

ONLINE PRIVACY AND INFORMATION  
DISCLOSURE BY CONSUMERS\*

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## Abstract

I study the question of what information consumers should disclose to firms. A consumer discloses information to a multi-product firm, which learns about his preferences, sets prices, and makes product recommendations. While the consumer benefits from disclosure as it enables the firm to make accurate recommendations, the firm may use the information to price discriminate. I show three main results. First, the firm prefers to commit to not price discriminate, which encourages the consumer to provide information that is useful for product recommendations. This result provides a new rationale for firms not to engage in price discrimination. Second, nondiscriminatory pricing hurts the consumer, who would be better off by precommitting to withhold some information. This provides a rationale for privacy regulations that limit the amount of personal data that firms can possess. Third, in contrast to single-product models, equilibrium is typically inefficient even if the consumer can disclose any information about his preferences.

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# 1 Introduction

This paper studies the question of what information consumers should disclose to firms, which is a first-order issue in the digital economy. Online firms can observe detailed information about consumers, such as their browsing histories, purchases, and characteristics; however, consumers can affect whether and to what extent this information is revealed. For instance, they can delete cookies to hide their web-browsing activities, and they can create multiple accounts on shopping websites to obfuscate their purchasing behavior. For policymakers, information disclosure by consumers is an important consideration in formulating policies concerning online privacy.

In this paper, I focus on the following economic trade-off: The benefit for consumers to disclose information is that firms can recommend or advertise appropriate products. The cost is that firms may use this information to price discriminate. For instance, Amazon, Netflix, Spotify, and other e-commerce firms use consumers' personal data to offer product recommendations, which help consumers discover items that they might not have found otherwise, and Facebook, Twitter, and YouTube display online ads tailored to consumers' interests. However, these firms could potentially use such information to obtain estimates of consumers' willingness to pay for recommended or advertised products and, in turn, set prices on this basis.<sup>1</sup>

I study a model that captures this trade-off. The baseline model consists of a monopolistic firm that sells  $K$  products and a consumer with unit demand. The consumer has limited attention: He is initially uninformed of the valuations of the products, and it is prohibitively costly for him to assess all available products. At the beginning of the game, the consumer chooses a “disclosure policy,” which determines what the firm learns about his willingness to pay for each product. For example, two disclosure policies could correspond to sharing his browsing history or not, where

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<sup>1</sup>Section 5 shows that we can interpret my model as a game between a consumer and an online advertising platform that holds ad auctions, where advertisers set prices for their products and join the auctions.

sharing it enables the firm to form a more accurate estimate of the valuations.

I consider two games that differ in the firm's ability to price discriminate. Under the *discriminatory pricing regime*, the firm sets prices after observing the information provided by the consumer. Under the *nondiscriminatory pricing regime*, the firm posts prices without observing the information. In either regime, after learning about the consumer's valuations, the firm selects one of the  $K$  products and recommends it. Finally, the consumer learns the value and the price of the recommended product and decides whether to buy it.

I model information disclosure by the consumer as in the literature on Bayesian persuasion ([Kamenica and Gentzkow, 2011](#)). In other words, without observing his valuations, the consumer chooses what information to be disclosed to the firm. The idea is that while it is difficult for consumers themselves to determine which item in a huge set of available products is most appropriate for them, firms can do this using consumers' personal data. For instance, firms might analyze consumers' browsing histories by using their knowledge of the products' characteristics, the prior experiences of other consumers, and their computing power. These enable firms to map a given consumer's personal data into estimates of his willingness to pay, even though the consumer himself cannot learn the whole vector of valuations for all products in the market.

The model has a second interpretation, in which a large number of consumers disclose information to the firm. Formally, I can interpret the model as information disclosure by a continuum of consumers, where the different pricing regimes specify whether the firm can charge different prices to different consumers. Under nondiscriminatory pricing in which the firm commits to set a single price for each product, each consumer is non-pivotal in the sense that his disclosure does not affect prices. Thus, it is as if the firm sets prices without observing information disclosed.

I obtain three main findings, which have implications for understanding observed facts, designing privacy regulations, and the literature on information design. First, the firm prefers to commit to not price discriminate. This commitment encourages the consumer to disclose information, which leads to more accurate recommendations and benefits the firm. This result gives a potential economic explanation of a somewhat puzzling observation: "The mystery about online

price discrimination is why so little of it seems to be happening” (Narayanan, 2013). Namely, price discrimination by online sellers seems to be uncommon despite their potential ability to use consumers’ personal data to obtain estimates of their willingness to pay and, in turn, vary prices on this basis to capture more of the surplus.<sup>2</sup>

The second main finding is that the consumer is worse off under nondiscriminatory pricing. Under discriminatory pricing, the consumer decides what information to reveal taking into account how disclosure affects prices. In contrast, under nondiscriminatory pricing, the consumer discloses much information to obtain better product recommendations because disclosure has no impact on prices. However, expecting this greater level of disclosure and resulting accurate recommendations, the firm prefers to set a high price for each product upfront.<sup>3</sup> As a result, the consumer discloses more information and obtains a lower payoff under nondiscriminatory pricing than under discriminatory pricing. The result highlights a place where consumers are worse off due to their lack of commitment power to withhold information from firms, and a regulator could improve consumer welfare by limiting the amount of personal data that firms can expect to acquire from consumers. Furthermore, the second interpretation of the model enables us to attribute this result to a negative externality associated with information sharing, which sheds light on the divergence between individual and collective incentives to disclose information.

The third main finding is that the equilibrium is often inefficient even if the consumer can disclose any information about his willingness to pay for each product. In particular, discriminatory pricing discourages the consumer from revealing information about his horizontal tastes, which makes product mismatch more likely to occur. This observation is in contrast to the single-product setting of Bergemann, Brooks, and Morris (2015), in which equilibrium is efficient under discriminatory pricing. I prove this inefficiency result without explicitly solving the consumer’s

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<sup>2</sup>There have been several attempts by researchers to detect price discrimination by e-commerce websites. For instance, Jordanou, Soriente, Sirivianos, and Laoutaris (2017) examine around two thousand e-commerce websites and they “conclude that the specific e-retailers do not perform PDI-PD (personal-data-induced price discrimination).”

<sup>3</sup>Formally, I show that if the consumer reveals more information about which product is more valuable, the valuation distribution for the recommended product shifts in the sense of a lower hazard rate, which gives the firm an incentive to charge a higher price. Note that the first-order stochastic shift is not sufficient to conclude that the firm prefers to set high prices.

equilibrium strategy, which is a solution of a multidimensional Bayesian persuasion problem. This approach could be useful to understand the qualitative nature of optimal information structures in complex information design problems.

The remainder of the paper is organized as follows. In [Section 2](#), after discussing related work, I present the baseline model with a single multi-product firm. I also provide a second interpretation of the model as information disclosure by a continuum of consumers. In [Section 3](#), I restrict the consumer to choosing from disclosure policies that reveal different amounts of information about which product is most valuable. I show that the firm is better off and the consumer is worse off under nondiscriminatory pricing. [Section 4](#) allows the consumer to choose any disclosure policy. I show that equilibrium is typically inefficient and different pricing regimes lead to qualitatively different kinds of inefficiency. I use these inefficiency results to show that the firm still prefers to commit to not price discriminate at the expense of the consumer's welfare. This section also shows that if there are a large number of products, nondiscriminatory pricing enhances total welfare. [Section 5](#) extends the model to consider competition between multi-product firms. I show that disclosure benefits the consumer by intensifying price competition. This section also explains that the model could apply to the context of an online advertising platform that runs ad auctions. [Section 6](#) discusses the results and concludes.

## **1.1 Related Work**

My work relates to two strands of literature: The literature on information design and that on the economics of privacy. In terms of modeling, the point of departure is [Bergemann, Brooks, and Morris \(2015\)](#). They consider a single-product monopoly pricing problem in which a monopolist has additional information about a consumer's taste. I extend their framework by considering a multi-product firm and a consumer with limited attention, which renders information useful not only for pricing but also for product recommendation. These additional components yield different welfare consequences of information disclosure and discriminatory pricing. In contrast to [Bergemann, Brooks, and Morris \(2015\)](#), who consider the entire set of attainable surplus, I restrict

attention to the case in which the consumer discloses information to maximize his own payoff. Part of my analysis employs their “greedy algorithm,” which generates a consumer-optimal information disclosure policy given any prior valuation distribution.

My work also relates to the economics of privacy literature. As a growing number of transactions are conducted online and based on data about consumers’ behavior and characteristics, recent strands of literature have devoted considerable attention to the relationships between personal data and intertemporal price discrimination ([Acquisti and Varian, 2005](#); [Conitzer, Taylor, and Wagman, 2012](#); [Fudenberg and Tirole, 2000](#); [Fudenberg and Villas-Boas, 2006](#); [Taylor, 2004](#); [Villas-Boas, 1999, 2004](#)). In these models, firms learn about a consumer’s preferences from his purchase history. Thus, personal data endogenously arises as a history of a game. A consumer’s attempt to conceal information is often formulated as delaying purchase or erasing purchase history. In contrast, I assume that the consumer is endowed with his personal data at the outset.

Several papers, such as [Conitzer, Taylor, and Wagman \(2012\)](#) and [Montes, Sand-Zantman, and Valletti \(2017\)](#), examine consumers’ endogenous privacy choices. [Braghieri \(2017\)](#) studies a consumer search model in which a consumer can choose to be targeted by revealing his horizontal taste to firms instead of engaging in costly search. In the model, targeting plays a similar role to information disclosure in this paper: It enables consumers to find their favored products at low cost, but it can also hurt them because of discriminatory pricing. A similar trade-off can be observed in [De Corniere and De Nijs \(2016\)](#), who study a platform’s choice of disclosing consumers’ preferences to advertisers.

This paper differs from these works in three ways. Most importantly, my main focus is not on how and when consumers choose to protect their privacy or how consumers are harmed by discriminatory pricing. Instead, I aim to provide a potential explanation for why discriminatory pricing seems to be uncommon on the Internet, and why consumers seem to share their information so casually despite the potential negative effect of price discrimination. The second difference is in the formulation. While these works assume that a consumer’s privacy choice is full or no disclosure, I assume that a consumer can choose what kind of information to disclose. This enables

me to compare the amount and type of information disclosed under different environments. Third, and relatedly, I emphasize the difference between individual and group incentives to share personal data, which would be useful for understanding the relationship between the actual market behavior of consumers and its welfare implications.

Beyond the context of online disclosure, my work relates to voluntary information disclosure in bilateral transactions (Glode, Opp, and Zhang, 2016). Also, as information disclosure with commitment can be interpreted as a combination of information gathering and truthful disclosure, my work also relates to information gathering by buyers before trade (Roesler, 2015; Roesler and Szentes, 2017).

Finally, several papers, such as Calzolari and Pavan (2006a,b), and Dworzak (2017), study the privacy of agents in mechanism design problems. In their models, a principal can commit to a mechanism, which elicits an agent's private type, and a disclosure policy, which reveals information about an outcome of the mechanism to other players. Relative to these works, the consumer in my model has more commitment power regarding what information to provide, and the firm has less commitment power in determining allocation and pricing.

## 2 Baseline Model

There is a monopolistic firm that sells multiple products  $k = 1, \dots, K$  with the set of the products denoted by  $\mathcal{K}$ . There is a single consumer who has unit demand, in that he eventually consumes one of  $K$  products or nothing. The consumer's value for product  $k$ , denoted by  $u_k$ , is drawn independently and identically across  $k \in \mathcal{K}$  according to probability distribution  $x_0$  that has a density and is supported on  $V \subset \mathbb{R}_+$ .<sup>4</sup> Let  $u := (u_1, \dots, u_K)$  denote the consumer's valuation vector. Appendix B contains the corresponding results for a more general  $x_0$ , which may not have a density.

The consumer's preferences are quasi-linear: If he purchases product  $k$  at price  $p$ , his ex post

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<sup>4</sup>See Remark 4 for how the results extend to correlated values.



payoff is  $u_k - p$ . If he does not purchase any product, he obtains a payoff of zero. The firm's payoff is its revenue. The consumer and the firm are risk-neutral.

