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RBC LiONS™ S&P 500 Buffered Protection Securities (USD) Series 4 Analysis

Option Pricing Analysis, Issuing Company Risk-hedging Analysis, and Recommended Investment Strategy

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5th Solution Fund
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Abstract

This paper will compute the value of the RBC financial derivative-RBC LiONS™ S&P 500 Buffered Protection Securities (USD), Series 4 by utilizing the Black-Scholes Option Pricing Model. In order conduct a thorough analysis of the securities, the paper will compare the model value with the actual price at which the security was issued and the price at which it was traded. This model will help establish a recommended strategy for the issuing company to hedge the liability incurred by the security issued, and provide a possible hedging strategy for the investors.

Introduction

The objective of the paper to compute the value of the RBC financial derivative-RBC LiONS™ S&P 500 Buffered Protection Securities (USD), Series 4 by using the Black-Scholes under Option Pricing Model. Further we analyze the security described in the product offering from the viewpoint of the issuing company and from the viewpoint of an investor considering whether or not to buy them. In this scenario Royal Bank of Canada (RBC) offered up to $10,000,000 US dollars of RBC LiONS™ S&P 500 Buffered Protection Securities (Macroption, 2013). Moreover, Series 4 is for investors that need to reach a large segment of the United States equity market and therefore this product is necessary for them (Macroption, 2013). Furthermore, principal of securities can protect when up against a decline of up to 30% when the price performance and maximum return is 41% (Macroption, 2013). This is due to the fact that the payment, which is paid at maturity on these specific securities, is based on the price performance of S&P 500® Index subject to a buffer of 30% and a cap of 41% (Macroption, 2013). If there is a negative performance that exceeds 30%, the amount of the principal could be lost in the maturity (Macroption, 2013).

Black Schole Model

In 1973, the Chicago Board of Options Exchange began trading options in exchanges, although previously financial institutions in had regularly traded options over the counter markets (Macroption, 2013). During the same year, Black and Scholes (1973), and Merton (1973), published their seminal papers on the theory of option pricing (Macroption, 2013). The time since the seminal papers have caused the growth of the derivative securities field to become incredible. In 1997, Scholes and Merton received a Nobel Prize within the Economics discipline in order to recognize their contributions to option valuations (Macroption, 2013). During the ceremony, Black was unable to receive his award because he had passed away but he was known as someone that left this world a little better than he found it. Overall, the Black-Scholes model is able to eliminate all market risks by simply continuously adjusting proportions of stocks and options in a portfolio, as this will allow an investor to craft a riskless hedge portfolio (Macroption, 2013). This portfolio would be dependent on the assumptions of continuous trading and sample paths of the asset price (Macroption, 2013). Further, all portfolios that exhibit a zero market risk must have an expected rate of return equal to the risk-free interest rate within an efficient market that showcases no riskless arbitrage opportunities. This is an approach that leads to a differential equation that is known as the “heat equation”. The solution in this scenario is the Black-Scholes formula for pricing European options on non-dividend paying stocks shown below (Macroption, 2013). Overall, in a study six different stocks were compared to the real market and the Black Scholes model and the result was striking similarities (Macbeth, J. D., & Merville, L. J., 1979). Moreover, this model’s “predicted prices are on average less (greater) than market prices for in the money (out of the money) options (Macbeth, J. D., & Merville, L. J., 1979). According to the research done by Black and Scholes by thorough empirical tests "actual prices at which options are bought and sold deviate in certain systematic ways from the values predicted by the formula (Black, F., & Scholes, M., 1979).

