac{E(\tilde{V}_1)}{\sigma(\tilde{V}_1)}$, is sufficiently small.

\footnote{Of course, spreads are not volatilities - spreads do not involve probabilities while volatilities do. But nevertheless, they are a component of volatilities and give some intuition as to the effect of volatility (and thereby uncertainty) on investment.}

\footnote{More precisely, assuming $T$ and $t_1$ change, and that $t_1$ changes in such a way to offset the differential changes on both sides of the inequality, the incentive to invest decreases as the ratio $\frac{T - t_1}{t_1 - 1}$ increases.}
2.2.2 Delay Investment Until After the Second Resolution of Uncertainty

Now I will consider when delay occurs and happens at a longer horizon as opposed to a shorter horizon. That is, when will the decision to invest be delayed until after the second resolution of uncertainty rather than taking place after the first resolution of uncertainty? This will happen if (C*) the net present value of waiting until after the second resolution of uncertainty (strategy 4* above) is higher than the net present value of waiting and investing just after the first resolution of uncertainty (strategy 3* above), and (D*) the net present value of waiting until after the second resolution of uncertainty (strategy 4* above) and investing is higher than the net present value of not investing at all (strategy 1* above).

As shown in the appendix, (C*) and (D*) being satisfied is mathematically equivalent to:

$$\frac{\sigma(\tilde{V}_2 | H) \sqrt{1 - p_2}}{\sqrt{p_2}} > \frac{t_1 - 1}{T - t_1 - 1} V_H + \frac{E(\tilde{V}_2 | H)}{T - t_1 - 1}$$  \hspace{1cm} (8)

$$\Leftrightarrow$$

$$\frac{\sigma(\tilde{V}_2 | H) \sqrt{1 - p_2}}{\sqrt{p_2}} (T - t_1 - 1) > (t_1 - 1)V_H + E(\tilde{V}_2 | H)$$  \hspace{1cm} (9)

This result is summarized in the following theorem.

**Theorem 3.** The decision to invest will be delayed until after the second resolution of uncertainty only if (8) holds.

Theorem ?? yields a number of results analogous to the solution of the model with one resolution of uncertainty.\[18\] The decision to invest will be delayed only if uncertainty about payoffs after the second resolution of uncertainty, \(\sigma(\tilde{V}_2 | H)\), is sufficiently large, the probability of the bad state being realized after the second resolution of uncertainty, \(1 - p_2\), is sufficiently large, or the period after the second resolution of uncertainty, \(T - t_1\), is sufficiently large relative to the period between the resolutions of uncertainty, \(t_1 - 1\) – essentially, the longer the effect of the state resulting from a resolution of uncertainty, the greater the effect it has on the decision to delay investment.

3 Data and Empirical Strategy

Having used the models in the previous section to illustrate qualitatively that both short- and long-term uncertainty about future prospects can dampen investment today, I now explore the magnitude of these effects empirically. As mentioned in the introduction, various studies have found a significant correlation between uncertainty and investment. However, none of these has studied the simultaneous correlations of short-term and long-term uncertainty with firm investment, as I do now.

A number of testable hypotheses are generated by the models of the previous section. These are that the decision to invest will be delayed only if:

\[18\] This is not surprising, since I am solving what is essentially a one-uncertainty model problem here.
1. Uncertainty is sufficiently high (classic real options result), both in the short-term and the in the long-term (new result)

2. The probability of a bad state in the future is high (classic “bad news principle” result)

3. The period of time affected by the resolution of uncertainty is large (new result)

4. Long-term uncertainty is large relative to short-term uncertainty (new result)

5. The period of time affected by the outcome of long-term uncertainty relative to the length of the period of time affected by the outcome of short-term uncertainty is large (new result)

While the models generated many new testable hypotheses, I focus on empirically measuring the magnitude of the effects of short- and long-term uncertainty on firm investment, and in turn only test hypotheses (1) and (4). I do this to directly address the argument that long-term uncertainty has caused economic recovery from the Great Recession to be slow. I leave testing the remainder of the hypotheses to future work.

To test hypotheses (1) and (4) I use the following empirical framework:

\[ I_{i,t} = \alpha_i + \gamma_t + \beta_1 \sigma^L_{i,t-1} + \beta_2 \sigma^S_{i,t-1} + \psi X_{i,t} + \epsilon_{i,t} \]  

(10)

where \( I_{i,t} \) is firm \( i \)’s investment over the period ending at time \( t \), \( \alpha_i \) is a firm fixed effect, \( \gamma_t \) is a time fixed effect, \( \sigma^L_{i,t-1} \) is a long-term uncertainty measure at time \( t-1 \), \( \sigma^S_{i,t-1} \) is a short-term uncertainty measure at time \( t-1 \), \( X_{i,t} \) is a vector of control variables at time \( t \), and \( \epsilon_{i,t} \) is a residual.

To obtain uncertainty measures I turn to options markets.\(^{19}\) An option implied volatility is a measure of the expected risk neutral variance\(^{20}\) of an asset price until the expiration of the option. It is obtained by taking market option prices and then inverting the option-pricing formula of Black and Scholes (1973) and iteratively solving for the volatility parameter that results in the observed price, given all other option characteristics. Thus, implied volatilities are a measure of market uncertainty about firms.\(^{21}\)

\(^{19}\)Ideally, I would use management expectations of uncertainty. However, such data does not broadly exist (see Guiso and Parigi (1999) for one case of such data being used). However, the uncertainty measures I use have been shown to be highly correlated with other measures of uncertainty used in the literature such as dispersion of TFP shocks across plants within a firm (Bloom, et al. (2012)).

\(^{20}\)As was mentioned earlier, while the risk-neutral variance, \( \text{Var}^Q(R_t) \), differs from the variance under the historical probability measure, \( \text{Var}^P(R_t) \), by the variance risk premium, it has been shown that the two measures of variance move together, as in Bollerslev, Tauchen and Zhou (2009). Hence, relative movements in uncertainty about firms should be captured by relative movements in risk neutral variance.

\(^{21}\)Although they have long been used in different models as volatility/uncertainty forecasts, recently “model-free” volatilities as in Jiang and Tian (2005) have become increasingly popular. Indeed, the CBOE began using such model-free volatilities to compute the VIX index in 2003, whereas prior to that they had used Black-Scholes implied volatilities. However, computing model-free volatilities requires a range of strike prices, which are not always available at the firm level. Hence, I use Black-Scholes at-the-money forward price implied volatilities. In my analysis all I need is for relative movements in uncertainty to be reflected by relative movements in the uncertainty measures. Practitioners and academics alike agree that Black-Scholes implied volatilities are measures that do exactly this. Disagreement has resulted about how much information they incorporate, but not whether or not they incorporate information.
options are available at various horizons, so are implied volatilities. Optionmetrics makes available data on all options listed on exchanges, including implied volatilities. Using various curve-fitting techniques Optionmetrics also makes available a set of standardized option implied volatilities, derived from the raw option data. These standardized implied volatilities are available for all firms with options meeting trading and liquidity criteria imposed by Optionmetrics and are for theoretical American put and call options with strike prices equal to at-the-money forward stock prices and maturities of 30, 60, 91, 122, 152, 182, 273, 365 and 730 days, conditional on the firm having options trading beyond or at these maturities. Since my empirical results are similar both qualitatively and quantitatively for put and call option implied volatilities, I take the average of the two to obtain my implied volatility measures.

In what follows I analyze correlations of firm investment with both firm and market uncertainty measures over different horizons. As my short-term firm uncertainty measure I use the average of put and call implied volatilities on standardized 30-day options with strike equal to the at-the-money forward price of the underlying common equity. I do this because this is the shortest horizon of standardized option available, and thus captures the level of uncertainty in the nearest term possible. Due to the collinearity of implied volatilities at different horizons (if you’re uncertain about a firm over the next 30 days then you may also be uncertain about the firm over the next year as well), for my long-term uncertainty measure I use the difference between the average of put and call implied volatilities on standardized 365-day and 30-day options, both with strike equal to the at-the-money forward price of the underlying common equity. This captures uncertainty in the long-term relative to uncertainty in the short-term. Using such “level” and “slope” measures to deal with collinearity is similar to what is often done in the term structure literature. Also, while 365 days is not the longest-term measure of uncertainty that I could use, it allows for more observations to be employed at the firm level since less than half of the firms with 365-day implied volatilities have 730-day implied volatilities. Additionally, 1-year measures of slope uncertainty have strong correlations with longer-term measures of slope uncertainty. To illustrate this Figure plots the difference between 2-year and 30-day firm implied volatilities against the difference between 1-year

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22 Optionmetrics obtains a set of standardized options of constant maturity through time by using all available options on the same security and weighting them by vega, maturity, delta and exercise style incorporated into a normal kernel weighting function and choosing bandwidth empirically. Details of this procedure are available at [http://www.optionmetrics.com/](http://www.optionmetrics.com/).