At the beginning of the game, before observing  $u$ , the consumer chooses a *disclosure policy*  $(M, \phi)$  from an exogenously given set  $\mathcal{D}$ . Each element of  $\mathcal{D}$  is a pair of a message space  $M$  and a function  $\phi : V^K \rightarrow \Delta(M)$ , where  $\Delta(M)$  is the set of the probability distributions over  $M$ . After the consumer chooses a disclosure policy  $(M, \phi)$ , Nature draws  $u \in V^K$  and a message  $m \in M$  according to  $\phi(\cdot|u) \in \Delta(M)$ . In the current application,  $\mathcal{D}$  consists of consumers' privacy choices, such as whether to share one's browsing history or not. As in [Section 4](#), if  $\mathcal{D}$  consists of all disclosure policies, information disclosure takes the form of Bayesian persuasion studied by [Kamenica and Gentzkow \(2011\)](#). Hereafter, I sometimes write a disclosure policy as  $\phi$  instead of  $(M, \phi)$ .

Next, I describe the firm's pricing. I consider two games that differ in whether the firm can price discriminate on the basis of information. Under the *discriminatory pricing regime*, the firm sets the price of each product *after* observing a disclosure policy  $(M, \phi)$  and a realized message  $m$ . Under the *nondiscriminatory pricing regime*, the firm sets the price of each product *simultaneously* with the consumer's choice of a disclosure policy  $(M, \phi)$ .<sup>5</sup> Note that under nondiscriminatory pricing, the firm not only does not base prices on a realized message  $m$  but also does not base prices on a disclosure policy  $\phi$ . For example, if firms adopt this regime, they set prices based on neither browsing history nor whether consumers share their browsing history.

Under both pricing regimes, after observing a disclosure policy  $(M, \phi)$  and a realized message  $m$ , the firm recommends one of the  $K$  products. The consumer observes the value and price of the recommended product and decides whether to buy it.

The timing of the game under each pricing regime, which is summarized in [Figure I](#), is as follows. First, the consumer chooses a disclosure policy  $(M, \phi) \in \mathcal{D}$ . Under the nondiscrimina-

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<sup>5</sup>I can alternatively assume that under nondiscriminatory pricing, the firm sets prices first and the consumer chooses a disclosure policy after observing the prices. This alternative assumption does not change equilibrium in [Section 3](#). In contrast, it could change equilibrium in [Section 4](#), because the firm may set different prices for different products to induce an asymmetric disclosure policy. However, it does not affect the main conclusion that the firm prefers nondiscriminatory pricing and the consumer prefers discriminatory pricing, because the firm benefits from its ability to influence a disclosure policy through prices. See the discussion after [Proposition 1](#).

tory pricing regime, the firm simultaneously sets the price of each product. Then Nature draws the consumer's valuations  $u$  and a message  $m \sim \phi(\cdot|u)$ . After observing  $(M, \phi)$  and  $m$ , the firm recommends a product. Under the discriminatory pricing regime, the firm sets the price of the recommended product at this point. Finally, the consumer decides whether to buy the recommended product.

My solution concept is subgame perfect equilibrium in which the firm breaks a tie in favor of the consumer whenever it is indifferent. Under nondiscriminatory pricing, I focus on equilibrium in which each product has the same price.

**Remark 1 (Discussion of Modeling Assumptions).** I assume that the consumer commits to a disclosure policy before observing  $u$ . This would be suitable, for instance, if the consumer is not informed of the existence of products or products' characteristics, but understands that his personal data enable the firm to learn about which product is valuable to him. In [Section 5](#), I provide a microfoundation for this idea in a model of two-sided private information in which the consumer is informed of his subjective taste and the firm is informed of its products' characteristics. The commitment assumption would be natural when studying information disclosure in online marketplaces, as consumers or regulators typically set disclosure rules up front and incentives to distort or misrepresent one's browsing history or characteristics seem to be less relevant.

There are also two substantial assumptions on the firm's recommendation and the consumer's purchasing decision. First, the firm recommends one product and the consumer decides whether to buy it. In other words, the firm cannot offer a more general contract, and the consumer cannot purchase products that are not recommended. This captures the consumer's limited attention: He can assess only a small number of products relative to the huge variety of products available in the market. Several papers, such as [Salant and Rubinstein \(2008\)](#) and [Eliaz and Spiegel \(2011\)](#), formulate limited attention in a similar manner: Consumer first constructs a "consideration set" and chooses the best element within this set. In my model, the consideration set is endogenously determined, because the set consists of "no purchase" and a product that the firm recommends, which depends on what information the consumer discloses.

Second, the consumer observes his willingness to pay for the recommended product when he decides whether to buy it. This is reasonable in settings in which a consumer can learn the value after the purchase and return it for a refund whenever the price exceeds the value. It would be interesting to consider a setting in which a firm, which has superior knowledge about valuations, can convey information about valuations through product recommendations.<sup>6</sup>

Finally, it is *not* without loss of generality to assume that production costs are equal across products. (Assuming that they are equal, it is without loss to normalize them to zero.) For example, if the firm incurs low production cost for product 1, it has a greater incentive to recommend product 1 even if it is less valuable to the consumer than other products. Correspondingly, heterogeneous production costs are likely to affect the consumer's incentive to disclose information. For instance, the consumer might be better off if the firm is less informed about the consumer's values for high-margin products as this can make it less profitable for the firm to recommend those products. I leave this extension for future research.

**Remark 2 (Other Applications).** I use consumers' privacy in online marketplaces as the main application. However, there are many other potential applications. For instance, the model could describe the following situation: An employer assigns his worker one of  $K$  tasks, the completion of which delivers a fixed value to the employer. The worker can disclose information about cost  $c_k$  that he incurs to complete each task  $k$ . In this application, a pricing regime could correspond to whether the wage should be contingent on the information revealed.

## 2.1 Interpretation as a Model with a Continuum of Consumers

We can interpret the current setting as the reduced form of a model in which a *continuum of consumers* disclose information about their preferences. This interpretation enables us to draw insights about the divergence between individual and collective incentives to share information.

Formally, consider a continuum of consumers, each of whom discloses information in the same

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<sup>6</sup>This extension is similar to Bayesian persuasion but is different in that product recommendations affect both the consumer's learning and choice set. [Rayo and Segal \(2010\)](#) study a related problem in a single-product setting. [Ichihashi \(2017\)](#) studies the question of what information should be at firms' disposal in a single-product setting.

way as the consumer in the baseline model. I assume that the valuation vectors are independent across consumers.<sup>7</sup> Under nondiscriminatory pricing, *after* observing the information provided, the firm sets a single price for each product. In other words, the firm cannot use individual-level information but can use aggregate information to set prices. Under discriminatory pricing, the firm can charge different prices to different consumers, which leads to exactly the same model as the baseline model. Under both regimes, the firm can recommend different products to different consumers.

To see that this alternative interpretation is equivalent to the original formulation, consider an equilibrium under nondiscriminatory pricing in which all consumers take the same disclosure policy  $\phi^*$ . In the equilibrium,  $\phi^*$  must be optimal for each consumer when he chooses a disclosure policy taking prices as given, because an individual is nonpivotal and cannot influence the firm's pricing.<sup>8</sup> Moreover, the firm sets the price of each product to maximize its revenue given the commonly chosen disclosure policy  $\phi^*$ . This mutual best-response condition is the same as the equilibrium condition of the original formulation.

I derive the main results using the model of a single consumer because I can obtain clean statements without considering technical details specific to a game with a continuum of players. However, I refer to the alternative interpretation whenever it provides additional insights. [Appendix C](#) formally provides an appropriate equilibrium notion for a continuum of consumers under which I can identify the original formulation as the model with a continuum of consumers.

## 2.2 Restricted and Unrestricted Information Disclosure

My model contains the following two kinds of information. One is information that is relevant to product recommendations. For instance, if the firm learns that product  $k$  is most valuable to the consumer, the firm would recommend it instead of other less valuable products. In contrast,

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<sup>7</sup>The independence of valuation vectors across a continuum of consumers might raise a concern about the existence of a continuum of independent random variables. [Sun \(2006\)](#) formalizes the notion of a continuum of IID random variables for which the “law of large numbers” holds.

<sup>8</sup>[Appendix D](#) formalizes this.

some information is used only to price discriminate. For instance, if the firm additionally learns that the consumer's value of product  $k$  is high, the firm might charge a higher price. [Bergemann, Brooks, and Morris \(2015\)](#) generally study how the latter kind of information affects welfare in a single-product setting.

One might expect that the consumer prefers to reveal information to the extent that it leads to better recommendation, but he does not prefer to disclose information that enables the firm to better price discriminate. However, as I show later, the consumer's informational incentive is subtler than this intuition. In particular, the consumer may prefer not to reveal information useful for product recommendations.

To clarify the intuition for why the consumer might not want to disclose information useful for recommendations, in [Section 3](#), I restrict  $\mathcal{D}$  to the set of disclosure policies that reveal different amounts of information about which product is most valuable. In [Section 4](#), I assume that  $\mathcal{D}$  consists of all disclosure policies.

### 3 Restricted Information Disclosure

In this section, I assume that the firm sells two products ( $K = 2$ ). Then, I identify  $\mathcal{D}$  with  $[1/2, 1]$  and call each  $\delta \in [1/2, 1]$  a *disclosure level*. Each disclosure level  $\delta$  corresponds to disclosure policy  $(\{1, 2\}, \phi_\delta)$  that sends message  $k \in \{1, 2\}$  with probability  $\delta$  whenever  $u_k = \max(u_1, u_2)$ . Note that the greater  $\delta$  is, the more informative  $\phi_\delta$  is in the sense of Blackwell. [Figure II](#) shows the disclosure policy corresponding to  $\delta$ .

#### 3.1 Equilibrium Analysis

I solve the game backwards. The following lemma describes the firm's equilibrium recommendation strategy, which is common between the two pricing regimes. The result is intuitive: The firm can maximize its revenue by recommending the product that the consumer is more likely to prefer. For the proof, see [Appendix A](#).

**Lemma 1.** *Fix a pricing regime and take any equilibrium. Suppose that the consumer has chosen a disclosure level  $\delta > 1/2$  and the firm follows the equilibrium strategy. Then, the firm recommends product  $k \in \{1, 2\}$  if message  $m = k$  realizes.*

This lemma gives the consumer an incentive to disclose information: The greater disclosure level  $\delta$  the consumer chooses, the more likely that he is recommended the most valuable product. Indeed, given Lemma 1, disclosure level  $\delta$  is precisely a probability that the consumer is recommended the preferred product.

Now, how does disclosing information affect product prices? To consider the firm's pricing problem, let  $F^{MAX}$  and  $F^{MIN}$  denote the cumulative distribution functions of  $\max(u_1, u_2)$  and  $\min(u_1, u_2)$ , respectively. Conditional on a disclosure level  $\delta$  and a realized message  $k \in \{1, 2\}$ , the valuation distribution of product  $k$ , which the firm recommends in equilibrium, is given by  $\delta F^{MAX} + (1 - \delta)F^{MIN}$ .

$\delta F^{MAX} + (1 - \delta)F^{MIN}$  is increasing in  $\delta$  in the first-order stochastic dominance. However, this is not sufficient to conclude that the firm prefers to charge a higher price, as the first-order stochastic shift has no implications on the behavior of the monopoly price. For example, suppose that distribution  $F_0$  puts equal probability on values 1 and 3 and that distribution  $F_1$  puts equal probability on 2 and 3. Though  $F_1$  first-order stochastically dominates  $F_0$ , the monopoly price under  $F_0$  is 3, while the one under  $F_1$  is 2.

It turns out that  $F^{MAX}$  is greater than  $F^{MIN}$  in a sense stronger than the usual stochastic order, which I introduce below.

**Definition 1.** Let  $G_0$  and  $G_1$  be two CDFs.  $G_1$  is greater than  $G_0$  in the *hazard rate order* if  $\frac{1-G_1(z)}{1-G_0(z)}$  increases in  $z \in (-\infty, \max(s_1, s_0))$ .<sup>9</sup> Here,  $s_0$  and  $s_1$  are the right endpoints of the supports of  $G_0$  and  $G_1$ , respectively.

