Estimating Earnings Adjustment Frictions: Method and Evidence from the Social Security Earnings Test

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Abstract

Recent literature has documented that individuals face frictions in adjusting their earnings with respect to policy, but this literature has not yet developed a method for estimating earnings adjustment costs. We introduce a method for estimating the cost of adjusting earnings, as well as the elasticity of earnings with respect to the incentive to earn. Our method uses information on the amount of bunching in the earnings distribution at convex budget set kinks before and after policy changes in the earnings incentives around the kinks: the larger is the adjustment cost, the smaller is the absolute change in bunching from before to after the policy change. We apply this method in the context of the convex kink created by the Social Security Annual Earnings Test (AET). Using a one percent sample of earnings histories from Social Security Administration micro-data, we show that individuals continue to bunch at the kink formerly created by the AET even when they are no longer subject to the AET, indicating that they must face adjustment frictions. We estimate in a baseline case that the earnings elasticity with respect to the implicit net-of-tax share is 0.35, and the fixed cost of adjustment is around $280. Our results demonstrate that the short-run impact of changes in the effective marginal tax rate can be substantially attenuated, which can inform projections of the timing of earnings responses to tax and transfer policies.
1 Introduction

In a traditional model of workers’ earnings or labor supply choices, individuals optimize their behavior frictionlessly. Recently, several papers have found that individuals face frictions in adjusting their behavior with respect to policy (Chetty, Looney, and Kroft, 2009; Chetty, Friedman, Olsen, and Pistaferri, 2011; Chetty, Guren, Manoli, and Weber, 2012; Chetty, 2012; Chetty, Friedman, and Saez, 2013; Kleven and Waseem, 2013). Adjustment frictions, which we interpret broadly to encompass factors preventing individuals from adjusting their earnings, may reflect a variety of elements including a lack of knowledge of a tax regime, the cost of negotiating a new contract with an employer, or the time and financial cost of job search. These frictions could impede immediate or long-term adjustments to tax policy changes. Such adjustment frictions can also affect the welfare consequences of taxation. For example, if taxes are not fully salient, this must be measured to calculate the welfare costs of taxation (Chetty et al., 2009; Farhi and Gabaix, 2015). Adjustment frictions also help to explain heterogeneity across contexts in the observed elasticity of earnings with respect to the net-of-tax rate (Chetty et al., 2011, 2012b; Chetty, 2012). Frictions in adjusting earnings may underlie other patterns in the data, such as the slow rise in retirement at age 62 subsequent to the introduction of the Social Security Early Retirement Age (Gruber and Wise, 2013), or the lack of “bunching” in the earnings distribution at many convex kink points in budget sets that lead to an incentive for many to locate near the kink (Chetty et al., 2011). Policy-makers often wish to estimate the magnitude and timing of the earnings or labor supply reaction to changes in tax and transfer policies (e.g. Congressional Budget Office, 2009), which could be greatly affected by adjustment frictions. However, the existing literature has not yet developed a method for estimating earnings adjustment costs.

We make three main contributions to understanding adjustment frictions in the earnings context. First, we introduce a method for documenting earnings adjustment frictions. In the absence of adjustment frictions, the removal of a convex kink in the effective tax schedule should result in the immediate dissolution of bunching at the former kink; thus, any observed delay in reaching zero bunching should reflect adjustment frictions.

Second, formalizing and generalizing this insight, we specify a model of earnings adjustment that allows us to estimate a fixed adjustment cost and the elasticity of earnings with respect to the effective net-of-tax rate. Adding adjustment frictions to the model of Saez (2010), we develop tractable methods that allow the estimation of elasticities and adjustment costs with kinked budget sets. When tax rates change around a kink in our framework, ceteris paribus the absolute change in the amount of bunching is decreasing in the adjustment cost, while the initial amount of bunching is increasing in the elasticity. Kinked budget sets are common across a wide variety of economic applications, including labor supply (e.g. Hausman, 1981), electricity demand (e.g. Ito, 2014), health insurance (e.g. Einav, Finkelstein, and Schrimpf, 2015), and retirement savings (e.g. Bernheim, Fradkin, and Popov, 2015). We suggest that our method for estimating

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1The net-of-tax rate is defined as one minus the marginal tax rate (MTR). Labor economics literature examines hours constraints in the context of labor supply (e.g. Cogan, 1981; Altonji and Paxson, 1990, 1992; Dickens and Lundberg, 1993).
elasticities and adjustment costs applies not only in the case of earnings, but also in the case of demand for other commodities.

Our method complements Kleven and Waseem (2013), who innovate a method to estimate elasticities and the share of the population that is inert in the presence of a notch in the budget set (but do not estimate adjustment costs). To our knowledge, our method is the first to allow the estimation of both elasticities and adjustment costs using bunching in earnings. The elasticity and adjustment cost are both necessary for welfare calculations in many applications (Chetty et al., 2009), as the adjustment cost can be used to perform counterfactual exercises across different contexts, whereas the inert share can vary across populations even holding the adjustment cost fixed. Our method is also applicable in an additional context to that of Kleven and Waseem (2013): the basic version of our method relies on bunching at kinks to perform the estimates. Indeed, such kinks are common, and Kleven’s (2016) survey of bunching papers argues that our method can be broadly applicable in such contexts. In fact, the method we develop has been used subsequently by follow-up papers to estimate elasticities in other tax and transfer contexts involving kinks (He, Peng, and Wang, 2016; Schächtele 2016), and has also been applied as the sole existing method to recover point estimates of adjustment costs in the context of notches (Gudgeon and Trenkle, 2016; Zaresani, 2016). Our method of documenting adjustment frictions – showing that bunching persists even when a policy no longer applies – has also been used in Mortenson et al. (2017). Finally, we present a dynamic version of our model, which extends current bunching techniques beyond the typical static approach and allows us to address gradual adjustment in bunching over time. Einav, Finkelstein, and Schrimpf (2015, 2017) also model dynamics in the context of bunching at kinks created by health insurance contracts, but they capture bunching patterns without explicitly modeling adjustment costs in their setting.

Our method builds on insights in Chetty (2012), who derives bounds on the “structural” elasticity as a function of the elasticity observed empirically, the size of the price change used for identification, and the degree of optimization frictions. Chetty (2012) uses these theoretical results, in combination with estimates of observed elasticities from prior empirical literature, to calculate bounds on the structural labor supply elasticity given an assumption about the utility cost of ignoring policy, but that paper does not document frictions or introduce methods for estimating structural elasticities and adjustment costs. That exercise naturally leads to the question of how to estimate structural elasticities and adjustment costs using data; our paper is the first in this literature to introduce a method that accomplishes this. Similarly, Chetty et al. (2011) model earnings adjustment frictions and document evidence consistent with their existence, but they do not estimate adjustment costs or structural elasticities, or develop methods for doing so.

Third, we apply our methods to document and estimate earnings adjustment costs, as well as elasticities. The U.S. Social Security Annual Earnings Test (AET) represents a promising environment for studying these questions, providing an illustration of our broadly applicable methods. The AET reduces Social Security Old Age and Survivors Insurance (OASI) benefits in a given year as a proportion of an OASI claimant’s earnings above an exempt amount in that year. For example, for OASI claimants aged 62 to 65 in 2017, current
OASI benefits are reduced by one dollar for every two dollars earned above $16,920. The AET may lead to large benefit reduction rates (BRRs) on earnings above the exempt amount, creating a strong incentive for many individuals to bunch at the convex budget set kink at the exempt amount (Burtless and Moffitt, 1985; Friedberg, 1998, 2000; Song and Manchester, 2007; Engelhardt and Kumar, 2014).

The AET is an appealing context for studying earnings adjustment for at least three reasons. First, bunching at the AET kink is easily visible on a graph, allowing credible documentation of behavioral responses. Second, the AET represents one of the few known kinks at which bunching in earnings (outside self-employment earnings) occurs in the U.S. Indeed, our paper represents the first study to find robust evidence of sharp bunching among the non-self-employed at any kink in the U.S. — though bunching is common in other contexts in which our method could also be applied (e.g. notches at which our method can be adapted as in the citations above, earnings kinks in other countries as in Chetty et al., 2011, or business taxes as in Mahon and Zwick, 2017). Third, the AET is important to policy-makers in its own right, as it is a significant factor affecting the earnings of older Americans. The importance of the AET is now increasing as the NRA gradually rises from 65 for those born in 1937 and earlier, to 67 for those born in 1960 and later, exposing more OASI claimants to the AET. Even when the NRA was 65.5 for the youngest OASI primary beneficiaries in the latest year of the available micro-data in 2003, the AET led to a total of $4.3 billion in current benefit reductions for around 538,000 beneficiaries, thus substantially affecting benefits and their timing.

Using Social Security Administration (SSA) administrative tax data on a one percent sample of the U.S. population, we begin by documenting clear evidence of adjustment frictions: after individuals no longer face the AET after age 69 in 1990 to 1999, they continue to bunch around the location of the former exempt amount. This demonstrates that earnings adjustment frictions exist in the U.S. Next, we apply our estimation method to data spanning the decrease in the AET BRR from 50 percent to 33.33 percent in 1990 for those aged 66 to 69, as well as moving from age 69 to ages 70 and older in 1990 to 1999. In a baseline specification examining the 1990 change, we estimate that the fixed adjustment cost is around $280 (in 2010 dollars). We also estimate that the earnings elasticity with respect to the net-of-tax share is 0.35. This specification examines data on individuals in 1989 and 1990; thus, our estimated adjustment cost represents the cost of adjusting earnings in the year of the policy change. Other strategies—including the version of our method that allows for gradual, dynamic adjustment—show results in the same range. Immediately following the reduction in the kink in 1990, if we constrain the adjustment cost to be zero and thus fail to account for excess bunching following the policy change due to inertia, we instead estimate a statistically significantly higher earnings elasticity of 0.58. The constrained estimate is 66 percent higher than the unconstrained estimate.

Many other papers have examined bunching in the earnings schedule, including Blundell and Hoyes (2004) and Saez (2010).


For consistency with the previous literature on kink points that has focused on the effect of taxation, we sometimes use “tax” as shorthand for “tax-and-transfer,” while recognizing that the AET reduces Social Security benefits and is not administered through the tax system. The “effective” marginal tax rate is potentially affected by the AET BRR, among other factors.
Our estimates suggest that although adjustment costs are modest in our setting, they have the potential to change earnings elasticity estimates significantly, illustrating that it can be important to incorporate adjustment costs when estimating earnings elasticities. It is particularly striking that we find evidence of adjustment frictions even among those initially bunching at kinks, whose initial bunching may indicate flexibility. By demonstrating that earnings adjustment frictions exist and substantially change elasticity estimates even in this setting, our results suggest the importance of taking them into account in other settings.

Moreover, our results show that even modest adjustment costs can have important implications for the timing of earnings reactions even to large policy changes. Simulations based on our elasticity and adjustment cost estimates show that the adjustment frictions we estimate can greatly attenuate the short-run earnings reaction even to a large change in the effective marginal tax rate, frustrating the goal of affecting short-run earnings as envisioned in many recent discussions of tax policy. This could have implications, for example, for policy-makers’ projections of the time frame over which earnings choices may respond to changes in tax and transfer policies.

Our paper follows a large existing literature on adjustment costs in areas outside labor and public economics. For example, adjustment costs have long been studied in inventory theory (e.g. Arrow et al., 1951, and subsequent literature), macroeconomics (e.g. Baumol, 1952, and subsequent literature), firm investment (e.g. Abel and Eberly, 1994), durable good consumption (e.g. Grossman and Laroque, 1990), pricing and inflation (e.g. Sheshinski and Weiss, 1977), and other settings. Relative to this literature, we make several contributions. First, we explore these issues in the context of earnings determination, and we exploit changes in policies creating effective tax rates. Second, we offer new, transparent evidence for earnings adjustment frictions by showing that bunching persists after a kink has been removed. This method for documenting the presence of adjustment frictions is broadly applicable in other economic contexts in which budget set kinks are removed. Such non-linear budget set kinks occur not only in the context of earnings determination but have also been studied in other economic applications with non-linear pricing, including electricity demand (e.g. Reiss and White, 2005) or subsidized retirement savings (Bernheim, Fradkin, and Popov, forthcoming). Third, we introduce methods for estimating adjustment costs and elasticities by exploiting bunching at non-linear budget set kinks. We argue that the changes in bunching at kinks in response to changes in incentives can be transparently mapped to elasticity and adjustment cost estimates.

The primary focus of the paper relates to developing methods for studying adjustment frictions, illustrating their implementation, and drawing insights from this. A secondary contribution is to provide new evidence on the effects of the AET in particular. We use SSA data with a main sample of 376,431 observations, building on previous studies of the AET that use survey data. Our study is the first to estimate bunching at the AET kink through a method similar to Saez (2010) or Chetty et al. (2011).

\[5\] Within public finance, Marx (2015) also examines bunching in the reporting of revenue by charities within a dynamic context, while Werquin (2015) derives a continuous-time, \"s-S\" model of taxable earnings in the presence of adjustment costs. The former paper does not feature adjustment frictions, while the latter abstracts from kinks in the tax schedule and bunching in earnings.
The remainder of the paper is structured as follows. Section 2 describes the policy environment. Section 3 describes our empirical strategy for quantifying bunching. Section 4 describes our data. Section 5 presents empirical evidence on the earnings response to changes in the AET. Section 6 specifies a tractable model of earnings adjustment. Section 7 estimates the fixed adjustment cost and elasticity simultaneously. Section 8 uses these estimates to demonstrate through simulations that due to the adjustment frictions we estimate, even large changes in the effective marginal tax rate can lead to little short-run response. Section 9 concludes.

2 Policy Environment

Figure 1 shows key features of the AET rules from 1961 to 2009. The AET became less stringent over this period. The dashed lines and right vertical axis show the BRR. From 1961 to 1989, an additional dollar of earnings above the exempt amount reduced OASI benefits by 50 cents (until OASI benefits reached zero). In 1990 and after, the BRR fell to 33.33 percent for beneficiaries at or older than the Normal Retirement Age (NRA); this change had been scheduled since the 1983 Social Security Amendments. The NRA, the age at which workers can claim their full OASI benefits, is 65 for those in the 1983 to 1999 period that we focus on. During this period, the AET applied to OASI beneficiaries aged 62-69. The solid lines and left vertical axis show the real exempt amount. Starting in 1978, beneficiaries younger than NRA faced a lower exempt amount than those at NRA or above.

When current OASI benefits are lost to the AET, future scheduled benefits are increased in some circumstances, which is sometimes called “benefit enhancement.” This can reduce the effective tax rate associated with the AET. For beneficiaries subject to the AET aged NRA and older, a one percent Delayed Retirement Credit (DRC) was introduced in 1972, meaning that each year of foregone benefits led to a one percent increase in future yearly benefits. The DRC was raised to three percent in 1982 and gradually rose to eight percent for cohorts reaching NRA from 1990 to 2008 (though the AET was eliminated in 2000 for those older than the NRA). An increase in future benefits between seven and eight percent is approximately actuarially fair on average, meaning that an individual with no liquidity constraints and average life expectancy should be indifferent between either claiming benefits now or delaying claiming and receiving higher benefits once she begins to collect OASI (Diamond and Gruber, 1999).

OASI claimants’ future benefits are only raised due to the DRC when annual earnings are sufficiently high that the individual loses an entire month’s worth of OASI benefits due to the reductions associated with the AET (Friedberg, 1998; Social Security Administration, 2012a). In particular, an entire month’s benefits are lost—and benefit enhancement occurs—one the individual earns $z^* + (MB/\tau)$ or higher, where $z^*$ is the annual exempt amount, $MB$ is the monthly benefit, and $\tau$ is the AET BRR. With a typical monthly benefit of $1,000 and a BRR of 33.33 percent, one month’s benefit enhancement occurs when the individual’s annual earnings are $3,000 (= $1,000/0.3333) above the exempt amount. As a result, benefit enhancement is only relevant to an individual considering earning substantially in excess of the exempt amount. Although the

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6Prior to 1983, the AET applied to beneficiaries aged 62 to 71.
AET withholds benefits at the monthly level, the AET is generally applied based on annual earnings—the object we observe in our data. We model the AET as creating a positive implicit marginal tax rate for some individuals—reflecting the reduction in current benefits—consistent with both the empirical finding that some individuals bunch at AET kinks and with the practice in previous literature.

For individuals considering earning in a region well above the AET exempt amount, thus triggering benefit enhancement, the AET could also affect decisions for several reasons. The AET was roughly actuarially fair only beginning in the late 1990s. Furthermore, those whose expected life span is shorter than average should expect to collect OASI benefits for less long than average, implying that the AET is more financially punitive. Liquidity-constrained individuals or those who discount faster than average could also reduce work in response to the AET. Finally, many individuals also may not understand the AET benefit enhancement or other aspects of OASI (Liebman and Luttmer 2012; Brown, Kapteyn, Mitchell, and Mattox, 2013). We follow previous work and do not distinguish among these potential reasons for a response to the AET in our main analysis (Gelber, Jones, and Sacks 2013 analyze certain reasons for the response).

For beneficiaries under NRA, the actuarial adjustment raises future benefits whenever an individual earns over the AET exempt amount (Social Security Administration, 2012, Section 728.2; Gruber and Orszag, 2003), by 0.55 percent per month of benefits withheld. Thus, beneficiaries in this age range do not face a pure kink in the budget set at the exempt amount. To address this, we limit the sample to ages above NRA in our estimates of elasticities and adjustment costs.

3 Initial Bunching Framework

As a preliminary step, we begin with a model with no frictions to illustrate our technique for estimating bunching, which is also suited to measuring the bunching at the kink arising in a model with frictions. This model is well-known and described in detail in Saez (2010), but we briefly describe it in preparation for our initial descriptive evidence. Agents maximize utility \( u(c, z; a) \) over consumption \( c \) and pre-tax earnings \( z \) (where greater earnings are associated with greater disutility due to the cost of effort), subject to a budget constraint \( c = (1 - \tau) z + R \), where \( R \) is virtual income. Agents can adjust earnings, for example, through a change in hours worked or effort, or in principle through a change in the reporting of earnings. The first-order condition, \( (1 - \tau) u_c + u_z = 0 \), implicitly defines an earnings supply function \( z((1 - \tau), R; a) \).

The parameter \( a \) reflects heterogeneous “ability,” i.e. the trade-off between consumption and earnings supply. Following previous literature, we assume rank preservation in earnings as a function of \( a \). Thus, \( a \) is isomorphic to the level of earnings that would occur in the absence of any tax. For example, if we assume a standard isoelastic and quasilinear utility function, \( u(c, z; a) = c^{(1 + 1/\epsilon)} (z/a)^{1/\epsilon} \), the optimal level of earnings is \( z((1 - \tau), R; a) = a (1 - \tau)^{\epsilon} \). Therefore, when \( \tau = 0 \), we have \( z = a \).
according a smooth CDF. Under a constant marginal tax rate of \( \tau_0 \), this implies a smooth distribution of earnings \( H_0(\cdot) \), with pdf \( h_0(\cdot) \).

Starting with a linear tax at a rate of \( \tau_0 \), suppose the AET is additionally introduced, so that the marginal net-of-tax rate decreases to \( 1 - \tau_1 \) for earnings above a threshold \( z^* \), where \( \tau_1 > \tau_0 \). Individuals earning in the neighborhood above \( z^* \) reduce their earnings. If ability is smoothly distributed, a range of individuals initially locating between \( z^* \) and \( z^* + \Delta z^* \) will “bunch” exactly at \( z^* \), due to the discontinuous jump in the marginal net-of-tax rate at \( z^* \). In practice, previous literature finds empirically that individuals locate in the neighborhood of \( z^* \) (rather than exactly at \( z^* \)).

To quantify the amount of bunching, i.e. “excess mass,” we use a technique similar to Chetty et al. (2011) and Kleven and Waseem (2013). For each earnings bin \( z_i \) of width \( \delta \) we calculate \( p_i \), the proportion of all people with annual earnings in the range \([z_i - \delta/2, z_i + \delta/2] \). We then estimate the following regression:

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p_i = \sum_{d=0}^{D} \beta_d (z_i - z^*)^d + \sum_{j=-k}^{k} \gamma_j \mathbb{1}\{z_i - z^* = j \cdot \delta\} + u_i
\]

This expresses the annual earnings distribution as a degree \( D \) polynomial, plus a set of indicators for each bin with a midpoint within \( k\delta \) of the kink.

Our measure of excess mass, or bunching, is \( \hat{B} = \sum_{j=-k}^{k} \hat{\gamma}_j \), the estimated excess probability of locating at the kink (relative to the polynomial fit). To obtain a measure of excess mass that is comparable across different kinks, we scale by the counterfactual density at \( z^* \), i.e. \( \hat{h}_0(z^*) = \hat{\beta}_0/\delta \). Thus, our estimate of “normalized excess mass” is \( \hat{b} = \hat{B} / \hat{h}_0(z^*) = \delta \hat{B} / \hat{\beta}_0 \). In our empirical application, we choose \( D = 7 \), \( \delta = 800 \) and \( k = 4 \) as a baseline, implying that our estimate of bunching is driven by individuals with annual earnings within $3,600 of the kink. We also show our results under alternative choices of \( D \), \( \delta \), and \( k \). We estimate bootstrapped standard errors.

4 Data

We use a one percent random sample of Social Security numbers from the restricted-access Social Security Administration Master Earnings File (MEF), linked to the Master Beneficiary Record (MBR). The data contain a complete longitudinal earnings history with information on earnings in each calendar year since 1951; year of birth; the year (if any) that claiming began; date of death; and sex (among other variables). Separate information is available on self-employment earnings and non-self-employment earnings. Starting in 1978, the earnings measure reflects total wage compensation, as reported on Internal Revenue Service forms. In a calendar year, “age” is defined as the highest age an individual attains in that calendar year.

In choosing our main sample, we take into account a number of considerations. It is desirable to show a constant sample in making comparisons of earnings densities. Meanwhile, the AET only affects people who claim OASI, and thus we wish to focus on claimants. However, many individuals claim OASI at ages older than the Early Entitlement Age (EEA) of 62. Thus, to investigate a constant sample, we cannot simply limit the sample to claimants at each age, as many people move from not claiming to claiming. To balance these
considerations, our main sample at each age and year consists of individuals who have ultimately claimed at an age less than or equal to 65. In our main analysis we exclude person-years with positive self-employment income. Because we focus on the intensive margin response, in our main analysis we further limit the sample in a given year to observations with positive earnings in that year.\footnote{We explore extensive margin decisions in Gelber, Jones, Sacks, and Song (2016).}

Several features of the data are notable. First, these administrative data allow large sample sizes and are subject to little measurement error. Second, earnings (as measured in the dataset) are taken from W-2 tax forms and are not subject to manipulation through tax deductions, credits, or exemptions. Third, because earnings are taken from W-2s, they are subject to third-party reporting among the non-self-employed. Fourth, the data do not contain information on hours worked or job amenities.

Table 1 shows summary statistics in our main sample, 62 to 69 year-olds in 1990 to 1999. The sample has 376,431 observations. 57 percent of the sample is male. Median earnings, $14,555.56, is not far from the AET exempt amount, which averages $16,738 for those NRA and older and $11,650 for those younger than NRA over this period. Mean earnings (conditional on positive earnings) is $28,892.63.