$$C = SN(d_1) - Xe^{-rt} N(d_2)$$

where

C = call option price
S = current stock price
X = exercise price
r = short-term risk-free interest rate
e = 2.718
ln = natural logarithm
t = time remaining to the expiration date (as a fraction of a year)
s = standard deviation of the stock price
$N(.)$ = the cumulative normal probability

$$d_1 = \frac{ln\,(S/X) + (r + 0.5s^2)\,t}{s\sqrt{t}}$$

$$d_2 = d_1 - s\sqrt{t}$$

As stated above, there are several assumptions that must be acknowledged in order to effectively utilize the model. The first assumption is that the price of the instrument is a geometric Brownian motion, which embodies constant drift and volatility (Macroption, 2013). Next, it is possible to short sell the underlying stock and there are absolutely no riskless arbitrage opportunities even though it may seem as this is the truth. Following this, it is vital to understand trading in the stock is continuous, there are no transaction costs, and every security is perfectly divisible. Finally, the risk-free interest rate is constant and the same for every maturity date (Macroption, 2013). Finally, Black and Scholes did another empirical study alongside Michael C. Jensen from Harvard Business School showcasing the power of this model and vital assumptions. Specifically, as shown above they relate to portfolios mean and variance, transaction costs, homogeneous views, and riskless rate (Black , F., Jensen, M., & Scholes, M., 1972).

SPX Index Data Collection

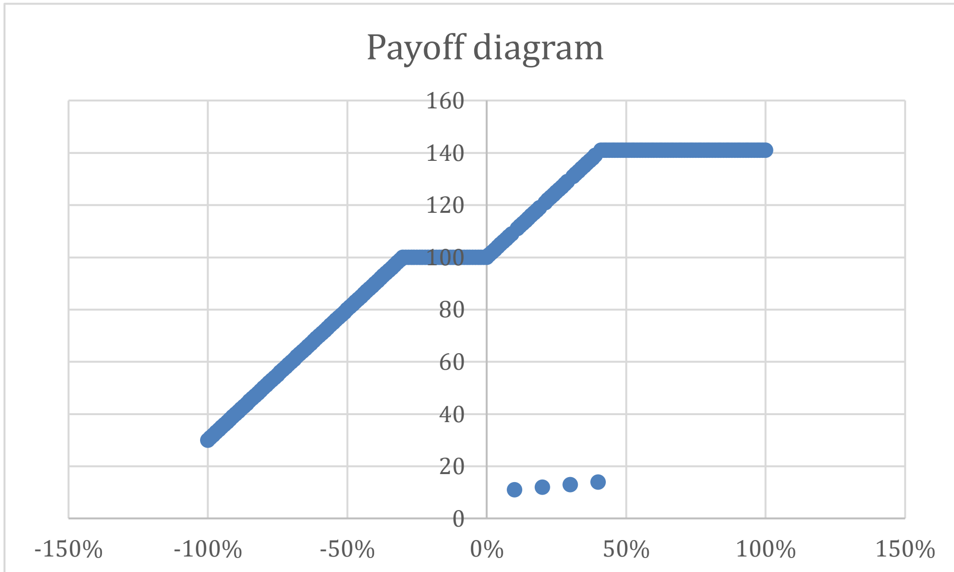
In order to value a derivative, it is vital to estimate the volatility of its underlying asset. This will require obtaining data on the weekly return on SPX Index for a period of about 6 months to 1 year preceding the date on which the security is priced. One is able to collect this date (from 2011,11,22 to 2016,11,22) by accessing the Bloomberg terminals and importing the data to Excel.

Cash Flows of Security

From an investor's point of view, this security has an initial cash flow, which is its Principal Amount, -$100 per security. On the mature date, the cash inflow has four possible outcomes dependent on the index market return: $141(Percentage Change>41%), $100+$100*percentage change (41%>Percentage Change>0), $100 (percentage change<-30%), or $100*percentage change (percentage change<30%) per security.

	T_0	T			
		$S_t = S_0 * \Delta$			
Index	S_0	$\Delta<0.7$	$\Delta=0.7\sim1$	$\Delta=0.7\sim1.41$	$\Delta>1.41$

Cash	-100	100*(Δ+0.3)	100	100*Δ	141



Fair Market Value of the Security at the Time of Issue and on the Present

Prior to addressing the fair market value it is important to outline the assumptions used during the calculations. The primary assumption is that the interest rate is risk free. Rf of the issuing day is set to be 5-year US Treasury bill rate of that day because the product has a 5-year investment horizon; and Rf of today is set to be 1-year US treasury bill rate of November 22, 2016 because there is 1 year from now to maturity. The second assumption is related to the dividend yield. Specifically, the annual dividend yield of the issuing day is set to be 2.14% constant (retrieved from pricing supplements); that of today is set to be 2.05% (retrieved from SP500). The third assumption is that the future stock price and return is normally distributed. Finally, the last assumption is related to implied volatility. Specifically, it is computed using the return of one year preceding the calculation day.