If  $G_0$  and  $G_1$  have densities  $g_0$  and  $g_1$ , the above definition is equivalent to

$$\frac{g_0(z)}{1 - G_0(z)} \geq \frac{g_1(z)}{1 - G_1(z)}, \forall z \in (-\infty, \max(s_1, s_0)).$$

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<sup>9</sup> $a/0$  is taken to be equal to  $+\infty$  whenever  $a > 0$ .

The next lemma states that the valuation distribution of the preferred product is greater than that of the less preferred product in the hazard rate order. The proof is in [Section 5](#), where I prove the same result for a more general formulation of “horizontal information,” the information that is useful for accurate recommendations.

**Lemma 2.**  $F^{MAX}$  is greater than  $F^{MIN}$  in the hazard rate order.

The intuition is as follows. Suppose that the firm recommends some product at price  $p$ , and the consumer is willing to pay at least  $p$ . Suppose that the firm marginally increases the price from  $p$  to  $p + \varepsilon$ . If the recommended product is the consumer’s preferred product, after observing the price increment, he gives up buying it only when both  $u_1$  and  $u_2$  are below  $p + \varepsilon$  as the value is given by  $\max(u_1, u_2)$ . Alternatively, if the recommended product is the less preferred product, he gives up buying whenever one of  $u_1$  and  $u_2$  is below  $p + \varepsilon$  as the value is given by  $\min(u_1, u_2)$ . That is, the consumer is less likely to give up buying after observing marginal price increment if the recommended product is his preferred one. This implies that the valuation distribution  $F^{MAX}$  of the most favored product has a lower hazard rate than  $F^{MIN}$  does.

As this intuition suggests, the hazard rate order relates to the comparison of price elasticity of demands. Namely, for two CDFs  $F_1$  and  $F_0$ ,  $F_1$  is greater than  $F_0$  in the hazard rate order if and only if the “demand curve” associated with  $F_1$  has a lower price elasticity of demand than the one associated with  $F_0$ . Here, as in [Bulow and Roberts \(1989\)](#), the demand curve for  $F$  is given by  $D(p) = 1 - F(p)$ . Then, the price elasticity of demand is given by  $-\frac{d \log D(p)}{d \log p} = \frac{f(p)}{1-F(p)}p$ . Thus, the comparison of hazard rates translates to that of the price elasticity of demands.

Next, I show the key comparative statics: Under discriminatory pricing, more information disclosure leads to higher prices for recommended products. Intuitively, the more information the consumer discloses, the less elastic demand he has for the recommended product, in the sense of a lower hazard rate. This gives a monopolistic firm an incentive to charge a high price. To state the result generally, define  $P(\delta) := \arg \max_{p \in \mathbb{R}} p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)]$ . The following result holds for any prior distribution  $x_0$ .

**Lemma 3.** Suppose that the consumer has chosen a disclosure level  $\delta$  under discriminatory pricing.

ing. Then, the firm sets price  $p(\delta) = \min P(\delta)$  in equilibrium. Furthermore,  $P(\delta)$  is increasing in  $\delta$  in the strong set order, and thus  $p(\delta)$  is increasing in  $\delta$ .<sup>10</sup>

*Proof.* By Lemma 1, the consumer is recommended his preferred product with probability  $\delta$ . Then, at price  $p$ , the consumer buys the recommended product with probability  $1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)$ . Thus, the expected revenue is  $p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)]$ , which is maximized at  $p \in P(\delta)$ . Because the firm breaks a tie in favor of the consumer, it sets price  $p(\delta)$  at equilibrium.

To show that  $p(\delta)$  is increasing in  $\delta$ , note that

$$\begin{aligned} & \log p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)] - \log p[1 - \delta' F^{MAX}(p) - (1 - \delta')F^{MIN}(p)] \\ &= \log \frac{1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)}{1 - \delta' F^{MAX}(p) - (1 - \delta')F^{MIN}(p)}. \end{aligned} \quad (1)$$

By Theorem 1.B.22 of Shaked and Shanthikumar (2007), if  $\delta > \delta'$ ,  $\delta F^{MAX} + (1 - \delta)F^{MIN}$  is greater than  $\delta' F^{MAX} + (1 - \delta')F^{MIN}$  in the hazard rate order. Then, (1) is increasing in  $p$ . This implies that  $\log p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)]$  has increasing differences in  $(p, \delta)$ . By Topkis (1978),  $P(\delta)$  is increasing in the strong set order.  $\square$

The following result states that the firm prefers to commit to not price discriminate, from which the consumer obtains a lower payoff.

**Theorem 1.** *Take any equilibrium of each pricing regime. The firm obtains a higher payoff and the consumer obtains a lower payoff under nondiscriminatory pricing than under discriminatory pricing. Under nondiscriminatory pricing, the consumer chooses the highest disclosure level 1 and is charged higher prices.*

*Proof.* If the consumer is recommended his preferred and less preferred products at price  $p$ , his expected payoffs are  $u^{MAX}(p) := \int_p^{+\infty} (v - p) dF^{MAX}(v)$  and  $u^{MIN}(p) := \int_p^{+\infty} (v - p) dF^{MIN}(v)$ , respectively.

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<sup>10</sup>  $A \subset \mathbb{R}$  is greater than  $B \subset \mathbb{R}$  in the strong set order if  $a \in A$  and  $b \in B$  imply  $\max(a, b) \in A$  and  $\min(a, b) \in B$ .



Consider nondiscriminatory pricing. Let  $p^*$  denote the equilibrium price. If the consumer chooses  $\delta$ , his expected payoff is

$$\delta u^{MAX}(p^*) + (1 - \delta)u^{MIN}(p^*),$$

which is uniquely maximized at  $\delta = 1$ . Thus, the firm sets price  $p(1)$  at equilibrium. The consumer's resulting payoff is  $u^{MAX}(p(1))$ .

Consider discriminatory pricing. If the consumer chooses  $\delta$ , his payoff is

$$\delta u^{MAX}(p(\delta)) + (1 - \delta)u^{MIN}(p(\delta)). \quad (2)$$

In equilibrium, the consumer obtains

$$\max_{\delta \in [1/2, 1]} \delta u^{MAX}(p(\delta)) + (1 - \delta)u^{MIN}(p(\delta)) \geq u^{MAX}(p(1)).$$

In contrast, the firm is better off under nondiscriminatory pricing. Note that  $\delta F^{MAX} + (1 - \delta)F^{MIN}$  is stochastically increasing in  $\delta$  and thus  $p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)]$  is increasing in  $\delta$  for any  $p$ . Then for any  $\delta$ ,

$$\begin{aligned} p[1 - F^{MAX}(p)] &\geq p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)], \forall p \\ \Rightarrow p(1)[1 - F^{MAX}(p(1))] &\geq p[1 - \delta F^{MAX}(p) - (1 - \delta)F^{MIN}(p)], \forall p \\ \Rightarrow p(1)[1 - F^{MAX}(p(1))] &\geq p(\delta)[1 - \delta F^{MAX}(p(\delta)) - (1 - \delta)F^{MIN}(p(\delta))]. \end{aligned}$$

□

The intuition is as follows. Under nondiscriminatory pricing, the consumer fully reveals whether  $u_1 > u_2$  to obtain a better recommendation without worrying about how the firm uses the information to set prices. Expecting this full disclosure and the resulting accurate recommendation, the firm prefers to set a high price for each product upfront. In contrast, under discriminatory

pricing, the consumer is the Stackelberg leader, who chooses a disclosure level taking into account the benefit of a more accurate recommendation and the cost from a higher price. As a result, the consumer typically withholds some information. This leads to lower prices and gives the consumer a greater payoff than under nondiscriminatory pricing.

The result gives an economic explanation of the observed puzzle—that is, online price discrimination based on personal data seems to be uncommon. My analysis suggests that if firms started price discriminating on the basis of past browsing or purchases, consumers would have an incentive to hide or obfuscate this history. This would lower the quality of the match between consumers and products and hurt firms. In particular, the result emphasizes the importance of precommitment to nondiscriminatory pricing. This is in contrast to other rationales to avoid price discrimination, such as the existence of menu costs or customer antagonism (Anderson and Simester, 2010). In practice, a firm might commit to nondiscriminatory pricing by its privacy policy or a public statement. For instance, in 2000 Amazon CEO Jeff Bezos said, “We never have and we never will test prices based on customer demographics.”<sup>11</sup>

**Theorem 1** also has policy implications: It highlights a place where consumers suffer from their lack of commitment power to withhold their information, and a regulator could improve their welfare by restricting the amount of information firms can expect to acquire from consumers. Namely, let  $\delta^*$  denote the disclosure level that maximizes equation (2).  $\delta^*$  is the amount of information that the consumer would disclose if the firm could price discriminate. Then, the consumer is better off if a regulator restricts the set  $\mathcal{D}$  of available disclosure policies to  $[1/2, \delta^*]$ . In the game with this restriction, the firm is indifferent between two pricing regimes, and the consumer chooses disclosure level  $\delta^*$  and obtains a greater payoff than without such regulation. In the next subsection, I show that we can interpret this observation as a tragedy of the commons in the model with a large number of consumers.

That the firm prefers nondiscriminatory pricing might seem to be specific to the current setting, in which  $\mathcal{D}$  only consists of disclosure policies that reveal information about which product is more

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<sup>11</sup><http://www.e-commercetimes.com/story/4411.html>

valuable. This is partly true: The set of disclosure policies available to the consumer is crucial to determine which pricing regime the firm and the consumer prefer. For instance, if  $\mathcal{D}$  only consists of the fully revealing disclosure policy, then the firm is better off and the consumer is worse off under discriminatory pricing. At the same time, the current assumption on  $\mathcal{D}$  is not necessary to derive [Theorem 1](#): In the next section, I show that the same conclusion holds even if the consumer can choose *any* disclosure policy.

### 3.2 Implication of [Theorem 1](#) for a Continuum of Consumers

[Theorem 1](#) is a classic tragedy of the commons in the alternative interpretation of the model, in which a continuum of consumers disclose information. First, as in [Theorem 1](#), each consumer in a continuum of population chooses the highest disclosure level under nondiscriminatory pricing, because the disclosure by an individual does not affect the price of each product but rather leads to a better recommendation. In contrast, under discriminatory pricing, each consumer chooses a disclosure level that balances the benefit and cost, as information directly affects the price offered to him.

Now, why are consumers worse off under nondiscriminatory pricing even though the firm sets prices after observing information? This is attributed to the presence of a negative externality associated with information sharing. Under nondiscriminatory pricing, if some consumers disclose more information, the firm sets a higher price for each product, which is shared by all consumers. Thus, information disclosure by some consumers lowers the welfare of other consumers through higher prices. As consumers do not internalize this effect, they prefer to fully reveal information in equilibrium. The full revelation results in higher prices for all consumers and lowers their joint welfare. [Appendix D](#) formalizes this observation.

**Remark 3.** Assuming  $K = 2$  is to simplify the analysis. For a general  $K$ , for instance, I can conduct the same analysis by considering the following disclosure policy  $\phi_\delta$  corresponding to each disclosure level  $\delta$ :  $\phi_\delta$  sends message  $k$  with probability  $\delta$  if  $k \in \arg \max_{k \in \mathcal{K}} u_k$ , and it sends a message uniformly randomly from  $\{1, \dots, K\}$  with probability  $1 - \delta$ .

Also, I can relax the assumption that the disclosure policy corresponding to  $\delta = 1$  deterministically sends message  $k$  whenever  $k \in \arg \max_k u_k$ . The identical result holds, for instance, if I consider the disclosure policy that sends message  $k$  whenever  $k \in \arg \max_k (u_k + \varepsilon_k)$  where  $\varepsilon_k$  is IID across  $k$ . [Section 5](#) more generally defines a disclosure policy that reveals horizontal information and shows that the identical results hold if the consumer can choose any garblings of such a disclosure policy.

**Remark 4.** [Theorem 1](#) is robust to a variety of extensions.

*Correlated Values:* If  $K$  products are different versions of a similar product (such as economics textbooks with different levels of difficulty), then values might be correlated across products. In such a case, [Theorem 1](#) holds as long as the vector  $u$  of values is drawn from an exchangeable distribution whose multivariate hazard rate satisfies a condition described in [Theorem 1.B.29 of Shaked and Shanthikumar \(2007\)](#). Their condition ensures that  $\max(u_1, u_2)$  dominates  $\min(u_1, u_2)$  in the hazard rate order, which is sufficient to derive the theorem. I revisit correlated  $u_1$  and  $u_2$  in the model of competition in [Section 5](#).