Our second data source is the Longitudinal Employer Household Dynamics (LEHD) of the U.S. Census (Abowd \textit{et al.}, 2009), which longitudinally follows workers’ earnings. In covered states, the data have information on around nine-tenths of workers and their employers. We are only able to use data on a 20 percent random subsample of these individuals from 1990 to 1999. We use these data primarily because the sample size in the LEHD is much larger than in the SSA data. We use the LEHD only in the context of one figure for which the larger sample is helpful; all of the other analysis is based on the SSA data.\footnote{The LEHD lacks information on whether a given individual is claiming OASI, but the importance of this is limited because we use the LEHD to study earnings from ages 69 to 71. In our SSA data, 94 percent of individuals claim by age 69.}

5 Documenting Earnings Adjustment Frictions

We first examine the pattern of bunching across ages, documenting several pieces of evidence for adjustment frictions. We focus on the period 1990 to 1999, when the AET applied from ages 62 to 69. The policy changes at ages 62 and 70—when the AET is imposed and removed, respectively, for OASI claimants—would be anticipated by those who have knowledge of the relevant policies. Figure 2 Panel A plots earnings histograms for each age from 59 to 73 (connected dots), along with the estimated smooth counterfactual polynomial density (smooth line). Earnings are measured along the x-axis, relative to the exempt amount, which is shown by a vertical line. For ages younger than 62, we define the (placebo) kink in a given year as the kink that applies to pre-NRA individuals in that year. For individuals 70 and older, we define the (placebo) kink in a given year as the kink that applies to post-NRA individuals in that year.

Figure 2 Panel A shows clear visual evidence of substantial bunching from ages 62 to 69, when the AET applies to claimants’ OASI benefits, and no excess mass at earlier ages. At ages 70 and 71, which are not subject to the AET, there is still clear visual evidence of bunching in the region of the kink. Figure 2 Panel B plots the estimates of normalized excess mass at each age. Bunching is statistically significantly different
from zero at each age from 62 to 71 ($p < 0.01$ at all ages). Normalized excess mass rises from 62 to 63 and remains around this level until age 69 (with a dip at age 65 that we discuss below). We estimate that there is substantial excess mass at ages 70 and 71, which are not subject to the AET. Thus, “de-bunching” does not occur immediately for some individuals, where “de-bunching” refers to movement away from the former kink among those initially bunching at the kink.

Figure 3 shows spikes near the exempt amount in the mean percentage change in earnings from ages 69 to 70 and 70 to 71, consistent with de-bunching from age 69 to 70, and from age 70 to 71, among those initially near the kink. This shows that bunchers are returning to higher earnings, as predicted by theory, and that this process continues at least until age 71. It is striking that we document adjustment frictions even among the group bunching prior to age 70, who were evidently able to adjust earnings to the kink initially.

We classify claimants as age 70 when they attain age 70 during that calendar year. As a result, some individuals will be classified as age 70 but will have been subject to the AET for a portion of the year (in the extreme case of a December 31 birthday, for all but one day). In principle, this is one potential explanation for continued bunching at age 70 that does not rely on earnings adjustment frictions. However, other evidence is sufficient to document earnings adjustment frictions, namely: (1) the continued bunching at age 71, which cannot be explained through the coarse measure of age; (2) the continued adjustment away from the kink from age 70 to age 71 documented in Figure 3 Panel B; and (3) the spike in the elasticity estimated using the Saez (2010) approach in 1990, documented in Figure 9 and explained below. Moreover, Appendix Table B.1 shows that those born in January to March—who are subject to the AET for only a small portion of the calendar year when they turn age 70—also show statistically significant bunching at ages 70 ($p<0.05$) and 71 ($p<0.10$) from 1983 to 1999. Finally, when we pool data from 1983 to 1999 in Figure 4—giving us more power than in our baseline sample over 1990 to 1999 when the AET does not change—bunching above age 70 is even more visually apparent, and excess mass at age 71 is highly significant and clearly positive.

The data also show additional patterns consistent with adjustment frictions. Figure 2 Panel B shows that bunching is substantially lower at age 65 than surrounding ages. The location of the kink changes substantially from age 64 to age 65; as Figure 1 shows, during this period the exempt amount is much higher for individuals NRA and older than for individuals younger than NRA. Individuals may have difficulty adjusting to the new location of the kink within one year. This delay suggests that individuals also face adjustment frictions in this context. This interpretation of the patterns around ages 64 and 65 is consistent with Figure 5, which shows that conditional on age 64 earnings near the age 64 exempt amount, the age 65 earnings density shows a large spike at the kink that prevailed at age 64 and a smaller spike at the current, age 65 kink. Also, conditional on age 65 earnings near the age 65 exempt amount, the density of age 64 earnings shows a spike near the exempt amount for age 64.

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13 Figure 2 shows that more individuals appear to “bunch” below the exempt amount than above it. Appendix A.1 explains using simulations how this can arise under our parameter estimates in combination with a downward sloping counterfactual earnings distribution similar to what we observe in our data. This is corroborated by a simulated distribution presented in Appendix Figure B.1.

14 Limiting the sample only to those born in January yields insignificant and imprecise results.

15 In principle, our coarse measure of age could affect these patterns: individuals turning 65 in a given calendar year face the age-65 exempt amount for only the part of the calendar year after they turn 65, which could serve as a partial explanation for continued
In our context, the only “appearance” of a new kink that we observe is the appearance of a kink at age 62. The amount of time since the appearance of the kink at age 62 is correlated with age, and elasticities and adjustment costs could also be correlated with age—thus confounding analysis of the time necessary to adjust to the appearance of a kink. While recognizing this caveat, it is worth noting that the amount of bunching slowly rises from age 62 to 63, again suggesting gradual adjustment.\textsuperscript{16}

We interpret the continued bunching at ages 70 and 71 as reflecting frictions preventing adjustment. If this is the case, those bunching after the kink is removed should have been bunching prior to the removal (and those bunching before the kink is removed should be disproportionately represented among those bunching after the removal). A degree of such inertia has already been shown at ages 64 and 65 in Figure 5. Figure 6 further shows that indeed, conditional on earnings at ages 70 or 71 within $1,000 of the exempt amount, the density of earnings at age 69 spikes at the exempt amount (and conditional on earnings at age 69 within $1,000 of the exempt amount, the density of age 70 or age 71 earnings spikes near the exempt amount).

Each of these several pieces of evidence points to adjustment frictions. In Appendix Table B.2, we probe the robustness of our results by varying the bandwidth, the degree of the polynomial, and the excluded region. We also conduct several additional analyses in Gelber, Jones, and Sacks (2013), including varying the time period examined. Overall, these additional analyses generally show similar patterns.

We find no evidence of adjustment in anticipation of future changes in policy, as those younger than 62 do not bunch. If the cost of adjustment in each year rose with the size of adjustment and this relationship were convex, we would expect anticipatory adjustment. Other literature on earnings adjustment frictions has shown that firms are important in coordinating bunching responses to taxation in Denmark (Chetty \textit{et al.}, 2011), by documenting bunching among individuals not subject to the taxes. In our context, individuals younger than 62—who are not subject to the AET—do not show noticeable bunching at the kink, nor do those 72 and older (Figure 2). Although we cannot rule out that firms play some role in our context, the available evidence in our sample does not directly support this hypothesis. Thus, we interpret continued bunching at ages 70 and 71 as relating to individuals\textsuperscript{\textsuperscript{1}}—not firms\textsuperscript{\textsuperscript{1}}—choices.

6 Method for Estimating Elasticities and Adjustment Costs

The results thus far suggest a role for adjustment frictions in individuals’ earnings choices. To develop a method to estimate such adjustment costs as well as earnings elasticities, we build upon the frictionless Saez (2010) model described in Section 3. There we considered a transition from a linear tax schedule with a constant marginal tax rate (MTR) $\tau_0$ to a schedule with a convex kink, where the MTR below the kink earnings level $z^*$ is $\tau_0$, and the MTR above $z^*$ is $\tau_1 > \tau_0$. We refer to this kink at $z^*$ as $K_1$. Next, as in our empirical context, consider a decrease in the higher MTR above $z^*$ to $\tau_2 < \tau_1$. We refer to this less bunching at age 65 at the exempt amount applying to age 64. However, we would then expect the age 64 and age 65 exempt amounts to display equal amounts of bunching, which is not the case.

\textsuperscript{16}In principle, this could also relate to the fact that these graphs show the sample of those who have claimed by age 65, and the probability of claiming at a given age (conditional on claiming by age 65) rises from age 62 to 63. To address this issue, Appendix Figure B.2 shows that when the sample at a given age consists of those who have claimed by that age, we still find a substantial increase in bunching from 62 to 63.
sharply bent kink as $K_2$. In the presence of a kink $K_j$ with marginal tax rate $\tau_j$ below $z^*$ and $\tau_j$ above $z^*$, $j \in \{1, 2\}$, the share of individuals bunching at $z^*$ in the frictionless model will be:

$$B_j^* = \int_{z^*}^{z^* + \Delta z_j^*} h_0(\zeta) d\zeta$$

(2)

For small tax rate changes, we can relate the elasticity of earnings with respect to the net-of-tax rate to the earnings change $\Delta z_j^*$ for the individual with the highest ex ante earnings who bunches ex post:

$$\varepsilon = \frac{\Delta z_j^*/z^*}{d\tau_j/(1 - \tau_j)}$$

(3)

where $d\tau_j = \tau_j - \tau_0$ and $\varepsilon$ is the elasticity of pre-tax earnings with respect to the net-of-tax rate, $\varepsilon \equiv - (\partial z/z) / (\partial \tau/(1 - \tau))$.

6.1 Fixed Cost of Adjustment

We now extend the model to include a fixed cost of adjusting earnings. Following recent public finance literature on bunching including Saez (2010) and Kleven and Waseem (2013), our model is stylized to illustrate the relevant forces as transparently as possible. We assume that to change earnings from an initial level, individuals must pay a fixed utility cost of $\phi$, similar to Chetty et al. (2011). This could represent the information costs associated with navigating a new tax-and-transfer regime if, for example, individuals only make the effort to understand their earnings incentives when the utility gains from doing so are sufficiently large (e.g. Simon, 1955; Chetty et al., 2007; Hoopes, Reck, and Slemrod, 2015). Alternatively, this cost may represent frictions such as the cost of negotiating a new contract with an employer or the time and financial cost of job search, assuming that these costs do not depend on the size of the desired earnings change.

Our model of fixed costs relates to labor economics literature on constraints on hours worked, as well as public finance literature that explores frictions in earnings. One common feature of models of earnings frictions in labor economics (e.g. Cogan, 1981; Altonji and Paxson, 1990, 1992; Dickens and Lundberg, 1993) and public finance (e.g. Chetty et al., 2011; Chetty, 2012) is that the decision-making setting is generally static. We begin by adopting this modeling convention.

There is an extensive literature on fixed costs and adjustment in other fields, including the “s-S” literature (see literature reviews in Leahy, 2008, or Stokey, 2008). In “s-S” models, agents adjust behavior when the value of a state variable falls outside a “region of inactivity” around its “target” level, within a dynamic optimization problem. Our model shares a similar process, but unlike existing literature, we develop our model in the context of kinked budget sets and the determination of earnings.

6.2 Bunching in a Single Cross-Section with Adjustment Costs

Figure 7 Panel A illustrates how a fixed adjustment cost attenuates the level of bunching, relative to equation (2), and obscures the estimation of $\varepsilon$ in a single cross-section that is possible in the Saez (2010) model. The figure shows the budget set before and after the kink, $K_1$, is introduced, as well as indifference curves.
through key earnings levels. Consider the individual at point 0, who initially earns \( z_1 \) along the linear budget constraint with tax rate \( \tau_0 \). This individual faces a higher marginal tax rate after the kink is introduced, which increases the marginal tax rate to \( \tau_1 \) above earnings level \( z^* \). Because she faces an adjustment cost, she may decide to keep her earnings at \( z_1 \) and locate at point 1. Alternatively, with a sufficiently low adjustment cost, she incurs the adjustment cost and reduces her earnings to \( z^* \) (point 2).

We assume that the benefit of relocating to the kink is increasing in the distance from the kink for initial earnings in the range \([z^*, z^* + \Delta z_1]\). This requires that the size of the optimal adjustment in earnings increases in \( a \) at a rate faster than the decrease in the marginal utility of consumption.\(^{17}\) This is true, for example, if utility is quasi-linear, which is assumed in related recent public finance literature (e.g. Saez, 2010; Chetty et al., 2011; Kleven and Waseem, 2013; Kleven, Landais, Saez, and Schultz, 2014).

These assumptions imply that above a threshold level of initial earnings, \( z_1 \), individuals adjust their earnings to the kink, and below this threshold individuals remain inert. In Figure 7, this individual is the marginal buncher who is indifferent between staying at the initial level of earnings \( z_1 \) (point 1) and moving to the kink earnings level \( z^* \) (point 2) by paying the adjustment cost \( \phi \).

Panel B of Figure 7 shows that the level of bunching is attenuated due to the adjustment cost. Panel B plots the counterfactual density of earnings, i.e., under a linear tax \( \tau_0 \). Only individuals with initial earnings in the range \([z_1, z^* + \Delta z_1]\) bunch at the kink \( K_1 \) (areas ii, iii, iv, and v)—whereas in the absence of an adjustment cost, individuals with initial earnings in the range \([z^*, z^* + \Delta z_1]\) bunch (areas i, ii, iii, iv, and v). Bunching is given by the integral of the initial earnings density, \( h_0(\cdot) \), over the range \([z_1, z^* + \Delta z_1]\):\(^{18}\)

\[
B_1(\tau_1, z^*; \varepsilon, \phi) = \int_{z_1}^{z^* + \Delta z_1} h_0(\zeta) \, d\zeta,
\]

where \( \tau_1 = (\tau_0, \tau_1) \) measures the tax rates below and above \( z^* \). The lower limit of the integral, \( z_1 \), is implicitly defined by the indifference condition shown in Figure 7, Panel A:

\[
\phi = u((1 - \tau_1)z^* + R_1, z^*; a_1) - u((1 - \tau_1)z_1 + R_1, z_1; a_1)
\]

where \( R_1 \) is virtual income and \( a_1 \) is the ability level of this marginal buncher.\(^{18}\)

Bunching therefore depends on the preference parameters \( \varepsilon \) and \( \phi \), the tax rates below and above the kink, \( \tau_1 = (\tau_0, \tau_1) \), and the density \( h_0(\cdot) \) near the exempt amount \( z^* \). With only one kink and without further assumptions, we cannot estimate both \( \varepsilon \) and \( \phi \), as the level of bunching depends on both parameters.

\(^{17}\)To see this, note that the utility gain from reoptimizing is \( u((1 - \tau_1)z_1 + R_1, z_1; a) - u((1 - \tau_1)z_0 + R_1, z_0; a) \approx u_c \cdot (1 - \tau_1)[z_1 - z_0] + u_c \cdot [z_1 - z_0] = u_c \cdot (\tau_1 - \tau_0)[z_0 - z_1] \), where in the first expression, we have used a first-order approximation for utility at \((1 - \tau_0)z_0 + R_0, z_0\), and in the second expression we have used the first order condition \( u_c = -u_c(1 - \tau_0) \). The gain in utility is approximately equal to an expression that depends on the marginal utility of consumption, the change in tax rates, and the size of the earnings adjustment. The first term, \( u_c \), is decreasing as \( a \) (and therefore initial earnings \( z_0 \)) increases. Thus, in order for the gain in utility to be increasing in \( a \), we need the size of earnings adjustment \([z_0 - z_1]\) to increase at a rate that dominates.

\(^{18}\)The threshold level of earnings \( z_1 \) is an increasing function of \( \phi \). If adjustment costs are large enough, we may have \( z_1 > z^* + \Delta z_1 \), in which case frictions eliminate bunching entirely. Since we observe bunching in our empirical setting, we ignore this case.
6.3 Estimation Using Variation in Kink Size

We can estimate elasticities and adjustment costs when we observe bunching at a kink both before and after a change in $d\tau$, as we observe in our empirical applications. Suppose we observe a population that moves from facing a more pronounced kink $K_1$, with a marginal tax rate $\tau_1$ above $z^*$, to facing a less pronounced kink $K_2$, with a marginal tax rate of $\tau_2 < \tau_1$ above $z^*$. To make progress, we assume that ability $a$ is fixed over time from $K_1$ to $K_2$. Some individuals will remain bunching at the kink, even though they would prefer to move away from the kink in the absence of an adjustment cost, because the gain from de-bunching is not large enough to outweigh the adjustment cost. The fixed adjustment cost therefore attenuates the reduction in bunching, relative to a frictionless case.\footnote{If $d\tau_2 > d\tau_1$ instead – i.e. the kink becomes larger – then additional individuals will be induced to bunch, but the change in bunching will in general still be attenuated (due to the adjustment cost). This is governed by an analogous set of formulas to the case $d\tau_2 < d\tau_1$ that we explore.}

Attenuation in the change in bunching is driven by those in area $iv$ of Panel B in Figure 7. Under a frictionless model, individuals in this range do not bunch under $K_2$. To see this, note that their counterfactual earnings are greater than $z^* + \Delta z_2^*$, i.e. the highest level of initial earnings among bunchers at $K_2$ when there are no frictions. However, when moving from $K_1$ to $K_2$ in the presence of frictions, those in area $iv$ continue to bunch, as shown in Panel C of Figure 7. At point 0, we show an individual’s initial earnings $\tilde{z}_0 \in [z^*; z^* + \Delta z_1^*]$ under a constant marginal tax rate of $\tau_0$. We now introduce the first kink, $K_1$. The individual responds by bunching at $z^*$ (point 1), since $\tilde{z}_0 > \tilde{z}_1$. Next, we transition to the less pronounced kink $K_2$. Since $\tilde{z}_0 > z^* + \Delta z_2^*$, this individual would have chosen earnings $\tilde{z}_2 > z^*$ (point 2) under $K_2$ in a frictionless setting. However, to move to point 2, this individual must pay a fixed cost of $\phi$. We have drawn this individual as the marginal buncher who is indifferent between staying at $z^*$ and moving to $\tilde{z}_2$. Similarly, all individuals with initial earnings in the range $[z^* + \Delta z_2^*, \tilde{z}_0]$ remain at the kink.

Thus, bunching under $K_2$ is:
\[
\tilde{B}_2(\tau_2, z^*; \varepsilon, \phi) = \int_{\tilde{z}_1}^{\tilde{z}_0} h_0(\zeta) d\zeta,
\]
where $\tilde{\tau}_2 = (\tau_0, \tau_1, \tau_2)$ reflects the tax rate below $z^*$, the initial tax rate above $z^*$, and the final tax rate above $z^*$, respectively, and the "\sim" indicates that $K_2$ was preceded by a larger kink $K_1$. The critical earnings levels for the marginal buncher, $\tilde{z}_0$ and $\tilde{z}_2$, are implicitly defined by these conditions: \footnote{We additionally require that $\tilde{z}_0 \leq z^* + \Delta z_1^*$. When this inequality is binding, none of the bunchers move away from the kink at $z^*$ when the kink is reduced from $K_1$ to $K_2$. Since we observe a reduction in bunching in our empirical setting, we ignore this inequality.}

\[
\frac{u_z(c_2; \tilde{z}_2; \tilde{a}_2)}{u_c(c_2; \tilde{z}_2; \tilde{a}_2)} = (1 - \tau_2)
\]
\[
u((1 - \tau_2) \tilde{z}_2 + R_2, \tilde{z}_2; \tilde{a}_2) - u((1 - \tau_2) z^* + R_2, z^*; \tilde{a}_2) = \phi
\]
\[
\frac{u_z(c_0; \tilde{z}_0; \tilde{a}_0)}{u_c(c_0; \tilde{z}_0; \tilde{a}_2)} = (1 - \tau_0).
\]
of earnings that this individual chooses when facing a constant marginal tax rate of $\tau_0$ and no kink. The earnings elasticity is again related to the potential adjustment of the marginal buncher: $\varepsilon = \frac{\tilde{z}_0 - \tilde{z}_2}{\tilde{z}_2} \frac{1 - \tau_0}{d\tau_0}.$

The equations in (4), (5), (6) and (7) together pin down four unknowns ($\Delta z_1^*, \tilde{z}_1, \tilde{z}_0$ and $\tilde{z}_2$), each of which is in turn a function of $\varepsilon$ and $\phi$. Bunching at each kink is therefore jointly determined by $\varepsilon$ and $\phi$. Ultimately, we draw on two empirical moments in the data, $B_1$ and $\tilde{B}_2$, to identify our two key parameters, $\varepsilon$ and $\phi$. We refer to this estimation strategy as the “comparative static method.”

Relative to the frictionless case in the Saez (2010) model, the change in bunching from the more pronounced kink $K_1$ to the less pronounced kink $K_2$ is now attenuated by the adjustment cost. As noted above, in the Saez (2010) model, bunching decreases by areas $iv$ and $v$ in Figure 7 when moving from $K_1$ to $K_2$. When moving sequentially from $K_1$ to $K_2$ in the presence of an adjustment cost, areas $ii$, $iii$, $iv$, and $v$ bunch under $K_1$, whereas areas $ii$, $iii$, and $iv$ bunch under $K_2$. Thus, bunching decreases only by area $v$, rather than by both areas $iv$ and $v$ as in the frictionless case.

The features of the data that help drive our estimates of the elasticity and adjustment cost are intuitive. In the frictionless model of Saez (2010), bunching at a convex kink is approximately proportional to $d\tau$; thus, when $d\tau$ falls in this model, bunching at the kink falls proportionately. In our model, adjustment costs help to explain deviations from this pattern. As we move from the more sharply bent kink to the less sharply bent kink in our model with adjustment costs, bunching falls by a less-than-proportional amount—consistent with our empirical observation that individuals continue to bunch at the location of a former kink. In the extreme case in which a kink has been eliminated, we can attribute any residual bunching to adjustment costs. Moreover, we show in Gelber, Jones, and Sacks (2013) that the absolute value of the decrease in bunching from $K_1$ to $K_2$ is decreasing in the adjustment cost—$\tilde{z}_0$ is increasing in the adjustment cost, and therefore area $v$ is decreasing in the adjustment cost. As in the frictionless case, the amount of bunching at $K_1$ is still increasing in the elasticity, ceteris paribus.

6.4 Dynamic Version of Model

By applying our approach thus far to study adjustment over a given time frame, the resulting parameters should be interpreted as meaning that bunching in this given time frame can be predicted if individuals behaved as if they faced the indicated adjustment cost and elasticity—in the spirit of Friedman (1953), who argued that economic models should predict behavior “as if” individuals followed the model. In practice, we apply our model to study the nature of immediate adjustment to a policy change, so the parameters we estimate pertain to the frictions faced in immediately adjusting to a policy change. This framework may be applied in each period to yield “as if” estimates separately for each period.