23 Options must have vegas greater than 0.5 and time to maturity greater than 10 days to be input into the standardization process.

24 That is, if a firm’s longest-dated option matures in 560 days, for example, then the firm will have standardized options for maturities up to and including 365 days, but not for a maturity of 730 days.

25 Note that the fact that results are qualitatively and quantitatively similar for put and call options lends credence to the argument that illiquidity and lack of information at the long end of the volatility curve are not issues for estimation in my sample. Were they issues (due to large hedging orders that put price pressure only on one type of exercise style and thus put corresponding pressure on implied volatilities for only one exercise style, for example) then differing results would be expected when different exercise style options are used. This indicates that, for my purposes, the put and call implied volatility curves appear to carry the same information.

26 All implied volatilities are annualized.

27 At the same time, 30 days is a long enough horizon that it is not affected by well-documented liquidity problems with options that are very near to maturity.
and 30-day firm implied volatilities. The plot shows that the 1-year - 30-day implied volatility slope is a good proxy for the 2-year - 30-day implied volatility slope as the two move together almost in lockstep (the correlation is 0.96).

Figure 7: 2-year - 30-day Firm Slopes Against 1-year - 30-day Firm Slopes

Notes: Data from Optionmetrics.

For my aggregate uncertainty measures I use implied volatilities on the average of put and call standardized options on the S&P 500 index with at-the-money forward prices. As in the case of firm uncertainty, for the short-term measure I use 30-day implied volatilities and for the long-term measure I use the difference between 1-year and 30-day implied volatilities. Again, using the 1-year - 30-day implied volatility slope is valid as it is highly correlated with longer horizon implied volatility slopes at the market level. For example, long-term data on aggregate uncertainty is available out to 5 years from variance swaps on the S&P 500 index. These variance swaps can be used to construct a measure of expected volatility of the S&P 500 index over different horizons, called the VIX term structure, that is highly correlated with the S&P 500 index implied volatility term structure (the correlation between the two is greater than 0.98 at any maturity). Figure ?? plots the difference between the 5-year and 30-day VIX against the difference between the 1-year and 30-day VIX. This plot shows that market uncertainty slopes

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28 Courtesy of Gregory and Rangel (2012), Goldman Sachs.
29 In regressions I will use the S&P 500 index implied volatility term structure for my aggregate uncertainty measures instead of the VIX term structure. I do this so the methodology generating firm and aggregate uncertainties is the same. There is not enough data on individual firms to apply the VIX methodology to all of the firms’ options in my dataset, otherwise I would generate a “firm VIX” term structure for each firm and use that for firm uncertainty measures. However, results are robust to using the VIX in place of the S&P 500 index implied volatilities.
30 The “true” VIX published by the Chicago Board Options Exchange and what is generally referred
from 1-year to 5-years move together (the correlation is 0.96), validating my use of the 1-year - 30-day slope as a measure of expectations about long-term uncertainty relative to the level of short-term uncertainty for the market as well.

Figure 8: 5-year - 30-day Firm Slopes Against 1-year - 30-day VIX Slopes

Notes: Data from Gregory and Rangel (2012), Goldman Sachs.

For my investment measure I use firm quarterly capital expenditure, obtained from Compustat. Given that I want to study the real options effects of uncertainty upon investment, I desire investments that have some irreversibility associated with them. Partial irreversibility for capital expenditures is due to the inability to resale plant, property, equipment and the like at full face value, in addition to installation or training costs pertaining to the equipment. Assuming irreversibility for capital expenditure is standard practice as in Bloom (2009). To account for differences in firm size I scale investment by total firm book assets. Additionally, firm fixed effects will control for differences across firms in average investment behavior. Date fixed effects will control for differences in investment behavior due to macroeconomic conditions.

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to as “the VIX” is a model free measure of the risk neutral implied volatility of the S&P 500 index over the next 30 days. It is computed specifically for that horizon. However, the formula can be generalized to compute a VIX for various horizons, using variance swaps or options. For details, see CBOE (2009).

Guiso and Parigi (1999) also show that using 3-year or 5-year measures of uncertainty produce similar results in their study relating management expectations of uncertainty to investment decisions that management make, so it is not surprising that I find these different long-term horizons producing values that are highly correlated.

Obviously date fixed effects will be dropped in the specifications using aggregate uncertainty. I will use sets of fiscal quarter dummies (4) and calendar quarter dummies (4) in those specifications instead of the date fixed effects.
As control variables I use firm Tobin’s Q, $Q_{i,t}$ (calculated as the sum of the market value of outstanding equity and the book value of preferred stock, current debt and long-term debt, divided by the total book value of assets), firm cash flow (measured as operating income) divided by total book assets, $\frac{CF_{i,t}}{A_{i,t}}$, and year-over-year proportional firm sales growth, $SG_{i,t}$. All components of these measures are obtained from Compustat. A literature beginning with Brainard and Tobin (1968) highlights the importance of including Tobin’s Q to control for the value of internal investment opportunities (firms with high Tobin’s Q have more market value per dollar of assets, presumably due to the existence of profitable growth and future investment opportunities). I am including average Tobin’s Q as opposed to marginal Tobin’s Q (which would be preferred), since it is unclear how to measure the latter. Cash flow is included to control for investment financing constraints. I include sales growth to account for standard accelerator models of investment such as those discussed in Jorgenson (1971), where investment results simply as a function of growth, and vice versa.

Since firms report investment and balance sheet data each fiscal quarter but fiscal quarters end at different times for different firms, the dataset is of monthly frequency, with any given firm having quarterly data. I use uncertainty measures that are the average of end-of-day implied volatilities over the last month of a firm’s fiscal quarter, but my results are robust to using uncertainty measures as of the close of markets on the last day of a firm’s fiscal quarter.

My sample is from January 1996 to December 2012 since the Optionmetrics data begins in January 1996 and is only available with a significant lag. I winsorize all variables at the 1st and 99th percentiles to reduce the influence of outliers. I drop observations with negative or zero book equity, total assets or sales. I also exclude observations with SIC codes corresponding to utilities and financials, as has been standard practice in similar work (Gulen and Ion (2013), for example). Results are robust to not making any combination of these exclusions. Summary statistics are in Table 1. 27,434 firm-quarter observations are in my dataset, from 1,445 firms. The mean capital expenditure investment rate is about 1.6% of assets. The average 30-day firm implied volatility is 44%, which is, as expected, significantly larger than the average 30-day implied volatility on the S&P 500 index (20.1%) due to diversification. Both the firm and market 1-year - 30-day implied volatility slopes are very close to zero (-1.5% and 0.98%, respectively), indicating that short- and long-term uncertainty have, on average, been similar. Tobin’s Q has an average close to 2, while quarterly cash flow over assets and year-over-year proportional sales growth have averages of 2.7% and 5.2%, respectively. Firm total assets are, on average, slightly over 10 billion dollars and have a positively skewed distribution (with median total assets being only about 4.7 billion dollars). To be in the dataset firms must be large enough to have to file their balance sheet data quarterly and have both 30-day and 1-year options actively trading, and so it is no surprise that firms in the sample are quite large. This size and the heterogeneity of total assets across firms (the standard deviation of total assets is almost 12 billion) lends credence to my scaling investment and other balance sheet variables by firm total assets. Finally, the Chicago

\[ I \text{ use proportional sales growth rather than sales growth to normalize sales growth between } -1 \text{ and } 1. \text{ The formula is } SG_{i,t} = \frac{Sales_{i,t} - Sales_{i,t-1}}{Sales_{i,t} + Sales_{i,t-1}}. \text{ I compute sales growth on a year-over-year rather than quarterly basis to account for seasonality. Results are robust to the using raw sales growth.} \]
Fed National Activity Index (CFNAI) 3-month moving average has an average of -0.359, but a median value of -0.09 and a significant standard deviation of 0.907, indicating that economic activity has been both above and below trend.\footnote{The CFNAI is standardized to have a mean of zero and a standard deviation of one over the entire series. Details of its construction can be found at \url{http://www.chicagofed.org/webpages/publications/cfnai/}. It corresponds to the index produced by Stock and Watson (1999).} The skewness indicates some very extreme periods have occurred below trend, such as the Great Recession.