*Costly disclosure:* The main insights continue to hold if the consumer incurs a “privacy cost”  $c(\delta)$  from a disclosure level  $\delta$ . While introducing the cost may change the equilibrium disclosure level, the consumer still discloses more information and is worse off under nondiscriminatory pricing. Finding an equilibrium under nondiscriminatory pricing requires a nontrivial fixed-point argument because the consumer may prefer different disclosure levels depending on the price he expects. The formal analysis is similar to the one presented in [Proposition 6](#) in [Section 5.1](#).

*Informational Externality:* In practice, firms would be able to learn about the preferences of some consumers from the information provided by other consumers. For instance, if two consumers share similar characteristics, the purchase history of one consumer might tell something about the preferences of the other consumer. I can capture this informational externality by using the model with a continuum of consumers. A simple way is to assume that a “true” disclosure level of consumer  $i$  is given by  $\lambda \delta_i + (1 - \lambda) \bar{\delta}$  with an exogenous parameter  $\lambda \in (0, 1)$ , where  $\bar{\delta} = \int_{i \in [0,1]} \delta_i di$  is the average disclosure level of the consumers. That is, the firm can learn about

consumer  $i$ 's preference not only from  $i$ 's disclosure  $\delta_i$  but also from other consumers'  $\bar{\delta}$ . In this particular specification, a result identical to [Theorem 1](#) holds.

## 4 Unrestricted Information Disclosure

In this section I assume that  $\mathcal{D}$  consists of all disclosure policies. Beyond theoretical curiosity, allowing the consumer to choose any disclosure policy is important for two reasons. First, it enables us to study whether consumer surplus maximization conflicts with efficiency. Note that in [Section 3](#), equilibrium is often inefficient, regardless of the pricing regime due to the standard monopoly price distortion. For instance, if the value of each product is distributed between 0 and 1, then the consumer does not purchase the recommended product whenever his value is sufficiently close to zero, as the firm always sets a positive price. However, in the single-product setting of [Bergemann, Brooks, and Morris \(2015\)](#), if the consumer can disclose any information about his willingness to pay and the firm can price discriminate, equilibrium is efficient. The unrestricted  $\mathcal{D}$  is one neutral way to study whether the consumer's informational incentive leads to inefficiency in my multi-product setting.

The second reason is to study the robustness of the finding that firms prefer to commit not to price discriminate. The previous restriction on  $\mathcal{D}$  favored nondiscriminatory pricing, in that the two pricing regimes yield equal revenue for any *fixed*  $\delta$ . This observation does not hold for a general disclosure policy: For a fixed disclosure policy, the firm typically achieves higher revenue under discriminatory pricing. For example, given a disclosure policy that fully discloses  $u$ , the firm can extract full surplus only by discriminatory pricing. Thus, the unrestricted  $\mathcal{D}$  enables us to study whether the firm's incentive to commit not to price discriminate persists in an environment that does not apriori favor nondiscriminatory pricing.

In this section, I assume that the firm sells  $K \geq 2$  products. For ease of exposition, I assume that the prior distribution  $x_0$  of the value of each product is fully supported on a finite set  $V = \{v_1, \dots, v_N\}$  with  $0 < v_1 < \dots < v_N$  and  $N \geq 2$ . For each  $x \in \Delta(V)$ ,  $x(v)$  denotes the

probability that  $x$  puts on  $v \in V$ . Abusing notation slightly, let  $p(x)$  denote the lowest monopoly price when the value of each product is distributed according to  $x \in \Delta(V)$ :

$$p(x) := \min \left\{ p \in \mathbb{R}_+ : p \sum_{v \geq p} x(v) \geq p' \sum_{v \geq p'} x(v), \forall p' \in \mathbb{R} \right\}.$$

In particular,  $p(x_0)$  is the price set by the firm if the consumer discloses no information. Note that  $p(x)$  does not depend on  $K$ . Finally, let  $X_{>v_1} \subsetneq \Delta(V)$  denote the set of the valuation distributions supported on  $V$  at which the optimal price is strictly above the lowest positive value:

$$X_{>v_1} := \{x \in \Delta(V) : p(x) > v_1\}.$$

## 4.1 Horizontal and Vertical Inefficiency

In the model of recommendation and pricing, there are two ways for an allocation to be inefficient. First, an allocation is inefficient if prices are so high that the consumer is sometimes excluded from the market. Second, an allocation is inefficient if the firm recommends some product other than the most valuable products. The following definition formalizes these two kinds of inefficiency.

**Definition 2.** An equilibrium is *vertically inefficient* if the consumer does not purchase any products with positive probability. An equilibrium is *horizontally inefficient* if the consumer purchases a product outside of  $\arg \max_k u_k$  with positive probability.

I show that equilibrium is inefficient in both regimes whenever the firm sets a nontrivial monopoly price at the prior. Furthermore, I show that different pricing regimes lead to different kinds of inefficiency. First, an equilibrium under nondiscriminatory pricing is horizontally efficient: The consumer is recommended the most valuable product. However, trade may fail to occur because the firm cannot discount prices when the consumer has low willingness to pay.

**Proposition 1.** *Under nondiscriminatory pricing, there exists a horizontally efficient equilibrium. If  $x_0 \in X_{>v_1}$ , this equilibrium is vertically inefficient.*

*Proof.* To show the first part, consider an equilibrium in which the consumer chooses  $(\phi^*, M^*)$  defined as follows:  $M^* = \mathcal{K}$  and  $\phi^*(k|u) = \frac{1}{|\arg \max_{k \in \mathcal{K}} u_k|} 1_{\{k \in \arg \max_{k \in \mathcal{K}} u_k\}}$ . Namely,  $\phi^*$  discloses the name of the product with the highest value, breaking a tie uniformly if there is more than one such product. In equilibrium, if message  $k$  realizes, the firm recommends product  $k$  and sets an optimal price given the posterior induced by  $\phi^*$ , which is the highest-order statistic of  $K$  samples drawn according to  $x_0$ . Due to the uniform tie-breaking rule, the optimal price does not depend on  $k$ . Under nondiscriminatory pricing, the consumer chooses a disclosure policy taking prices as given. Because all products have the same price, it is optimal for the consumer to disclose which product he likes most so that the firm recommends product  $k \in \arg \max_{k \in \mathcal{K}} u_k$ . Thus,  $\phi^*$  consists of an equilibrium.

To show the vertical inefficiency, let  $x^{MAX} \in \Delta(V)$  denote the distribution of the highest-order statistic  $\max_k u_k$ . It follows that

$$p(x_0) \sum_{v \geq p(x_0)} x^{MAX}(v) \geq p(x_0) \sum_{v \geq p(x_0)} x_0(v) > v_1,$$

where the strict inequality follows from  $x_0 \in X_{>v_1}$ . Thus, the firm sets prices strictly greater than  $v_1$  in the equilibrium, which implies vertically inefficiency.  $\square$

Under nondiscriminatory pricing, the consumer discloses his favorite product without worrying how the firm uses the information to price discriminate. Such disclosure leads to better product match conditional on trade—but because the firm cannot tailor prices to the consumer’s willingness to pay, he is excluded from the market if he has low values for all products.

Does discriminatory pricing eliminate the inefficiency? The next result shows that there could be *no* efficient equilibrium in the multi-product setting. In particular, trade occurs whenever it is efficient to trade, but it occurs with product mismatch. [Appendix E](#) contains the proof.

**Proposition 2.** *Under discriminatory pricing, there exists a vertically efficient equilibrium. There is a measure-zero set  $X_0 \subset \Delta(V)$  such that, for any  $x_0 \in X_{>v_1} \setminus X_0$ , any equilibrium is horizontally inefficient.*

The vertical efficiency of equilibrium relates to [Bergemann, Brooks, and Morris \(2015\)](#), which show that equilibrium is efficient if the firm sells a single product. In contrast to [Bergemann, Brooks, and Morris \(2015\)](#), which directly construct a disclosure policy that achieves an efficient outcome, I prove the vertical efficiency by showing that any vertically inefficient equilibrium can be modified to create another equilibrium with a greater probability of trade. To see this, suppose that an equilibrium is vertically inefficient: At some realized message, the firm posts price  $p_0$ , which the consumer rejects with positive probability. Suppose that the consumer additionally discloses whether his value exceeds  $p_0$ . This additional disclosure does not reduce his payoff, and thus the new disclosure policy continues to be an equilibrium.<sup>12</sup> However, it strictly increases the probability of trade because if the value is below  $p_0$ , the firm can either set a lower price or recommend another product. Although this argument does not explain how to construct an equilibrium disclosure policy, it guarantees that there is at least one equilibrium in which trade occurs for sure.

In contrast, as long as the firm sets a non-trivial monopoly price at the prior, generically, no equilibrium involves horizontal efficiency. Under discriminatory pricing, the consumer can affect prices by controlling what information to disclose. [Proposition 2](#) points out that this gives the consumer an incentive to obfuscate his horizontal taste whenever the monopoly price at the prior exceeds the lowest possible value.

Under discriminatory pricing, an equilibrium disclosure policy maximizes consumer surplus among all the disclosure policies. Thus, an equilibrium disclosure policy would also be chosen by a regulator or an Internet intermediary who cares about consumers and wants to release their information to firms in order to maximize consumer welfare. [Proposition 2](#) shows that such a regulator or an intermediary has to balance consumer welfare and total welfare, as the former cannot be maximized without sacrificing the latter.

While the horizontal inefficiency in [Proposition 2](#) is conceptually similar to the consumer's choice of a low disclosure level in [Theorem 1](#), the actual proof is more complicated for three

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<sup>12</sup>In particular, if the firm learns that the value exceeds  $p_0$ , it continues to post price  $p_0$ , as the firm's new objective function is  $p \cdot \frac{\mathbf{P}(u_1 \geq p)}{\mathbf{P}(u_1 \geq p_0)}$ , which has the same maximizer as the original objective  $p \cdot \mathbf{P}(u_1 \geq p)$ .



reasons. First, a more informative disclosure policy does not necessarily lead to higher prices, and thus there is no analogue to the comparative static that a greater disclosure level leads to higher prices. Second, it is challenging to directly characterize an equilibrium policy, which is a solution of a multidimensional Bayesian persuasion problem. Third, even though equilibrium under discriminatory pricing is unique up to the consumer’s payoff, there can be multiple equilibria that give different total surplus.

I prove the horizontal inefficiency in two steps. First, I solve the following constrained Bayesian persuasion problem: I characterize disclosure policy  $\phi^G$  that maximizes the consumer’s payoff among all the disclosure policies that lead to horizontally efficient outcomes, given the firm’s equilibrium behavior. Second, I modify  $\phi^G$  in a horizontally inefficient way to strictly increase the consumer’s payoff. This completes the proof, as it shows that no horizontally efficient disclosure policy maximizes the consumer’s payoff.

To construct  $\phi^G$ , I apply a procedure similar to the greedy algorithm in [Bergemann, Brooks, and Morris \(2015\)](#) to each realized posterior of the disclosure policy that reveals  $\arg \max_{k \in \mathcal{K}} u_k$ . Then, I show that  $\phi^G$  can be modified in a horizontally inefficient way to strictly increase consumer surplus. To modify  $\phi^G$  in an inefficient way, I show that  $\phi^G$  generically sends messages  $m^*$  and  $m_1$  with the following properties. First, at  $m^*$ , the consumer obtains payoff zero, has the lowest value  $v_1$  for all nonrecommended products, and has value  $v > v_1$  for recommended product  $k$ . At  $m_1$ , the firm is willing to recommend product  $k' \neq k$  and is indifferent between any prices in  $V$ . I modify  $\phi^G$  so that it sends message  $m_1$  with a small probability whenever  $\phi^G$  is supposed to send  $m^*$ . Note that this modification yields a horizontally inefficient disclosure policy. While such modification does not change the consumer’s payoff or the firm’s action, at the “new”  $m_1$ , the firm strictly prefers to set the lowest price  $v_1$  for product  $k'$ . Finally, I further modify the disclosure policy by pooling the new  $m_1$  and another message  $m_2$  at which the consumer obtains a payoff of zero but has a value greater than  $v_1$  for product  $k'$ . In this way, the consumer can strictly increase his payoff. The following example illustrates this idea in a binary environment.