However, the comparative static model does not give an account of how bunching may evolve over time. Section 5 shows evidence of lagged adjustment up to two years following a change in incentives. To capture such features of the data, we can nest our comparative static model within a framework incorporating more dynamic elements that allow us to account for this subsequent adjustment. We use a Calvo (1983) or
“CalvoPlus” framework (e.g. Nakamura and Steinsson, 2010), in which there is a positive probability in each period of facing a finite, fixed adjustment cost.

Thus, we will assume that the adjustment cost in any period is drawn from a discrete distribution \( \{0, \phi\} \). This generates a gradual response to policy, as agents may adjust only when a sufficiently low value of the fixed cost is drawn. This element of the model is necessarily stylized, for the sake of tractability and consistency with previous literature. Such variation over time in the size of the adjustment cost from this discrete distribution could capture, for example, the job search process following a recent policy change: when a job offer arrives exogenously, the adjustment cost reaches zero, but when it is not available, search costs are positive and equal to \( \phi \). Alternatively, this discrete distribution could capture the slow diffusion of information regarding a recent policy change: when information becomes available to an individual, the adjustment cost reaches zero, but when it is not available, it is positive and equal to \( \phi \). Future work could generalize this or other aspects of the model.\(^{21}\)

How we model dynamics is also influenced by a key feature observed in the data: the lack of an anticipatory response to policy changes. In Appendix A.2, we solve a completely forward-looking model in which agents anticipate a policy change. This model which nests the models presented in the main text. In this forward-looking model, the key results are less parsimonious, and the identification of the key parameters is less transparent. In practice, the data drive this unrestricted version of the model to place little to no weight on the future by estimating discount factors of zero or near zero. If agents were to place weight on the future in our forward-looking model, they should begin to bunch in anticipation of facing a kink, and they should begin to de-bunch in anticipation of the disappearance of a kink—neither of which we have observed in the data, as shown for example in Figure 2. Meanwhile, we observe a degree of delayed response to policy changes. We can capture both of these features of the data by assuming that a stochastic process determines whether an agent faces the cost of adjustment, but agents do not anticipate the policy change. In general, this could either be because agents are forward-looking but the change is unanticipated even for agents who are fully informed, or because agents are not forward looking. We therefore focus on the case without anticipatory behavior in the main text, as it is sufficient to explain the patterns in our data. In our particular context, the lack of anticipatory behavior can be rationalized, for example, if agents are myopic or only learn about the AET through experience with it, consistent with the results in Gelber, Isen, and Song (2016).\(^{22}\)

Formally, our main dynamic model (without anticipatory behavior) extends the notation from above as follows. As before, we assume that agents begin with their optimal frictionless level of earnings in period 0. Flow utility in each period is \( v(c_{a,t}, z_{a,t}; a, z_{a,t-1}) = u(c_{a,t}, z_{a,t}; a) - \phi_t \cdot \mathbf{1}(z_{a,t} \neq z_{a,t-1}) \), where \( \mathbf{1}(\cdot) \) is

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\(^{21}\)We have alternatively modeled intertemporal shocks via a time-varying ability, \( a \). This model will likewise generate delayed response, as agents will only reoptimize when the ability draw generates a preferred level of earnings far enough away from current earnings to justify paying the fixed cost of adjustment. Analogously, agents in our model reoptimize when they draw a low enough cost of adjustment.

\(^{22}\)Our model without anticipatory adjustment can be interpreted as meaning that bunching can be predicted if individuals behaved as if they faced the estimated parameters, in the spirit of Friedman (1953). Moreover, our static model can be interpreted as consistent with a degree of forward-looking behavior, if people take into account their future earnings when they consider adjusting.
the indicator function. Individuals are again indexed by a time-invariant heterogeneity parameter, \( a \), which captures ability. In each period, an individual draws a cost of adjustment, \( \tilde{\phi}_t \), from a discrete distribution, which equals \( \phi \) with probability \( \pi_{t-1} \) and equals 0 with probability \( 1 - \pi_{t-1} \). To capture the observed features of the data, in which the probability of adjusting (conditional on initially locating at the kink) appears to vary over time, we allow the probability \( \pi_{t-1} \) to be potentially a function of the time lapsed since the most recent policy change, \( i.e. \ t - t^* \).

Individuals make decisions over a finite horizon. In period 0, individuals face a linear tax schedule, \( T_0(z) = \tau_0 z \), with marginal tax rate \( \tau_0 \). In period 1, a kink, \( K_1 \), is introduced at the earnings level \( z^* \). This tax schedule is implemented for \( T_1 \) periods, after which the tax schedule features a smaller kink, \( K_2 \), at the earnings level \( z^* \). As before, the kink \( K_j \), \( j \in \{1, 2\} \), features a marginal tax rate of \( \tau_j \) for earnings above \( z^* \). For simplicity, we abstract from income effects, to focus on the dynamics created by the presence of adjustment costs. In particular, \( u(c, z; a) = c - \frac{a}{1+1/\varepsilon} \left( \frac{z}{z^*} \right)^{1+1/\varepsilon} \), as in Saez (2010), Chetty et al. (2011), Kleven and Waseem (2013), or Kleven, Landais, Saez, and Schultz (2014). In each period, individuals draw a cost of adjustment, \( \tilde{\phi}_t \), and then maximize flow utility, \( u(c_{a,t}, z_{a,t}; a, z_{a,t-1}) \) subject to a per-period budget constraint \( z_{a,t} - T_j(z_{a,t}) - c_{a,t} \geq m \), where \( j = 1 (t \geq 1) + 1 (t > T_1) \), and \( m \) reflects a borrowing constraint.\(^{23}\)

These assumptions generate a simple decision rule for agents in each period. Let \( \tilde{z}_{a,t} \) be the optimal frictionless level of earnings for an individual with ability \( a \) in period \( t \), which is a function of the tax schedule in that period. An agent will choose this level of earnings provided that the utility gain of moving from \( z_{a,t-1} \) to \( \tilde{z}_{a,t} \) exceeds the currently-drawn cost of adjustment, \( \tilde{\phi}_t \). Otherwise, the agent remains at \( z_{a,t-1} \). The agent only considers current payoffs because we have abstracted from anticipatory behavior.

The model yields two types of transitions following a policy change. First, there are agents who adjust immediately following a policy change, because the gain in utility exceeds the highest possible adjustment cost, \( \phi \). These are the same agents who adjust in our model in Section 6.3. Second, there are agents who only adjust once a zero cost of adjustment is drawn, which happens in each period with probability \( 1 - \pi_{t-1} \).

We can now generalize our expressions for bunching under \( K_1 \) and \( K_2 \), (4) and (6). Denote \( B_1^t \) as bunching at \( K_1 \) in period \( t \in [1, T_1] \). We have the following dynamic version of (4):

\[
B_1^t = \int_{\tilde{z}_{1}}^{z_{t} + \Delta z_{1}} h_0(\zeta) \, d\zeta + (1 - \Pi_{j=1}^{t} \pi_j) \int_{z^*}^{\tilde{z}_{1}} h_0(\zeta) \, d\zeta = \Pi_{j=1}^{t} \pi_j \cdot B_1 + (1 - \Pi_{j=1}^{t} \pi_j) B_1^* \tag{8}
\]

where \( h_0(\cdot) \) is again the density of earnings under a linear tax \( \tau_0 \), \( \tilde{z}_{1} \) is implicitly defined in equation (5), \( B_1 \) is defined in (4) and \( B_1^* \) is the frictionless level of bunching defined in (2) when \( j = 1 \). We see on the first line of (8) that bunching in period \( t \) at \( K_1 \) is composed of two parts. First, there are individuals who immediately adjust in period 1, \( i.e. \) areas ii to v in Figure 7, Panel B. Second, there are individuals in area i of Figure 7, who only adjust if they have drawn a zero cost of adjustment. The probability that this occurs

\(^{23}\)The quasilinearity assumption implies that the borrowing constraint does not directly affect the earnings decision. However, when agents are not forward looking, the borrowing constraint is necessary to rule out infinite borrowing.
by period $t$ is $1 - \Pi_{j=1}^t \pi_j$. Finally, we see in the second line of (8) that as $t$ grows, $\Pi_{j=1}^t \pi_j$ converges to zero, and bunching converges to the frictionless level of bunching $B_2^*$, i.e. areas $i$ to $v$ in Figure 7, Panel B.

We can similarly derive an expression for $B_2^t$, bunching at $K_2$ in period $t > \mathcal{T}_1$:

$$B_2^t = \int_{\xi_1}^{z^* + \Delta z^*_2} h_0(\xi) d\xi + \Pi_{j=1}^t \pi_j \int_{z^* + \Delta z^*_2}^{\xi_0} h_0(\xi) d\xi + \left(1 - \Pi_{j=1}^t \pi_j \cdot \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right) \int_{z^*}^{\xi_1} h_0(\xi) d\xi = \Pi_{j=1}^{\mathcal{T}_1} \pi_j \cdot \left(\bar{B}_2 + \left(1 - \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right) [B_1^t - B_1]\right) + \left(1 - \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right) B_2^*$$

(9)

where $\xi_0$ is implicitly defined in (7), $\bar{B}_2$ is defined in (6), and $B_2^*$ is the frictionless level of bunching defined in (2) when $j = 2$. In the first two lines, bunching in period $t$ at $K_2$ consists of three components. First, there are individuals who immediately bunched in period 1, and remain bunching at the smaller kink, i.e. areas $ii$ to $iii$ in Figure 7, Panel B. Second, there are “excess bunchers” who immediately bunched in period 1, and now “de-bunch” when a zero cost of adjustment is drawn, i.e. individuals in area $iv$ of the same figure. The probability of not having drawn a zero cost of adjustment between periods $\mathcal{T}_1 + 1$ and $t$ is $\Pi_{j=1}^{t-\mathcal{T}_1} \pi_j$. Finally, there are individuals who would like to bunch under both $K_1$ and $K_2$, but will only do so once a zero cost of adjustment is drawn, i.e. those in area $i$ of Figure 7, Panel B. A fraction of these agents, $\left(1 - \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right)$, have drawn a zero cost of adjustment by period $\mathcal{T}_1$, and of the remaining $\Pi_{j=1}^{\mathcal{T}_1} \pi_j$, the probability of drawing a zero cost from period $\mathcal{T}_1 + 1$ to $t$ is $\left(1 - \Pi_{j=1}^{t-\mathcal{T}_1} \pi_j\right)$, yielding a total share of $\left(1 - \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right) + \Pi_{j=1}^{\mathcal{T}_1} \pi_j \cdot \left(1 - \Pi_{j=1}^{t-\mathcal{T}_1} \pi_j\right) = \left(1 - \Pi_{j=1}^{t-\mathcal{T}_1} \pi_j \cdot \Pi_{j=1}^{\mathcal{T}_1} \pi_j\right)$. On the third line, we once again see that as the time between period $t$ and $\mathcal{T}_1$ grows, $\Pi_{j=1}^{t-\mathcal{T}_1} \pi_j$ converges to zero, and the level of bunching converges to the frictionless amount, $B_2^*$, i.e. areas $i$ to $iii$ in Figure 7, Panel B.

This dynamic version of the model nests the comparative static model from Sections 6.2-6.3. When $\pi_j = 1, \forall j$, (8) shows that $B_1^t = B_1$, and (9) shows that $B_2^t = \bar{B}_2$. Furthermore, when $\pi_j = 0, \forall j$, or $\phi = 0$, the model returns the predictions from the frictionless, Saez (2010) model.

Relative to the comparative static model, the dynamic model has both strengths and weaknesses. As noted in Einav, Finkelstein, and Schrimpf (2017), both dynamic and static models can fit observed bunching patterns in their context. The comparative static model transparently illustrates the basic forces determining the elasticity and adjustment cost. The estimation of the more dynamic model requires more moments from the data to estimate additional parameters. We assume that ability is fixed throughout the window of estimation, which may be more plausible in the case of the comparative static model—when we only use two cross-sections from adjacent time periods—than when we use a dynamic model and study a longer time frame. However, this affords us tractability in the dynamic model, while allowing us to account for the time pattern of bunching.
6.5 Extensions

Our framework can be extended, including by allowing for heterogeneity in our key parameters, or by allowing for frictions to affect the distribution of earnings in the initial period. We also discuss the claiming decision. In Gelber, Jones, and Sacks (2013), we discuss estimation of a model with a cost of adjustment that has both a fixed cost component and a component that is linear in the size of the earnings adjustment.

6.5.1 Heterogeneity in Elasticities and Costs of Adjustment

The previous analysis assumed homogeneous elasticities and adjustment costs, but we can extend the model to accommodate heterogeneity. In our static model, suppose $(\varepsilon_i, \phi_i, a_i)$ is jointly distributed according to a smooth CDF, which translates into a smooth, joint distribution of elasticities, fixed adjustment costs, and earnings in the presence of a linear tax, $h^*_0(\varepsilon, \phi, z_0)$. In Appendix A.3 we derive generalized formulae for bunching that allow us to interpret our estimates as average parameters among the set of bunchers. Appendix A.3.2 further discusses how the dynamic model can be interpreted in the presence of heterogeneity in these parameters and the vector $\pi_i$.

Our estimates of elasticities and adjustment costs, and our earlier descriptive evidence documenting the speed of adjustment, are specific to the population that is observed bunching at the kinks. At the same time, note that for any value of $\phi_i$, there exists a value of $\varepsilon_i$ that generates positive bunching. Thus, although our estimates are local to the observed set of bunchers, they need not be confined to a subpopulation with small values of $\phi_i$—for example, if $\varepsilon_i$ and $\phi_i$ are positively correlated. Nevertheless, there may be a set of individuals for whom $\varepsilon_i$ is small enough relative to $\phi_i$ to preclude bunching under either $K_1$ or $K_2$, and who therefore do not contribute to our parameter estimates.

Although our estimates are local in this sense, with sufficiently large variation in tax rates it may be possible to estimate population average parameters. Our estimation procedure relies on estimating bunching at more than one kink; over all such kinks, the limits of the integrals used to calculate bunching could jointly cover much of the earnings distribution. Loosely speaking, the greater the variation in tax rates, the more of the population we will observe who bunch at kinks and therefore contribute to our estimates. In that light, our policy variation is useful because it varies over a large range of BRRs (from 50 percent to 33.33 percent to zero percent). Moreover, it is perhaps reassuring that we will find similar elasticity and adjustment cost estimates when we examine a larger change in the BRR (from 33.33 percent to zero percent) as when we examine a smaller change (from 50 percent to 33.33 percent). Extrapolating our estimates from bunchers to non-bunchers would require assumptions on the joint distribution of $\varepsilon$ and $\phi$.

Special cases of our model have implications for other moments of the earnings distribution, but it is not possible to use these moments without more stringent distributional assumptions. As an example of such implications, in our model in Section 6.3, the set of non-bunchers (e.g. area $i$ in Figure 7, Panel B) will tend to overlap with relatively high earners (those to the right of area $v$) who adjust down just to the right of the kink $K_1$ after it is introduced. This will tend to generate excess mass just above $z^*$ and a sharp drop.
in the \textit{ex post} density at $z_{1}$. However, if the observed earnings distribution is averaged over subpopulations with different values of $z_{1}$ under the heterogeneity described in this section, the predicted excess mass just above $z^*$ will tend to be smoothed. Thus, such additional predictions from the comparative static model with homogenous $\varepsilon$ and $\phi$ are not as robust as our use of the amount of bunching for estimation.

6.5.2 Frictions in Initial Period

Our model above assumes that initial earnings under $\tau_{0}$ are located at the frictionless optimum. However, it is also possible to assume that individuals may find themselves away from their frictionless optimum in period 0, due to the same adjustment costs that attenuate bunching under $K_{1}$ and $K_{2}$. In Appendix A.4, we specify an extended model that allows for individuals to be arbitrarily located in a neighborhood of their frictionless optimum. As in Chetty (2012), we only require that earnings are close enough to the optimum to preclude any further utility gains that outweigh the adjustment cost $\phi^*$. We derive generalized versions of equations (4) and (6) that hold for arbitrary initial conditions consistent with a fixed adjustment cost. We then show that estimation now requires structure on the distribution of initial earnings within the neighborhood of the frictionless optimum. We demonstrate how our estimator is amended and report estimates under this method below.

6.5.3 Accounting for the Claiming Decision

In our model and estimation we abstract from the claiming decision by examining a sample of those who have already claimed OASI (\textit{i.e.} our estimates use a sample of those older than age 65 who had claimed by age 65); our model thus applies more broadly to understanding responses to kink points where the claiming decision is not relevant (as in, for example, most other tax or transfer contexts, including those where Gudgeon and Trenkle (2016), He, Peng, and Wang (2016), and Zaresani (2016) have applied our method). In the context of the AET, this modeling choice follows previous literature such as Friedberg (1998, 2000).

This decision is only a trivial abstraction in our context because nearly everyone has claimed by the ages we study in our main evidence, 69 to 71, implying minimal scope for the claiming decision—or conditioning on those who have claimed by 65, as we do in our primary estimates—to affect our results. By age 69, fully 94 percent of the sample has claimed OASI, and nearly everyone has claimed by 71. Moreover, the bunching we observe at ages after the AET ceases to apply is a tell-tale sign of adjustment frictions, regardless of the fraction claiming at these ages. By ages 66 to 68, which we later examine in the context of the 1990 change in the AET BRR, 92 percent have claimed OASI on average across ages in the time period we study. Moreover, over the period we examine from before to after 1990, the proportion claiming is stable at 92 percent in each year separately from 1988 to 1992, implying that the results should not be materially affected by changes in claiming over time. As the percent of the sample claiming approaches 100, our bunching estimates—and therefore the resulting parameter estimates using our estimator explained below—will converge to those we have calculated conditional on claiming. Consistent with this, we estimate very similar, and insignificantly different, elasticities and adjustment costs when bunching is estimated from the sample of only those who...
have claimed, as when the sample includes both claimants and non-claimants.

Moreover, empirically our evidence suggests the claiming decision during our ages of interest does not appear to interact notably with the AET. In particular, we add to previous literature by showing in Appendix Figure B.3 that the hazard of claiming at year \( t + 1 \) is smooth around the exempt amount at year \( t \), indicating no evidence that claimants come disproportionately from close to or far from the kink. Building on these results, Gelber, Jones, Sacks, and Song (2016) are studying claiming responses in greater depth (as studied using a different strategy in Gruber and Orszag, 2003).  

### 6.6 Econometric Estimation of the Model

#### 6.6.1 Comparative Static Model

We estimate the model we have described using a minimum distance estimator. We begin by describing our econometric estimation procedure under our basic comparative static model of Sections 6.2 and 6.3. Let \( B = (B_1, B_2, \ldots, B_L) \) be a vector of (estimated) bunching amounts, using the method described in Section 3. Let \( \tau = (\tau_1, \ldots, \tau_L) \) be the tax schedule at each kink. The triplet \( \tau_l = (\tau_{0,l}, \tau_{1,l}, \tau_{2,l}) \) denotes the tax rate below the kink \( \tau_{0,l} \), above the kink \( \tau_{1,l} \), and the \textit{ex post} marginal tax rate above the kink after it has been reduced \( \tau_{2,l} \), as in Section 6.3. Let \( z^* = (z_{1,l}^*, \ldots, z_{L,l}^*) \) be the earnings levels associated with each kink. In principle, it would be possible to estimate bunching separately for each age group at a given kink. In practice and for simplicity, we pool across a constant set of ages to estimate bunching at a given kink—for example, when examining the 1990 policy change we examine 66-68 year-olds both before and after the change. Thus, the bunching amounts are not indexed by age.  

In our baseline, we use a non-parametric density for the counterfactual earnings distribution, \( H_0 \). Once \( H_0 \) is known, we use (4) and (6) to obtain predicted bunching from the model. To recover \( H_0 \) non-parametrically we take the empirical earnings distribution for 72 year-olds in $800 bins as the counterfactual distribution. 72 year-olds’ earnings density represents a reasonable counterfactual because they no longer face the AET, no longer show bunching, and are close in age to those aged 70 or 71. Letting \( z_i \) index the bins, our estimate of the distribution is \( \hat{H}_0(z_i) = \sum_{j < i} Pr(z \in z_j) \). This function is only defined at the midpoints of the bins, so we use linear interpolation for other values of \( z \). In a robustness check, we instead assume that the earnings distribution over the range \( [z^*, z^* + \Delta z] \) is uniform, a common assumption in the literature (e.g. Chetty et al., 2011, Kleven and Waseem, 2013). Using the nonparametrically-estimated distribution of earnings from age 72 is helpful because it does not entail distributional assumptions, but relative to assuming a uniform distribution, using the age-72 distribution comes at the cost of using a different age (i.e. 72) to generate the earnings distribution.  

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24 In Gelber, Jones, and Sacks (2013), we discuss further extensions of the model, including certain assumptions under which we can express the elasticity and adjustment cost as functions of observed levels of bunching that can be easily solved in closed form.

25 Analogously, when we examine bunching at each age around 70 when the AET is eliminated, we pool across calendar years (namely 1990-1999) to estimate bunching, so that we do not also have to index the bunching amounts by calendar year. We find comparable results when we estimate bunching separately at each age and year.

26 Because we use the age-72 density as our counterfactual density—unlike most bunching papers bunching that estimate the counterfactual from the same density that is used to estimate bunching—our method is not subject to the Blomquist and Newey (2017) point that the functional form of preference heterogeneity cannot be simultaneously estimated with the taxable income elasticity.
To estimate \((\varepsilon, \phi)\), we seek the values of the parameters that make predicted bunching \(\hat{B}\) and actual (estimated) bunching \(B\) as close as possible on average. Letting \(\hat{B}(\varepsilon, \phi) \equiv (\hat{B}(\tau_1, z^*_1, \varepsilon, \phi), \ldots, \hat{B}(\tau_L, z^*_L, \varepsilon, \phi))\), our estimator is:

\[
(\varepsilon, \phi) = \arg\min_{(\varepsilon, \phi)} \left( \hat{B}(\varepsilon, \phi) - B \right)^TW\left( \hat{B}(\varepsilon, \phi) - B \right),
\]

where \(W\) is a \(K \times K\) identity matrix. This estimation procedure runs parallel to our theoretical model, as the bunching amounts \(\hat{B}\) are those predicted by the theory (and the estimated counterparts \(B\) are found using the procedure outlined in Section 3).\(^{27}\) When we pool data across multiple time periods, we assume that \(\varepsilon\) and \(\phi\) are constant across these time periods.

We obtain our estimates by minimizing (10) numerically. Solving this problem requires evaluating \(\hat{B}\) at each trial guess of \((\varepsilon, \phi)\).\(^{28}\) Our estimator assumes a quasilinear utility function, \(u(c, z; a) = c - \frac{a}{1 + \varepsilon z} \left( \frac{z}{a} \right)^{1+1/\varepsilon}\), following Saez (2010), Chetty et al. (2011) and Kleven and Waseem (2013). Note that because we have assumed quasilinearity, \(\Delta z_{1,l} = z^*_l \left( \left( \frac{1}{1-\tau_{0,l}} \right)^\varepsilon - 1 \right)\) and \(a = z(\tau) / (1 - \tau)^\varepsilon\), where \(z(\tau)\) are the optimal, interior earnings under a linear tax of \(\tau\). Typically there is no closed form solution for \(\tilde{z}_{1,l}\) or \(\tilde{z}_{0,l}\). Instead, given \(\varepsilon\) and \(\phi\), we find \(\tilde{z}_{1,l}\) and \(\tilde{z}_{0,l}\) numerically as the solution to the relevant indifference conditions in (5) and (7).