The following is a thorough description of the calculations. The initial index level is $1417.27 (the price of the fourth exchanging day preceding the issuing day). Therefore, the payoff of one share of index can be represented with a 41% cap equals to $1998.34 and a 30% buffer at the level $992.08. And then we replicate the securities by constructing a portfolio using a combination of put and a bond.

The replicated portfolio consists of short position of two put options at strike price of $1998.34 and $992.08, respectively, and long position of a put at $1417.27, and selling a bond with its future value equals to $581.08.

The portfolio pricing was calculated to be $63.98 for issuing day, and $534.05 for today, for each share of the underlying asset. As one security represents $100, the fair values per security for the issuing day and present day would be $104.52 and $137.68, respectively.

Pricing issuing day						
σ	14.74%					
T	5					
div	2.14%					
5 yr treasury rate on nov 9 2012	0.65%					
Replication						
	K	**d1**	**N-d1**	**d2**	**N-d2**	**Put**
Short put cap	1998.337	-1.103	0.865	-1.433	0.924	685.950
Short put buffer	992.082	1.021	0.154	0.691	0.245	39.342
Long put initial	1417.260	-0.061	0.524	-0.391	0.652	226.780

	FV	PV
Lend	581.077	562.495
PV of portfolio	$ (63.98)	
PV 1 unit of security	$ (4.51)	
PV pricing	$ 104.51	

Pricing today						
σ	14.01%					
T	0.976190476					
Div	2.05%					
1 yr treasury rate on nov 22 2016	0.78%					
S today	2202.94	(246 days to maturity)				
Replication						
	K	**d1**	**N-d1**	**d2**	**N-d2**	**Put**
Short put cap	1998.337	0.684	0.247	0.546	0.293	47.095
Short put buffer	992.082	5.744	0.000	5.606	0.000	0.000
Long put initial	1417.260	3.167	0.001	3.028	0.001	0.065
	FV	**PV**				
Lend	581.077	581.077				
PV of portfolio	$ (534.05)					
PV 1 unit of security	$ (37.68)					
PV pricing	$ 137.68					

Comparison of Model to Actual Price During Issue Date and Present

	Issue day	Today
BS model	104.5146	137.6816
Actual price	100	133.54
Difference	4.514595	4.14161

When we compare the model value with the actual price, the model prices are slightly higher both in 2012 and 2016 (4.5 and 4.1 respectively). For the issue price, the difference can be caused by two possibilities including the bank may not use Black Scholes model for option pricing, or that the inputs are different.

Binomial model and Monte Carlo simulation also can be appropriate choices of valuation. And also, sometimes banks make small tweaks to the model because they are fully aware of the limitation of Black Schole method. Similar to return, it is assumed as normally distributed. However, in reality, the normal distribution is very unlikely to happen. As for the inputs, there have been a few simple assumptions included (see previous section) for calculation purposes, which could be inaccurate.

Also, Black Scholes model assumes a constant risk free rate, and normal distribution for stock price, which are not realistic given that the market is very dynamic and complex (NYU, 2013). Therefore, when pricing the options at the bank, individuals notice these problems and slightly change the valuation.

The difference from today's price is because the model price is not the same as market price. The market price is "determined in the market by the interaction of supply and demand for the particular option." In comparison, the model price is created based on different assumptions, such as constant interest rate. However, it ignores the market interactions, which results in difference from the current market price.