**Example 1.** Suppose that  $K = 2$ ,  $V = \{1, 2\}$ , and  $(x_0(1), x_0(2)) = (1/3, 2/3)$ . Consider disclosure

policy  $\phi$  in [Table I](#).  $\phi$  only discloses which product the consumer weakly prefers. Because such information is necessary to achieve a horizontally efficient allocation, any horizontally efficient disclosure policy is weakly more informative than  $\phi$ .<sup>13</sup>

I characterize  $\phi^G$  as a disclosure policy that maximizes consumer surplus among those more informative than  $\phi$ . To find  $\phi^G$ , I apply a procedure similar to [Bergemann, Brooks, and Morris's \(2015\)](#) greedy algorithm to the posterior distribution conditional on each message  $k \in \{1, 2\}$  drawn by  $\phi$ . Note that the greedy algorithm generates a disclosure policy that maximizes consumer surplus given any prior. Thus, my procedure generates a disclosure policy that maximizes consumer surplus among the policies that induce the same valuation distribution (e.g., the distribution of  $\max_k u_k$ ) for each recommended product as  $\phi$  does.

[Table II](#) shows disclosure policy  $\phi^G$  obtained by this procedure, which decomposes message  $k$  of  $\phi$  into messages  $k1$  and  $k2$  of  $\phi^G$ .<sup>14</sup> For each message  $kj \in \{11, 12, 21, 22\}$ , the firm recommends product  $k$  at price  $j$ . Note that the firm is indifferent between prices 1 and 2 at message  $k1$ .

Now, I show that the consumer can strictly increase his payoff by modifying  $\phi^G$  to create  $\phi^I$  in [Table III](#). At  $(u_1, u_2) = (2, 1)$ ,  $\phi^I$  sends message 21 (corresponding to  $m_1$ ) with probability  $\varepsilon$  instead of message 12 (corresponding to  $m^*$ ). This does not change consumer surplus but gives the firm a strict incentive to set price 1 at message 21. At  $(u_1, u_2) = (2, 2)$ ,  $\phi^I$  sends message 21 with probability  $\varepsilon'$  instead of message 22 (corresponding to  $m_2$ ). For a small  $\varepsilon'$ , this modification does not affect the firm's pricing and recommendation at message 21. However, this modification strictly increases consumer surplus because consumer surplus conditional on  $(u_1, u_2) = (2, 2)$  increases from zero to a positive number.

**Remark 5.** The equilibrium in [Proposition 1](#) is not a unique subgame perfect equilibrium. To see this, suppose that  $V = \{L, H\}$  with  $L < H$ . [Table IV](#) and [V](#) show two equilibrium disclosure

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<sup>13</sup>Precisely, I am restricting attention to “symmetric” disclosure policies, which are shown to be without loss of generality.

<sup>14</sup>As I discuss in the proof, the greedy algorithm does not pin down the valuation distribution of the nonrecommended products. [Table II](#) is derived based on the procedure I define in the proof, which uniquely pins down the joint distribution of  $(u_1, \dots, u_K)$ .

policies, where  $\phi^S$  is the disclosure policy in [Proposition 1](#). Recall that  $\phi(k|u_1, u_2)$  denotes the probability of sending message  $k$  conditional on values  $(u_1, u_2)$ .

For any prior  $x_0$ , there is an equilibrium in which the consumer chooses  $\phi^A$ , the firm recommends product  $k$  given message  $k$ , and the firm sets price  $H$  for product 2. Furthermore, the equilibria associated with  $\phi^A$  and  $\phi^S$  can give different consumer and total surplus. For instance, if  $x_0(L)$  is sufficiently large, the firm sets prices  $H$  and  $L$  for product 2 under  $\phi^A$  and  $\phi^S$ , respectively. In this case, both  $\phi^A$  and  $\phi^S$  achieve the efficient outcome but  $\phi^A$  gives strictly lower consumer surplus. As another example, there can be  $x_0$  such that the firm sets prices  $L$  and  $H$  for product 1 given  $\phi^A$  and  $\phi^S$ , respectively, while it always sets price  $H$  for product 2.<sup>15</sup> In this case,  $\phi^A$  yields strictly greater total surplus than  $\phi^S$ .<sup>16</sup>

This example illustrates the complementarity between information disclosure and pricing: If the firm sets a high price only for one product, then it could be a best response for the consumer to disclose information so that the firm can learn whether the consumer has a high value for the product or not, which in turn rationalizes the asymmetric pricing.

## 4.2 Firm (Still) Prefers Nondiscriminatory Pricing

In the model of the restricted disclosure set, [Theorem 1](#) shows that the firm prefers to commit to nondiscriminatory pricing, under which the consumer chooses the most informative disclosure policy, which corresponds to disclosure level 1. This argument does not apply here, because equilibrium disclosure policies under the two pricing regimes might not be ordered in terms of informativeness.

The following result, however, shows that the firm still prefers to commit to not price discriminate, and this commitment hurts the consumer. The proof utilizes the previous inefficiency results. To state the result, let  $R_{ND}$  and  $CS_{ND}$  be the payoffs for the firm and the consumer, respectively,

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<sup>15</sup>Consider  $(L, H) = (3, 4 + \varepsilon)$  and  $x_0(L) = x_0(H) = 1/2$ . For a small  $\varepsilon$ , the firm strictly prefers price 3 for product 1 at  $\phi^A$ , while it strictly prefers price 4 under  $\phi^S$ .

<sup>16</sup>The existence of multiple equilibria is not specific to the binary valuation in that I can construct a similar example for a general  $V = \{v_1, \dots, v_N\}$  using priors that put sufficiently small weights on values other than  $v_1$  and  $v_2$ .

in the horizontally efficient equilibrium under nondiscriminatory pricing in [Proposition 1](#). Also, let  $R_D$  and  $CS_D$  denote the payoffs of the firm and the consumer, respectively, in any equilibrium under discriminatory pricing.

**Theorem 2.** *Suppose that the consumer can choose any disclosure policy. Then, the firm is better off and the consumer is worse off under nondiscriminatory pricing:  $R_{ND} \geq R_D$  and  $CS_{ND} \leq CS_D$  hold. Furthermore, there is a measure-zero set  $X_0 \subset \Delta(V)$  such that, for any  $x_0 \in X_{>v_1} \setminus X_0$ ,  $R_{ND} > R_D$  and  $CS_{ND} < CS_D$  hold.*

*Proof.* Let  $\phi^H$  denote the disclosure policy associated with the horizontally efficient equilibrium in [Proposition 1](#). Also, let  $\phi^G$  denote the symmetric disclosure policy that maximizes consumer surplus, subject to the constraint that it leads to a horizontally efficient outcome under discriminatory pricing. As the proof of [Proposition 2](#) shows,  $\phi^G$  achieves full efficiency. Because the consumer is recommended his most favored product under both  $\phi^H$  and  $\phi^G$ , conditional on the event that product  $k$  is recommended, the valuation distribution of product  $k$  is identical between  $\phi^H$  and  $\phi^G$ .

Let  $R_{ND}$  and  $p^*$  denote the firm's expected revenue and the optimal price for each product in the equilibrium with  $\phi^H$ . First,  $R_{ND}$  is given by  $R_{ND} = p^* \sum_{v \geq p^*} x^{MAX}(v)$  with  $x^{MAX}$  being the distribution of  $\max_k u_k$ . Second, under  $\phi^G$ ,  $p^*$  is optimal for any posterior that can realize with positive probability. This observation follows from the fact that I construct  $\phi^G$  using the [Bergemann, Brooks, and Morris's \(2015\)](#) greedy algorithm. Denoting the firm's expected revenue from  $\phi^G$  under discriminatory pricing by  $R_G$ , I obtain  $R_{ND} = R_G$ . Now, under discriminatory pricing, the equilibrium payoff of the consumer is weakly greater than the one from  $\phi^G$ . Because  $\phi^G$  leads to an efficient allocation,  $R_{ND} \geq R_D$  must hold. As [Proposition 2](#) shows, for a generic prior  $x_0 \in X_{>v_1}$ , the consumer chooses an inefficient disclosure policy  $\phi^I$  to obtain a payoff strictly greater than the payoff from  $\phi^G$ , which implies  $CS_D > CS_{ND}$ . Because the allocation under  $\phi^G$  is efficient,  $R_{ND} > R_D$  holds. □

A rough intuition is as follows. As [Proposition 2](#) shows, under discriminatory pricing, the consumer withholds information about his horizontal taste to induce the firm to set lower prices. In

this model, the firm could still benefit from discriminatory pricing because it can tailor prices based on the consumer's willingness to pay, which nondiscriminatory pricing does not allow. However, it turns out that the consumer can increase the probability of trade without increasing the firm's payoff by disclosing coarse information about his willingness to pay. This implies that the welfare gain created by the firm's ability to price discriminate accrues to the consumer. As a result, under discriminatory pricing, the firm loses due to horizontal inefficiency without benefiting from vertical efficiency.

Although consumers in reality might not be endowed with the unrestricted  $\mathcal{D}$  as their privacy choices, [Theorem 2](#) still has the following economic content. Suppose that a sophisticated third party, such as a regulator or an Internet intermediary, wants to reveal its information about consumers to firms in order to maximize consumer welfare. The result states that if the third party tries to achieve the best possible way to reveal information, it in turn gives firms an incentive to commit to nondiscriminatory pricing, which hurts consumers.

### **4.3 Does Nondiscriminatory Pricing Enhance Efficiency?**

The previous analysis suggests that different pricing regimes have different advantages in enhancing total welfare: Nondiscriminatory pricing leads to efficient recommendations, and discriminatory pricing leads to the greatest probability of trade. Indeed, whether nondiscriminatory pricing achieves greater total surplus depends on the prior valuation distribution  $x_0$  and the number  $K$  of the products.

It is useful to consider extreme values of  $K$  to illustrate this point. First, if  $K = 1$ , then vertical efficiency is equivalent to full efficiency. Thus, discriminatory pricing always achieves greater total surplus.

In contrast, if a large number of products are available in the market, the welfare gain from improving product match quality seems to be significant. I formalize this idea by showing that for a large  $K$ , nondiscriminatory pricing dominates discriminatory pricing in terms of efficiency. This result relies on the following proposition, which describes the firm's and the consumer's payoffs

in the limiting case of  $K \rightarrow +\infty$ . The proof is in [Appendix F](#).

**Proposition 3.** *Suppose that the value of each product is drawn from a probability distribution that has a density and a compact interval support  $[a, b]$ . Under nondiscriminatory pricing, as  $K \rightarrow +\infty$ , the firm's equilibrium payoff converges to  $b$  and the consumer's equilibrium payoff converges to 0. Under discriminatory pricing, there exists  $\underline{u} > 0$  such that the consumer's equilibrium payoff is greater than  $\underline{u}$  for any  $K$ .*

The intuition is as follows. Under discriminatory pricing, the consumer can always choose to disclose no information, which guarantees a lower bound of his payoff that is independent of the number of the products. In contrast, under nondiscriminatory pricing, the consumer discloses which product has the highest value. If  $K$  is large and the prior valuation distribution has the bounded support, this information reveals that the consumer is likely to have a value close to the maximum possible valuation. As a result, the firm can charge prices close to this maximum value upfront to extract most of the surplus.

[Proposition 3](#) implies that nondiscriminatory pricing achieves the first-best outcome as  $K \rightarrow +\infty$ , because total surplus is greater than the firm's payoff, which converges to the highest possible value in the limit. However, this does not exclude the possibility that discriminatory pricing is more efficient and achieves the first-best outcome in the limit as well. The next proposition states that this does not occur: Under discriminatory pricing, total surplus is uniformly bounded away from the highest possible value  $b$ . In other words, if there are a large number of products, the firm's strategic commitment to not price discriminate is welfare-enhancing. The proof is in [Appendix F](#).