For example, \(\tilde{z}_{1,l}\) is defined implicitly by:

\[
\frac{u((1 - \tau_{1,l})z^*_l + R_{1,l}, z^*_l; \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l})}{(1 - \tau_{0,l})} - \frac{u((1 - \tau_{1,l})\tilde{z}_{1,l} + R_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l}, \tilde{z}_{1,l})}{(1 - \tau_{0,l})} = \phi, \tag{11}
\]

This equation is continuously differentiable and has a unique solution for \(\tilde{z}_{1,l}\).\(^{29}\)

### 6.6.2 Dynamic Model

Our estimation method is easily amended to accommodate the dynamic extension of our model in Section 6.4. As in (8) and (9), the bunching expressions in the dynamic model are weighted sums of \(B_1\) and \(\bar{B}_2\), which are calculated as in Section 6.6.1, and two measures of frictionless bunching, \(B_1^*\) and \(B_2^*\). Frictionless bunching under either kink can be calculated conditional on \(H_0\) and \(\varepsilon\) using (2).

We must also estimate the probability of drawing a positive fixed cost as a function of the time since the last policy shock, \(\pi_{t-1,t}^*\).\(^{30}\) For given values of \(\varepsilon\), \(\phi\), and the vector \(\pi\) of \(\pi_{t-1,t}^*\)’s, we can evaluate (8) and (9). Our vector of predicted bunching, \(\tilde{B}\), will now be a function of these additional parameters, as well as the relevant time indices:

\(\tilde{B}(\varepsilon, \phi, \pi) \equiv (\hat{B}(\tau_1, z^*_1, t_1, T_{1,l}, \varepsilon, \phi, \pi), \ldots, \hat{B}(\tau_L, z^*_L, t_L, T_{L,l}, \varepsilon, \phi, \pi))\), where \(t_l\) is the time elapsed since the first kink, \(K_{1,l}\), was introduced, and \(T_{1,l}\) is the length of time before the second kink, \(K_{2,l}\), is introduced. Once again we use the minimum distance estimator (10).

Equations (8) and (9) illustrate how we estimate the elasticity and adjustment cost in this richer setting.

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\(^{27}\) Without loss of generality, we use normalized bunching, \(\tilde{b} = \delta \tilde{B}/h_0(\tilde{z}^*)\), so that the moments are identical to what is reported elsewhere in the text.

\(^{28}\) In solving (10), we impose that \(\phi \geq 0\). When \(\phi < 0\), every individual adjusts her earnings by at least some arbitrarily small amount, regardless of the size of \(\phi\). This implies that \(\phi\) is not identified if it is less than zero.

\(^{29}\) Note that some combinations of \(\pi_l, z^*_l\), \(\varepsilon\), and \(\phi\) imply \(\tilde{z}_{1,l} > z^*_l + \Delta z_{1,l}\). In this case, the lowest-earning adjuster does not adjust to the kink. Whenever this happens, we set \(\hat{B}_l = 0\).

\(^{30}\) We have also tried using a flexible, logistic functional form, \(\pi_j = \exp(\alpha + \beta \cdot j) / (1 + \exp(\alpha + \beta \cdot j))\), and we found comparable results (available upon request).
We require as many observations of bunching as the parameters, \((\varepsilon, \phi, \pi_1, \ldots, \pi_J)\), and these moments must span a change in \(d\tau\).\(^{31}\) Suppose we observe the pattern of bunching over time around two or more different policy changes. Loosely speaking, the \(\pi\)’s are estimated relative to one another from the time pattern of bunching over time: a delay in adjustment in a given period will generally correspond to a higher probability of facing the adjustment cost (all else equal). Note that the relationship is linear; the degree of “inertia” in bunching in (for example) period 1 increases linearly in \(\pi_1\). Meanwhile, a higher \(\phi\) implies a larger amount of inertia in all periods until bunching has fully dissipated (in a way that depends on the earnings distribution, the elasticity, and the size of the tax change). Finally, a higher \(\varepsilon\) will correspond to a larger amount of bunching once bunching has had time to adjust fully to the policy changes. Intuitively, these features of the data help us to identify the parameters using our dynamic model.

6.7 Inference and Identification

We again estimate bootstrapped standard errors to perform inference about our parameters. For example, in our comparative static model, the estimated vector of parameters \((\hat{\varepsilon}, \hat{\phi})\) is a function of the estimated amount of bunching. We use the bootstrap procedure of Chetty et al. (2011) to obtain 200 bootstrap samples of \(B\). For each bootstrap sample, we compute \(\hat{\varepsilon}\) and \(\hat{\phi}\) as the solution to (10). We determine whether an estimate of \(\phi\) is significantly different from zero by assessing how frequently the constraint \(\phi \geq 0\) binds in our estimation. This share is doubled in order to construct \(p\)-values from a two-sided test.

We have already discussed the features of the data that intuitively help us identify the parameters in the comparative static and dynamic models. It is also possible to demonstrate identification more formally. In Appendix A.5 we give formal conditions for identification in both the comparative static and dynamic cases, and we show that these conditions hold in our data in both cases. These conditions essentially require that we have sufficient tax variation relative to the adjustment cost. Intuitively, without sufficient tax variation, there will not be any bunching or de-bunching, and so we cannot identify the parameters—a case that is not relevant in our data, as we observe both bunching and de-bunching.

7 Estimates of Elasticity and Adjustment Cost

7.1 Estimates using the Comparative Static Method

To estimate \(\varepsilon\) and \(\phi\) using our “comparative static” method, we first examine the reduction in the rate in 1990 and next turn to the elimination of the AET at ages 70 and older. We use the 1990 change as a baseline because this allows us to compare our method to the Saez (2010) method, whereas we cannot apply the Saez method at age 70 or later because the BRR is zero above the exempt amount.

We begin with Figure 8, a graphical depiction of the patterns driving the parameter estimates for the 1990 change. We follow a group of 66-68 year-olds, so that we can examine an age group that moved over time from being affected by the 50 percent tax rate before the policy change to the 33.33 percent tax rate

\(^{31}\)The number of moments is not itself sufficient. We also require non-trivial variation in bunching before and after the tax change in order to point identify \(\phi\). As in footnote 20, this requires \(z_0 < z^* + \Delta z^*_1\).
after the policy change. Figure 8 shows bunching among 66-68 year-olds, for whom the BRR fell from 50 percent to 33.33 percent in 1990. Bunching fell negligibly from 1989 to 1990 but fell more subsequent to 1990.

Estimating these parameters requires estimates of the implicit marginal tax rate: both the “baseline” marginal tax rate, $\tau_0$—the rate that individuals near the AET threshold face in the absence of the AET due to federal and state taxes—and the implicit marginal tax rate associated with the AET. We begin by using a marginal tax rate incorporating the AET BRR as well as the average federal and state income and payroll marginal tax rates, calculated using the TAXSIM calculator of the National Bureau of Economic Research (Feenberg and Coutts, 1993) and information on individuals within $\pm 2,000 of the kink in the Statistics of Income data in the years we examine.

Table 2 presents estimates of our static model, examining 66-68 year-olds in 1989 and 1990. We estimate an elasticity of 0.35 in Column (1) of Table 2 and an adjustment cost of $278 in Column (2), both significantly different from zero ($p < 0.01$). This specification examines data in 1989 and 1990; thus, our estimated adjustment cost represents the cost of adjusting earnings in the first year after the policy change. We interpret our estimates as meaning that when considering a given time frame (in this particular case, from 1989 to 1990), bunching amounts can be predicted if individuals behaved as if they had the estimated elasticity and adjustment cost (in this particular case, 0.35 and $278, respectively).

When we constrain the adjustment cost to zero using 1990 data in Column (3), as most previous literature has implicitly done, we estimate a substantially larger elasticity of 0.58. Consistent with our discussion above, the estimated elasticity is higher when we do not allow for adjustment costs than when we do, because adjustment costs keep individuals bunching at the kink even though tax rates have fallen. The difference in the constrained and unconstrained estimates of the elasticity is substantial (66 percent higher in the constrained case) and statistically significant ($p < 0.01$).

Other specifications in Table 2 show similar results. We adjust the marginal tax rate to take account of benefit enhancement, following the calculations of the effective OASI tax rate net of benefit enhancement in Coile and Gruber (2001). This raises the estimated elasticity but yields similar qualitative patterns across the constrained and unconstrained estimates. The next rows show that our estimates are similar under other specifications: excluding FICA taxes from the baseline tax rate; using a locally uniform density; other bandwidths; and other years of analysis. Returning to the baseline specification, the point estimates in Appendix Table B.3 show that across groups, elasticities tend to be similar, but women have higher adjustment costs than men, those with low prior lifetime real earnings have higher adjustment costs than those with high prior earnings, and those with high and low volatility of prior earnings have similar adjustment costs.

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Although 69 year-olds are also subject to the AET, an individual who is 69 in 1989 would have turned 70 by 1990 and therefore would not have been affected by the 33.33 percent tax rate in 1990—preventing us from examining those aged 69 if we wish to examine a constant age group. Examining a constant age group (i.e. those aged 66-68 in each calendar year) is crucial because different age groups have persistently different amounts of bunching, which we do not wish to confound with the effect of the policy change.

Friedberg (2000) finds uncompensated elasticity estimates of 0.22 and 0.32 in different samples. However, differences in the estimation strategies imply that these results are not directly comparable to ours.

The estimates by group are comparable for the dynamic model estimated below.
Appendix Table B.4 we apply our method to the 1990 policy change but allow individuals to be initially located away from their frictionless optimum, as described above and in Appendix A.4. We again estimate similar results to the baseline.35

In Table 2, the variation driving our identification is not confounded with age variation, because we hold ages constant from before to after the policy change (examining 66-68 year-olds in both cases). Our identification rests on a comparison of bunching over time; we believe that three additional observations make this identification strategy credible. First, Figure 8 shows that in a “control group” of 62-64 year-olds who do not experience a policy change in 1990, bunching is very stable in the years before and after 1990, suggesting that the 66-68 year-old group will be sufficient to pick up changes in bunching due to the policy change. Appendix Table B.5 verifies that in a “differences-in-differences specification” comparing 66-68 year-olds to 62-64 year-olds, bunching among 66-68 year-olds falls insignificantly in 1990 relative to before 1990, bunching is significantly smaller among 66-68 year-olds in years after 1990, and these estimates are very similar to the time series estimates comparing only 66-68 year-olds over time.

Second, Table 3 shows comparable evidence of frictions when we examine the removal of the kink at age 70 (pooling years 1990-1999), in which case the results cannot be driven by shocks across ages because there is no reason for bunching at ages over 70 except delayed adjustment to the removal of the kink. When comparing adjustment at age 70 to adjustment in 1990, a key pattern in the data consistent with our model is that the decrease in normalized excess mass from 1989 to 1990 shown in Figure 8 is much smaller in absolute and percentage terms than the decrease in normalized excess mass from age 69 to age 70 shown in Figure 2 Panel B. With an adjustment cost preventing immediate adjustment as in our model, normalized excess mass should fall less when the jump in marginal tax rates at the kink falls less (in the change from a 50 percent to a 33.33 percent BRR in 1990) than when the jump in marginal tax rates at the kink falls more (in the change from a 33.33 percent to a 0 percent BRR at age 70). Table 4 veriﬁes that we estimate similar results when we pool data from the ages 69 to 71 transition with the 1989 to 1990 transition.

Third, Figure 9 shows that the elasticity we estimate among 66-68 year-olds using the Saez (2010) method—constraining the adjustment cost to be zero—shows a sharp, sudden upward spike in bunching in 1990 but subsequently reverts to near its previous level. This relates directly to our theory, which predicts that following a reduction in the change in the MTR at the kink, there may be excess bunching due to inertia (reflected in area $iv$ in Figure 7, Panel B). Once we allow for an adjustment cost, this excess bunching is attributed to optimization frictions.36 Thus, the spike in 1990 is consistent with the excess bunching in 1990 reflecting delayed adjustment to the policy change, rather than another shock.

35Gelber, Jones, and Sacks (2013) ﬁnd similar estimates when examining the 1990 policy change but assuming that bunching in 1989 is not attenuated by adjustment frictions, under the rationale that bunching could have reached a “steady state” in 1989. To explain the 1990 spike in the Saez (2010) elasticity using changes in the probability of claiming, this probability would also have to spike in 1990, but it is constant at 92 percent before, during, and after 1990.

36This ﬁgure also serves as additional evidence that adjustment frictions drive our results, rather than the probability of claiming.
7.2 Estimates using the Dynamic Method

Table 5 shows the estimates of the dynamic model described in Sections 6.4 and 6.6.2. There are several parameters to estimate—ε, φ, and the vector of observed π₁, j’s—but a limited number of years in the data with useful variation in bunching. Namely, bunching varies little from year to year prior to the policy changes in 1990 or at age 70, and bunching fully dissipates by at most three years after the policy changes. As in Table 4, we pool data on bunching at ages 67, 68, 69, 70, 71, and 72 (pooling 1990 to 1999), with data on bunching among 66-68 year-olds in 1987, 1988, 1989, 1990, 1991, and 1992. This gives us twelve moments (six moments for each of two policy changes) with which to estimate seven parameters (ε, φ, π₁, π₁π₂, π₁π₂π₃, π₁π₂π₃π₄, and π₁π₂π₃π₄π₅). We pool the 1990 and age 70 transitions so that we have a sufficient number of moments to estimate the parameters.

In the baseline dynamic specification, we estimate ε = 0.36 and φ = $243. The estimates of ε are remarkably similar—usually within several percent for a given specification—under the static and dynamic models applied to comparable data, i.e. Table 4 and Table 5, respectively. The estimates of φ are also in the same range. The point estimate of π₁ varies across specifications from 0.64 in the baseline to 1, indicating that at most a minority of individuals are able to adjust in the year of the policy change. This mirrors our earlier finding that while some individuals adjust in the year of a policy change, particularly to the change from age 69 to 70, there are still many who do not (both from age 69 to 70 and from 1989 to 1990). The point estimate of π₁π₂ varies across specifications from 0.00 to 0.47 (with an estimate of 0.22 in the baseline), indicating that a majority of individuals are able to adjust by the year following a policy change. Again, this mirrors our earlier finding that substantial adjustment occurs from 1990 to 1991. In all specifications, π₁π₂π₃ is estimated to be zero, with confidence intervals that rule out more than a modest positive value. Thus, our estimates indicate that individuals are fully able to adjust by the third year after a policy change. Again, this mirrors our earlier finding that adjustment has fully taken place by three years after the policy change, both in 1990 and at age 70. It therefore makes sense that π₁π₂π₃π₄ and π₁π₂π₃π₄π₅ are also estimated to be zero, with confidence intervals that rule out more than modest positive values.

Given our estimates of the πₖ’s, it makes sense that we estimate comparable results from the static and dynamic models. If, hypothetically, adjustment were completely constrained in years 1 and 2 after the policy change and subsequently completely unconstrained, then we should estimate essentially identical results in the static and dynamic models because the static model effectively assumes that the only barrier to adjustment is the adjustment cost φ (effectively similar to assuming that πₖ = 1 for the periods over which adjustment is estimated). The dynamic model shows results that are not very different from this.
hypothetical scenario: \( \pi_1 \) is well over 50 percent, and \( \pi_1 \pi_2 \) is substantial but under 50 percent. Meanwhile, subsequent probabilities of facing the adjustment cost are zero. Thus, the immediate adjustment to policy in the first one to two years is substantially constrained, and our estimates of the static model during this time frame should show results that are not far from the dynamic model—as the estimates bear out.

We can simulate the amount of excess normalized mass we should observe at each age from 61 to 73 from 1990 to 1999, and in each year from 1983 to 1993 among ages 66 to 68, both under our dynamic model estimates and the Saez (2010) model estimates. Specifically, we simulate this using the estimates above of \((\varepsilon, \phi, \pi)\) from our dynamic model applied to the pooled ages 67 to 72 (from 1990 to 1999) and years 1987 to 1992 (at ages 66 to 68), and under the Saez (2010) model applied to these pooled ages and years. These ages and years form the basis for our “in-sample” predictions, while the “out-of-sample” predictions are at ages 61, 62, 63, 64, 65, 66, and 73 (from 1990 to 1999) and years 1983, 1984, 1985, 1986, and 1993 (at ages 66 to 68). These simulations result in an in-sample root-mean-square error (RMSE) from the Saez (2010) model that is 22.6 percent larger than our model (575 vs. 469), and show an out-of-sample RMSE from the Saez (2010) model that is 16.9 percent larger than our model (1,121 vs. 959). For context, the mean excess normalized mass is 3,003.6 during the ages and years when the AET applies to earnings, and is 2,449.9 across all the ages and years we consider. Thus, our dynamic model achieves a substantially better fit by capturing slow adjustment.

8 Simulations of the Effect of Policy Changes

Our parameter estimates imply that incorporating adjustment costs into the analysis can have important implications for predicting the short-run impact of policy changes on earnings, as policy-makers often seek to do. In particular, the adjustment costs we estimate greatly attenuate the predicted short-run impact of policy changes on earnings.

We use our estimates of the static model, using the year before through the year after a policy change as in our baseline, to simulate the effect in our data of two illustrative policy changes. Further details are provided in Appendix A.6 and Appendix Table B.6. Reducing the marginal tax rate above the kink by 50 percentage points (as could be implied by a policy like eliminating the AET for 62-64 year-olds) would cause a large, 23.4 percent rise in earnings at the intensive margin. However, a less large change—in particular, any cut in the marginal tax rate of 17.22 percentage points or smaller above the exempt amount—would cause no change in earnings within a one-year time horizon because the potential gains from adjusting are not large enough to outweigh the adjustment cost. Although this result relies on the extreme assumption of a homogeneous elasticity and adjustment cost, it illustrates the principle that because the gains to relocation are second-order near the kink, even a modest adjustment cost around $280 can prevent individuals’ adjustment—and even following a substantial cut in marginal tax rates (up to 17.22 percentage points). Moreover, the lack of response predicted with a change of 17.22 percentage points makes sense in light of the empirical patterns we observe, in particular the negligible change in bunching seen in the data from 1989 to 1990 when the
marginal tax rate falls by 17 percentage points.

These estimates are intended to suggest the potential for adjustment costs to be important in the analysis of the earnings effects of tax changes in such contexts. This conclusion is robust to other assumptions that we discuss in the Appendix. Under our estimates of the dynamic model we would still find that the short-run reaction even to large taxes changes is greatly attenuated, since the dynamic model estimates show that most individuals are constrained from adjusting immediately.

9 Conclusion

We introduce a new method for documenting adjustment frictions: examining the speed of adjustment to the disappearance of convex kinks in the effective tax schedule. We document delays in earnings adjustment to the Social Security Earnings Test, consistent with the existence of earnings adjustment frictions in the U.S. The lack of immediate response suggests that the short-run impact of changes in the effective marginal tax rate can be substantially attenuated, even with large policy changes like the 1990 change in the benefit reduction rate from 50 to 33 percent that was accompanied by little immediate change in bunching.

Next, we develop a method to estimate earnings elasticities and adjustment costs, using transparent identification relying on bunching at convex budget set kinks, which are commonly encountered in public programs and other economic applications. Examining data in the year of a policy change, we estimate that the elasticity is 0.35 and the adjustment cost is around $280. When we constrain adjustment costs to zero in the baseline specification, the elasticity of 0.58 we estimate in 1990 is substantially (66 percent) larger, demonstrating the potential importance of taking account of adjustment costs. We extend our methods to a dynamic context and continue to find comparable results.

Our estimates demonstrate the applicability of the methodology and the potential importance of allowing for adjustment costs when estimating elasticities. The method we develop has subsequently been applied outside our particular setting, and adjustment costs and elasticities may be different in other contexts. For example, individuals may pay particular attention to policy rules in the context of Social Security and retirement, particularly given that the 50 percent BRR may be very salient. The finding that adjustment costs matter even among those flexible enough to bunch at the kink, and even in the context of a salient policy, suggests the importance of taking frictions into account in other contexts.

As demonstrated by the data showing little immediate reaction even to large policy changes, even modest fixed adjustment costs—like the $280 cost we estimate in our baseline—can greatly impede short-run adjustment to large reforms because the costs of deviating from the frictionless optimum are second order. Adjustment costs can therefore make a dramatic difference in the predictions, as our simulations confirm. This could frustrate the goal of immediately impacting short-run earnings, as envisioned in many recent policy discussions, and could have important implications for policy-makers’ projections of the magnitude and timing of the earnings reaction to changes in tax and transfer policies.

Further analysis could enrich our findings. First, we consider our framework for estimating elasticities
and adjustment costs to be a natural first step (following papers such as Saez, 2010; Chetty et al., 2009, 2011, 2012a,b; Chetty, 2012; and Kleven and Waseem, 2013), but a next step could involve estimating a model that incorporates additional dynamic considerations (including the potential for anticipatory behavior as we have begun to model). Second, following most previous literature, in our formal model we have treated the adjustment cost as a “black box,” without modeling the process that underlies this cost, such as information acquisition or job search. Further work distinguishing among the possible reasons for reaction to the AET, including misperceptions, remains an important issue. Third, further investigation of claiming responses to the AET would be valuable, as Gelber, Jones, Sacks, and Song (2016) are investigating. Fourth, an important outstanding question, also suggested by Best and Kleven (forthcoming), is why bunching does not take long to disappear after the removal of the kink, despite the existence of frictions.

Finally, although we have modeled the earnings decision as a key first context, our method could be used to estimate elasticities and adjustment costs in the context of other consumption decisions in response to kinked budget sets. Since kinked budget sets are found in many economic applications, our method should have scope to be adapted in other economic contexts.