In order to measure the correlation between investment and uncertainty at different horizons, I specify my initial regression as

\[
\frac{\text{CAPX}_{it}}{A_{it}} = \alpha_i + \gamma_t + \beta_1 \sigma^L_{it-1} + \beta_2 \sigma^S_{it-1} + \psi_1 Q_{it} + \psi_2 \frac{CF_{it}}{A_{it}} + \psi_3 SG_{it} + \epsilon_{i,t} \tag{11}
\]

where a lagged variable indicates the variable at the end of the prior fiscal quarter.\footnote{Results are robust to lagging either cash flow over assets or sales growth in this specification.} For ease of economic interpretation I scale each of my variables by their standard deviations. Additionally, in all specifications I will cluster standard errors by firm.\footnote{Similar results are obtained if standard errors are clustered by date, or by date and firm.}

To qualitatively tie back what I am doing to the model, 1-year is analogous to time \( t = t_1 \) and 30-days is analogous to time \( t = 1 \), both relative to time \( t = 0 \). Thus, \( V \) is similar to the implied volatility level (the 30-day measure) while \( \frac{E(V_2[H]) - E(V_2[L])}{\sigma(V_1)} \) is analogous to the implied volatility slope.

### 4 Empirical Results

#### 4.1 Investment is Negatively Correlated with Both Short-Term and Long-Term Firm Uncertainty

Table 2 presents results of regression \((??)\). Column (1) reports results of regressing firm quarterly capital expenditure on the firm implied volatility level and the firm implied volatility slope, and includes firm and date fixed effects. Column (2) adds Tobin’s Q, column (3) adds cash flow over assets, and column (4) adds year-over-year proportional sales growth. Coefficient estimates do not change significantly from column (1) to column (4), indicating that regardless of the controls that are being included, the correlation between investment and both current and future uncertainty is robust. Table 1 indicates the mean firm investment rate is 0.0158 and the standard deviation of the investment rate is 0.0184. Hence, coefficient estimates in column (4) of Table 2 imply that a one standard deviation increase in lagged 30-day firm implied volatility correlates with a change in investment equal to 8.3% of the mean firm investment rate. A one standard deviation increase in the lagged difference between the 1-year and 30-day firm implied volatility correlates with a decrease in investment equal to 2.9% of the mean firm investment rate. While the coefficients are all statistically significant, these magnitudes are also economically meaningful. While short-term implied volatility measures have fallen since the crisis, at the end of 2012 long-term implied volatility measures were roughly one empirical standard deviation higher than their empirical mean in my sample. This heightened long-term
uncertainty predicts roughly a 3% shortfall in investment relative to what it would be if long-term uncertainty had fallen to its historical mean. In addition, Tobin’s Q, cash flow/assets and sales growth are all positively correlated with investment, as expected, and their coefficients do not change much from column to column.

One potential issue with these results is that I do not perfectly control for other variables that may be correlated with investment and uncertainty. For example, unobservable investment opportunities or marginal Tobin’s Q may be both negatively correlated with uncertainty and also positively correlated with investment. However, while including Tobin’s Q, cash flow/assets and sales growth may not perfectly control for such variables, including these effectively restricts the channels by which endogeneity can bias coefficients. For example, in order for an omitted variable to bias coefficients it would need to, first, be positively (negatively) correlated with uncertainty and negatively (positively) correlated with investment and second, have variation that is unexplained by the fixed effects, cash flow, sales growth and average Tobin’s Q. A number of omitted variables may satisfy the former condition, but the control variables included span much of the space one might think the omitted variables occupy. For example, today’s cash flow and sales growth are informative about future investment opportunities of a firm (firms that are not growing or generating revenue are likely struggling to identify profitable investments), and average Tobin’s Q incorporates the market’s perception of the amount of profitable investment opportunities a firm has through its equity price. If profitable opportunities did not exist, average Tobin’s Q would fall as a company’s equity lost value in the face of diminished growth prospects. In addition, for unobservable investment opportunities to be driving the correlation I observe between uncertainty and investment, such opportunities need to be correlated with the market perception of uncertainty (the implied volatility) and also have independent variation from that of average Tobin’s Q, which incorporates the market’s perception of the value of firm and is a control variable. This will not occur. If the market is aware of opportunities enough to incorporate them into implied volatility, it will also incorporate them into the value of equity and hence, average Tobin’s Q. By choosing the uncertainty measures and control variables I have, the latter argument addresses many such endogeneity concerns.

Another criticism of this specification is that it does not account for the fact that many investments take many years to build. A firm announces a building and proceeds to work on it for many years, not just for one quarter. However, this paper is about firms delaying investment, which firms have autonomy to do in a given quarter even if the project is scheduled to take many years to complete. In addition, when lagged investment is included as a variable in my specification to account for the longevity of some investment projects (column (5) of Table 2), coefficients fall in magnitude but remain both economically and statistically significant.\textsuperscript{37, 38}

\textsuperscript{37}I do not include lagged investment in all my specifications because, under the null, it is highly correlated with uncertainty. Thus, including lagged investment could just be proxying for uncertainty in previous periods. My goal is to estimate the correlation of uncertainty last period with investment this period, without incorporating information about uncertainty in the more distant past.

\textsuperscript{38}My specification also addresses standard reverse-causality concerns. Low investment this period cannot cause high implied volatility last period, even if investment is persistent or previously announced since I control for investment last period in column (5) and coefficients remain economically and statistically significant.
4.2 Short- and Long-Term Aggregate Uncertainty are Negatively Correlated with Investment

Table 3 presents results showing that aggregate uncertainty (measured using the implied volatility on the S&P 500 index) is also negatively correlated with firm investment. Column (1) reports results of a regression of firm quarterly capital expenditure on the 30-day S&P 500 index implied volatility and the difference between the 1-year and 30-day S&P 500 index implied volatilities, including firm fixed effects and calendar and fiscal quarter dummies. Column (2) adds firm sales growth, cash flow/assets and lagged Tobin’s Q, as well as the lagged Chicago Fed National Activity Index 3-month moving average, a lagged default spread (the difference between Moody’s Seasoned AAA and BAA Corporate Bond Yields, available via FRED) and a lagged term spread (the difference between the 1-year U.S. Treasury yield and the 3-month U.S. Treasury yield) as macroeconomic control variables since I obviously cannot include date fixed effects. Column (3) includes all control variables and only firm uncertainty measures, while column (4) includes all control variables and both firm and market uncertainty measures.

To focus on the correlations of firm and aggregate uncertainty on investment while controlling for each other, I focus on the estimates in column (5). These coefficient estimates imply that, after controlling for aggregate uncertainty, a one standard deviation increase in lagged 30-day firm implied volatility correlates with a decrease in firm investment equal to 5.1% of the mean firm investment rate, and a one standard deviation increase in the lagged difference between the 1-year and 30-day firm implied volatilities correlates with a decrease in firm investment equal to 3.1% of the mean firm investment rate. Further, in this specification a one standard deviation increase in the lagged difference between the 1-year and 30-day market implied volatilities correlates with a decrease in firm investment equal to 4.4% of the mean firm investment rate. A one standard deviation increase in short-term market uncertainty (i.e. 30-day market implied volatility) correlates with a decrease in firm investment equal to 1% of the mean firm investment rate, but it is not statistically significant. In addition and as expected, higher long-term interest rates (which are likely the most relevant for investment decisions) relative to short-term interest rates are correlated with lower investment rates, and higher default spreads are correlated with lower investment rates.

The above results make clear that both short- and long-term uncertainty are negatively correlated with firm investment rates, both at the aggregate and firm levels. These findings verify hypotheses (1) and (4) of the qualitative model I solved earlier: (1) when investment is depressed, uncertainty is high either in the short-term or the long-term and (4) investment is depressed when uncertainty is high in the long-term relative to the short-term. Indeed, a simultaneous one standard deviation increase in my measures of long-term uncertainty above their historical means (as has been seen during the recovery

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39 There are 4 calendar quarter dummies total (Q1, Q2, Q3, Q4). Similarly, there are 4 fiscal quarter dummies total.
40 A default spread is included as a standard business cycle factor, as is a term spread. A term spread also captures the cost of long-term financing relative to the cost of short-term financing, since most capital expenditures are typically longer-term investments. I have also estimated both firm and market specifications including lagged firm and market stock returns to capture firm-specific financing costs, but these surprisingly do not carry any additional explanatory power beyond that given by the set of variables I already employ.
from the Great Recession, such as in Figure 2) predicts a fall in the firm investment rate equal to 7.5% of its mean. Figure 1 indicated that the U.S. investment to GDP ratio at the end of 2012 was still 20% below its pre-crisis peak. These results indicate that over 30% of that shortfall is predicted by the rise in long-term uncertainty alone.  

4.3 Robustness

As a robustness check for the results, in Table 4 I report regression results where I include interaction terms between each uncertainty measure and a dummy variable equal to 1 if an observation period began during an NBER recession and 0 otherwise. This will indicate whether the recessionary periods in the data, which are typically high uncertainty and low investment times, are driving the results. Columns (1) and (2) present results of the firm specification, first without recession interactions and then with recession interactions. Columns (3) and (4) present results of the specification with both firm and aggregate uncertainty included, first without recession interactions and then with recession interactions included. As Table 4 indicates, two of the recession interaction terms are statistically significant, both in column (4). The short-term firm-recession interaction is negative while the short-term aggregate-recession interaction is positive. The two results somewhat cancel each other out leading me to draw no robust conclusions from them. Additionally, coefficients on uncertainty measures in columns (2) and (4) are largely unchanged from the coefficients on uncertainty measures in columns (1) and (3). Recessions do not appear to be driving the results.