**Proposition 4.** *Consider the same distributional assumption as in [Proposition 3](#). Under nondiscriminatory pricing, the equilibrium total surplus converges to  $b$  as  $K \rightarrow +\infty$ . Under discriminatory pricing, there exists  $\varepsilon > 0$  such that the equilibrium total surplus is at most  $b - \varepsilon$  for any  $K$ .*

**Remark 6 (Welfare Impact of Nondiscriminatory Pricing with Restricted  $\mathcal{D}$ ).** I can study which pricing regime is more efficient in the restricted environment in [Section 3](#). As [Appendix G](#)

shows, nondiscriminatory pricing can increase or decrease total welfare depending on  $x_0$ .

When the consumer is restricted to choosing a disclosure level, the question posed here is equivalent to the question of whether an outward shift in a demand curve increases total surplus in the standard monopoly pricing problem. In this respect, the observation that total surplus can be either increasing or decreasing in disclosure level is consistent with the observation that an outward shift in demand can increase or decrease total surplus in the monopoly pricing problem.<sup>17</sup>

**Remark 7 (Welfare Impact of Nondiscriminatory Pricing with General  $\mathcal{D}$ ).** As the proofs of [Proposition 3](#) and [4](#) suggest, I do not need to assume that  $\mathcal{D}$  contains all disclosure policies. Indeed, the following milder assumption suffices: For each  $K$ ,  $\mathcal{D}$  consists of a disclosure policy that reveals no information and a disclosure policy which is weakly more informative than the one that discloses  $\arg \max_k u_k$ .

#### 4.4 Characterizing Consumer-Optimal Disclosure Policy

While the general characterization of a “consumer-optimal” disclosure policy is beyond the scope of this paper, I provide full characterization assuming that the firm sells two products with binary valuation. Under discriminatory pricing, a consumer-optimal disclosure policy is an equilibrium disclosure policy. Under nondiscriminatory pricing, it is a disclosure policy that maximizes consumer surplus, assuming that the firm optimally sets (nondiscriminatory) prices against the disclosure policy. In the model of a continuum of consumers, the consumer-optimal policy under nondiscriminatory pricing is a collective deviation by consumers that maximizes their joint surplus given the firm’s best response.

The characterization provides two insights. First, I illustrate that disclosing correlated information is necessary to maximize consumer surplus. In particular, though valuations are independent across products, it is suboptimal to disclose information about the value of each product separately. Second, I show that the insight of [Theorem 1](#) continues to hold. That is, equilibrium disclosure

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<sup>17</sup>[Cowan \(2004\)](#) provides a necessary and sufficient condition under which additive and multiplicative shifts of the demand and inverse demand function increase total surplus under monopoly. To the best of my knowledge, a more general condition has not been found in the industrial organization literature.

under nondiscriminatory pricing could be consumer-suboptimal, in that the consumer is better off committing to withhold some information.

First, I describe the consumer-optimal disclosure policy under discriminatory pricing. Let  $K = 2$  and  $V = \{L, H\}$  with  $0 < L < H$ . To begin with, I define three distributions. First, given the prior  $x_0 \in \Delta(V)$  for the value of each product, let  $x^{MAX}$  be the distribution of the highest-order statistic  $\max(u_1, u_2)$  of two IID draws from  $x_0$ . Second, let  $x^{OPT} \in \Delta(V)$  satisfy  $x^{OPT}(H) = \frac{L}{H}$ . At  $x^{OPT}$ , the firm is indifferent between prices  $L$  and  $H$ . Thus,  $x^{OPT}$  puts the largest probability on  $H$  subject to the constraint that the firm is willing to set price  $L$ , which implies that  $x^{OPT}$  maximizes consumer surplus among  $\Delta(V)$  given the firm's best response. Third, let  $\bar{x} \in \Delta(V)$  satisfy  $\bar{x}^{MAX} = x^{OPT}$ .  $\bar{x}$  is the valuation distribution such that the highest-order statistic of two IID draws from  $\bar{x}$  coincides with  $x^{OPT}$ .

(Case 1:  $x^{MAX}(H) \leq x^{OPT}(H)$ ) This occurs when the prior  $x_0$  puts a sufficiently large weight on value  $L$ . Consider  $\phi^*$ , which sends messages 1 and 2 according to [Table VI](#) with  $\alpha = 1$ . Note that the firm recommends product  $k$  following message  $k$ . In the present case, even at  $\alpha = 1$ , the firm prefers to set price  $L$  at each posterior because the valuation distribution for the recommended product is given by  $x^{MAX}$ , and thus  $Hx^{MAX}(H) \leq Hx^{OPT}(H) \leq L$ . This is optimal for the consumer, because the allocation is efficient and the price is always  $L$ .

(Case 2:  $x_0(H) \leq x^{OPT}(H) < x^{MAX}(H)$ ) This occurs when  $x_0(L)$  is not too small but also not too large. The optimal disclosure policy is described in [Table VI](#) with  $\alpha < 1$ . In this case, the disclosure policy is horizontally inefficient.  $\alpha$  is chosen so that at each message  $k$ , the valuation distribution  $x^k$  of the recommended product  $k$  is  $x^{OPT}$ . Such  $\alpha$  exists because  $x^k(H) = x_0(H) \leq x^{OPT}(H)$  at  $\alpha = 1/2$ , and  $x^{OPT}(H) < x^{MAX}(H) = x^k(H)$  at  $\alpha = 1$ . This is an equilibrium, because the consumer obtains  $(H - L)x^{OPT}(H)$ , which is the maximum payoff he can obtain among all of the distributions over  $V$  given the firm's best response. Note that equilibrium consumer surplus and total surplus are constant for priors such that  $x_0(H) \leq x^{OPT}(H) < x^{MAX}(H)$ , which is in contrast to the single-product case in which consumer and total surplus always vary across priors.



(Case 3:  $x^{OPT}(H) < x_0(H)$ ) In this case, an equilibrium disclosure is horizontally inefficient. Indeed, the consumer reveals no information about which product is more valuable. [Appendix H](#) shows the specific form of the equilibrium disclosure policy.

It turns out that the equilibrium disclosure policy under discriminatory pricing is also consumer-optimal under nondiscriminatory pricing. Note that the disclosure policies shown above send only two messages, each of which corresponds to each product. Thus, the firm can set the price of each product without observing a particular message realization. This implies that if the consumer commits to  $\phi^*$ , it maximizes consumer surplus even under nondiscriminatory pricing.

Comparing  $\phi^*$  and the horizontally efficient equilibrium in [Proposition 1](#), I can conclude that the consumer-optimal disclosure policy cannot be sustained in equilibrium of nondiscriminatory pricing if the prior puts a high probability on value  $H$ , i.e.,  $x^{MAX}(H) > x^{OPT}(H)$ . This is consistent with the idea of [Theorem 1](#), that the consumer could increase his payoff by committing to withhold some information.

## 5 Extensions

### 5.1 Competition between Firms

A natural question would be how the welfare consequences of information disclosure depend on market structure. In this section I study the model of competition assuming that, as in [Section 3](#), the consumer chooses a disclosure level from  $[1/2, 1]$ . I show that the consumer has a stronger incentive to provide information than in the monopolistic market, because disclosure intensifies price competition.

The game is a natural extension of [Section 3](#). There are two firms, Firm A and Firm B, both of which sell products 1 and 2. The game proceeds similarly as before (see [Figure III](#)). First, the consumer chooses a disclosure level  $\delta$ . Under nondiscriminatory pricing, Firms  $A$  and  $B$  simultaneously set prices at this point. Second, Nature draws a type  $u$  and messages  $(m^A, m^B)$ .  $m^j$  denotes the message sent from the consumer to Firm  $j \in \{A, B\}$ . I assume that, conditional on

$u$ ,  $m^A$  and  $m^B$  are stochastically independent. Third, after observing  $(\delta, m^j)$ , each firm  $j \in \{A, B\}$  simultaneously recommends one of two products to the consumer. Under discriminatory pricing, each firm also sets the price of the recommended product. The consumer has three choices. He can choose to buy the recommended product from Firm  $A$  or  $B$ , or he can reject both. To facilitate the analysis, I replace the distributional assumption on  $u_1$  and  $u_2$ . The following assumption implies that trade never occurs if the firm recommends a less preferred product. See [Remark 4](#) for how the results under a monopolist change if  $u_1$  and  $u_2$  are correlated.

**Assumption 1.** The joint distribution of  $(u_1, u_2)$  satisfies the following: The distribution  $F^{MIN}$  of  $\min(u_1, u_2)$  puts probability 1 on 0, and the distribution  $F^{MAX}$  of  $\max(u_1, u_2)$  has no probability mass and  $p[1 - F^{MAX}(p)]$  is strictly concave for  $p > 0$ .

I solve the game backwards. Let  $\Pi(p) := p[1 - F^{MAX}(p)]$ ,  $p^M := p(1) = \arg \max_{p \geq 0} \Pi(p)$ , and  $\Pi^M := \Pi(p^M)$ . Given  $\delta$ , let  $p^*(\delta)$  satisfy  $\Pi(p^*(\delta)) = (1 - \delta)\Pi^M$ .<sup>18</sup> The following lemma describes an equilibrium price distribution given any disclosure levels. The proof is in [Appendix I](#).

**Lemma 4.** *Define the following CDF parametrized by  $\delta$ :*

$$G_\delta^*(p) = \begin{cases} 0 & \text{if } p < p^*(\delta), \\ \frac{1}{\delta} - \frac{1 - \delta}{\delta} \cdot \frac{\Pi^M}{\Pi(p)} & \text{if } p \in [p^*(\delta), p^M], \\ 1 & \text{if } p > p^M. \end{cases}$$

*Suppose that it is publicly known that the consumer has chosen a disclosure level  $\delta$ . Under both pricing regimes, there is an equilibrium in which each firm  $j$  recommends product  $k$  whenever  $m^j = k$ . There is a unique equilibrium price distribution consistent with this recommendation rule: Each firm sets the price of each product according to  $G_\delta^*$ .*

Note that whenever  $\delta > 1/2$ , there is no equilibrium in which different firms specialize in recommending different products. For instance, suppose that Firm  $A$  and  $B$  recommend only

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<sup>18</sup>Such  $p^*(\delta)$  exists, because  $\Pi(p) = p[1 - F^{MAX}(p)]$  is concave and thus continuous.

products 1 and 2, respectively. Then one firm, say Firm  $B$ , has an incentive to recommend product 1 at a price slightly lower than the price set by Firm  $A$  if the consumer is more likely to prefer product 1.

The next lemma states that information disclosure intensifies price competition.

**Lemma 5.** *Take any  $\delta$  and  $\delta'$  such that  $\delta > \delta'$ .  $G_{\delta'}^*$  first-order stochastically dominates  $G_{\delta}^*$ .*

*Proof.* By differentiating  $G_{\delta}^*$  in  $\delta$ , I obtain

$$\frac{\partial G_{\delta}^*(p)}{\partial \delta} = -\frac{1}{\delta^2} \left( 1 - \frac{\Pi^M}{\Pi(p)} \right) \geq 0, \forall p,$$

where the inequality is strict for all  $p \in (p^*(\delta), p^M)$ . □

Intuitively, the more information the consumer discloses, the more likely that both firms will recommend the same product. This intensifies price competition and pushes prices downward. In the model with a continuum of consumers, the result highlights a “positive externality” associated with information sharing: Information disclosure by some consumers benefits other consumers through lower prices.

In the equilibrium of the entire game, the consumer fully discloses horizontal information and the first-best outcome is attained.

**Proposition 5.** *Suppose that the firms’ equilibrium strategies are given by [Lemma 4](#). Then, under both pricing regimes, the consumer chooses  $\delta = 1$  and the firms set the price of zero for each product.*

*Proof.* If the firms’ equilibrium strategies are given by [Lemma 4](#), then under discriminatory pricing the highest disclosure level  $\delta = 1$  gives the highest possible payoff for the consumer, because he is recommended his preferred product at price 0. Under nondiscriminatory pricing,  $\delta = 1$  is a unique best response for the consumer because it uniquely maximizes the probability that he is recommended his preferred product without affecting prices. □

Under nondiscriminatory pricing, the consumer still fails to consider the impact of disclosure on prices. In this case, the impact is beneficial in the sense that disclosure not only improves the accuracy of recommendations but also lowers prices. However, because the consumer has an incentive to disclose information to obtain a better recommendation, there is no gap between equilibrium disclosure levels under nondiscriminatory and discriminatory pricing.