References


Figure 1: Key Earnings Test Rules, 1961-2009

Notes: The right vertical axis measures the benefit reduction rate (BRR) in OASI payments for every dollar earned beyond the exempt amount. The left vertical axis measures the real value of the exempt amount over time.
Figure 2: Earnings Histograms and Normalized Excess Mass by Age

A. Histograms by Age

Notes: The sample is a one percent random sample of all Social Security numbers, among individuals who claim OASI benefits by age 65, over calendar years 1990 to 1999. We exclude person-years with self-employment income or with zero non-self-employment earnings. The bin width is $800. In Panel A, the earnings level zero, shown by the vertical lines, denotes the kink. The dots show the histograms using the raw data, and the polynomial curves show the estimated counterfactual densities estimated using data away from the kink. Panel B shows normalized bunching at the AET kink, calculated as described in Section 3. Dashed lines denote 95% confidence intervals. The vertical lines show the ages at which the AET first applies (62) and ceases to apply (70).
Figure 3: Mean Percentage Change in Earnings from Age \( t \) to \( t+1 \), by Earnings at Age \( t \), 1990-1998

A. Growth from Age 69 to 70

B. Growth from Age 70 to 71

Notes: The figure shows the mean percentage change in earnings from age \( t \) to age \( t \) (y-axis), against earnings at age \( t \) (x-axis). In Panel A, \( t=69 \), and in Panel B, \( t=70 \). Dashed lines denote 95 percent confidence intervals. Earnings are measured relative to the kink, shown at zero on the x-axis. The data are a 20 percent random sample of 69-year-olds in the LEHD in 1990-1998. We exclude 1999 as a base year in this and similar graphs because the AET is eliminated for those older than NRA in 2000. Higher earnings growth far below the kink reflects mean reversion visible in this part of the earnings distribution at all ages.
Figure 4: Normalized Excess Mass of Claimants, Ages 69 to 72, 1983 to 1999

Note: See notes from Figure 2. Panel A of this figure differs from Figure 2 because here we pool 1983 to 1999 to gain extra statistical power. The continued bunching at age 71 is more evident. In the main sample, we pool only 1990 to 1999 because the BRR was constant over this period, avoiding issues relating to the transition to a lower BRR in 1990. Panel B of the figure shows normalized excess mass by age, demonstrating that excess normalized mass remains significant until age 71 and smoothly decreases from age 69 to age 72.
Figure 5: Inertia in Bunching from 64 to 65

Notes: Panel A of the figure shows that when they are age 65, those previously bunching at age 64 tend to either (a) remain near the age 64 exempt amount or (b) move to the age 65 exempt amount. A greater fraction remains near the age 64 exempt amount than the fraction that moves to the age 65 exempt amount. Panel B of the figure shows that those bunching at age 65 were usually bunching at age 64 in the previous year (or were near the age 65 exempt amount in the previous year). Having earnings “near the kink” at a given age is defined as having earnings within $1,000 of the kink at that age; in other words, “near kink at 64” means that the individual has age 64 earnings within $1,000 of the exempt amount applying at age 64, and “near kink at 65” means that the individual has age 65 earnings within $1,000 of the exempt amount applying at age 65. The first vertical line at zero shows the location of the age 64 exempt amount (normalized to zero), and the second vertical line shows the average location of the age 65 exempt amount (relative to the age 64 exempt amount that has been normalized to zero). The years of data used are 1990 to 1999.
Notes: Using data from 1990 to 1999, the figure shows that those bunching at age 69 tend to remain near the kink at ages 70 and 71, and that those bunching at ages 70 and 71 were also bunching at age 69. Specifically, the figure shows the density of earnings at age 69 conditional on having earnings near the kink at age 70 (Panel A), the density of earnings at age 69 conditional on having earnings near the kink at age 71 (Panel B), the density of earnings at age 70 conditional on having earnings near the kink at age 69 (Panel C), and the density of earnings at age 71 conditional on having earnings near the kink at age 69 (Panel D). Having earnings “near the kink” is defined as having earnings within $1,000 of the exempt amount applying to that age. See also notes from Figure 5.
Figure 7: Bunching Responses to a Convex Kink, with Fixed Adjustment Costs

Panel A: Adjustment from a linear tax to a kink ($K_1$)

Panel B: Counterfactual earnings under a linear tax

Panel C: Adjustment from a more pronounced ($K_1$) to a less pronounced kink ($K_2$)

Note: See Section 6 for an explanation of the figures.
Figure 8: Comparison of Normalized Excess Mass Among 62-64 Year-Olds and 66-68 Year-Olds, 1982-1993

Notes: The figure shows normalized bunching among 62-64 year-olds and 66-68 year-olds in each year from 1982 to 1993. See other notes from Figure 2.

Figure 9: Elasticity Estimates by Year, Saez (2010) Method, 1982-1993

Notes: The figure shows elasticities estimated using the Saez (2010) method, by year from 1982 to 1993, among 66-68 year-old OASI claimants. Dashed lines denote 95 percent confidence intervals. We use our methods for estimating normalized excess mass but use Saez’ (2010) formula to calculate elasticities, under a constant density. This method yields the following formula: \( \varepsilon = \frac{\log \left( \frac{\theta}{\tau} + 1 \right)}{\log \left( \frac{1 - \tau_0}{1 - \tau_1} \right)} \).
Table 1: Summary Statistics, Social Security Administration Master Earnings File

<table>
<thead>
<tr>
<th></th>
<th>Ages 62-69</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Earnings</td>
<td>28,892.63</td>
</tr>
<tr>
<td>(78,842.99)</td>
<td></td>
</tr>
<tr>
<td>10th Percentile</td>
<td>1,193.64</td>
</tr>
<tr>
<td>25th Percentile</td>
<td>5,887.75</td>
</tr>
<tr>
<td>50th Percentile</td>
<td>14,555.56</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>35,073.00</td>
</tr>
<tr>
<td>90th Percentile</td>
<td>64,647.40</td>
</tr>
<tr>
<td>Fraction Male</td>
<td>0.57</td>
</tr>
<tr>
<td>Observations</td>
<td>376,431</td>
</tr>
</tbody>
</table>

Notes: The data are taken from a one percent random sample of the SSA Master Earnings File and Master Beneficiary Record. The data cover those in 1990-1999 who are aged 62-69, claim by age 65, do not report self-employment earnings, and have positive earnings. Earnings are expressed in 2010 dollars. Numbers in parentheses are standard deviations.

Table 2: Estimates of Elasticity and Adjustment Cost: Variation Around 1990 Policy Change

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ε</td>
<td>φ</td>
<td>ε</td>
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<tr>
<td></td>
<td>1990</td>
<td>1989</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
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<td>$278</td>
<td>0.58</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>[0.31, 0.43]***</td>
<td>[58, 391]***</td>
<td>[0.45, 0.73]***</td>
<td>[0.24, 0.39]***</td>
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<tr>
<td>Uniform Density</td>
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<td>$162</td>
<td>0.36</td>
<td>0.19</td>
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<tr>
<td></td>
<td>[0.18, 0.24]***</td>
<td>[55, 211]***</td>
<td>[0.30, 0.43]***</td>
<td>[0.16, 0.23]***</td>
</tr>
<tr>
<td>Benefit Enhancement</td>
<td>0.58</td>
<td>$151</td>
<td>0.87</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>[0.50, 0.72]***</td>
<td>[17, 226]***</td>
<td>[0.69, 1.11]***</td>
<td>[0.41, 0.66]***</td>
</tr>
<tr>
<td>Excluding FICA</td>
<td>0.49</td>
<td>$318</td>
<td>0.74</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>[0.44, 0.59]***</td>
<td>[60, 364]***</td>
<td>[0.58, 0.94]***</td>
<td>[0.33, 0.54]***</td>
</tr>
<tr>
<td>Bandwidth = $400</td>
<td>0.45</td>
<td>$103</td>
<td>0.62</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>[0.36, 0.58]***</td>
<td>[0, 478]*</td>
<td>[0.47, 0.81]***</td>
<td>[0.32, 0.56]***</td>
</tr>
<tr>
<td>Bandwidth = $1,600</td>
<td>0.33</td>
<td>$251</td>
<td>0.55</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>[0.29, 0.43]***</td>
<td>[34, 407]***</td>
<td>[0.43, 0.72]***</td>
<td>[0.23, 0.40]***</td>
</tr>
</tbody>
</table>

Notes: The table shows estimates of the elasticity and adjustment cost using the method described in Section 6.3, investigating the 1990 reduction in the AET BRR from 50 percent to 33.33 percent. We report bootstrapped 95 percent confidence intervals in parentheses. The baseline specification uses a nonparametric density taken from the age 72 earnings distribution, calculates the effective MTR by including the effects of the AET BRR and federal and state income and FICA taxes, uses data from 1989 and 1990, and calculates bunching using a bin width of $800. The estimates that include benefit enhancement use effective marginal tax rates due to the AET based on the authors’ calculations relying on Coile and Gruber (2001) (assuming that individuals are considering earning just enough to trigger benefit enhancement), which imply the BRR falls from 36% to 24% due to the 1990 policy change. Columns (1) and (2) report joint estimates with φ ≥ 0 imposed (consistent with theory), while Columns (3) and (4) impose the restriction φ = 0. The constrained estimate in Column (3) only uses data from 1990, Column (4) uses only data from 1989. *** indicates that the left endpoint of the 99% confidence interval (CI) is greater than zero, ** the 95% CI and * the 90% CI.
Table 3: Estimates of Elasticity and Adjustment Cost: Disappearance of Kink at Age 70

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>( \varepsilon</th>
<th>\phi</th>
<th>\varepsilon + \phi = 0, \text{Age 69}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.42</td>
<td>$90</td>
<td>0.38</td>
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</tr>
<tr>
<td>Uniform Density</td>
<td>0.28</td>
<td>$90</td>
<td>0.25</td>
<td></td>
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</tr>
<tr>
<td>Benefit Enhancement</td>
<td>0.62</td>
<td>$59</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Excluding FICA</td>
<td>0.53</td>
<td>$83</td>
<td>0.49</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Bandwidth = $400</td>
<td>0.39</td>
<td>$62</td>
<td>0.36</td>
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<td></td>
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</tr>
<tr>
<td>Bandwidth = $1,600</td>
<td>0.45</td>
<td>$100</td>
<td>0.41</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>68-70 year-olds</td>
<td>0.44</td>
<td>$42</td>
<td>0.37</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>69, 71 year-olds</td>
<td>0.45</td>
<td>$175</td>
<td>0.38</td>
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<td></td>
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<tr>
<td>Born January-March</td>
<td>0.48</td>
<td>$86</td>
<td>0.49</td>
<td></td>
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</tr>
</tbody>
</table>

Notes: The table estimates parameters using the removal of the AET at age 70, using data on 69-71 year-olds in 1990-1999. The estimates of bunching at age 70 are potentially affected by the coarse measure of age that we use, as explained in the main text. Thus, we use both age 70 and age 71 in estimating these results, and alternatively use only ages 69 and 71, which shows very similar results. The final row shows the results only for those born in January to March, again to address this issue. For this sample, we pool 1983-1989 and 1990-1999 (accounting for the different benefit reduction rates in each period) to maximize statistical power. See also notes from Table 2.

Table 4: Estimates of Elasticity and Adjustment Cost: Pooling 69/70 Transition and 1989/1990 Transition

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.39</td>
<td>$160</td>
</tr>
<tr>
<td>Uniform Density</td>
<td>0.22</td>
<td>$105</td>
</tr>
<tr>
<td>Benefit Enhancement</td>
<td>0.62</td>
<td>$100</td>
</tr>
<tr>
<td>Excluding FICA</td>
<td>0.41</td>
<td>$67</td>
</tr>
<tr>
<td>Bandwidth = $400</td>
<td>0.46</td>
<td>$94</td>
</tr>
<tr>
<td>Bandwidth = $1,600</td>
<td>0.37</td>
<td>$135</td>
</tr>
</tbody>
</table>

Notes: This table implements our “comparative static” method, applied to pooled data from two policy changes: (1) around the 1989/1990 transition analyzed in Table 2, and (2) around the age 69/70 transition analyzed in Table 3. The table shows extremely similar results to the dynamic specification in Table 5, where we also pool data from around these two policy changes. See also notes from Tables 2 and 3.
Table 5: Estimates of Elasticity and Adjustment Cost Using Dynamic Model

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\varepsilon$</td>
<td>0.36</td>
<td>$\phi$</td>
<td>$243$</td>
<td>$[0.34, 0.40]***$</td>
</tr>
<tr>
<td>Baseline</td>
<td>$\pi_1$</td>
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</tr>
<tr>
<td>Uniform Density</td>
<td>$\pi_{12}$</td>
<td>0.22</td>
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<td></td>
</tr>
<tr>
<td>Benefit Enhancement</td>
<td>$\pi_{13}$</td>
<td>0.31</td>
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<tr>
<td>Excluding FICA</td>
<td>$\pi_{14}$</td>
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</tr>
<tr>
<td>Bandwidth = $400$</td>
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<td>Bandwidth = $1,600$</td>
<td>$\pi_{16}$</td>
<td>0.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The table shows estimates of the elasticity $\varepsilon$, the adjustment cost $\phi$, and the cumulative probability in each period of having drawn $\phi_i > 0$ for each period following the policy change, i.e. $\pi_1$ as well as $\pi_{12}$. The model is estimated by matching predicted and observed bunching, using bunching on 66-68 year-olds (pooled) for each year 1987-1992, and bunching on 1990-1999 (pooled) for each age 67-72. Estimates of $\pi_{1234}$ are statistically significantly different from zero, even though the reported point estimates are 0.00, because the point estimates are positive but round to zero. $\pi_{12345}$ are always estimated to 0.00, with a confidence interval that rules out more than a small value (results available upon request). The results are comparable when we investigate only the 1989/1990 or 69/70 policy changes alone using the dynamic specification (results available upon request). *** indicates $p<0.01; ** p<0.05; * p<0.10$. 


A Appendix (for online publication)

A.1 Explaining Bunching Below and Above the Kink

Figure 2 shows an intriguing pattern: bunching appears asymmetric around the exempt amount. In particular, it is evident that more excess mass appears just below the exempt amount, relative to just above it. We show in this appendix that our current model can explain this pattern. In particular, the observed pattern can result simply from a downward-sloping counterfactual density and symmetric noise in realized earnings, relative to desired earnings.

We demonstrate this in a simple model without adjustment costs or heterogeneity. In the upper-left panel of Appendix Figure B.1, we show the earnings density prior to the introduction of a kink and partition earners into three groups: A, B, and C. In the bottom-left panel, the kink has been introduced. Group A, who earns below $z^*$, does not respond. Group B is the set of bunchers, who locate near $z^*$. Finally, group C is comprised of earners who reduce their earnings in response to the kink, but do not bunch at $z^*$. To match the data, we assume that bunching is diffuse around $z^*$, rather than occurring only at $z^*$—i.e. realized earnings are equal to desired earnings plus some random, mean-zero noise (Friedberg, 2000). As can be seen, the downward slope of the initial density causes the ex-post density, net of bunchers, to be much higher just below the exempt amount, relative to just above the it. As a result, when the set of bunchers create excess mass near $z^*$, it appears that more excess mass is located just below the exempt amount, relative to just above it.

We confirm this in the right panel of Appendix Figure B.1, where we have plotted a counterfactual density and a simulated earnings density. We use the age 72 earnings density as the counterfactual density under a linear budget set, as in our main results. Next, we simulate the density that would be predicted at age 69 with the baseline elasticity of 0.36 from our dynamic model. We ignore the adjustment cost under the assumption—for illustrative purposes—that we are operating in a steady state in which enough time has elapsed for full adjustment to take place, which is plausible by age 69. Finally, we assume, for illustrative purposes, that relative to desired earnings, a normally distributed, mean-zero error is added, with a standard deviation equal to $1,800, i.e. half of the baseline excluded region above and below the exempt amount. This ensures that among those intending to locate at the exempt amount, over 99 percent locate within the baseline excluded region of $3,600 on either side of the exempt amount.40

It is evident in Appendix Figure B.1 that more excess mass appears just below the exempt amount than just above it, consistent with the observed pattern at age 69 in Figure 2. Indeed, we can calculate measures of excess mass in the regions below and above the exempt amount, respectively. We do so by estimating excess mass, limiting the sample only to observations below and above the exempt amount, respectively. This calculation on the actual density for age 69 shows that normalized excess mass below the exempt amount accounts for 63 percent of the total amount of excess mass (in the bins centered below, at, and above the exempt amount). This is insignificantly different from the 55 percent predicted in the simulation. Note that in both the actual and simulated distributions, the percent bunching in the bin centered at the kink is substantial and includes some individuals who have earnings below the kink (but still within the central bin’s width).

These qualitative conclusions are robust to alternative assumptions about the parameters (results available upon request). Thus, we can reproduce the patterns of bunching below and above the kink using only the fact that the counterfactual density is downward sloping, and following previous literature in adding noise to realized earnings, relative to desired earnings.

A.2 Dynamic Model with Forward-Looking Behavior

We present in this appendix a version of the dynamic model in Section 6.4 in which we allow for forward-looking behavior. The key difference in implications is that in addition to a gradual, lagged response to policy changes, this version of the model also predicts anticipatory adjustment by agents when policy changes are anticipated in advance. We have essentially the same setting as in Section 6.4, except that we will alter three of the assumptions. First, in each period, an individual draws a cost of adjustment, $\phi_t$, from a discrete distribution, which takes a value of $\phi$ with probability $\pi$ and a value of 0 with probability $1 - \pi$.41 Second,

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40 The results are comparable throughout a range of assumptions about this standard deviation.
41 For expositional purposes, we constrain the probability of drawing a nonzero fixed costs to be $\pi$ in all periods. Thus, the terms from Section 6.4 of the form $\prod \pi_j$ simplify to $\pi^2$ in this appendix. All results go through with the more flexible
individuals make decisions over a finite horizon, living until Period $T$. In period 0, the individuals face a linear tax schedule, $T_0(z) = \tau_0 z$, with marginal tax rate $\tau_0$. In period 1, a kink, $K_1$, is introduced at the earnings level $z^*$. This tax schedule is implemented for $T_1$ periods, after which the tax schedule features a smaller kink, $K_2$, at the earnings level $z^*$. The smaller kink is present until period $T_2$, after which we return to the linear tax schedule, $T_0$. As before, the kink $K_j$, $j \in \{1, 2\}$, features a top marginal tax rate of $\tau_j$ for earnings above $z^*$.\footnote{In Section 6.4, we do not specify time $T_2$, when the smaller kink, $K_2$, is removed, as it is not relevant to the case where individuals are not forward-looking.} Finally, in each period, individuals solve this maximization problem:

$$\max_{(c_{a,t}, z_{a,t})} v(c_{a,t}, z_{a,t}; a, z_{a,t-1}) + \delta V_{a,t+1}(z_{a,t}, A_{a,t}),$$  \hspace{1cm} (A.1)

where $v(c_{a,t}, z_{a,t}; a, z_{a,t-1}) \equiv u(c_{a,t}, z_{a,t}; a) - \hat{\phi}_t \cdot 1(z_{a,t} \neq z_{a,t-1})$, $\delta$ is the discount factor, and $V_{a,t+1}$ is the value function moving forward in Period $t + 1$:

$$V_{a,t+1}(\zeta, A_{a,t}) = \mathbb{E}_\phi \left[ \max_{(c_{a,t+1}, z_{a,t+1})} v(c_{a,t+1}, z_{a,t+1}; a, \zeta) + \delta V_{a,t+2}(z_{a,t+1}, A_{a,t+1}) \right].$$  \hspace{1cm} (A.2)

$V_{a,t+1}$ is a function of where the individual has chosen to earn in Period $t$ and assets $A_{a,t}$. The expectation $\mathbb{E}_\phi [\cdot]$ is taken over the distribution of $\hat{\phi}_t$. The intertemporal budget constraint is:

$$A_{a,t} = (1 + r)(A_{a,t-1} + z_{a,t} - T(z_{a,t}) - c_{a,t}).$$  \hspace{1cm} (A.3)

We assume that $\delta (1 + r) = 1$. Because individuals have quasilinear preferences, this implies that consumption can be set to disposable income in each period: $c_{a,t} = z_{a,t} - T(z_{a,t})$. We therefore use the following shorthand:

$$u^0_a(z) = u(z - T_j(z), z; a)$$
$$V_{a,t}(z) = V_{a,t}(z, A_{a,t-1})$$  \hspace{1cm} (A.4)

Next, we define two operators that measure the utility gain (or loss) following a discrete change in earnings:

$$\Delta u^0_a(z, z') = u^0_a(z) - u^0_a(z')$$
$$\Delta V_{a,t}(z, z') = V_{a,t}(z) - V_{a,t}(z')$$  \hspace{1cm} (A.5)

In each case above, the utility and utility differential depend on the tax schedule. We define $z^0_a$ as the optimal level of earnings under a frictionless, static optimization problem, facing the tax schedule $T_j$. We will refer to the frictionless, dynamic optimum in any given period as $\hat{z}_{a,t}$.\footnote{In a model with no forward-looking behavior, $z^0_a = \hat{z}_{a,t}$.} This is the optimal level of earnings when there is a fixed cost of zero drawn in the current period, but a nonzero fixed cost may be drawn in future periods. We will also make a distinction between two types of earnings adjustments: active and passive. An active earnings adjustment takes place in the presence of a nonzero fixed cost, while a passive earnings adjustment takes place only when a fixed cost of zero is drawn. We solve the model recursively, beginning in the regime after time $T_2$, when the smaller kink, $K_2$, has been removed, continuing with the solution while the kink $K_2$ is present between times $T_1$ and $T_2$, and finally considering the first regime when the kink $K_1$ is present between time period 1 and $T_1$.\footnote{Our recursive method can be extended to the case of multiple, successive kinks. The effect on bunching of a sequence of more kinks depends on the relative size of the successive kinks.}

### A.2.1 Earnings between $T_2$ and $T$

We will now derive the value function $V_{a,T_2+1}(z)$. We begin with the following result: If an individual with initial earnings $z$ makes an active adjustment in period $t > T_2 + 1$, then it must be the case that

$$\frac{1 - (\delta \pi)^{T_{T_2+1-t}}} {1 - \delta \pi} \Delta u^0_a(z^0_a, z) \geq \phi.$$  \hspace{1cm} (A.6)
We demonstrate this result with a constructive proof, showing the result for periods $\mathcal{T}$ and $\mathcal{T} - 1$. Because the tax schedule is constant throughout this terminal period, the frictionless, dynamic optimum is equal to the static optimum: $z_{a,t} = z_a^0$. First, consider an agent in period $\mathcal{T}$, with initial earnings $z$, who is considering maintaining earnings at $z$ or paying the fixed cost $\phi$ and making an active adjustment to $z_a$, the frictionless, dynamic optimum in period $\mathcal{T}$. The agent will make the adjustment if:

$$\Delta u_a^0 (z_a, z) \geq \phi = \frac{1 - \delta \pi}{1 - \delta \phi}. \quad (A.7)$$

Rearranging terms, we have satisfied the inequality in (A.6).

Now consider agents in period $\mathcal{T} - 1$ with initial earnings $z$. There are two types, those who would make an active adjustment to $z_a^0$ in period $\mathcal{T}$ if the earnings $z$ are carried forward and those who would not. Consider those who would not. If the agent remains with earnings of $z$, then utility will be $u_a^0 (z) + \delta V_a, \mathcal{T} (z) = u_a^0 (z) + \delta \left[ \pi (u_a^0 (z)) + (1 - \pi) u_a^0 (z_a^0) \right]$. If the agent actively adjusts to $z_a^0$, then utility will be $u_a^0 (z_a^0) - \phi + \delta u_a^0 (z_a^0)$. The agent will actively adjust in period $\mathcal{T} - 1$ if:

$$\Delta u_a^0 (z_a^0, z) \geq \frac{1}{1 + \delta \phi} \frac{1 - \delta \pi}{1 - (\delta \pi)^2} \phi. \quad (A.8)$$

Once again, rearranging terms confirms that (A.6) holds. Finally, consider agents who would actively adjust from $z$ to $z_a^0$ if earnings level $z$ is carried forward. In this case, the agent’s utility when remaining at $z$ is:

$$u_a^0 (z) + \delta V_a, \mathcal{T} (z) = u_a^0 (z) + \delta \left[ \pi (u_a^0 (z_a^0) - \phi) + (1 - \pi) u_a^0 (z_a^0) \right] \quad (A.9)$$

Intuitively, the agent will receive the optimal level of utility in the next period, and with probability $\pi$ the agent will have to pay the fixed cost to achieve it. Similarly, the agent’s utility after actively adjusting to $z_a^0$ in period $\mathcal{T} - 1$ is $u_a^0 (z_a^0) - \phi + \delta u_a^0 (z_a^0)$. The agent will therefore adjust in period $\mathcal{T}$ if:

$$\Delta u_a^0 (z_a^0, z) \geq (1 - \delta \pi) \phi. \quad (A.10)$$

However, we know from (A.7) that this already holds for the agent who actively adjusts in period $\mathcal{T}$. Finally, note that (A.7) implies (A.8). It follows that in period $\mathcal{T} - 1$, adjustment implies (A.7). We can similarly show the result for earlier periods by considering separately: (a) those who would actively adjust in the current period, but not in any future period; and (b) those who would adjust in some future period. Both types will satisfy the key inequality. As a corollary, note that if an individual with initial earnings $z$ makes an active adjustment in period $t > \mathcal{T} + 1$, then she will also find it optimal to do so in any period $t'$, where $\mathcal{T} < t' < t$. To see this, note that if (A.6) holds for $t$, then it also holds for $t' < t$. It follows that the agent would also actively adjust in period $t'$.