Recommended Strategy to Hedge the Liability

Completely Covered Position

Replicating to Hedge			Payoff	708.63	992.082	1417.26	1998.337	1999
Negative payoff offset	short call at	992.082	$-\max(S_t - 992,0)$	0	0	-425.178	-1006.25	-1006.92
	lend FV	425.178	425.178	425.178	425.178	425.178	425.178	425.178
	short call at	1998.3366	$-\max(S_t - 1998,0)$	0	0	0	0	-0.6634
Positive payoff offset	long call at	1417.26	$\max(S_t - 1417,0)$	0	0	0	581.0766	581.74
Total payoff				**425.178**	**425.178**	**0**	**0**	**-0.6634**

One of the recommended strategies is the completely covered position. In this position, the issuing bank's payoffs are completely offset. The negative payoff is hedged through a combination of shorting two call options at $992.08 and $1998.34, respectively, and lending an amount equals to a future value of $425.18. The positive payoff will be offset by a long call position at a strike price of $1417.26. However, we believe the hedging of the positive side is not necessary.

Through the completely hedged position, regardless of the future stock price, the payoff is going to be consistent for the issuing bank.

Delta Dynamic Hedging

Replicating Security							
		Payoff	708.63	992.082	1417.26	1998.337	200000
Long call at	992.082	max(S_t -992,0)	0	0	425.178	1006.255	199007.9
Borrow FV	-425.178	-425.178	-425.178	-425.178	-425.178	-425.178	-425.178
Long call at	1998.3366	max(S_t -1998,0)	0	0	0	0	198001.7
Short call at	1417.26	-max(S_t -1417,0)	0	0	0	-581.077	-198583
Total payoff			**-425.178**	**-425.178**	**0.00**	**0**	**198001.7**

Delta Dynamic Hedging is another effective strategy. Dynamic hedging through constructing a delta neutral portfolio is another way for the issuing bank to hedge against the volatility of the index. This strategy is known as an options strategy that is able to reduce, or hedge, the risk with price movement in the underlying asset, which is performed by offsetting long and short positions (NYU, 2013). For example, a long call position may be delta hedged by shorting the underlying stock. This strategy is based on the change in premium, or price of option, caused by a change in the price of the underlying security (NYU, 2013).

From the RBC's perspective, the security payoff was replicated with two long call options at 992.08, and 1998.34 respectively, a short call at 1417.26, as well as a bond purchasing (money borrowing) with a future value equals to 425.18. To illustrate the strategy, it is assumed the index follows a Geometric Brownian Motion and simulated a complete path for the index over its 5 years life span.

Stock price	1417.26		put initial	1417.26
			put cap	1998.3366
Expect. Return	0.00050254		put buffer	992.082
T	5			
Volatility	14.74%			
Risk free rate	0.10%			
dividend	2.14%			
M	0.00396825			