As an additional robustness check I estimate specifications where I use 91 days as my short-term horizon and 6 months as my long-term horizon. Columns (1) and (2) of Table 5 report estimates from regressions using 30-day implied volatility as the level (short-term) uncertainty measure and the difference between 6-month and 30-day implied volatilities as the slope (long-term) uncertainty measure. Columns (3) and (4) report estimates from regressions using 91-day implied volatility as the level (short-term) uncertainty measure and the difference between the 1-year and 91-day implied volatilities as the slope (long-term) uncertainty measure. Estimates are similar to those in Tables 2 and 3, retaining their statistical and economic significance where applicable. This indicates that my finding is not simply a function of the horizon I chose, but is a function of measuring long-term uncertainty relative to short-term uncertainty.

41 Of course, the “effect” of uncertainty on investment is much less identified in the market specification relative to the firm specification. This is because date fixed effects can be included in the firm specification while they cannot be in the aggregate specification, due to obvious collinearity with the macro variables and the aggregate uncertainty measures themselves. The exclusion of these fixed effects allows for many other omitted variables to be potential drivers of the results. While I have included some standard business cycle controls, I can never account for all macroeconomic variables that could be driving the results. Thus, going forward I present results from both the firm specification and the market specification since I have better identification when I include date fixed effects.

42 Investment is reported as of the end of a fiscal quarter. If the start of that fiscal quarter was during an NBER recession then the dummy variable is equal to 1. It is 0 otherwise.

43 While the NBER formally defines a recession, it does not formally define a “recovery.” Nevertheless, under many arbitrary definitions of recoveries the data also show results are not driven solely by recoveries.
4.4 Alternative Investment Measures

A firm has many avenues for investment besides capital expenditure. Two other investments firms commonly undertake are research and development (R&D) and acquisitions of other firms. Data on each of these measures is available at quarterly frequency for many firms via Compustat. Thus, in Table 6 I report results of estimating equation (??? using firm acquisitions and R&D ratios as dependent variables. In columns (3) and (4) of Table 6 I use research and development expenditure scaled by firm assets as my dependent variable, and in columns (5) and (6) I use firm acquisitions scaled by firm assets as my dependent variable. In addition, in columns (7) and (8) I use cash holdings scaled by firm assets as the dependent variable in my estimation: if firms are delaying investment, we might expect to see them hoard cash until the time they decide to undertake the projects. For sake of comparison, I also report estimation results using capital expenditure as the dependent variable in columns (1) and (2).{44}

Columns (3) and (4) reveal that both short- and long-term firm uncertainty are positively correlated with R&D spending (from column (4) a one standard deviation increase in level (slope) firm uncertainty is correlated with a 2.90% (1.90%) increase in R&D spending{45}, while column (4) reveals that both horizons of market uncertainty are negatively correlated with firm R&D expenditure. The former result I interpret as evidence that R&D spending in the face of uncertainty is essentially buying a call option on future growth opportunities (see Bar-Ilan and Strange (1996)), resulting in a positive correlation between the two. When uncertainty is resolved, a firm does not want to miss out on opportunities if the new state of the world is good simply because it did not pay a small cost before the resolution of uncertainty. In the event a bad state of the world results for the firm and the R&D spending is worthless, the firm has only lost a small amount. But if a good state of the world results for the firm, the firm stands to lose a great deal of potential upside by not being prepared by previous R&D. The latter result I interpret as possible evidence of a risk premium effect. In times of market uncertainty firm financing is more costly to obtain and thus, spending generally decreases. The growth option story pertains only to firm-specific uncertainty.

Columns (5) and (6) of Table 6 reveal that the correlation between uncertainty and firm acquisitions is negative, particularly in the short-term. Since acquisitions are not reversible investments, this is another channel through which real options effects may be acting. Additionally, precautionary savings and the “pausing” of potential acquisitions deals may occur until short-term uncertainty about a firm’s prospects are resolved. Both of those channels are potential explanations for this empirical finding{46}.

Finally, columns (7) and (8) reveal that cash holdings are positively correlated with long-term firm uncertainty, but negatively correlated with market short-term and long-term uncertainty. This is intuitive. Market uncertainty is generally high when the economy is contracting and therefore less cash is being generated. When less cash is being generated, there is less to hold. However, when uncertainty specifically facing the firm is

---

{44}The sample differs from Table 6 to Tables 2 and 3 since not all firms in the original sample report the amount of their R&D expenditures, cash holdings or acquisitions. Hence, results are not directly comparable.

{45}The mean R&D spending to total assets ratio is 0.022, with a standard deviation of 0.026.

{46}This confirms the earlier finding of Stein and Stone (2013).

{47}The mean acquisitions to total assets ratio is 0.008, with standard deviation 0.0522.
high, particularly over the long-run, the firm will wait to invest and stockpile cash until the uncertainty is resolved and it is easy to determine what the most optimal investments are. Rather than make a mistake with long-term consequences, firms will hold the cash and wait to invest until long-term uncertainty is resolved.\textsuperscript{48}

4.5 Firm Heterogeneity

While I have identified a robust correlation across the panel of firms, I have not identified heterogeneity in the magnitude of the correlation across firms. Are small or large firms’ investments affected more by uncertainty? Does uncertainty have a larger negative correlation with investment if the firm is young as opposed to when the firm is more established? And, are faster-growing firms more affected than slower-growing firms because they are more rapidly expanding. To address these questions I include interaction terms of uncertainty with three different dummy variables included in my regression specification in turn. The first dummy variable is equal to one for firm-quarters with total assets above the sample median of total assets and equal to zero for firm-quarters with total assets below the sample median of total assets. The second dummy variable is equal to one for firm-quarters with sales growth above the sample median of sales growth and equal to zero for firm-quarters with sales growth below the sample median of sales growth. The last dummy variable is equal to one for firm-quarters with an associated IPO date after the sample median IPO date and equal to zero for firm-quarters with an associated IPO date before the sample median IPO date. If interaction terms are negative (positive) then investment of firms with more assets, higher sales growth, or more youth (i.e. a later IPO date) are more negatively (positively) affected by uncertainty relative to firms with less.

Columns (1) and (2) of Table 7 report results of regression estimation that include uncertainty measures interacted with the total assets dummy variable, columns (3) and (4) of Table 7 report results of regression estimations that include uncertainty measures interacted with the sales growth dummy variable, and columns (5) and (6) of Table 7 report results of regression estimations that include uncertainty measures interacted with the IPO date dummy variable. Correlations indicate that large firms appear to have a significantly more negative correlations between firm uncertainty and investment than do small firms. Indeed, these point estimates are roughly 1-3 times larger in magnitude for large firms than for small firms. However, no such result exists for sales growth or firm age (as measured by IPO date). While large firms appear to be more affected by uncertainty than are small firms, the rate at which the firm is expanding and it’s maturity in the market do not matter for the size of investment-uncertainty correlations.\textsuperscript{49} This is rationalizable. A mis-step in an irreversible investment by fast-growing, young firms could change their opportunity set far more than it could a mature firm that is already established and has lower sales growth, and through this channel uncertainty may depress investment more relative to other firms. At the same time, young, fast-growing firms have a strong need to grow now to satisfy investors since they are not yet established, and through this channel uncertainty may affect investment less relative to other firms. Hence, it is not clear what direction the correlation of investment and uncertainty should

\textsuperscript{48} The mean cash holdings to assets ratio is 0.193, with standard deviation .201.

\textsuperscript{49} IPO date is not a perfect measure of firm age and sales growth is not a perfect measure of the firm’s rate of expansion, but these are two of the most reasonable measures based on available data.
governments relative to other firms, which is precisely what the results show.