Now consider an agent who earns $z$ in period $\mathcal{T}_2$. Note that our results above imply that any active adjustment that takes place after $\mathcal{T}_2$ will only happen in period $\mathcal{T}_2 + 1$. These agents will receive a stream of discounted payoffs of $u_a^0 (z_a^0)$ for $\mathcal{T} - \mathcal{T}_2$ periods, i.e. $\sum_{j=0}^{\mathcal{T} - \mathcal{T}_2} \delta^j u_a^0 (z_a^0) = \frac{1 - \delta^{\mathcal{T} - \mathcal{T}_2}}{1 - \delta \pi} u_a^0 (z_a^0)$, and pay a fixed cost of $\phi$ in period $\mathcal{T}_2$ with probability $\pi$. Otherwise, an agent will adjust to the dynamic frictionless optimum $z_a^0$ only when a fixed cost of zero is drawn. In the latter case, the agent receives a payoff of $u_a^0 (z_a^0)$ until a fixed cost of zero is drawn, after which, the agent receives $u_a^0 (z_a^0)$. We can therefore derive the following value function:

$$V_a, \mathcal{T}_2 + 1 (z) = \begin{cases} \frac{1 - \delta^{\mathcal{T} - \mathcal{T}_2}}{1 - \delta \pi} u_a^0 (z_a^0) - \pi \phi & \text{if } \frac{1 - (\delta \pi)^{\mathcal{T} - \mathcal{T}_2}}{1 - \delta \phi} \Delta u_a^0 (z_a^0, z) \geq \phi \\
\frac{1 - \delta^{\mathcal{T} - \mathcal{T}_2}}{1 - \delta \pi} u_a^0 (z_a^0) - \pi \frac{1 - (\delta \pi)^{\mathcal{T} - \mathcal{T}_2}}{1 - \delta \phi} \Delta u_a^0 (z_a^0, z) & \text{otherwise} \end{cases} \quad (A.11)$$

The expected utility for passive adjusters is constructed recursively, working backward from period $\mathcal{T}$ to period $\mathcal{T}_2 + 1$. 

45 The expected utility for passive adjusters is constructed recursively, working backward from period $\mathcal{T}$ to period $\mathcal{T}_2 + 1$. 

43
To gain some intuition for (A.6), note that the left side of (A.6) is the net present value of the stream of
the utility differential once the agent adjusts from $z$ to $z_0$. If this exceeds the up-front cost of adjustment, $\phi$,
then the agent actively adjusts. The discount factor for $j$ periods in the future, however, is $(\delta \pi)^j$, instead of
only $\delta^j$. The reason is that current adjustment only affects future utility $j$ periods from now if $j$ consecutive
nonzero fixed costs are drawn, which happens with probability $\pi^j$. To better understand our second result
regarding the timing of active changes, note that if the gains from adjustment over $T_t$ periods exceed the
up-front cost, then the agent should also be willing to adjust in period $t' < t$ and accrue $T - t'$ periods of
this gain, for the same up-front cost of $\phi$.

A.2.2 Earnings between $T_1$ and $T_2$

We now derive the value function $V_{a,T_2+1}(z)$. In this case, the dynamic frictionless optimum in each period,
$\tilde{z}_{a,t}$, is not constant. Intuitively, the agent trades off the gains from adjusting earnings in response to $K_2$
with the effect of this adjustment on the value function $V_{a,T_2+1}$. In general, the optimum is defined as:

$$
\tilde{z}_{a,t} = \arg \max_{z \in [z_a^2, z_a^0]} \frac{1 - (\delta \pi)^{T_2+1-t}}{1 - \delta \pi} u_a^2(z) + \delta^{T_2+1-t} \pi^{T_2-t} V_{a,T_2+1}(z).
$$

(A.12)

We restrict the maximization to the interval $[z_a^2, z_a^0]$, since reducing earnings below $z_a^2$ or raising earnings
above $z_a^0$ weakly reduces utility in any current and all future periods for $t > T_1$. From (A.11), we know that
$V_{a,T_2+1}$ is continuous, and thus the solution in (A.12) exists. We present two results analogous to those in
Section A.2.1, without proof. The proofs, nearly identical to those in the previous section, are available upon
request. First, if an individual with initial earnings $z$ makes an active adjustment in period $t$, $T_1 < t \leq T_2$,
then:

$$
1 - (\delta \pi)^{T_2+1-t} \Delta u_a^2(\tilde{z}_{a,t}, z) + \delta^{T_2+1-t} \pi^{T_2-t} \Delta V_{a,T_2+1}(\tilde{z}_{a,t}, z) \geq \phi.
$$

(A.13)

Furthermore, if an individual with initial earnings $z$ makes an active adjustment in period $t$, $T_1 < t \leq T_2$,
then she will also find it optimal to do so in any period $t'$, where $T_1 < t' < t$.

The condition in (A.13) differs from that in (A.6) because the effect of adjustment on the utility beyond
period $T_2$ is taken into account, in addition to the up-front cost of adjustment, $\phi$. Any adjustment in this
time interval, active or passive, will be to the dynamic, frictionless optimum for the current period, $\tilde{z}_{a,t}$.
As before, (A.13) implies that all active adjustment occurring between $T_1 + 1$ and $T_2$ takes place in period $T_2+1$. Those who adjust in period $T_2+1$ will earn $\tilde{z}_{a,T_1+1}$. Thereafter, they only adjust to $\tilde{z}_{a,t}$ when a fixed
cost of zero is drawn. Likewise, those who only adjust passively earn $z_a,T_1$ in period $T_1+1$, and thereafter

46 Technically, we can see from (A.11) that while the function $V_{a,T_2+1}$ is continuous, it is kinked, which creates a nonconvexity.
Thus, the solution in (A.12) may not always be single-valued. In such cases, we define $\tilde{z}_{a,t}$ as the lowest level of earnings that
maximizes utility.
adjust to \( \tilde{z}_{a,t} \) when a fixed cost of zero is drawn. We can therefore derive the following value function:

\[
V_{a,T_1+1}(z) = \begin{cases} 
\sum_{j=0}^{T_2-T_1-1} \delta^j u^2_a(\tilde{z}_{a,T_1+1+j}) + \delta^{T_2-T_1} V_{a,T_2+1}(\tilde{z}_{a,T_2}) \\
\sum_{j=0}^{T_2-T_1-2} \left( \frac{\delta \pi}{\pi^j} \right) \delta^{T_2-T_1} V_{a,T_2+1}(\tilde{z}_{a,T_1+2+j}, \tilde{z}_{a,T_1+1+j}) \\
\sum_{j=0}^{T_2-T_1-2} \left( 1 - \frac{\delta \pi}{\pi^j} \right) \delta^{T_2-T_1-1-j} \pi \delta^{j+1} u^2_a(\tilde{z}_{a,T_1+2+j}, \tilde{z}_{a,T_1+1+j}) \\
\sum_{j=0}^{T_2-T_1-1} (\pi \delta)^j \pi \delta^{T_2-T_1-1} V_{a,T_2+1}(\tilde{z}_{a,T_1+1}, z) \\
\sum_{j=0}^{T_2-T_1-1} (\pi \delta)^j \pi \delta^{T_2-T_1-1} \delta^{j+1} u^2_a(\tilde{z}_{a,T_1+2+j}, \tilde{z}_{a,T_1+1+j}) \\
-\pi \bigg\{ \sum_{j=0}^{T_2-T_1-1} \left( \frac{\delta \pi}{\pi^j} \right) u^2_a(\tilde{z}_{a,T_1+1+j}) \bigg\} \\
\end{cases} \tag{A.14}
\]

The first case in (A.14) applies to those who actively adjust in period \( T_1 + 1 \) and passively adjust thereafter. The first line is the utility that would accrue if a fixed cost of zero were drawn in each period. The next two lines represent the deviation from this stream of utility, due to nonzero fixed costs potentially drawn in periods \( T_1 + 1 \) through \( T_2 \). The final line represents the fixed cost that is paid in period \( T_1 + 1 \) with probability \( \pi \). The second case in (A.14) applies to those who only passively adjust. The first three lines remain the same. The final two lines represent a loss in utility attributed to fact that earnings in period \( T_1 + 1 \) may not be \( \tilde{z}_{a,T_1+1} \). Note that earnings in period \( T_1 \) can only affect utility through this last channel.

### A.2.3 Earnings between Period 1 and \( T_1 \)

Earnings during the first period, when the kink \( K_1 \) is present, can be derived similarly. The dynamic, frictionless optimum is now defined as:

\[
\tilde{z}_{a,t} = \arg \max_{z \in [z^0_a, z^1_a]} \frac{1 - (\pi \delta)^{T_1+1-t}}{1 - \delta \pi} u^1_a(z) + \delta^{T_1+1-t} \pi \delta^{T_1-t} V_{a,T_1+1}(z) \tag{A.15}
\]

Similar to the other cases, if an individual with initial earnings \( z \) makes an active adjustment in period \( t \), \( 0 < t \leq T_1 \), then it must be the case that

\[
\frac{1 - (\pi \delta)^{T_1+1-t}}{1 - \delta \pi} u^1_a(\tilde{z}_{a,t}, z) + \delta^{T_1+1-t} \pi \delta^{T_1-t} \delta V_{a,T_1+1}(\tilde{z}_{a,t}, z) \geq \phi. \tag{A.16}
\]

Furthermore, if an individual with initial earnings \( z \) makes an active adjustment in period \( t \), \( 0 < t \leq T_1 \), then she will also find it optimal to do so in any period \( t' \), where \( 0 < t' < t \). Again, this implies that all active adjustment will take place in period 1. Since individuals begin with earnings of \( z^0_a \), we know that all active adjustment will be downward. Thereafter, it can be shown that \( \tilde{z}_{a,t} \) is weakly increasing, and upward adjustment will occur passively.

### A.2.4 Characterizing Bunching

Given these results, we can now derive expressions for excess mass at \( z^* \) analogous to (8) and (9). For notational convenience, we define \( A_j(z) \) as the set of individuals, \( a \), with initial earnings \( z \) who actively

\[\text{Note, the objective function now features two potential nonconvexities. In cases where the solution is multi-valued, we again define } \tilde{z}_{a,t} \text{ as the lowest earnings level from the set of solutions.}\]
adjust in period $j$. Again, denote $B^t_1$ as bunching at $K_1$ in period $t \in [1, T_1]$. We have the following generalized version of (8):

$$
B^t_1 = \int_{z^*}^{z^* + \Delta z^*} \left[ 1 \{ \tilde{z}_{a,1} = z^*, a \in A_1(\zeta) \} 
+ \sum_{j=1}^{t} (1-\pi^j) \pi^{t-j} 1 \{ \sup \{ l | l \leq t, \tilde{z}_{a,t} = z^* \} = j, a \notin A_1(\zeta) \}
- \sum_{j=1}^{t-1} (1-\pi^{t-j}) 1 \{ \sup \{ l | l \leq t, \tilde{z}_{a,t} = z^* \} = j, a \in A_1(\zeta) \} \right] h_0(\zeta) d\zeta.
$$

(A.17)

We have partitioned the set of potential bunchers into three groups in (A.17). In the first line, we have the set of active bunchers in period 1. In the second line, we capture individuals who are passive bunchers, i.e. $a \notin A_1(\zeta^0)$. For $j \in [1, t-1]$, the indicator function selects the individual who has $\tilde{z}_{a,j} = z^*$ but $\tilde{z}_{a,j+1} \neq z^*$. Since $\tilde{z}_{a,t}$ is weakly increasing, the optimal earnings for this individual is $z^*$ in periods 1 through $j-1$. The probability that the individual bunches by period $j$ is $1 - \pi^j$. Thereafter, the individual will de-bunch if a fixed cost of zero is drawn. The probability of only drawing nonzero fixed costs thereafter is $\pi^{t-j}$. For $j = t$, the indicator function selects agents for whom $\tilde{z}_{a,t} = z^*$. Their probability of passively bunching by period $t$ is $1 - \pi^t$. The third line captures the outflow of active bunchers, for whom $\tilde{z}_{a,t}$ ceases to be $z^*$ starting in period $j$. The probability of having drawn a nonzero fixed cost and de-bunching since period $j$ is $1 - \pi^{t-j}$.

Equation (A.17) differs from (8) in three key ways. First, the set of active bunchers in period 1 is different, as can be seen by comparing (A.16) and the relevant condition for active bunchers in Section 6.4, $\Delta u_0^t(z^*, \zeta^0) \geq \phi$. The utility gain accrues for multiple periods in the forward-looking case, increasing the probability of actively bunching, but the effect of adjustment on future payoffs via $V_{a,T_1+1}$ may either reinforce or offset this incentive. Furthermore, passive bunchers are (weakly) less likely to remain bunching, as they de-bunch in anticipation of policy changes in future periods. To see this, note that the $\pi^{t-j}$ factor is decreasing in $t$. Finally, the set of active bunchers similarly de-bunch passively, in anticipation of future policy changes. The model therefore predicts a gradual outflow from the set of bunchers, in anticipation of the shift from $K_1$ to $K_2$. Nonetheless, the overall net change in bunching over time is ambiguous.

We now turn to bunching starting in period $T + 1$. It can be shown, similarly to the cases above, that if an agent would be willing to actively bunch in period $T_1 + 1$, she will also be willing to actively bunch in earlier periods. Thus, the only active adjustment occurring that affects bunching will be de-bunching. The set of individuals who actively de-bunch, $A_{T_1+1}(z^*)$, are those for whom (A.13) is satisfied, when evaluated at $t = T_1 + 1$ and $z = z^*$. The remaining changes in bunching between $T_1$ and $T_2$ consist of passive adjustment among those who were bunching at the end of period $T_1$. We can thus characterize $B^t_2$, bunching at $K_2$ in
period \( t \in [T_1 + 1, T] \), in a manner analogous to (9):\(^{48}\)

\[
B^t_2 = \int_{z^*}^{z^* + \Delta z_i^*} \left[ \{ a \notin A_{T_{i+1}} (z^*) \} \right. \\
\times \left\{ \pi^{t-T_i} \mathbf{1} \{ \tilde{z}_{a,T_i+1} \neq z^* \} + \sum_{j=T_i+1}^{t} \pi^{t-j} \mathbf{1} \{ \sup \{ l \mid l \leq t, \tilde{z}_{a,l} = z^* \} = j \} \right\} \\
\times \left\{ \mathbf{1} \{ \tilde{z}_{a,1} = z^*, a \in A_1 (\zeta) \} + \sum_{j=1}^{T_i} (1 - \pi^j) \pi^{T_i-j} \mathbf{1} \{ \sup \{ l \mid l \leq T_i, \tilde{z}_{a,l} = z^* \} = j, a \notin A_1 (\zeta) \} \\
- \sum_{j=1}^{T_i-1} (1 - \pi^{T_i-j}) \mathbf{1} \{ \sup \{ l \mid l \leq T_i, \tilde{z}_{a,l} = z^* \} = j, a \in A_1 (\zeta) \} \right\} h_0 (\zeta) d\zeta.
\]

(A.18)

The first line of this expression selects only those agents who do not actively de-bunch immediately in period \( T_1 + 1 \). The second line selects the set of agents who would like to passively de-bunch beginning at some period \( j > T_1 + 1 \). They are weighted by the probability of continuing to bunch due to consecutive draws of nonzero fixed costs. The final three lines select agents from the set of bunchers at the end of period \( T_1 \). As with our simpler model in Section 6.4, bunching gradually decreases following a reduction in the size of the kink from \( K_1 \) to \( K_2 \). However, in this case, the reduction is due to both fixed costs of adjustment and anticipation of the removal of the kink \( K_2 \) in period \( T_2 + 1 \).

As in Section 6.4, the richer model in this appendix nests the dynamic model without forward looking behavior when we set \( \delta = 0 \), collapses to the comparative static model of Sections 6.2-6.3 if we additionally assume that \( \pi = 1 \) and is equivalent to the frictionless model when either \( \phi = 0 \) or \( \pi = 0 \).

### A.3 Derivation of Bunching Formulae with Heterogeneity

#### A.3.1 Comparative Static Model

Under heterogenous preferences, our estimates can be interpreted as reflecting average parameters among the set of bunchers (as in Saez, 2010, and Kleven and Waseem, 2013). As described in Section 6.5.1, suppose \((\varepsilon_i, \phi_i, a_i)\) is jointly distributed according to a smooth CDF, which translates to a smooth, joint distribution of earnings, adjustment costs and elasticities, fixed costs and earnings. Let the joint density of earnings, adjustment costs and elasticities be \( h_0^i (z, \varepsilon, \phi) \) under a linear tax of \( \tau_0 \). Assume that the density of earnings is constant over the interval \([z^*, z^* + \Delta z^*] \), conditional on \( \varepsilon \) and \( \phi \). When moving from no kink to a kink, we derive a formula for bunching at \( K_1 \) in the presence of heterogeneity as follows:

\[
B_1 = \iint_{\Xi_1} h_0^i (\zeta, \varepsilon, \phi) d\zeta d\varepsilon d\phi \\
= \int_{\Xi_1} [z^* + \Delta z_i^* - \Xi_i] h_0^i (z^*, \varepsilon, \phi) d\varepsilon d\phi \\
= h_0 (z^*) \cdot \int_{\Xi_1} [z^* + \Delta z_i^* - \Xi_i] \frac{h_0^i (z^*, \varepsilon, \phi)}{h_0 (z^*)} d\varepsilon d\phi \\
= h_0 (z^*) \cdot \mathbb{E} [z^* + \Delta z_i^* - \Xi_i],
\]

(A.19)

where we have used the assumption of constant \( h_0^i (\cdot) \) in line two, \( h_0 (z^*) = \iiint h_0^i (z^*, \varepsilon, \phi) d\varepsilon d\phi \), and \( \zeta, \varepsilon \) and \( \phi \) are dummies of integration. The expectation \( \mathbb{E} [\cdot] \) is taken over the set of bunchers, under the various combinations of \( \varepsilon \) and \( \phi \) throughout the support. It follows that normalized bunching can be expressed as

\(^{48}\)When \( T_1 = 1 \), we set the very last summation to zero.
follows:

\[ b_1 = z^* + \mathbb{E}[\Delta z^*_1] - \mathbb{E}[\bar{z}_1]. \] (A.20)

Under heterogeneity, the level of bunching identifies the average behavioral response, \( \Delta z^* \), and threshold earnings, \( \bar{z}_1 \), among the marginal bunchers under each possible combination of parameters \( \varepsilon \) and \( \phi \). Under certain parameter values, there is no bunching, and thus, the values of the elasticity and adjustment cost in these cases do not contribute our estimates.

When we move sequentially from a larger kink, \( K_1 \), to a smaller kink, \( K_2 \), our formula for bunching under \( K_2 \) in the presence of heterogeneity is likewise derived as follows:

\[
\hat{B}_2 = \iint \int z_0 \cdot h_0^*(\zeta, \varepsilon, \phi) \, d\zeta d\varepsilon d\phi \\
= \iint \int [\bar{z}_0 - \bar{z}_1] \cdot h_0^*(z^*, \varepsilon, \phi) \, d\varepsilon d\phi \\
= h_0(z^*) \cdot \iint \int [\bar{z}_0 - \bar{z}_1] \cdot h_0^*(z^*, \varepsilon, \phi) \, d\varepsilon d\phi \\
= h_0(z^*) \cdot \mathbb{E} [\bar{z}_0 - \bar{z}_1]. \] (A.21)

Similarly, normalized bunching can now be expressed as follows:

\[ \hat{b}_2 = \mathbb{E} [\bar{z}_0] - \mathbb{E} [\bar{z}_1]. \] (A.22)

Once again, the expectations are taken over the population of bunchers.

Following the approach in Kleven and Waseem (2013, pg. 682), the average value of the parameters \( \Delta z^*_1 \), \( \bar{z}_1 \) and \( \bar{z}_0 \) can then be related to \( \varepsilon \) and \( \phi \), assuming a quasi-linear utility function and using (5) and (7) and the identities \( \Delta z^*_1 = \varepsilon z^* d\tau_1 / (1 - \tau_0) \) and \( \bar{z}_0 - \bar{z}_1 = \varepsilon \bar{z}_2 d\tau_2 / (1 - \tau_0) \).