			short call initial			long call cap			long call buffer			bond								
days	Stock Price	T-t	d1	d2	call	d1	d2	cal	d1	d2	call		potfolio	Delta	Gamma	No. Shares	Additional sharess	Additional cost	Cost (borrow)	Value of hedged portfolio
0	1417.260	5.000	-0.145	-0.474	115.561	-1.187	-1.516	21.175	0.937	0.608	332.565	425.178	186.998	0.450	0.000	-0.450	-0.450	-637.792	-637.792	-450.794
1	1396.589	4.996	-0.189	-0.519	107.483	-1.232	-1.561	19.051	0.893	0.564	317.376	425.180	196.236	0.448	0.000	-0.448	0.002	3.316	-634.478	-438.242
2	1389.228	4.992	-0.205	-0.535	104.672	-1.248	-1.578	18.320	0.878	0.548	312.044	425.181	199.489	0.447	0.000	-0.447	0.001	1.162	-633.319	-433.830
3	1405.272	4.988	-0.170	-0.499	110.788	-1.214	-1.543	19.875	0.913	0.584	323.831	425.183	192.264	0.449	0.000	-0.449	-0.002	-2.845	-636.167	-443.902
4	1405.070	4.984	-0.171	-0.500	110.689	-1.214	-1.544	19.835	0.913	0.584	323.724	425.185	192.314	0.449	0.000	-0.449	0.000	-0.067	-636.236	-443.922
5	1412.969	4.980	-0.154	-0.483	113.762	-1.198	-1.527	20.623	0.931	0.602	329.600	425.186	188.725	0.450	0.000	-0.450	-0.001	-1.394	-637.633	-448.908
6	1411.866	4.976	-0.156	-0.485	113.307	-1.201	-1.529	20.488	0.929	0.600	328.827	425.188	189.180	0.450	0.000	-0.450	0.000	0.081	-637.555	-448.375
7	1405.458	4.972	-0.170	-0.498	110.778	-1.215	-1.544	19.814	0.915	0.587	324.135	425.190	192.019	0.449	0.000	-0.449	0.001	0.942	-636.616	-444.596
8	1404.400	4.968	-0.172	-0.500	110.347	-1.217	-1.546	19.687	0.914	0.585	323.397	425.191	192.454	0.449	0.000	-0.449	0.000	0.073	-636.545	-444.091
9	1420.539	4.964	-0.137	-0.466	116.692	-1.183	-1.511	21.336	0.949	0.620	335.399	425.193	185.150	0.451	0.000	-0.451	-0.002	-2.699	-639.246	-454.096
10	1412.495	4.960	-0.154	-0.483	113.474	-1.201	-1.529	20.472	0.932	0.604	329.464	425.195	188.733	0.450	0.000	-0.450	0.001	1.173	-638.076	-449.343
11	1424.810	4.956	-0.128	-0.456	118.371	-1.175	-1.503	21.752	0.959	0.631	338.676	425.197	183.139	0.452	0.000	-0.452	-0.001	-2.035	-640.114	-456.975
12	1425.019	4.952	-0.127	-0.455	118.436	-1.174	-1.503	21.754	0.960	0.632	338.877	425.198	183.003	0.452	0.000	-0.452	0.000	-0.124	-640.241	-457.238
13	1454.645	4.948	-0.064	-0.392	130.725	-1.112	-1.440	25.103	1.023	0.695	361.290	425.200	169.532	0.455	0.000	-0.455	-0.003	-4.561	-644.804	-475.273
14	1431.531	4.944	-0.113	-0.441	121.045	-1.161	-1.489	22.422	0.975	0.647	343.850	425.202	179.975	0.452	0.000	-0.452	0.002	3.313	-641.494	-461.519
15	1415.310	4.940	-0.148	-0.476	114.487	-1.196	-1.524	20.661	0.941	0.613	331.770	425.203	187.259	0.451	0.000	-0.451	0.002	2.378	-639.118	-451.859
16	1399.236	4.937	-0.183	-0.510	108.185	-1.232	-1.559	19.015	0.906	0.579	319.924	425.205	194.451	0.449	0.000	-0.449	0.002	2.442	-636.678	-442.228
17	1385.801	4.933	-0.212	-0.540	103.065	-1.261	-1.589	17.711	0.877	0.550	310.127	425.207	200.434	0.448	0.000	-0.448	0.002	2.109	-634.572	-434.138
18	1380.496	4.929	-0.224	-0.551	101.068	-1.274	-1.601	17.203	0.866	0.539	306.306	425.208	202.767	0.447	0.000	-0.447	0.001	0.792	-633.782	-431.015
19	1380.883	4.925	-0.223	-0.550	101.189	-1.273	-1.600	17.219	0.867	0.540	306.625	425.210	202.555	0.447	0.000	-0.447	0.000	-0.177	-633.961	-431.406
20	1369.943	4.921	-0.247	-0.574	97.159	-1.298	-1.625	16.223	0.843	0.516	298.747	425.212	207.401	0.446	0.000	-0.446	0.001	1.785	-632.179	-424.778
21	1356.446	4.917	-0.278	-0.604	92.319	-1.329	-1.655	15.056	0.814	0.487	289.107	425.213	213.369	0.444	0.000	-0.444	0.002	2.326	-629.856	-416.487
22	1351.829	4.913	-0.288	-0.615	90.681	-1.339	-1.666	14.660	0.804	0.477	285.855	425.215	215.381	0.444	0.000	-0.444	0.001	0.746	-629.113	-413.732
23	1371.212	4.909	-0.244	-0.571	97.552	-1.296	-1.623	16.280	0.848	0.521	299.769	425.217	206.720	0.446	0.000	-0.446	-0.003	-3.676	-632.791	-426.070
24	1369.149	4.905	-0.249	-0.575	96.782	-1.301	-1.628	16.081	0.843	0.517	298.317	425.218	207.603	0.446	0.000	-0.446	0.000	0.247	-632.546	-424.944
25	1374.890	4.901	-0.236	-0.563	98.849	-1.289	-1.615	16.568	0.857	0.530	302.501	425.220	205.001	0.447	0.000	-0.447	-0.001	-1.115	-633.664	-428.663

As delta of an option being the rate of change of the option price with respect to the price of the underlying asset, assuming no arbitrage, it means that the delta for the security should be consistent with that of its replicated portfolio.