I show that this observation is specific to the assumption that information disclosure is costless. To see this, suppose the consumer incurs cost  $c(\delta)$  from a disclosure level  $\delta \in [1/2, 1]$ . I assume that  $c(\cdot)$  satisfies the following.

**Assumption 2.**  $c(\cdot)$  is convex and differentiable with  $c'(1/2) \leq 0 < c'(1)$ .

$c(\cdot)$  can be nonmonotone, and if  $c(\cdot)$  satisfies the assumption,  $\gamma c(\cdot)$  also satisfies it for any  $\gamma > 0$ . The condition  $c'(1/2) \leq 0 < c'(1)$  is rather a technical restriction, which ensures that there exists an equilibrium with an interior disclosure level. One interpretation for  $c'(1) > 0$  might be that  $c(\cdot)$  represents an intrinsic privacy cost, and a consumer prefers not to disclose full information about his taste given the potential cost of having too much information disclosed. Practically, the specific shape of disclosure cost could be influenced by other factors, such as a default privacy setting.

The following result is a converse of [Theorem 1](#): At any equilibrium of nondiscriminatory pricing regime, the consumer could be better off by precommitting to disclose *more* information. In contrast to the previous model, in which  $\delta = 1$  is always a best response, this model requires a fixed-point argument to find an equilibrium under nondiscriminatory pricing. The proof is in [Appendix J](#).

**Proposition 6.** *Suppose information disclosure is costly. An equilibrium exists under each pricing regime, and the consumer is strictly worse off under nondiscriminatory pricing. Furthermore, for any equilibrium disclosure level  $\delta^*$  under nondiscriminatory pricing, there is  $\varepsilon > 0$  such that if the consumer chooses  $\delta^* + \varepsilon$  under discriminatory pricing, he obtains a strictly greater payoff.*

The interpretation of the result is clear once we consider a continuum of consumers. Under

nondiscriminatory pricing, each consumer ignores the fact that information disclosure collectively benefits consumers through intensifying price competition. Thus, in any equilibrium, if consumers collectively deviate to disclose more information, they could be better off.

**Remark 8.** The monopolist’s model in [Section 3](#) and this competition model are special cases of the following mixed-population model. Suppose there is a unit mass of consumers. Fraction  $\alpha \in (0, 1)$  of consumers are savvy and can compare recommendations from both firms as they do in this section. The remaining fraction  $1 - \alpha$  of consumers are non-savvy and randomly choose one of the two firms. While price competition occurs with respect to mass  $\alpha$  of consumers, each firm is a monopolist to mass  $\frac{1-\alpha}{2}$  of consumers. Under nondiscriminatory pricing, information disclosure by savvy consumers benefits all other consumers through intensifying price competition. In contrast, information disclosure by non-savvy consumers harms all other consumers because the firm, which is a monopolist for them, charges higher prices that are shared by everyone.

## 5.2 A Model of Two-Sided Private Information

I assume that the consumer does not observe the valuations of products when he chooses a disclosure policy. Focusing on the model of the restricted disclosure in [Section 3](#), I provide a micro-foundation for this assumption. I consider a model in which the consumer is privately informed of his taste, the firm is privately informed of the products’ characteristics, and the two pieces of information are necessary to determine the value of each product.

Formally, let  $\theta \in \{1, 2\}$  denote the consumer’s *taste*.  $\theta = 1$  and  $\theta = 2$  are equally likely and the consumer privately observes a realized  $\theta$ . The firm is privately informed of the *products’ characteristics* denoted by  $\pi \in \{0, 1\}$ .  $\pi$  and  $\theta$  jointly determine which product is more valuable. If  $\pi = 0$ , the consumer with  $\theta$  draws values of products  $\theta$  and  $-\theta$  from  $\max\{u_1, u_2\}$  and  $\min\{u_1, u_2\}$ , respectively.<sup>19</sup> If  $\pi = 1$ , the consumer with  $\theta$  draws values of products  $-\theta$  and  $\theta$  from  $\max\{u_1, u_2\}$  and  $\min\{u_1, u_2\}$ , respectively.

The game proceeds similarly as before. *After privately observing his taste  $\theta$* , the consumer

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<sup>19</sup>For  $\theta \in \{1, 2\}$ ,  $-\theta \in \{1, 2\} \setminus \{\theta\}$ .

(publicly) chooses a disclosure policy  $(M, \phi)$ , which is now a pair of a message space and a probability distribution  $\phi \in \Delta(M)$ . As in the baseline model, the interpretation of a disclosure policy here is statistical information about his taste, such as browsing history. After observing  $(M, \phi)$ , a realized  $m \sim \phi$ , and  $\pi$ , the firm recommends a product. The games under nondiscriminatory pricing and discriminatory pricing are defined in the same way as in the baseline model.

Note that being uninformed of the products' characteristics  $\pi$ , the consumer learns nothing about the values of products from  $\theta$ . This model formalizes a natural setting in which combining consumers' subjective tastes and the products' characteristics is necessary for firms to give good product recommendations.

In this model, results identical with [Theorem 1](#) hold: Under nondiscriminatory pricing, the consumer with taste  $\theta$  chooses a disclosure policy that sends message (say)  $\theta$  with probability 1, because it maximizes the probability that he is recommended his preferred product. Under discriminatory pricing, the consumer chooses  $\phi$ , which sends messages  $\theta$  and  $-\theta$  with probabilities  $\delta^*$  and  $1 - \delta^*$ , respectively, where  $\delta^*$  is an equilibrium disclosure level under discriminatory pricing in [Section 3](#).

### 5.3 General Formulation of Horizontal Information

In [Section 3](#), I assume that the most informative disclosure policy available to the consumer discloses whether  $u_1 > u_2$  or  $u_1 < u_2$ . Here, I provide a more general formulation of horizontal information under which all of the results in [Section 3](#) hold. I denote the prior CDF for the value of each product by  $x_0$ .

Consider the following  $(\bar{M}, \bar{\phi})$  with  $\bar{M} = \{1, 2\}$ . First,  $\bar{\phi}(1|u_1, u_2) = \bar{\phi}(2|u_2, u_1)$  for any  $(u_1, u_2) \in V^2$ . Second,  $\int_{u_2 \in V} \bar{\phi}(1|u_1, u_2) dx_0(u_2)$  is strictly increasing in  $u_1 \in V$ . One disclosure policy that satisfies this condition is the following: It sends message 1 with probability  $h(u_1 - u_2)$ , where  $h(\cdot)$  is strictly increasing and  $h(x) + h(-x) = 1$ . (If  $h$  is a step function, I obtain the disclosure policy in [Section 3](#).) Intuitively, the higher value the consumer has for product  $k \in \{1, 2\}$ , the more likely message  $k$  is sent by  $\bar{\phi}$ .

Because of the symmetry, the posterior distribution for the value of product  $k$  conditional on message  $j$  depends only on whether  $k = j$ . Let  $F_1$  and  $F_0$  denote the posterior distributions for the value of product  $k$  conditional on messages  $k$  and  $-k$ , respectively. The next lemma extends [Lemma 2](#).

**Lemma 6.**  $F_1$  is greater than  $F_0$  in the hazard rate order.

*Proof.* Because  $\int_{u_2} \bar{\phi}(1|u_1, u_2) dx_0(u_2)$  is increasing in  $u_1$ , for any  $u^+ \geq u^-$ , I obtain

$$\begin{aligned} \frac{\int_{u_1 > u^+} \int_{u_2} \phi(1|u_1, u_2) dx_0(u_2) dx_0(u_1)}{1 - x_0(u^+)} &\geq \frac{\int_{u_1 > u^-} \int_{u_2} \phi(1|u_1, u_2) dx_0(u_2) dx_0(u_1)}{1 - x_0(u^-)} \\ \iff \frac{1 - F_1(u^+)}{1 - x_0(u^+)} &\geq \frac{1 - F_1(u^-)}{1 - x_0(u^-)}. \end{aligned}$$

Replacing  $\phi(1|u_1, u_2)$  by  $\phi(2|u_1, u_2)$ , I obtain

$$\frac{1 - F_0(u^+)}{1 - x_0(u^+)} \leq \frac{1 - F_0(u^-)}{1 - x_0(u^-)}.$$

These inequalities imply

$$\frac{1 - F_1(u^+)}{1 - F_0(u^+)} \geq \frac{1 - F_1(u^-)}{1 - F_0(u^-)}$$

whenever the fractions are well-defined. Therefore,  $F_1$  dominates  $F_0$  in the hazard rate order.  $\square$

Note that the derivation of [Theorem 1](#) only uses the property that the value of the consumer's preferred product dominates that of the less preferred product in the hazard rate order. Thus, I can conduct the same analysis under the assumption that consumers choose any disclosure policy less informative than  $(\bar{M}, \bar{\phi})$ .

## 5.4 Alternative Interpretation of the Model: Online Advertising Platform

I reinterpret the model of [Section 3](#) as a game between the consumer, an online advertising platform (such as Google or Facebook), and two advertisers. Advertisers 1 and 2 sell products 1 and 2,

respectively. The platform auctions off the consumer's impression using a second price auction with an optimal reserve price.

In this interpretation, first, the consumer chooses his disclosure level  $\delta$  (e.g., whether to accept a cookie) and visits the platform. Each advertiser  $k \in \{1, 2\}$  sets a price of product  $k$  and a bidding rule  $b_k : \{1, 2\} \rightarrow \mathbb{R}$ . Here,  $b_k(j)$  is the bid that Advertiser  $k$  places for the impression of consumers with a realized message  $j \in \{1, 2\}$ .<sup>20</sup> If Advertiser  $k \in \{1, 2\}$  wins the auction for the impression, the ad of product  $k$  is shown to the consumer. The consumer sees the ad of the winning advertiser, learns the value and the price of the product, and decides whether to buy it.

The ad auction works as the product recommendation. Assume that each advertiser can set a price after observing a disclosure level. Suppose that the consumer chooses a disclosure level  $\delta$ . In this subgame, the following equilibrium exists. Each advertiser sets price  $p(\delta)$  and a bidding rule such that  $b_x(x) = p(\delta)[1 - \delta F^{MAX}(p(\delta)) - (1 - \delta)F^{MIN}(p(\delta))]$  and  $b_x(-x) < b_x(x)$ . The platform sets a reserve price  $p(\delta)[1 - \delta F^{MAX}(p(\delta)) - (1 - \delta)F^{MIN}(p(\delta))]$  to extract full surplus from the winning advertiser. If the advertisers and the platform adopt these strategies, the consumer is shown the ad of a product he prefers more with probability  $\delta$  at price  $p(\delta)$ . In contrast, if each advertiser has to set a price before observing  $\delta$ , then there is an equilibrium in which the consumer chooses disclosure level 1 and each advertiser sets price  $p(1)$ . Thus, I obtain the same result as [Theorem 1](#).

Now, if an advertiser cannot credibly commit to nondiscriminatory pricing, how can the platform prevent discriminatory pricing to promote information disclosure? One way is to adopt the following "privacy policy": The platform commits to disclose advertisers which product the consumer is more likely to prefer, but without disclosing the exact likelihood. In other words, the platform only discloses the message realization and not  $\delta$  itself. This privacy policy implements nondiscriminatory pricing: Even if Advertiser  $k$  can set a product price after learning about the consumer's preference, it cannot set a price based on  $\delta$ , which is not disclosed by the platform. This in turn implies that the consumer chooses the highest disclosure level without worrying about

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<sup>20</sup>The analysis does not depend on whether an advertiser can place a bid contingent on  $\delta$ . For simplicity, I assume that an advertiser cannot do so.



how the information is used by the advertisers. Expecting this, each advertiser sets a price of  $p(1)$  in equilibrium. [Theorem 1](#) states that such a privacy policy benefits the platform and hurts the consumer.

## 6 Concluding Discussion

In this paper, I focus on consumers' privacy in online marketplaces and its welfare implications, which would be a first-order issue in today's economy. The key of the analysis is the following economic trade-off: Disclosing information could benefit consumers, because firms can provide personalized offerings such as product recommendations, online ads, and customized products. However, disclosure could hurt consumers if firms use the information to price discriminate. I capture this trade-off in a model in which a consumer discloses information about his preferences to a multi-product firm. The consumer has limited attention, in that he does not yet know his vector of values for the products and can evaluate only a small number of products relative to the huge variety of products available. As a result, the firm can use the provided information not only to extract surplus through pricing, but also to create surplus through product recommendations. This new modeling feature is important for studying the role of consumers' information in online marketplaces, in which firms often use consumers' personal data to make direct product recommendations and deliver ads to consumers, who are unable to view and assess all available products.