5 Policy Implications and Further Research

5.1 Policy Implications

The fact that both short-term uncertainty and long-term uncertainty are negatively correlated with firm investment activity has important implications for economic policy. Economic policies that will create uncertainty rather than mitigate it are predicted to lower firm investment. My findings are particularly informative for government policy during the economic recovery. While short-term volatility measures have fallen, long-term uncertainty has remained elevated. My results predict that when this is the case, firm investment will be depressed as it has been during the recovery. Indeed I have provided empirical and theoretical evidence consistent with the arguments made by Greenspan (2012) and Ohanian and Taylor (2012) that long-term uncertainty leads to reduced economic activity, particularly with regard to long-lived investment. What are governments to do? Since a large portion of long-term uncertainty may be attributed to regulatory and policy uncertainty, my results predict that if governments reduce long-term uncertainty by credibly committing to long-term policy paths, they will stimulate the economy. However, this is precisely what governments do not do in times of crisis and recovery. Typically they take short-term, unpredictable actions in the name of stimulus that then increase long-term uncertainty about their future behavior. In the U.S., for example, since the Great Recession the Federal Reserve has taken unprecedented actions to “stimulate” the economy and in so doing has created a great deal of long-term uncertainty about what its future actions will be. Additionally, questions about how government policies such as the Affordable Care Act will be implemented have created long-term uncertainty for firms. Similarly, the increase in tax code expirations per year (as documented by Baker, et al. (2013)) has created long-term uncertainty for firms about their future profitability after taxes, and how their investments will be taxed. Any of these examples may cause firms to wait to build and invest in long-lived assets until they have more information about the path of future interest rates (and thus financing costs), the implications of new laws for their business, and the rules by which new investments will be taxed and thereby influence profits. By committing to and making known long-term policy paths, governments can reduce the disincentive of firms to withhold investment and thereby stimulate the economy.

5.2 Further Research

While I have given arguments for causality in the correlations I have presented, performing instrumental variables estimation or creating a structural model would establish a causal effect even more clearly. The former is difficult because a set of instruments is valid only if they (1) are correlated with uncertainty but not correlated with unobservables such as marginal Tobin’s Q and (2) have independent covariation with uncertainty.

The sample differs from Table 7 to Tables 2, 3 and 6 since not all firms in the original sample report their IPO date. Hence, results are not directly comparable.
measures across the uncertainty term structure. Previous work such as Stein and Stone (2013) claims to have found instruments satisfying the first condition \(^{51}\), but their instruments do not have sufficient independent covariation with different uncertainties along the term structure to satisfy the second condition. Indeed, even the existence of satisfactory instruments is questionable.

As a result, future work in the direction of creating a structural model that illustrates the quantitative dynamics of the effects on investment of changes in uncertainty at different horizons would be meaningful. The simplest case would be a model of a firm making a partially irreversible investment in a single capital good, where the productivity process associated with production has two horizons of uncertainty instead of one, which is standard in current models. For example, a productivity process \(A_t\) could be simply parameterized as follows:

\[
\log(A_t) = \rho \log(A_{t-1}) + \sigma_t^S \epsilon_t^S + \sigma_t^{L} \epsilon_t^{L}
\]

where \(\epsilon_t^S\) and \(\epsilon_t^L\) are both i.i.d. \(N(0,1)\) with correlation \(\gamma\), and the uncertainty processes are 2-state Markov processes. The volatility process \(\sigma_t^i\), with \(i \in \{S, L\}\), would take values in \(\{\sigma_L^i, \sigma_H^i\}\), with \(\sigma_L^i < \sigma_H^i\). Additionally, in the transition matrix the persistence of \(\sigma^L\) would be set to be higher (say, 0.8), while the persistence of \(\sigma^S\) would be set to be lower (say, 0.3). In addition, the long-term volatility component, \(\sigma^L\) is known a period ahead of the short-term volatility component, \(\sigma^S\).

These two processes capture the idea that both short term uncertainty (i.e. the distribution of outcomes today) and longer-term uncertainty (i.e. news today about the future distribution of outcomes) are relevant for productivity and demand shocks. The persistence of the two processes captures the fact that long-term uncertainty is more permanent and therefore is much more persistent than short-term uncertainty, which is more transitory.

A model such as this would provide a direct framework for analyzing the effects of different horizon uncertainties on firm investment behavior. Correlations from simulated model data could then be compared to observed empirical correlations. As extensions to the structural model outlined above, decomposing real option, growth option, risk premium and Oi-Hartman-Abel effects (outlined in Bloom (2014)) at different horizons would be insightful in order to know how each of the different channels contributes to the correlations I present in this paper. This could be done through a general equilibrium model, which would also allow for a stock market to be introduced so that data from equity option markets could be mapped more directly to the model. Analyzing the differential effects of horizons of uncertainty on capital types with different irreversibilities and depreciations, similar to Bloom, Bond, Van Reenen (2007), Tuzel (2010) and Kim and Kung (2014), would also be informative as it would reveal what goods and industries are most affected by what horizons of uncertainty \(^{52}\).

\(^{51}\)They use commodity and foreign exchange rate prices and uncertainty to instrument for average Tobin’s Q and uncertainty. Their instruments may not be valid, however, because their instruments may still be correlated with investment through marginal Tobin’s Q, for example.

\(^{52}\)While I focus on structural quantitative model extensions in this section, extensions to the simple model of this paper that could be considered are that of allowing for multiple periods of investment (e.g. each period the firm has the opportunity to undertake a new investment in addition to the previous not yet undertaken investments, rather than the firm just having one shot) and depreciation of the project.
6 Conclusion

In this paper I have presented a simple model showing that uncertainty at different horizons can differentially influence a firm’s decision to invest through real options channels. I then empirically verified two new predictions of the model: (1) both short-term and long-term uncertainty are negatively correlated with investment, and these effects exist for each differentially, and (2) the larger long-term uncertainty is relative to short-term uncertainty, the more depressed investment is today. These correlations are robust to a number of perturbations, are not driven by subsamples such as recessions, and are particularly strong for large firms. These results suggest that reducing long-term uncertainty by having governments credibly commit to long-term policy paths may be more beneficial for economic growth than having governments take short-term, unpredictable actions, even if those short-term actions are stimulating. This is particularly relevant in light of the slow recovery from the Great Recession and the heightened levels of long-term uncertainty we have observed during that period. It appears that both short- and long-term uncertainty need to be kept low in order to stimulate economic activity, investment and growth.

payoffs over time, which should have similar results to assuming a positive interest rate in the model. Empirical extensions such as using other measures of uncertainty that have a term structure, such as the dispersion of analyst earnings forecasts, could be used to confirm the findings of this paper as well.
7 References


Gulen, Huyesin and Mihai Ion (2013). “Policy Uncertainty and Corporate Investment.” Purdue University working paper.


8 Appendix

8.1 Economists Surveyed

The following economists were surveyed between August 17 and September 14, 2012 by foreignpolicy.com in the survey referenced in the Introduction: Daron Acemoglu, Daniel Altman, Martin Baily, Dean Baker, Stacie Beck, Barry Bosworth, Richard Burdekin, Richard Burkhauser, Edwin Burton, Mark Calabria, Menzie Chinn, John Coleman, Tyler Cowen, David Cutler, Uri Dadush, Steven Davis, Tad DeHaven, Peter Diamond, Karen Dynan, Mohamed A. El-Erian, Michael Ettlinger, Eugene Fama, Jeffrey Frankel, Jeffry Frieden, Benjamin Friedman, James Galbraith, William Gale, Mark Gertler, Ann Harrison, David Henderson, Tim Kane, Nicholas Kiefer, Randall Kroszner, Deborah Lucas, David Madland, Donald Marron, John Matsusaka, Allan Meltzer, Jeffrey Miron, Daniel Mitchell, Michael Munger, Maurice Obstfeld, Lee Ohanian, Sam Peltzman, John Pencavel, Ricardo Reis, Christina Romer, David Romer, Amity Shlaes, David Smick, Irwin Steizer, Phillip Swagel, Richard Thaler, Mark Thoma, Edwin Truman, Laura Tyson, Gianluca Violante, John Weicher, Matthew Weinzierl, Christian Weller, Daniel Wilson and Justin Wolfers.

8.2 One Uncertainty Resolution Model

8.2.1 Computation of Variance

\[ \sigma^2(\tilde{V}_1) = E(\tilde{V}_1^2) - E(\tilde{V}_1)^2 \]

\[ = pV_H^2 + (1 - p)V_L^2 - [p^2V_H^2 + 2p(1 - p)V_LV_H + (1 - p)^2V_L^2] \]

\[ = pV_H^2(1 - p) + (1 - p)V_L^2(1 - (1 - p)) - 2p(1 - p)V_LV_H \]

\[ = p(1 - p)[V_H^2 - 2V_LV_H + V_L^2] \]

\[ = p(1 - p)(V_H - V_L)^2 \]

8.3 Two Uncertainty Resolutions Model

8.3.1 Strategies

Since the firm is risk neutral, the firm will implement a strategy based on net present values of the different strategies at time \( t = 0 \). Thus, to determine the optimal strategy of the firm I simply need to compute the net present values (NPVs) associated with each potential strategy of the firm. Note that the (unreduced) strategy space is quite large. However, this can be reduced significantly by noting that the firm will only consider investing in periods \( t = 0 \), \( t = 1 \) and \( t = t_1 + 1 \). Why? Suppose this is not the case. Then, consider the firm finding it optimal to invest at some time \( t^* \) not in the set of periods above. Note that if \( t^* > t_1 + 1 \) then the firm could have done better by investing at time \( t_1 + 1 \) since by doing so the firm will collect additional periods of payoff. Since the cost of investing is static (\( I \)) and since there is not time discounting and no information revelation except at times \( t = t_1 + 1 \) and \( t = 1 \), the only difference between investing at \( t_1 + 1 \) and \( t^* > t_1 + 1 \) is the additional payoff from investing earlier. Hence, it cannot be optimal to invest at some time \( t^* > t_1 + 1 \). The same argument applies if we consider \( t^* \)
such that $1 < t^* < t_1 + 1$: additional payoff could be obtained by investing at $t = 1$, with no difference in cost or information revelation. Hence, the only potentially optimal times of investment on the part of the firm are $t = 0$, $t = 1$ and $t = t_1 + 1$.