Therefore, we obtained the security's delta value D through computing the delta value for its replicated portfolio. And then short D shares of index to achieve delta neutral.

The number of shares will be adjusted over time to ensure zero sensitivity to stock price change.

Interesting Features in the Issued Security

The security is suitable for the investors who prepared to hold the security to maturity over the 5-year investment horizon and do not expect to regular payments. The minimum investment is 50 securities or US $5000. The principal amount per security is US$100, so is the minimum increment of the investment.

The initial index level is the closing index level on the fourth Exchange Day immediately preceding the issue date of the securities. The final index level is the closing index level on the third Exchange Day immediately preceding the maturity date of the securities.

Factors that Influence the Pricing on the Securities

Change of Factor	Securities
Increase in Index level	↑
Decrease in time to maturity	↑
Increase in volatility	↓
Increase in U.S. interest rates	↓
Increase in dividend/income yield	↓
Increase in Bank's credit rating	↑

Tax Treatment of Capital Gain and Losses

The taxable capital gain income equals to one-half of any capital gain realized, whereas one-half of any capital loss incurred will constitute an allowable capital loss that is deductible against taxable capital gains of the Resident Holder. Also, noting the security is eligible for RRSPs, RRIFs, RESPs, RDSPs, DPSPs and TFSA, in which case the Resident Holder can arrange the tax expenses of the year accordingly.

Risk Factors

The first risk is credit risk. Since structured notes are an IOU from the issuer, the investors bear the risk that the investment bank forfeits on the debt. A structured note adds a layer of credit risk on top of the market risk. Market risks are vital to address in this scenario. The partially protected security has large exposure to the large-cap segment of the US equity market. The amount being repaid on the maturity date is not fixed, which means the return could be positive or negative. Despite there are 30% buffers on the price drop, the principal amount of the security is still fully exposed, and the investor could lose a significant amount on the investment. Finally, the last risk to consider is liquidity risk. According to the prospect, we notice that the securities will not be listed on any stock exchange. It may be resold using the FundSERV network, however there is no assurance that a secondary market will develop or be sustained. In case of resale, the price is determined at the time of sale by the Calculation Agent. The price is likely to be lower than the Principal Amount and it will be subject to specified early trading charges, depending on the timing. Noted the bank may have the right to redeem or "call" the securities prior to their maturity, which terminate the investor's entitlement to any appreciation in index level or any regular payment of interest or principal.

Possible Hedging Strategy for the Investors

Investor is likely to hedge the situation when the price drops below 30% of the index. There are two methods to address this. The first method is using the completely covered position, so that the bank's payoffs are completely offset. The negative payoff are hedged through a combination of shorting two call options at $992.08 and $1998.34, respectively, and lending an amount equals to a future value of $425.18. The second method is hedging the negative payoff using short put position with a strike price of $992.08. The credit risk of the security coming from the issuing bank RBC, although we believe it is extremely unlikely, in the case of default; investor could hedge it by utilizing credit insurance or default swap.

Conclusion

In conclusion, the RBC security is a sound investment that given the thorough analysis described above. The Black Scholes model has showcased similar numbers with little variances allowing one to understand the power of this model. There may be other models that would be able to get closer to the actual amount; however, this is difficult to overcome without the assumptions that have been outlined. Although there are various risk factors associated with this investment it is a rare occurrence and therefore the suggestion is to take part in this security. Moreover, in order to ensure success it is necessary to follow the various recommendations outlined above as well as understanding the precautions outlined. Overall, the RBC investment has showcased the probability of a positive investment.

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