### A.3.2 Dynamic Model

A similar interpretation of our results holds when we turn to our more dynamic framework in Section 6.4. Suppose now that \( (\varepsilon_i, \phi_i, a_i, \pi_i) \) is jointly distributed according to a smooth CDF, which results in a smooth, joint distribution of elasticities, fixed costs, earnings, and probabilities of drawing a positive fixed cost. In order to gain tractability, we assume that the profile \( \pi_i \) is independent of the parameters \( (\varepsilon_i, \phi_i, a_i) \). The result is that the joint density of these parameters, under a linear tax of \( \tau_0 \), can be expressed as a product of two densities: \( h_0^i(z^*, \varepsilon, \phi, g(\pi_i)) \). We maintain the assumption that the density of earnings is constant over the interval \([z^*, z^* + \Delta z^*], \) conditional on \( \varepsilon \) and \( \phi \). Bunching at \( K_1 \) in period \( t \in [1, T_1] \) will now be:

\[
B_1^t = \iint \int \int z^* + \Delta z^*_1 \cdot h_0^i(\zeta, \varepsilon, \phi) \, g(\pi) \, d\zeta d\varepsilon d\pi \\
+ \iint \int \int \int z^* \cdot (1 - \Pi^t_j = 1) \cdot h_0^i(\zeta, \varepsilon, \phi) \, g(\pi) \, d\zeta d\varepsilon d\pi \\
= \iint \int [z^* + \Delta z^*_1 - \bar{z}_1] \cdot h_0^i(z^*, \varepsilon, \phi) \left( \int g(\pi) d\pi \right) \, d\varepsilon d\phi \\
+ \iint \int [\bar{z}_1 - z^*] \cdot h_0^i(z^*, \varepsilon, \phi) \left( \int (1 - \Pi^t_j = 1) \cdot g(\pi) d\pi \right) \, d\varepsilon d\phi \\
= h_0(z^*) \left\{ \iint [z^* + \Delta z^*_1 - \bar{z}_1] \cdot \frac{h_0^i(z^*, \varepsilon, \phi)}{h_0^i(z^*)} \, d\varepsilon d\phi \\
+ (1 - \mathbb{E} [\Pi^t_j = 1]) \cdot \iint [\bar{z}_1 - z^*] \cdot \frac{h_0^i(z^*, \varepsilon, \phi)}{h_0^i(z^*)} \, d\varepsilon d\phi \right\} \\
= h_0(z^*) \left\{ \mathbb{E} [\Delta z^*_1] - \mathbb{E} [\Pi^t_j = 1] \cdot (\mathbb{E} [\bar{z}_1] - z^*) \right\} \\
= h_0(z^*) \left\{ \mathbb{E} [\Delta z^*_1] - \mathbb{E} [\Pi^t_j = 1] \cdot (\mathbb{E} [\bar{z}_1] - z^*) \right\}. \] (A.23)
where now \( h_0(z^*) = \iint h_0^1(z^*, \epsilon, \varphi) g(\pi) \, d\epsilon d\varphi d\pi \). In the second line, we have again made use of a constant \( h_0^1(\cdot) \) and also the independence of \( \pi_i \). Normalized bunching at \( K_1 \) in period \( t \) will then be:

\[
b_1^t = E[\Delta z_1^t] - E[\Pi_{j=1}^t \pi_j] (E[z_1] - z^*). \tag{A.24}
\]

Using similar steps, we can show that bunching in period \( t > T_1 \) at \( K_2 \), when moving sequentially from \( K_1 \), can be written as:

\[
B_2^t = \iint \int z^* + \Delta z_2^t h_0^1(\zeta, \epsilon, \varphi) g(\pi) \, d\zeta d\epsilon d\varphi d\pi \nonumber \\
+ \iint \int z^* \Pi_{j=1}^{t-T_1} \pi_j h_0^1(\zeta, \epsilon, \varphi) g(\pi) \, d\zeta d\epsilon d\varphi d\pi \\
+ \iint \int z^* (1 - \Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j) h_0^1(\zeta, \epsilon, \varphi) g(\pi) \, d\zeta d\epsilon d\varphi d\pi \\
= h_0(z^*) \left\{ (1 - E[\Pi_{j=1}^{t-T_1} \pi_j]) E[\Delta z_2^t] + E[\Pi_{j=1}^{t-T_1} \pi_j] E[z_2^t] \right. \\
- E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] E[z_1] - E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] z^* \right\}. \tag{A.25}
\]

Likewise, normalized bunching at \( K_2 \) will be:

\[
b_2^t = \left( 1 - E[\Pi_{j=1}^{t-T_1} \pi_j] \right) E[\Delta z_2^t] + E[\Pi_{j=1}^{t-T_1} \pi_j] E[z_2^t] - E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] E[z_1] \\
- \left( E[\Pi_{j=1}^{t-T_1} \pi_j] - E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] \right) z^*. \tag{A.26}
\]

The levels of bunching at the kink before and after the transition are now functions of average behavioral responses, \((\Delta z_1^t, \Delta z_2^t)\), the average thresholds for marginal bunchers, \((z_1, z_0)\), and average survival probabilities, \(\left(\Pi_{j=1}^{t-T_1} \pi_j, \Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j\right)\). Relative to our baseline dynamic model in Section 6.4, the number of intermediate parameters to be identified is increasing in the number of post-transition periods, due to the terms of the form \( E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] \). A sufficient condition that allows us to retain identification while only using two transitions in kinks is that the expectation of this product simplifies to a product of expectations: \( E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] = E[\Pi_{j=1}^{t-T_1} \pi_j] E[\Pi_{j=1}^{T_1} \pi_j] \). There are two cases of interest that satisfy this condition. First, if \( \pi_j = 0 \) for some \( j < T_1 \), then \( \Pi_{j=1}^{t-T_1} \pi_j = 0 \), and the condition holds. This empirically appears to be the case in our context: adjustment takes roughly two years, while \( T_1 \geq 3 \) in our two main applications. Second, if there is no heterogeneity in \( \pi \) across agents, the condition also holds.

If we relax the assumption that \( E[\Pi_{j=1}^{t-T_1} \pi_j \cdot \Pi_{j=1}^{T_1} \pi_j] = E[\Pi_{j=1}^{t-T_1} \pi_j] E[\Pi_{j=1}^{T_1} \pi_j] \), we will require additional transitions in kinks in order to achieve identification. Furthermore, if we relax the assumption that the profile \( \pi_i \) is independent of \( (\epsilon_i, \phi_i, a_i) \), identification is more complicated, as the expectations in the above expressions will then feature weights that vary with \( t \). In that case, more parametric structure on the joint distribution of \( (\epsilon_i, \phi_i, a_i, \pi_i) \) is needed to achieve identification. We discuss identification further in section A.5 of the Appendix.

### A.4 Allowing for Frictions in Initial Earnings

In the initial period 0 (prior to the policy change), under a linear tax of \( \tau_0 \), we have assumed that individuals are located at their frictionless optimum, while we have assumed in subsequent periods adjustment costs may preclude individuals from reaching their exact, interior optimum. Here, we extend the model to allow for agents to be away from their optimum in period 0, in a way that is consistent with our model of a fixed adjustment cost.

We now analyze the thought experiment previously discussed in Section 6.3. That is, we demonstrate this extension in the context of the “comparative static” model. From a linear tax of \( \tau_0 \) in period 0, in
period 1 we introduce a kink, $K_1$, at $z^*$, and let the marginal tax rate increase to $\tau_1$ for earnings above $z^*$. Finally, in period 2 we replace the first kink with a second, smaller kink, $K_2$, at $z^*$, where the marginal tax rate only increases to $\tau_2$.

Again, agents are indexed by $a$. Let $z_{a,j}$ be actual earnings for individual $a$ in period when facing tax schedule $T_j(z)$, and let $\bar{z}_{a,j}$ be the optimal level of earnings she would choose in the absence of adjustment frictions. As in Chetty (2012), assume that earnings are not “too far” from the frictionless optimum; that is, assume that earnings are within a set such that the utility gain of adjusting to the optimum does not exceed the adjustment cost. Formally:

$$z_{a,j} \in [\bar{z}_{a,j}^-, \bar{z}_{a,j}^+]$$

where $\bar{z}_{a,t} \leq \bar{z}_{a,j} \leq \bar{z}_{a,t}^+$ and $u(\bar{z}_{a,j} - T_j(\bar{z}_{a,j}), \bar{z}_{a,j}; a) - \phi^* = u(z_{a,j} - T_j(z_{a,j}), z_{a,j}; a)$

$$= u(z_{a,j}^+ - T_j(z_{a,j}^+), z_{a,j}^+; a)$$

(A.27)

where $T_j(\cdot)$ represents a linear tax of $\tau_0$ in period 0, reflects the kink $K_1$ in period 1, and reflects the kink $K_2$ in period 2. In words, $\bar{z}_{a,j}^-$ and $\bar{z}_{a,j}^+$ are the lowest and highest level of earnings, respectively, that would be acceptable before an individual chooses to adjust to their optimal earnings level. Note that we have defined $z_{a,j}(\bar{z}_{a,0})$ as a function of the optimal level of earnings for individual $a$ in period 0 for notational convenience. Let the actual earnings, conditional on optimal earnings in period 0, be distributed according to the cumulative distribution function $F_{a,j}(z_{a,j} | \bar{z}_{a,0})$, with probability density function $f_{a,j}(z_{a,j} | \bar{z}_{a,0})$. Thus, individuals are distributed around their frictionless optimum in period 0.

First, consider the level of bunching at $K_1$. Relative to our baseline model with frictions (that assumes individuals are initially located at their frictionless optimum), there will be two differences in who bunches. First, individuals in Figure 7 Panel B area i did not bunch in the baseline because they were sufficiently close to the kink. These are agents for whom $z^* < \bar{z}_{a,0} < \bar{z}_1$. Now, with some probability, a fraction of these agents will be sufficiently far from $z^*$ in period 0 to justify moving to the kink in Period 1—formally, those for whom $z_{a,0} \in [\bar{z}_{a,1}, \bar{z}_{a,0}^+]$. Their initial earnings are above their interior optimum in period 0, but not far enough to outweigh the fixed cost of adjustment in Period 0. Now that the optimum in period 1 has moved to $z^*$, the utility gain to readjusting exceeds the fixed cost of adjustment. These individuals will now bunch under $K_1$. The second difference in this version of the model relative to our baseline model is that some individuals who had bunched under $K_1$ in the baseline model, i.e. areas ii, iii, iv, and v in Figure 7, may find themselves already close enough to $z^*$ in period 0 that they do not bunch at $z^*$ in period 0 (because relocating to $z^*$ in period 0 does not have sufficient benefit to outweigh the fixed adjustment cost). Formally, these are individuals for whom $z_{a,0} < \bar{z}_{a,1}$. These cases are illustrated in Appendix Figure A.4.

Define bunching under this modified model as $B'_1$. Bunching under $K_1$ can be expressed as:

$$B'_1 = \int_{z^*}^{z^* + \Delta z_1} \left[ \int_{z_{a,0}^+}^{z_{a,0}^+} f_{a,0}(v | \zeta) \, dv \right] h_0(\zeta) \, d\zeta$$

$$= \int_{z^*}^{z^* + \Delta z_1^*} \left[ 1 - F_{a,0}(z_{a,1}^+ | \zeta) \right] h_0(\zeta) \, d\zeta$$

$$= \int_{z^*}^{z^* + \Delta z_1^*} \Pr(z_{a,0} \geq z_{a,1}^+ | \bar{z}_{a,0} = \zeta) \, h_0(\zeta) \, d\zeta$$

where $\nu$ and $\zeta$ are dummies of integration.

We now turn to bunching in period 2, under $K_2$. Note that because this kink is smaller, anyone sufficiently close to $z^*$ that they did not bunch under $K_1$ will continue not to bunch under $K_2$. Thus, the only change in bunching in period 2 will be those who now move away from the kink. Under the baseline model, these were individuals for whom $\bar{z}_0 \leq \bar{z}_{a,0} < z^*$, i.e. area v in Figure 7, Panel B. These individuals will still find it worthwhile to move away from the kink, but the difference from the baseline model is that only a subset of them bunched in period 1. Thus, the decrease in bunching will be related to the share of people in area $v$ who actually bunched under $K_1$. What remains are those individuals with $z^* \leq \bar{z}_{a,0} < z^* + \Delta z_1^*$ who
We can rewrite the level of bunching in this setting in terms of bunching amounts derived above:

\[
\hat{B}_2' = \int_{z^*}^{z_0} \left[ \int_{z_{a,1}^-}^{z_{a,0}^+} f_{a,0}(v|\zeta) \, dv \right] h_0(\zeta) \, d\zeta
\]

\[= \int_{z^*}^{z_0} \left[ 1 - F_{a,0}(z_{a,1}^+|\zeta) \right] h_0(\zeta) \, d\zeta
\]

\[= \int_{z^*}^{z_0} \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) h_0(\zeta) \, d\zeta
\]

We can rewrite the level of bunching in this setting in terms of bunching amounts derived above:

\[B'_1 = \int_{z^*}^{z^{*+\Delta z_1}} \Pr \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) h_0(\zeta) \, d\zeta
\]

\[= \int_{z^*}^{z^{*+\Delta z_1}} h(\zeta) \, d\zeta \cdot \int_{z^*}^{z^{*+\Delta z_1}} \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) \frac{h_0(\zeta)}{\int_{z^*}^{z^{*+\Delta z_1}} h(\zeta) \, d\zeta} \, d\zeta
\]

\[= B_1^* \cdot \int_{z^*}^{z^{*+\Delta z_1}} \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) h(\zeta|z^* < \zeta \leq z^* + \Delta z_1^*) \, d\zeta
\]

\[= B_1^* \cdot \mathbb{E} \left[ \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | z^* < \hat{z}_{a,0} \leq z^* + \Delta z_1^* \right) \right]
\]

where \(B_1^* = \int_{z^*}^{z^{*+\Delta z_1}} h_0(\zeta) \, d\zeta\) is defined in equation (2) when \(j = 1\). This is the bunching that would occur in a model of no frictions under \(K_1\), i.e. areas \(i - v\) in Figure 7, Panel B. Likewise, we have:

\[B_2' = \int_{z^*}^{z_0} \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) h_0(\zeta) \, d\zeta
\]

\[= \left[ \hat{B}_2 + B_1^* - B_1 \right] \cdot \mathbb{E} \left[ \text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | z^* < \hat{z}_{a,0} \leq z_0 \right) \right]
\]

where \(\hat{B}_2\) is defined in equation (6), and \(B_1\) is defined in equation (4). It follows that \(\hat{B}_2 + B_1^* - B_1 = \int_{z^*}^{z_0} h_0(\zeta) \, d\zeta\), i.e. areas \(i - iv\) in Figure 7.

Without further restrictions on the distribution of optimal earnings under a linear tax, \(H_0(z)\), or distribution of earnings about the frictionless optimum in period 0, \(F_{a,j}(z_{a,j}|\hat{z}_{a,0})\), we cannot make further simplifications of these expressions. However, if we assume that the initial actual earnings level is distributed uniformly about optimal earnings in period 0, following Chetty et al. (2011) or Kleven and Waseem (2013), then we have:

\[z_{a,0} \sim U \left[ z_{a,0}^-, z_{a,0}^+ \right]
\]

which implies that:

\[\text{Pr} \left( z_{a,0} \geq z_{a,1}^+ | \hat{z}_{a,0} = \zeta \right) = \min \left( \frac{z_{a,0}^+(\zeta) - z_{a,1}^+(\zeta)}{z_{a,0}^+(\zeta) - z_{a,0}^-(\zeta)}, 1 \right)
\]

Using our definitions above for \(z_{a,0}^+(\cdot)\), \(z_{a,0}^-(\cdot)\) and \(z_{a,1}^+(\cdot)\) we can calculate this probability conditional on initial frictionless earnings in period 0, the elasticity \(\varepsilon\) and the adjustment cost \(\phi\). Note that the uniform distribution of actual earnings is not generally centered at the optimal earnings level in period 0, since the lower and upper limits of the support in period 0, i.e. \([z_{a,0}^-, z_{a,0}^+]\), will tend to be different distances from the frictionless optimum. We can also calculate \(B_1^*, B_1, \text{and} \, \hat{B}_2\), conditional on the counterfactual distribution \(H_0(z)\) and a value of \(\varepsilon\) and \(\phi\). We are therefore able to calculate predicted values for \(B_1'\) and \(\hat{B}_2\) and use these in an verified version of the estimation procedure outlined in Section 6.6.1.

Although it is not necessary for our estimation procedure, if we further assume that the optimal earnings density, \(h_0(\cdot)\), is constant over the range \([z^*, z^* + \Delta z_1^*]\), as is common in the literature (e.g., Chetty et al.
2011 or Kleven and Waseem 2013), then we have the following:

\[ B_0' = B_1' \cdot \mathbb{E} \left[ \Pr \left( z_{a,0} \geq z_{a,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq z^* + \Delta z_1^* \right] \]

\[ = \Delta z_1^* h_0 (z^*) \cdot \mathbb{E} \left[ \Pr \left( z_{a,0} \geq z_{a,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq z^* + \Delta z_1^* \right] \]

and likewise:

\[ B_2' = [\tilde{B}_2 + B_1' - B_1] \cdot \mathbb{E} \left[ \Pr \left( z_{n,0} \geq z_{n,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq \tilde{z}_0 \right] \]

\[ = [\tilde{z}_0 - z^*] h_0 (z^*) \cdot \mathbb{E} \left[ \Pr \left( z_{n,0} \geq z_{n,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq \tilde{z}_0 \right] \]

It also follows that bunching normalized by the height of the density at the kink will be:

\[ b_0' = \Delta z_1^* \cdot \mathbb{E} \left[ \Pr \left( z_{a,0} \geq z_{a,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq z^* + \Delta z_1^* \right] \]

\[ b_2' = [\tilde{z}_0 - z^*] \cdot \mathbb{E} \left[ \Pr \left( z_{n,0} \geq z_{n,1}^+ \right) | z^* < \tilde{z}_{a,0} \leq \tilde{z}_0 \right] \]

A.5 Identification

Our estimator is a minimum distance estimator (MDE); Newey and McFadden (1994) give conditions for identification, consistency, and asymptotic normality. An MDE is defined as:

\[ \hat{\theta} = \arg \min_{\theta} \hat{Q}(\theta) \]

\[ \hat{Q}(\theta) = [B - m(\theta)]' W [B - m(\theta)] \]

In our case, \( B \) is a vector of \( L \) estimated bunching amounts from before and after a policy change, and \( m(\theta) \) is a vector of predicted bunching amounts. \( W \) is a weighting matrix. We consider our comparative static, and dynamic, models, in turn.

A.5.1 Comparative Static Model

We focus on the exactly identified case with two bunching moments, which is relevant in our empirical application of the comparative static model. We have:

\[ m(\theta) = (B_1(z, \phi), \tilde{B}_2(z, \phi)) \]

\[ B_1 = \int_{z_1}^{z^* + \Delta z_1^*} h(\xi) d\xi \]

\[ \tilde{B}_2 = \int_{z_1}^{\tilde{z}_0} h(\xi) d\xi \]

where \( B_1 \) and \( \tilde{B}_2 \) refer to bunching before and after the policy change, and \( \theta \equiv (z, \phi) \).

The upper cutoff in \( B_1 \) is defined as

\[ z^* + \Delta z_1^* = z^* \left( \frac{1 - \tau_0}{1 - \tau_1} \right)^z \]

A necessary condition for identification is that solutions for \( \tilde{z}_1 \) and \( \tilde{z}_0 \) exist; if they do not, then no bunching occurs. It is straightforward to show that a solution for \( \tilde{z}_1 \) exists if

\[ z^* \left[ (1 - \tau_1) - \left( \frac{1 - \tau_0}{1 - \tau_1} \right)^z (1 - \tau_1 - \varepsilon (\tau_1 - \tau_0)) \right] > \phi (\varepsilon + 1) \]

This ensures that the “top” buncher wants to adjust to the kink. A solution for \( \tilde{z}_0 \) exists as long as some debunching occurs. It is straightforward to show that this requires that:
As long as \( \tau_0 < \tau_2 < \tau_1, \varepsilon > 0 \), and \( \phi > 0 \), there exists a range of values of \( \varepsilon \) and \( \phi \) for which these inequalities hold.

Provided that \( \tilde{z}_0 \) and \( \tilde{z}_1 \) exist, identification requires that \( m(\theta) = B \) has a unique solution. Following previous literature (e.g. Kline and Walters 2016), we establish local uniqueness by linearizing \( m(\cdot) \) around a solution \( m(\theta_0) = B \). Let \( \theta_0 \) be a solution to \( m(\theta) = B \). Linearizing \( m(\cdot) \) around \( \theta_0 \), we have:

\[
m(\theta) \approx m(\theta_0) + \nabla m(\theta_0)(\theta - \theta_0).
\]

It follows that a unique solution requires \( J_m(\theta_0) \) to have full rank, where \( J_m(\theta_0) \) is the Jacobian of \( m(\cdot) \) evaluated at \( \theta_0 \):

\[
J_m(\theta_0) = \begin{bmatrix}
\frac{\partial B_1}{\partial \varepsilon} & \frac{\partial B_1}{\partial \phi} \\
\frac{\partial B_2}{\partial \varepsilon} & \frac{\partial B_2}{\partial \phi}
\end{bmatrix}.
\]

We calculate the elements of this matrix analytically by differentiating the expressions above for \( B_1 \) and \( \hat{B}_2 \), which is straightforward.\(^{49}\) Thus, given \( \theta, \tilde{z}_1, \) and \( \tilde{z}_0 \), we can calculate the Jacobian analytically (although \( \tilde{z}_1 \) and \( \tilde{z}_0 \) must be found numerically).

\( J_m \) has full rank only if it has a non-zero determinant. We find in all of our bootstrap iterations that \( \det(J_m) < 0 \), demonstrating that the determinant is significantly different from zero. We have also shown analytically that the determinant is generically non-zero (results available upon request).

### A.5.2 Dynamic Model

To identify the dynamic model, we need to observe at least as many moments as the number of parameters we seek to estimate. In our case this means that we must observe bunching across multiple policy changes, specifically the reductions in the BRR above the exempt amount in 1990 and at age 70. Let \( l \) index different such policy changes (in our case, \( l \in \{1990, 70\} \)). Let \( B^l_{1,1} \) be bunching at kink \( l \) and period \( t \) before the policy change, let \( B^l_{2,1} \) be bunching at kink \( l \) and period \( t \) after the policy change, let \( t \) measure the time since the introduction of the first kink, \( K_{1,l} \), and let the policy change at kink \( l \) take place at time \( T_{1,l} \). The parameter vector \( \theta \) now consists of \((\varepsilon, \phi, \pi_1, \pi_2, \ldots, \pi_5)\). We match 12 bunching amounts in our estimates: 1987 to 1992 (pooling 66 to 68 year olds) and ages 67 to 72 (pooling years 1990 to 1999).

Bunching before the policy change is

\[
B^l_{1,1} = \Pi^l_{j=1} \pi_j : B_{1,1} + (1 - \Pi^l_{j=1} \pi_j) B^*_{1,1}
\]

where \( B_{1,1} = \int_{\tilde{z}_1}^{z_1 + \Delta z_{1,l}} h(\xi) \, d\xi \) and \( B^*_{1,1} = \int_{\tilde{z}_1}^{z_1 + \Delta z_{1,l}} h(\xi) \, d\xi \), and the limits of integration are defined similarly to the static case (but with the additional subscript \( l \) to allow for analysis across multiple policy changes, as in our empirical application of the dynamic model). If the policy change happens \( T_{1,l} \) periods after the kink is initially introduced, then bunching under the new policy in period \( t \) is

\[
B^l_{2,1} = \Pi^{(T_{1,l} - 1)}_{j=1} \pi_j : \hat{B}_{2,1} + (1 - \Pi^{(T_{1,l} - 1)}_{j=1} \pi_j) B^*_{2,1} + \Pi^{(T_{1,l} - 1)}_{j=1} \pi_j \left(1 - \Pi^{(T_{1,l} - 1)}_{j=1} \pi_j \right) \left( B^*_{1,1} - B_{1,1} \right)
\]

where \( \hat{B}_{2,1} = \int_{\tilde{z}_0}^{z_0 + \Delta z_{1,l}} h(\xi) \, d\xi \), \( B^*_{2,1} = \int_{\tilde{z}_1}^{z_1 + \Delta z_{1,l}} h(\xi) \, d\xi \), and the limits of integration again are defined similarly to the static case but with the additional subscript \( l \).

We calculate the elements of the resulting Jacobian analytically by differentiating the expressions above for \( B^l_{1,1} \) and \( B^l_{2,1} \) with respect to \( \varepsilon, \phi, \pi_1, \pi_2, \pi_3, \pi_4, \) and \( \pi_5 \), which is again straightforward. Thus, given \( \theta, \tilde{z}_{1,l} \) and \( \tilde{z}_{0,l} \), we can again calculate the Jacobian analytically.