This reduction results in the following strategies as potentially being optimal for the firm:

1. Never invest
2. Invest at time $t = 0$
3. Invest at time $t = 1$ but only if the state is $H$
4. Invest at time $t = 1$ but only if the state is $L$
5. Invest at time $t = 1$ regardless of the state of world
6. Invest at time $t = t_1 + 1$ but only if the state is $HH$
7. Invest at time $t = t_1 + 1$ but only if the state is $HL$
8. Invest at time $t = t_1 + 1$ but only if the state is $LH$
9. Invest at time $t = t_1 + 1$ but only if the state is $LL$
10. Invest at time $t = t_1 + 1$ but only if the state is $HH$ or $HL$
11. Invest at time $t = t_1 + 1$ but only if the state is $HH$ or $LH$
12. Invest at time $t = t_1 + 1$ but only if the state is $HH$ or $LL$
13. Invest at time $t = t_1 + 1$ but only if the state is $HH$, $HL$ or $LH$
14. Invest at time $t = t_1 + 1$ but only if the state is $HH$, $HL$ or $LL$
15. Invest at time $t = t_1 + 1$ but only if the state is $HH$, $LH$ or $LL$
16. Invest at time $t = t_1 + 1$ but only if the state is $HL$ or $LH$
17. Invest at time $t = t_1 + 1$ but only if the state is $HL$ or $LL$
18. Invest at time $t = t_1 + 1$ but only if the state is $HL$, $LH$ or $LL$
19. Invest at time $t = t_1 + 1$ but only if the state is $LH$ or $LL$
20. Invest at time $t = t_1 + 1$ regardless of the state of the world

The strategies involving combinations of states are somewhat laborious to deal with. However, we can, in fact, reduce the strategy space even further by noting that (1) the strategies to invest “regardless” of the state of the world cannot be optimal and (2) waiting to invest and then investing if the bad state of the world is realized ($L$, $HL$ or $LL$) can never be optimal. (1) cannot be true because if it were optimal to invest at $t = 1$ or $t_1 + 1$ regardless of the state of the world, then the firm should have just invested at time $t = 0$ or $t = 1$, respectively, and collected the additional payoff. (2) cannot be true for similar
reasoning. At any given time \( t \), if it is optimal to invest in the bad state of the world then it must also be optimal to invest in the good state of the world since good state payoffs dominate bad state payoffs. This implies it is optimal to invest at time \( t \) regardless of the state of the world. Thus, investing at a time \( t \) but only in the bad states of the world cannot be an optimal strategy.

Additionally, since by assumption it cannot be optimal to invest in any states of the world where a state \( L \) has been realized, the strategy space is reduced even further. This results in the space of potentially optimal strategies for the firm being dramatically reduced to the set reported in the paper.

### 8.3.2 Delay Investment Until After the First Resolution of Uncertainty

Mathematically, \((A^*)\) being satisfied is equivalent to:

\[
p_1[(t_1 - 1)V_H + (T - t_1)(p_2V_{HH} + (1 - p_2)V_{HL}) - I] >
\]

\[
t_1[p_1V_H + (1 - p_1)V_L] + (T - t_1)[p_1(p_2V_{HH} + (1 - p_2)V_{HL}) + (1 - p_1)(p_2V_{LH} + (1 - p_2)V_{LL})] - I
\]

Cancellation of common components and rearranging results in:

\[
(1 - p_1)I > p_1V_H + (1 - p_1)V_L + (T - t_1)(1 - p_1)(p_2V_{LH} + (1 - p_2)V_{LL})
\]

\[
\iff \quad \frac{(1 - p_1)I}{(1 - p_1)(t_1 - 1)} > \frac{p_1V_H + (1 - p_1)V_L}{(1 - p_1)(t_1 - 1)} + \frac{(1 - p_1)(t_1 - 1)V_L}{(1 - p_1)(t_1 - 1)} + \frac{(T - t_1)(1 - p_1)(p_2V_{LH} + (1 - p_2)V_{LL})}{(1 - p_1)(t_1 - 1)}
\]

\[
\iff \quad \frac{I}{(t_1 - 1)} > \frac{1}{(t_1 - 1)(1 - p_1)}E(\tilde{V}_1) + V_L + \frac{T - t_1}{t_1 - 1}E(\tilde{V}_2|H) \quad (13)
\]

This simply amounts to saying that if the project cost is sufficiently large relative to the expected cumulative project payoffs then the firm will wait to invest rather than invest now, since the project is only worth having if the good state is realized and costly to have undertaken if the bad state is realized.

\((B^*)\) being satisfied is equivalent to:

\[
p_1[(t_1 - 1)V_H + (T - t_1)(p_2V_{HH} + (1 - p_2)V_{HL}) - I] > 0
\]

\[
\iff \quad \frac{(t_1 - 1)V_H}{t_1 - 1} + \frac{(T - t_1)(p_2V_{HH} + (1 - p_2)V_{HL})}{t_1 - 1} > \frac{I}{t_1 - 1}
\]

\[
\iff \quad V_L + V + \frac{T - t_1}{t_1 - 1}E(\tilde{V}_2|H) > \frac{I}{t_1 - 1} \quad (14)
\]

This simply amounts to saying that the expected cumulative project payoffs must be sufficiently large relative to the cost of the project for the project to be undertaken.

Combining (??) and (??) thus yields
8.3.3 Delay Investment Until After the Second Resolution of Uncertainty

Mathematically, \((C^*)\) being satisfied is equivalent to:

\[
p_1p_2[(T - t_1 - 1)V_{HH} - I] > p_1[(t_1 - 1)V_H + (T - t_1)(p_2V_{HH} + (1 - p_2)V_{HL}) - I] \]

\[
\iff 
(1 - p_2)I > (t_1 - 1)V_H + E(V_2|H) + (1 - p_2)(T - t_1 - 1)V_{HL} \]

\[
\iff 
\frac{I}{T - t_1 - 1} > \frac{1}{T - t_1 - 1}(t_1 - 1)(1 - p_2)V_H + \frac{E(V_2|H)}{1 - p_2} + V_{HL} \tag{17} \]

This simply amounts to saying that if the project cost is sufficiently large relative to the expected cumulative project payoffs then the firm will wait to invest until after the second resolution of uncertainty rather than invest after the first resolution of uncertainty, since the project is only worth having if the good state is realized and costly to have undertaken if the bad state is realized.

\((D^*)\) being satisfied is equivalent to:

\[
p_1p_2[(T - t_1 - 1)V_{HH} - I] > 0 \]

\[
\iff 
V^H + V_{HL} > \frac{I}{T - t_1 - 1} \tag{18} \]

This simply amounts to saying that the expected cumulative project payoffs must be sufficiently large relative to the cost of the project for the project to be undertaken.
Combining (??) and (??) thus yields

\[
V^H + V_{HL} > \frac{I}{T - t_1 - 1} > \frac{1}{T - t_1 - 1} \left( t_1 - 1 \frac{V_H}{1 - p_2} + \frac{E(\tilde{V}_2|H)}{1 - p_2} \right) + V_{HL}
\]

\[\Rightarrow \]

\[
\frac{\sigma(\tilde{V}_2|H) \sqrt{1 - p_2}}{\sqrt{p_2}} > \frac{t_1 - 1}{T - t_1 - 1} V_H + \frac{E(\tilde{V}_2|H)}{T - t_1 - 1}
\]  \hspace{1cm} (19)

\[
\Leftrightarrow \]

\[
\frac{\sigma(\tilde{V}_2|H) \sqrt{1 - p_2}}{\sqrt{p_2}} (T - t_1 - 1) > (t_1 - 1) V_H + E(\tilde{V}_2|H)
\]  \hspace{1cm} (20)

since \( \sigma(\tilde{V}_2|H) = V^H p_2 (1 - p_2) \).
The table reports summary statistics of the main variables used in my analysis, as well as descriptive firm and macroeconomic variables. The data are quarterly and extend from January 1996 to December 2012. I calculate means, medians and standard deviations of all the variables I use, where the sample is all observations with non-missing values for all of the variables listed. 27,434 firm-quarter observations are in the sample, from 1,445 firms. Capital investment and cash flow are deflated by total firm assets at the beginning of the quarter. The uncertainty measures are the 30-day firm implied volatility and 30-day market (S&P 500 index) implied volatility, also referred to as the level, as well as the differences between the 30-day and 1-year firm implied volatilities and market (S&P 500 index) implied volatilities, also referred to as the slope. Implied volatilities are from Optionmetrics and are the average over the previous month of standardized put and call options with strikes equal to the at-the-money forward price. Utilities, financials and firm-quarters with negative book equity, total assets or sales observations are excluded. Tobin’s Q is the market capitalization of common stock plus the book value of long-term debt, current debt and preferred stock, divided by book assets. Cash flow is firm operating income. Proportional sales growth is sales in the most recent year minus sales the prior year, all divided by the sum of the two years of sales (effectively scaling sales growth to lie in the interval [-1,1]). The data have been winsorized at the 1% and 99% levels to reduce the influence of outliers.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Num. Obs.</th>
<th>Mean</th>
<th>Median</th>
<th>Std. Dev.</th>
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</thead>
<tbody>
<tr>
<td>Capital Investment/Assets</td>
<td>27,434</td>
<td>0.0158</td>
<td>0.0100</td>
<td>0.0184</td>
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<td>30-day firm impl vol</td>
<td>27,434</td>
<td>0.442</td>
<td>0.389</td>
<td>0.213</td>
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<td>1-year - 30-day firm impl vol slope</td>
<td>27,434</td>
<td>-0.015</td>
<td>-0.0040</td>
<td>0.0608</td>
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<tr>
<td>30-day market impl vol</td>
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<td>0.201</td>
<td>0.189</td>
<td>0.085</td>
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<tr>
<td>1-year - 30-day market impl vol slope</td>
<td>27,434</td>
<td>0.0098</td>
<td>0.0161</td>
<td>0.037</td>
</tr>
<tr>
<td>Lagged Tobin's Q</td>
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<td>1.49</td>
<td>1.82</td>
</tr>
<tr>
<td>Cash flow/Assets</td>
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<td>0.0267</td>
<td>0.0272</td>
<td>0.0418</td>
</tr>
<tr>
<td>Year-on-year proportional sales growth</td>
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<td>0.0519</td>
<td>0.0432</td>
<td>0.122</td>
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<tr>
<td>Total Assets (Millions)</td>
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<td>10,247</td>
<td>4,765</td>
<td>11,971</td>
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<td>Lagged CFNAI 3-month MA</td>
<td>27,434</td>
<td>-0.359</td>
<td>-0.09</td>
<td>0.907</td>
</tr>
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</table>
This is a regression of firm quarterly corporate investment (scaled by total firm assets) on measures of short-term firm uncertainty, long-term firm uncertainty and control variables. The short-term uncertainty measure (also referred to as the level) is the lagged 30-day firm implied volatility. The long-term uncertainty measure is the difference between the lagged 1-year and 30-day firm implied volatilities, also referred to as the slope. Uncertainty measures are taken as the average of the measures over the previous month. Control variables include lagged Tobin’s Q, contemporaneous quarterly cash flow (scaled by firm assets), and year-on-year proportional sales growth. The sample is quarterly from January 1996 to December 2012. All variables have been scaled by their sample standard deviations for ease of economic interpretation. Also included but not reported in each column are firm fixed effects, as well as date fixed effects. Below coefficients in parentheses are standard errors based on robust standard errors clustered by firm. The data has been winsorized at the 1% and 99% levels to reduce the influence of outliers. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively.

### Table 2

#### Investment is Negatively Correlated with Short- and Long-Term Firm Uncertainty

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<td>-0.0467***</td>
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<td>-0.0113**</td>
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<td>0.0806***</td>
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Firm Fixed Effects | Yes | Yes | Yes | Yes | Yes |
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Table 3
Investment is Negatively Correlated with Short- and Long-Term Aggregate Uncertainty

This is a regression of firm quarterly corporate investment (divided by total firm assets) on measures of short-term uncertainty, long-term uncertainty and control variables. The short-term uncertainty measure (also referred to as the level) is the lagged 30-day firm implied volatility or the lagged 30-day market (S&P 500 index) implied volatility. The long-term uncertainty measure is the difference between the lagged 1-year and 30-day firm implied volatilities or the lagged 1-year and 30-day market (S&P 500 index) implied volatilities, also referred to as the slope. Uncertainty measures are taken as the average of the measures over the previous month. Control variables include lagged Tobin’s Q, contemporaneous quarterly cash flow (scaled by firm assets), year-on-year proportional sales growth, the lagged 3-month moving average of the Chicago Fed National Activity Index (CFNAI), a lagged default spread (BAA-AAA yield from FRED) and a lagged term spread (difference in 1-year and 3-month Treasury yields). The sample is quarterly from January 1996 to December 2012. All variables have been scaled by their sample standard deviations for ease of economic interpretation. Also included but not reported are firm fixed effects, as well as time dummies (calendar and fiscal quarter dummies). Below coefficients in parentheses are standard errors based on robust standard errors clustered by firm. The data has been winsorized at the 1% and 99% levels to reduce the influence of outliers. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively.

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<td>Clustered by Firm</td>
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This is a regression of firm quarterly corporate investment (divided by total firm assets) on measures of short-term uncertainty, long-term uncertainty and control variables. Short- and long-term uncertainty measures vary across specifications, as indicated. Uncertainty measures are taken as the average of the measures over the previous month. Control variables include lagged Tobin's Q, contemporaneous quarterly cash flow (scaled by firm assets), year-on-year proportional sales growth, the lagged 3-month moving average of the Chicago Fed National Activity Index (CFNAI), a lagged default spread (BAA-AAA yield from FRED) and a lagged term spread (difference in 1-year and 3-month Treasury yields). The sample is quarterly from January 1996 to December 2012. All variables have been scaled by their sample standard deviations for ease of economic interpretation. Also included but not reported are firm fixed effects, as well as time dummies (date fixed effects for firm uncertainty regressions (1) and (3), and calendar and fiscal quarter dummies for firm & aggregate uncertainty regressions (2) and (4)). Below coefficients in parentheses are standard errors based on robust standard errors clustered by firm. The data has been winsorized at the 1% and 99% levels to reduce the influence of outliers. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively.

### Table 5

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</table>
Uncertainty has a Negative Correlation with Acquisitions, Positive with R&D and Cash Holdings