\(^{49}\)We can specify functions implicitly defining the lower and upper cutoffs \( \tilde{z}_{1,l} \) and \( \tilde{z}_{0,l} \), respectively, as functions of the other parameters, given our quasilinear and isodestic case. These enter the expressions for each element of the Jacobian (more details are available upon request).
Identification requires that this Jacobian have full rank. To test for full rank of the Jacobian, we use the method of Kleibergen and Papp (2006). We use the bootstrap to obtain an estimate of $\text{Var}[J_m(\theta)]$. In each iteration of our bootstrap, we also calculate $J_m(\hat{\theta})$, and we estimate $\text{Var}[J_m(\theta)]$ from the bootstrap variance-covariance matrix. The RK test easily rejects under-identification, with $p < 0.001$.

### A.6 Policy Simulations

In this Appendix, we describe how we simulate the effect of various policy changes on earnings. These calculations are designed to be illustrative of the attenuation of earnings responses to policy changes that can result from incorporating adjustment frictions in the analysis. Nonetheless, we highlight that these calculations are done in the context of a highly stylized model making a number of assumptions, as well as a particular sample of earners. One key (extreme) assumption is that everyone has the same elasticity and adjustment cost. Moreover, these estimates are specific to a particular context, and they are not intended to be an exhaustive account of the implications of adjustment costs for earnings responses to taxation. Rather, they are intended simply to illustrate the attenuation of earnings responses to policy changes that can result from incorporating adjustment frictions in the analysis in such contexts.

We assume that utility is isoelastic and quasi-linear with elasticity $\varepsilon$. Individuals must pay an adjustment cost $\phi$ to change their earnings. Individuals are heterogeneous in their ability $n_i$. Individuals are therefore distributed according to their “counterfactual” earnings $z_{0i}$ that they would have under a linear tax schedule. (Despite the absence of heterogeneity in the elasticity and adjustment cost, there is still heterogeneity in the gains from re-optimizing earnings, due to heterogeneity in $z_{0i}$.) We use the 1989 earnings distribution for 60-61 year-olds (from the MEF data) as the counterfactual earnings distribution, i.e. the earnings distribution under a linear tax schedule in the region of the exempt amount. We incorporate the key features of the individual income tax code, including individual federal income taxes, state income taxes, and FICA (all from Taxsim applied in 1989), and the AET. Our estimates of elasticities and adjustment costs apply to a population earning near the exempt amount; to avoid extrapolating too far out of sample, our simulations examine only those whose counterfactual earnings is from $10,000 under to $10,000 over the exempt amount (and is greater than $0$). (While the AET should only affect people whose counterfactual earnings are over the exempt amount, we also include the group earning up to $10,000 under the exempt amount in order to illustrate the fact that some individuals could be unaffected by a policy change.)

We consider two periods, 1 and 2. In period 1, in the region of the AET exempt amount, the mean tax rate below the exempt amount is 27.21 percent, and the mean tax rate above the exempt amount is 77.21 percent. Note that these tax rates mimic those faced by 62-64 year-old OASI claimants. In period 2, the tax rate below the exempt amount remains 27.21 percent, but the tax rate above the exempt amount changes according to the policy changes we specify below. (We assume that in the counterfactual individuals face a linear schedule with a mean tax rate of 27.21 percent.)

For a given counterfactual earnings level $z_{0i}$, we calculate optimal frictionless earnings $z^*_{1i}$ in period 1, and we calculate whether the individual with counterfactual earnings $z_{0i}$ wishes to adjust her earnings from the frictionless optimum because the gains from doing so outweigh the adjustment cost. (Optimal “frictionless” earnings refers to the individual’s optimal earnings in the absence of adjustment costs.) We then determine the individual’s optimal frictionless earnings $z^*_{2i}$ under the new tax schedule in period 2. We assess whether given the adjustment cost, the individual obtains higher utility by staying at her period 1 earnings level, or by paying the adjustment cost and moving to a new earnings level in period 2.

We perform these calculations alternatively under the assumptions that (a) the elasticity $\varepsilon$ is 0.35 and the adjustment cost $\phi$ is $280$ (our baseline estimates); or (b) the elasticity $\varepsilon$ is 0.35 and the adjustment cost $\phi$ is zero. Thus, our simulations illustrate the difference between incorporating adjustment costs and not incorporating them, holding the elasticity constant.

Under these alternative assumptions, we can perform a number of experiments to simulate the effects of changing the effective tax schedule. These calculations are shown in Appendix Table B.6 below.

We calculate that if the marginal tax rate above the exempt amount were reduced by 17.22 percentage points, so that the tax rate above the exempt amount were reduced from 77.21 percent to 59.99 percent, 50

---

50 As we note elsewhere, 62-64 year-olds technically face a notch in the budget constraint at the exempt amount, as opposed to a kink. However, we find no evidence that they behave as if they faced a notch, as the earnings distribution for this age group 1) does not show bunching just above the exempt amount and 2) does not show a "hole" in the earnings distribution just under the exempt amount.
mean earnings in the population under consideration would be unchanged at $9,371.9 under our baseline estimates of the elasticity and adjustment cost. In this case, adjustment is not optimal for anyone when we assume the adjustment cost. In fact, earnings would be unchanged for any reduction in the marginal tax rate above the exempt amount up to 17.22 percentage points; 17.22 percentage points is the largest percentage point marginal tax rate decrease above the exempt amount for which there is no adjustment. Since the gains are second-order near the kink, even a modest adjustment cost of $280 prevents adjustment with an 17.22 percentage point (or smaller) cut in marginal tax rates. By contrast, when assuming \( \varepsilon = 0.35 \) and \( \phi = 0 \), we predict that mean earnings would rise from $9,340.3 to $10,166.3, an increase of 8.84 percent.

At the same time we calculate that if the 50 percent AET above the exempt amount were eliminated, so that the tax rate above the exempt amount were reduced from 77.21 percent to 27.21 percent, mean earnings in the population under consideration would rise from $9,371.9 to $11,566.7, or 23.4 percent, under our baseline estimates of the elasticity and adjustment cost. When assuming \( \varepsilon = 0.35 \) and \( \phi = 0 \), we predict that mean earnings would rise from $9,340.3 to $11,639.2, a nearly identical increase of 24.6 percent. The slight discrepancy between the two estimates arises because there are individuals whose counterfactual earnings is just above the exempt amount who choose to adjust without adjustment costs, but for whom the gains from adjustment do not outweigh the adjustment cost when we assume the friction.

It is worth noting an additional caveat to these results: they apply to those with counterfactual earnings in the range from $10,000 below to $10,000 above the exempt amount. If we allowed unbounded counterfactual earnings, there would be some individuals with very large counterfactual earnings for whom the gains from adjustment would outweigh the adjustment cost, even in the presence of adjustment costs. However, this is less relevant to the AET because as we have noted, the OASI benefit phases out entirely at very high earnings levels. Moreover, considering such individuals would involve extrapolating the estimates much farther out of sample. Finally, the results are qualitatively robust to considering other earnings ranges within the range we measure in our study, such as the range of individuals earning from $10,000 below to $30,000 above the exempt amount. In fact, under all of the other choices we have explored, the results always show that the maximum tax cut that leads to no earnings change is quite substantial (and larger than the changes in marginal tax rates envisioned in most tax reform proposals)—including when we use other ages to specify the counterfactual earnings density; use a different baseline marginal tax rate; and use the constrained estimate of the elasticity (0.58) when performing the simulations (which actually leads to still starker results).

All of these simulations use the static model. If we were to use our estimates of the dynamic model instead to perform these simulations, we would still find that the immediate reaction even to large taxes changes is greatly attenuated, since the estimates of the dynamic model still show that most individuals are constrained from adjusting immediately.
Figure B.1: Simulated earnings distribution at age 69

Note: The left-hand side shows how a downward sloping, counterfactual density can lead to apparently asymmetric bunching when bunching is diffuse. The right-hand side shows a simulated distribution of earnings at age 69, given the estimated model parameters. The y-axis shows the simulated density of earnings in each bin, and the x-axis shows the distance to the exempt amount. The figure demonstrates that we simulate more excess mass below the exempt amount than above it, consistent with the empirical distribution of earnings at age 69 (and other ages) shown in Figure 2. See Appendix A.1 for details.
Figure B.2: Normalized Excess Mass of Claimants, Ages 59 to 73, 1990 to 1999

Note: See notes to Figure 2 Panel B. This figure differs from Figure 2 Panel B because here the sample in year $t$ consists only of people who have claimed OASI in year $t$ or before (whereas in Figure 2 Panel B it consists of those who claimed by age 65).
Figure B.3: Probability of claiming OASI in year $t+1$ among 61-68 year-olds in year $t$ who are not claiming, 1990-1998

Note: The figure shows the probability that an individual claims OASI in year $t+1$, conditional on not claiming OASI in year $t$, for those ages 61-68 in year $t$ from 1990 to 1998.
Figure B.4: Bunching Response to a Convex Kink, with Frictions in Initial Earnings

Note: See Section A.4 for an explanation of the figure.
### Table B.1: Robustness of normalized bunching to alternative birth month restrictions

<table>
<thead>
<tr>
<th></th>
<th>$b_{68}$</th>
<th>$b_{69}$</th>
<th>$b_{70}$</th>
<th>$b_{71}$</th>
<th>$b_{72}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Born January-March</td>
<td>3545.4</td>
<td>4036.2</td>
<td>565</td>
<td>881.7</td>
<td>-236.8</td>
</tr>
<tr>
<td></td>
<td>[2681.1, 4409.7]***</td>
<td>[2934.2, 5138.2]***</td>
<td>[42,1088.1]**</td>
<td>[-14.3, 1777.6]***</td>
<td>[-890.4, 416.8]</td>
</tr>
<tr>
<td>B) Born any month</td>
<td>3992.2</td>
<td>3552.3</td>
<td>1203.9</td>
<td>941.4</td>
<td>-231.4</td>
</tr>
<tr>
<td></td>
<td>[3386.8, 4597.7]***</td>
<td>[3092.4, 4012.2]***</td>
<td>[929,1478.9]***</td>
<td>[453,1429.8]***</td>
<td>[-510.7, 47.8]</td>
</tr>
</tbody>
</table>

Notes: The table shows excess normalized bunching and its confidence interval at each age from 68 to 72 for two samples: those born January to March (Row A), and those born in any month (Row B). The data are pooled over the period from 1983-1999. The table shows that we continue to estimate significant bunching at age 70 (and in some cases 71) when the sample is restricted to those born in January to March. Limiting the sample only to those born in January yields insignificant results, with little statistical power. *** indicates $p<0.01$; ** $p<0.05$; * $p<0.10$. 
<table>
<thead>
<tr>
<th>Binsize</th>
<th>Degree</th>
<th>Excluded Bins</th>
<th>$b_{68}$</th>
<th>$b_{69}$</th>
<th>$b_{70}$</th>
<th>$b_{71}$</th>
<th>$b_{72}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$800$</td>
<td>7</td>
<td>4</td>
<td>3442.3</td>
<td>2868.4</td>
<td>657.9</td>
<td>1068.8</td>
<td>70.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[2763.5, 4121.2]***</td>
<td>[2763.5, 4121.2]***</td>
<td>[195.5, 1120.4]***</td>
<td>[527.2, 1610.4]***</td>
<td>[-328.5, 468.7]***</td>
</tr>
<tr>
<td>$400$</td>
<td>7</td>
<td>8</td>
<td>3107.8</td>
<td>2606.3</td>
<td>462.2</td>
<td>923.0</td>
<td>-55.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[2653.6, 3561.9]***</td>
<td>[2090.4, 3122.2]***</td>
<td>[111.1, 813.2]***</td>
<td>[541.5, 1304.4]***</td>
<td>[-391.4, 280.7]***</td>
</tr>
<tr>
<td>$1,600$</td>
<td>7</td>
<td>2</td>
<td>3047.0</td>
<td>2941.0</td>
<td>601.4</td>
<td>1210.9</td>
<td>241.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[2362.4, 3731.7]***</td>
<td>[2458.5, 3423.5]***</td>
<td>[48.2, 1154.5]**</td>
<td>[581.5, 1840.3]***</td>
<td>[-363.6, 846.1]***</td>
</tr>
<tr>
<td>$800$</td>
<td>6</td>
<td>4</td>
<td>3677.2</td>
<td>3267.3</td>
<td>1310.6</td>
<td>993.7</td>
<td>224.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[3117.4, 4237.0]***</td>
<td>[2810.5, 3721.0]***</td>
<td>[853.3, 1767.9]***</td>
<td>[527.8, 1459.6]***</td>
<td>[-162.0, 611.2]***</td>
</tr>
<tr>
<td>$800$</td>
<td>8</td>
<td>4</td>
<td>3535.1</td>
<td>2948.0</td>
<td>710.8</td>
<td>1084.3</td>
<td>82.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[2944.7, 4125.6]***</td>
<td>[2529.1, 3366.9]***</td>
<td>[296.3, 1125.2]***</td>
<td>[554.5, 1614.1]***</td>
<td>[-393.4, 558.2]***</td>
</tr>
<tr>
<td>$800$</td>
<td>7</td>
<td>3</td>
<td>2170.2</td>
<td>2182.4</td>
<td>202.2</td>
<td>191.2</td>
<td>-55.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[1605.1, 2735.4]***</td>
<td>[1697.1, 2667.7]***</td>
<td>[-126.9, 531.4]</td>
<td>[-243.9, 626.2]</td>
<td>[-390.8, 280.8]</td>
</tr>
<tr>
<td>$800$</td>
<td>7</td>
<td>5</td>
<td>3610.6</td>
<td>2651.3</td>
<td>298.1</td>
<td>1103.3</td>
<td>579.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>[2672.5, 4548.6]***</td>
<td>[1972.2, 3330.4]***</td>
<td>[-304.8, 901.0]</td>
<td>[229.6, 1977.1]**</td>
<td>[-161.8, 1321.6]</td>
</tr>
</tbody>
</table>

Notes: The table shows the estimated bunching amount at each age from 68 to 72, varying the bin size, degree of the polynomial of the smooth density, or number of excluded bins around the exempt amount. Note that varying the bin size but fixing the number of excluded bins automatically changes the width of the excluded region, so to (approximately) fix the width of the excluded region when changing the bin size, we also change the number of excluded bins. *** indicates $p<0.01$; ** $p<0.05$; * $p<0.10$. 

Table B.2: Robustness to alternative empirical choices
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ε</td>
<td>p-value for ε equality</td>
<td>φ</td>
<td>p-value for φ equality</td>
</tr>
<tr>
<td>Men</td>
<td>0.44</td>
<td>0.39</td>
<td>$62</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[0.38, 0.52]***</td>
<td></td>
<td>[14, 167]***</td>
<td></td>
</tr>
<tr>
<td>Women</td>
<td>0.42</td>
<td></td>
<td>$489</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.32, 0.50]***</td>
<td></td>
<td>[165, 720]***</td>
<td></td>
</tr>
<tr>
<td>High lifetime earnings</td>
<td>0.48</td>
<td>0.05</td>
<td>$24</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>[0.41, 0.58]***</td>
<td></td>
<td>[2, 90]***</td>
<td></td>
</tr>
<tr>
<td>Low lifetime earnings</td>
<td>0.44</td>
<td></td>
<td>$538</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.32, 0.51]***</td>
<td></td>
<td>[217, 688]***</td>
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</tr>
<tr>
<td>High lifetime earnings variability</td>
<td>0.39</td>
<td>0.25</td>
<td>$116</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>[0.35, 0.46]***</td>
<td></td>
<td>[37, 315]***</td>
<td></td>
</tr>
<tr>
<td>Low lifetime earnings variability</td>
<td>0.38</td>
<td></td>
<td>$178</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[0.33, 0.46]***</td>
<td></td>
<td>[55, 378]***</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table implements our “comparative static” method separately in each of several groups shown in each row. “High/low lifetime earnings” refers to the group of individuals with mean real earnings from 1951 (when the data begin) to 1989 that are above/below the median level in our study population. “High/low lifetime earnings variability” refers to the group of individuals for whom the standard deviation of real earnings from 1951 to 1989 is above/below the median level in our study population. Columns 2 and 4 show the p-values for the two-sided test of equality in the estimates between each set of groups (i.e. men vs. women, high vs. low lifetime earnings, and high vs. low earnings variability), for ε and φ, respectively. We pool data from two policy changes: (a) around the 1989/1990 transition analyzed in Table 2, and (b) around the age 69/70 transition analyzed in Table 3. We pool the transitions because this gives us the maximum power to detect differences across groups. The results are generally comparable when we investigate each transition separately. See also notes from Tables 2 and 3.
Table B.4: Estimates of Elasticity and Adjustment Cost 1990 Policy Change, Assuming Pre-Period Bunching may not be at Frictionless Optimum

<table>
<thead>
<tr>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon )</td>
<td>( \phi )</td>
<td>( \varepsilon</td>
<td>\phi = 0 )</td>
</tr>
<tr>
<td>Baseline</td>
<td>0.28</td>
<td>$187.78</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>[0.25, 0.43]***</td>
<td>[58.97, 1303.68]***</td>
<td>[0.36, 0.54]***</td>
</tr>
<tr>
<td>Uniform Density</td>
<td>0.24</td>
<td>$162.26</td>
<td>0.39</td>
</tr>
<tr>
<td></td>
<td>[0.22, 0.39]***</td>
<td>[55.27, 1556.01]***</td>
<td>[0.33, 0.48]***</td>
</tr>
<tr>
<td>Benefit Enhancement</td>
<td>0.40</td>
<td>$86.45</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>[0.36, 0.53]***</td>
<td>[20.31, 561.81]***</td>
<td>[0.49, 0.73]***</td>
</tr>
<tr>
<td>Excluding FICA</td>
<td>0.33</td>
<td>$139.03</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>[0.29, 0.41]***</td>
<td>[38.30, 525.91]***</td>
<td>[0.42, 0.62]***</td>
</tr>
<tr>
<td>Bandwidth = $400</td>
<td>0.26</td>
<td>$104.10</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>[0.22, 0.33]***</td>
<td>[7.01, 440.80]***</td>
<td>[0.30, 0.50]***</td>
</tr>
</tbody>
</table>

Note: The table examines the 1990 policy change, using data from 1989 and 1990, but assumes that bunching in 1989 may not be at the frictionless optimum, as described in the text. See also notes to Table 2.
### Table B.5: Estimates of Changes in Bunching Around 1990

<table>
<thead>
<tr>
<th>Sample</th>
<th>Old only linear trend</th>
<th>DD</th>
<th>DD, separate linear trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>old x 1990 dummy</td>
<td>28.9</td>
<td>-107.3</td>
<td>-69.2</td>
</tr>
<tr>
<td></td>
<td>(249.1)</td>
<td>(306.7)</td>
<td>(411.7)</td>
</tr>
<tr>
<td>old x 1991 dummy</td>
<td>-1728.9</td>
<td>-1966.0</td>
<td>-1824.5</td>
</tr>
<tr>
<td></td>
<td>(500.6)***</td>
<td>(306.7)***</td>
<td>(481.3)***</td>
</tr>
<tr>
<td>old x 1992 dummy</td>
<td>-1648.8</td>
<td>-1928.9</td>
<td>-1130.2</td>
</tr>
<tr>
<td></td>
<td>(594.9)***</td>
<td>(306.7)***</td>
<td>(558.1)*</td>
</tr>
<tr>
<td>old x 1993 dummy</td>
<td>-2123.8</td>
<td>-2447.1</td>
<td>-2131.2</td>
</tr>
<tr>
<td></td>
<td>(692.1)***</td>
<td>(306.7)***</td>
<td>(639.7)***</td>
</tr>
<tr>
<td>Ages</td>
<td>66-68</td>
<td>66-68</td>
<td>62-64, 66-68</td>
</tr>
<tr>
<td>Year FE?</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Linear time trend (in year)</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Separate linear trend for “old”</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Notes: The table shows that the estimated change in bunching amounts from before to after 1990 in the age 66-68 age group are similar under several specifications. The dummy variable “old” indicates the older age group (66-68). The sample in Columns (1) and (2) includes only 66-68 year-olds, and in Columns (3) and (4) it also includes 62-64 year-olds. Additional controls include a linear time trend (in year) in column (2), year fixed effects in columns (3) and (4), and the linear time trend interacted with the “old” dummy in column (4). Robust standard errors are in parentheses. Under all the specifications, the coefficient on old x 1990 is insignificantly different from zero: bunching in 1990 is not significantly different from prior bunching, indicating that adjustment does not immediately occur. However, the coefficients on old x 1991, old x 1992, old x 1993 are negative and significant, indicating that bunching falls significantly after 1990—i.e. a reduction in bunching does eventually occur (but not immediately in 1990). The fact that the results are similar under all these various specifications indicates that the results are little changed by controlling for a linear trend (Column 2), comparing 66-68 year-olds to a reasonable control group of 62-64 year-olds (Column 3), and additionally controlling for a separate linear trend for the older group (Column 4). In Columns 1 and 3, the standard errors are the same across all of the interaction coefficients shown because there is only one observation underlying each dummy, and the dummies are exactly identified. See also notes from Table 2.
### Table B.6: Policy Simulations

<table>
<thead>
<tr>
<th>Panel A: Eliminate AET for 62-64 year olds</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>With adjustment costs</strong></td>
</tr>
<tr>
<td>Period 1 mean earnings</td>
</tr>
<tr>
<td>Mean earnings change</td>
</tr>
<tr>
<td>Share affected</td>
</tr>
<tr>
<td>Share who adjust</td>
</tr>
<tr>
<td>Mean change among adjusters</td>
</tr>
<tr>
<td>Percent change among adjusters</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Reduce AET BRR by 17.22 percentage points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>With adjustment costs</strong></td>
</tr>
<tr>
<td>Period 1 mean earnings</td>
</tr>
<tr>
<td>Mean earnings change</td>
</tr>
<tr>
<td>Percent earnings change</td>
</tr>
<tr>
<td>Share who adjust</td>
</tr>
<tr>
<td>Mean change among adjusters</td>
</tr>
<tr>
<td>Percent change among adjusters</td>
</tr>
</tbody>
</table>

Note: Each panel shows the results of a different policy simulation. Column 1 shows the results when we assume $\varepsilon = 0.35$ and $\phi = $280, and Column 2 shows the results when we assume $\varepsilon = 0.35$ and $\phi = 0$. “Mean earnings change” refers to the change in mean earnings from Period 1 to Period 2 predicted in the full study population (i.e. the population with counterfactual earnings between -$10,000 below and $10,000 above the exempt amount). “Percent earnings change” is the percent change in mean earnings predicted in the full study population. “Share who adjust” refers to the percent of the full study population whose earnings does not change in response to the policy change. Note that only 50.4 percent of the full study population has counterfactual earnings above the exempt amount and therefore has incentives that are potentially affected by the policy change in our model. “Mean change among adjusters” refers to the change in mean earnings predicted among those who change earnings in response to the policy change. “Percent change among adjusters” refers to the percent change in mean earnings among those who change earnings in response to the policy change. See Appendix A.6 for further explanation.