This is a regression of firm quarterly corporate investment (scaled by total firm assets) on measures of short-term uncertainty, long-term uncertainty and control variables. Investment is measured as capital expenditures/assets in columns (1) and (2), research and development expenses/assets in columns (3) and (4), and acquisitions/assets in columns (5) and (6). Cash holdings/assets is used as the dependent variable in columns (7) and (8). The short-term uncertainty measure (also referred to as the level) is the lagged 30-day firm implied volatility or the lagged 30-day market (S&P 500 index) implied volatility. The long-term uncertainty measure is the difference between the lagged 1-year and 30-day firm implied volatilities or the lagged 1-year and 30-day market (S&P 500 index) implied volatilities, also referred to as the slope. Uncertainty measures are taken as the average of the measures over the previous month. Control variables include lagged Tobin’s Q, contemporaneous quarterly cash flow (scaled by firm assets), year-on-year proportional sales growth, the lagged 3-month moving average of the Chicago Fed National Activity Index (CFNAI), a lagged default spread (BAA-AAA yield from FRED) and a lagged term spread (difference in 1-year and 3-month Treasury yields). The sample is quarterly from January 1996 to December 2012. All variables have been scaled by their sample standard deviations for ease of economic interpretation. Also included but not reported are firm fixed effects, as well as time dummies (date fixed effects for firm uncertainty regressions (1), (3), (5) and (7), and calendar and fiscal quarter dummies for firm & aggregate uncertainty regressions (2), (4), (6) and (8)). The data has been winsorized at the 1% and 99% levels to reduce the influence of outliers. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Investment/Assets</th>
<th>R&amp;D/Assets</th>
<th>Acquisitions/Assets</th>
<th>Cash Holding/Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specification</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>30-day firm impl vol (level)</td>
<td>-0.0685***</td>
<td>-0.0178</td>
<td>0.0478***</td>
<td>0.0245*</td>
</tr>
<tr>
<td></td>
<td>(0.0159)</td>
<td>(0.0149)</td>
<td>(0.0161)</td>
<td>(0.0139)</td>
</tr>
<tr>
<td>1-year firm impl vol - 30-day firm impl vol (slope)</td>
<td>-0.0212***</td>
<td>-0.0164**</td>
<td>0.0227***</td>
<td>0.0161**</td>
</tr>
<tr>
<td></td>
<td>(0.00770)</td>
<td>(0.00679)</td>
<td>(0.0074)</td>
<td>(0.00698)</td>
</tr>
<tr>
<td>30-day market impl vol (level)</td>
<td>-</td>
<td>-0.0271*</td>
<td>-</td>
<td>-0.0152</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.0152)</td>
<td>-</td>
<td>(0.0132)</td>
</tr>
<tr>
<td>1-year market impl vol - 30-day market impl vol (slope)</td>
<td>-</td>
<td>-0.0535***</td>
<td>-</td>
<td>-0.0227**</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.0115)</td>
<td>-</td>
<td>(0.00979)</td>
</tr>
<tr>
<td>Lagged Tobin’s Q</td>
<td>0.0932***</td>
<td>0.132***</td>
<td>0.103***</td>
<td>0.0988***</td>
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<tr>
<td></td>
<td>(0.0159)</td>
<td>(0.0156)</td>
<td>(0.0194)</td>
<td>(0.0185)</td>
</tr>
<tr>
<td>Cash flow/Assets</td>
<td>0.0392**</td>
<td>0.0308*</td>
<td>-0.0408***</td>
<td>-0.0436***</td>
</tr>
<tr>
<td></td>
<td>(0.0158)</td>
<td>(0.0164)</td>
<td>(0.0157)</td>
<td>(0.0160)</td>
</tr>
<tr>
<td>Year-on-year proportional sales growth</td>
<td>0.0261***</td>
<td>0.0298***</td>
<td>-0.00402</td>
<td>-0.00346</td>
</tr>
<tr>
<td></td>
<td>(0.00723)</td>
<td>(0.00730)</td>
<td>(0.00794)</td>
<td>(0.00762)</td>
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<tr>
<td>Lagged CFNAI 3-month moving average</td>
<td>-</td>
<td>-0.00381</td>
<td>-</td>
<td>-0.00748</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.00737)</td>
<td>-</td>
<td>(0.00627)</td>
</tr>
<tr>
<td>Lagged default spread</td>
<td>-</td>
<td>-0.0316***</td>
<td>-</td>
<td>-0.00288</td>
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<tr>
<td></td>
<td>-</td>
<td>(0.0101)</td>
<td>-</td>
<td>(0.00740)</td>
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<tr>
<td>Lagged term spread</td>
<td>-</td>
<td>-0.00418</td>
<td>-</td>
<td>-0.00216</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>(0.00428)</td>
<td>-</td>
<td>(0.00325)</td>
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<td>Firm Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Date Fixed Effects</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Fiscal Quarter Dummies (4)</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Calendar Quarter Dummies (4)</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
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<td>Clustered by Firm</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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Firms 963 963 963 963 963 963 963 963
Observations 14,998 14,998 14,998 14,998 14,998 14,998 14,998 14,998
This is a regression of firm quarterly corporate investment (scaled by total firm assets) on measures of short-term uncertainty, long-term uncertainty and control variables. The short-term uncertainty measure (also referred to as the level) is the lagged 30-day firm implied volatility or the lagged 30-day market (S&P 500 index) implied volatility. The long-term uncertainty measure is the difference between the lagged 1-year and 30-day firm implied volatilities or the lagged 1-year and 30-day market (S&P 500 index) implied volatilities, also referred to as the slope. Uncertainty measures are taken as the average of the measures over the previous month. In columns (1) and (2) the interaction is with a dummy variable equal to one if the observation has total firm assets greater than the sample median firm assets and zero otherwise. In columns (3) and (4) the interaction is with a dummy variable equal to one if the observation has proportional sales growth greater than the sample median proportional sales growth and zero otherwise. In columns (5) and (6) the interaction is with a dummy variable equal to one if the observation firm has an IPO date later than the sample median IPO date and zero otherwise. Control variables include lagged Tobin’s Q, contemporaneous quarterly cash flow (scaled by firm assets), year-on-year proportional sales growth, the lagged 3-month moving average of the Chicago Fed National Activity Index (CFNAI), a lagged default spread (BAA-AAA yield from FRED) and a lagged term spread (difference in 1-year and 3-month Treasury yields). The sample is quarterly from January 1996 to December 2012. All variables have been scaled by their sample standard deviations for ease of economic interpretation. Also included but not reported are firm fixed effects, as well as time dummies (date fixed effects for firm uncertainty regressions (1), (3) and (5), and calendar and fiscal quarter dummies for firm & aggregate uncertainty regressions (2), (4) and (6)). Below coefficients in parentheses are standard errors based on robust standard errors clustered by firm. The data has been winsorized at the 1% and 99% levels to reduce the influence of outliers. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively.

<table>
<thead>
<tr>
<th>Dep. Var: CAPX/Assets</th>
<th>Total Assets Interaction</th>
<th>Sales Growth Interaction</th>
<th>IPO Date Interaction</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5) (6)</td>
<td>(1) (2) (3) (4) (5) (6)</td>
<td>(1) (2) (3) (4) (5) (6)</td>
</tr>
<tr>
<td><strong>Specification</strong></td>
<td>Firm</td>
<td>Firm &amp; Market</td>
<td>Firm</td>
</tr>
<tr>
<td>30-day firm impl vol (level)</td>
<td>0.0936*** (-0.0262)</td>
<td>-0.0312 ((0.0209))</td>
<td>-0.112*** ((0.0258))</td>
</tr>
<tr>
<td>1-year firm impl vol - 30-day firm impl vol (slope)</td>
<td>-0.0243*** (0.0102)</td>
<td>-0.0125 ((0.00982))</td>
<td>-0.0255*** (0.00974)</td>
</tr>
<tr>
<td>30-day market impl vol (level)</td>
<td>- (-0.0297)</td>
<td>- (-0.0235)</td>
<td>- (-0.02097)</td>
</tr>
<tr>
<td>1-year market impl vol - 30-day market impl vol (slope)</td>
<td>- -0.0594*** (0.0187)</td>
<td>- -0.0669*** ((0.0171))</td>
<td>- -0.0534*** (0.0171)</td>
</tr>
<tr>
<td>30-day firm vol interaction</td>
<td>-0.0647*** (0.0156)</td>
<td>-0.0763** (0.0315)</td>
<td>0.00760 (0.0120)</td>
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<td>(1y firm vol - 30d firm vol) interaction</td>
<td>-0.0231 (0.0138)</td>
<td>-0.0332** (0.0168)</td>
<td>-0.00667 (0.0116)</td>
</tr>
<tr>
<td>30-day market vol interaction</td>
<td>- 0.00162 (0.0194)</td>
<td>- 0.0115 (0.0194)</td>
<td>- 0.00162 (0.0194)</td>
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<td>(1y market vol - 30d market vol) interaction</td>
<td>- -0.00118 (0.0215)</td>
<td>- -0.00203 (0.0215)</td>
<td>- -0.00118 (0.0215)</td>
</tr>
<tr>
<td>Lagged Tobin's Q</td>
<td>0.118*** (0.0192)</td>
<td>0.155*** (0.0189)</td>
<td>0.125*** (0.0193)</td>
</tr>
<tr>
<td>Cash flow/Assets</td>
<td>0.0450*** (0.0120)</td>
<td>0.0338*** (0.0124)</td>
<td>0.0445*** (0.0122)</td>
</tr>
<tr>
<td>Year-on-year proportional sales growth</td>
<td>0.0218** (0.00833)</td>
<td>0.0314*** (0.00903)</td>
<td>0.0161 (0.0114)</td>
</tr>
<tr>
<td>Lagged CFNAI 3-month moving average</td>
<td>- -0.00593 (0.0110)</td>
<td>- -0.00277 (0.0108)</td>
<td>- -0.00593 (0.0110)</td>
</tr>
<tr>
<td>Lagged default spread</td>
<td>- -0.0182 (0.0128)</td>
<td>- -0.0152 (0.0127)</td>
<td>- -0.0182 (0.0128)</td>
</tr>
<tr>
<td>Lagged term spread</td>
<td>- -0.00567 (0.00623)</td>
<td>- -0.00363 (0.00625)</td>
<td>- -0.00567 (0.00623)</td>
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<tr>
<td>Firm Fixed Effects</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>Date Fixed Effects</td>
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<td>No</td>
<td>Yes</td>
</tr>
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<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
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<td>Calendar Quarter Dummies (4)</td>
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<td>No</td>
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<tr>
<td>Clustered by Firm</td>
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<td>Yes</td>
</tr>
</tbody>
</table>

Firms 846 846 846 846 846 846
Observations 12,513 12,513 12,513 12,513 12,513 12,513

Table 7: Larger Firms Have Stronger Correlation Between Investment and Uncertainty