The paper's contributions are threefold. One is to give an economic explanation of a somewhat puzzling observation in the Internet economy: Firms seem to not use individual data to price discriminate, and consumers seem to casually share their information with online sellers. The model explains this phenomenon as firms' strategic commitment and consumers' best response. I show that this outcome robustly arises in two settings that differ in the information-disclosure technologies available to consumers.

The second contribution is to provide a framework for use in the design of privacy regulations. For instance, the model shows that nondiscriminatory pricing and the resulting full information

revelation are consumer-suboptimal. Restricting the amount of information firms can possess could benefit consumers, even if consumers are rational and can decide on their own what information to disclose.

The third contribution is to expand the theory of information disclosure by consumers. The model of unrestricted disclosure reveals that even with fine-grained control of information, consumers or a regulator cannot simultaneously achieve efficient price discrimination and efficient matching of products without sacrificing consumer welfare. I also extend the model to study the impact of competition on information disclosure. This extension demonstrates that the market structure and consumer attentiveness are two factors that affect what information consumers should disclose.

There are various interesting directions for future research. For example, the models could be extended to consider information sharing between firms or the presence of data brokers, which is likely to add new policy implications. Moreover, this paper highlights the value of the equilibrium analysis to study consumers' privacy choices in the Internet economy. It would also be fruitful to study how consumers' information disclosure collectively affects welfare in other aspects of online privacy.

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## Appendix

### A Proof of Lemma 1

Without loss of generality, suppose that message 1 realizes. If the firm recommends products 1 and 2 to the consumer, he draws values from distributions  $\delta F^{MAX} + (1 - \delta)F^{MIN}$  and  $\delta F^{MIN} + (1 - \delta)F^{MAX}$ , respectively. Because  $\delta > 1/2$  and  $F^{MAX}$  strictly first-order stochastically dominates  $F^{MIN}$ ,  $\delta F^{MAX} + (1 - \delta)F^{MIN}$  strictly first-order stochastically dominates  $\delta F^{MIN} + (1 - \delta)F^{MAX}$ . This implies that it is a unique best response for the firm to recommend products 1 in both pricing

regime: otherwise, the firm can change recommendation and set the price optimally to strictly increase its revenue.

## B A Prior Distribution without a Density Function

I show the analogue of [Theorem 1](#) when  $x_0$  is a general non-degenerate distribution. In this case, I can no longer say that the consumer chooses disclosure level 1 in the unique equilibrium. This is because the consumer may get a payoff of zero for any disclosure level. (Consider  $x_0$  that puts probability almost 1 on  $u_k = 100$  and the remaining probability on  $u_k = 1$ . The firm sets price 100 for any  $\delta$  and thus the consumer is indifferent between any disclosure levels.)

I show that the consumer still discloses more information under nondiscriminatory pricing in the strong set order. Let  $\Delta_N \subset [1/2, 1]$  denote the set of disclosure levels  $\delta_N$  in symmetric equilibrium under nondiscriminatory pricing. Similarly, let  $\Delta_D \subset [1/2, 1]$  denote the set of disclosure levels sustained in equilibrium under discriminatory pricing.

**Theorem 3.**  *$\Delta_N$  is greater than  $\Delta_D$  in the strong set order. Take the most informative equilibrium under each pricing regime. Then, the firm is better off and the consumer is worse off under nondiscriminatory pricing than under discriminatory pricing. Similarly, take the least informative equilibrium under each pricing regime. Then, the firm is better off and the consumer is worse off under nondiscriminatory pricing.*

*Proof.* Take any  $(\delta_N, \delta_D) \in \Delta_N \times \Delta_D$  with  $\delta_D > \delta_N$ . By [Lemma 1](#), under the nondiscriminatory pricing, the payoff of the consumer is

$$\delta_N u^{MAX}(p(\delta_N)) + (1 - \delta_N) u^{MIN}(p(\delta_N)).$$

If he deviates to  $\delta_D > \delta_N$ , he obtains

$$\delta_D u^{MAX}(p(\delta_N)) + (1 - \delta_D) u^{MIN}(p(\delta_N)) \leq \delta_N u^{MAX}(p(\delta_N)) + (1 - \delta_N) u^{MIN}(p(\delta_N)).$$

Then, I obtain  $u^{MAX}(p(\delta_N)) - u^{MIN}(p(\delta_N)) = 0$ . Because  $u^{MAX}(p) - u^{MIN}(p)$  is non-negative and non-increasing in  $p$ , I also obtain  $u^{MAX}(p(\delta_D)) - u^{MIN}(p(\delta_D)) = 0$ . Thus,

$$\delta_D u^{MAX}(p(\delta_D)) + (1 - \delta_D) u^{MIN}(p(\delta_D)) \geq \delta u^{MAX}(p(\delta_D)) + (1 - \delta) u^{MIN}(p(\delta_D))$$

for any  $\delta$ . Thus,  $\delta_D \in \Delta_N$ . Also,

$$\begin{aligned} \delta_N u^{MAX}(p(\delta_N)) + (1 - \delta_N) u^{MIN}(p(\delta_N)) &\geq \delta_D u^{MAX}(p(\delta_N)) + (1 - \delta_D) u^{MIN}(p(\delta_N)) \\ &\geq \delta_D u^{MAX}(p(\delta_D)) + (1 - \delta_D) u^{MIN}(p(\delta_D)). \end{aligned}$$

Thus,  $\delta_N \in \Delta_D$ .

□

## C Equivalence to the Model of a Continuum of Consumers

This subsection consists of two parts. First, I formally describe the model with a continuum of consumers with an appropriate equilibrium notion. Second, I show that for *any* set of disclosure policies  $\mathcal{D}$ , the set of equilibria with a single consumer is equal to the set of symmetric equilibria with a continuum of consumers. For ease of exposition I call the model with a continuum of consumers as model (C). Similarly, I call the original model with a single consumer as model (S).

First, I describe the model (C). There is a continuum of consumers uniformly distributed on  $[0, 1]$ . Each consumer is endowed with the identical preference and the strategy space as the consumer in model (S). The firm has enough units of each of  $K$  products to serve all the consumers. At the game beginning of the game, each consumer  $i \in [0, 1]$  chooses a disclosure policy  $\phi_i \in \mathcal{D}$ . Then, the valuation vector  $u^i$  and a message  $m_i \sim \phi_i(\cdot | u^i)$  realize for each  $i$ . Then, the firm observes  $(\phi_i, m_i)_{i \in [0, 1]}$ , sets prices, and recommends one of the  $K$  products to each consumer. Under nondiscriminatory pricing, the firm sets a single price on each product. Under discriminatory pricing, the firm can charge different prices to different consumers.

My solution concept for model (C) is subgame perfect equilibrium (hereafter, SPE) which satisfies the following four properties. First, at each information set, the firm recommends products to maximize not only her total revenue but also the expected payment of each consumer, who has measure zero. Furthermore, under discriminatory pricing, I impose the same requirement on the firm's pricing. Without this restriction, any disclosure policies are sustained in some SPE because the firm can "punish" the unilateral deviation of a consumer, who has measure zero, by offering a completely random recommendation. The second property requires that the firm chooses prices and recommendations to maximize consumers' payoffs whenever the firm is indifferent given the first requirement. This excludes the multiplicity of equilibrium due to the firm's tie-breaking rule. The third property requires that the firm recommends each of  $K$  products to a positive mass of consumers on the equilibrium path. The fourth property requires the symmetry: each consumer takes the same disclosure policy on the equilibrium path. I use equilibrium (C) to mean a SPE with these four properties.

Note that under discriminatory pricing, model (S) and (C) differ only in the number of consumers. Thus, the equivalence is trivial. The following result shows the equivalence under nondiscriminatory pricing.

**Proposition 7.** *Take any  $\mathcal{D}$ . Consider nondiscriminatory pricing. Take any equilibrium disclosure policy  $\phi^*$  and product prices  $p^* = (p_1^*, \dots, p_K^*)$  of model (S) in which each product is recommended with positive probability. Then,  $(\phi^*, p^*)$  consists of an equilibrium (C) in model (C). Similarly, if  $(\phi^*, p^*)$  consists of an equilibrium (C) of model (C), then it also consists of an equilibrium of model (S).*

*Proof.* Take any  $(\phi^*, p^*)$  consisting of an equilibrium (C) in model (C). I show that  $(\phi^*, p^*)$  is an equilibrium under model (S). First, if everyone chooses  $\phi^*$  in models (C) and (S), then the firm's revenue as a function of prices and recommendation strategy is the same between model (C) and model (S) with the only difference being whether the expectation is taken based on mass of consumers or on probability. Thus,  $p^*$  and the associated recommendation strategy in model (C) continues to be optimal in model (S). Next, each consumer in model (C) is choosing a disclosure

policy from  $\mathcal{D}$  taking prices as given, which is the same best response condition as in model (S). Note that the fact that each consumer is a price taker relies on the first, second, and third requirements I impose in equilibrium (C). The equivalence of the other direction proceeds in the same way.  $\square$

## D Presence of “Negative Externality” with a Continuum of Consumers

I show that in the second interpretation of the model, information disclosure by a positive mass of consumers lowers the welfare of other consumers. To see this, note that if each consumer  $i$  chooses a disclosure level  $\delta_i$  and the firm sets price  $p$  for each product, then the total revenue is given by

$$\begin{aligned} & \int_{i \in [0,1]} p[1 - \delta_i F^{MAX}(p) - (1 - \delta_i) F^{MIN}(p)] di \\ & = p[1 - \bar{\delta} F^{MAX}(p) - (1 - \bar{\delta}) F^{MIN}(p)] \end{aligned}$$

where  $\bar{\delta} = \int_{i \in [0,1]} \delta_i di$  is the average disclosure level. This implies that the optimal price under nondiscriminatory pricing is given by  $p(\bar{\delta})$ . If a positive mass of consumers disclose more information,  $\bar{\delta}$  increases. This increases  $p(\bar{\delta})$  and decreases the payoffs of other consumers who have not changed disclosure levels.

## E Proof of Proposition 2

First, I show that there is a vertically efficient equilibrium. Take any  $(M^*, \phi^*)$  which leads to a vertically inefficient allocation given the firm’s best response. Let  $x \in \Delta(V^K)$  denote a realized posterior at which trade may not occur.<sup>21</sup> Without loss of generality, suppose that product 1 is recommended at price  $v_\ell$  at  $x$ . I show that there is another disclosure policy  $\phi^{**}$  which gives a weakly greater payoff than  $\phi^*$  to the consumer and achieves a strictly greater total surplus. Suppose that  $\phi^{**}$  discloses whether the value for product 1 is weakly greater than  $v_\ell$  or not whenever posterior

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<sup>21</sup>Because  $|V^K| < +\infty$ , without loss of generality, I can assume  $|M^*| < +\infty$ . Then, each message realizes with a positive probability from the ex-ante perspective. Thus, there must be a posterior  $x \in \Delta(V^K)$  which realizes with a positive probability and trade may fail to occur given  $x$ .

$x$  realizes, in addition to the information disclosed by  $\phi^*$ . Let  $x^+$  and  $x^- \in \Delta(V^K)$  denote the posterior beliefs of the firm after the consumer discloses that the value for product 1 is weakly above and strictly below  $v_\ell$ , respectively. Then,  $x = \alpha x^+ + (1 - \alpha)x^-$  holds for some  $\alpha \in (0, 1)$ .

I show that  $\phi^{**}$  weakly increases the payoff of the consumer. First, conditional on the event that the value is below  $v_\ell$ , the consumer gets a greater payoff under  $\phi^{**}$  than under  $\phi^*$  because the consumer obtains a payoff of zero under  $\phi^*$ . Second, I show that, conditional on the event that the value is weakly above  $v_\ell$ , the firm continues to recommend product 1 at price  $v_\ell$ . To show this, suppose to the contrary that the firm strictly prefers to recommend another product  $m$  at price  $v_k$ . If  $m = 1$ ,  $v_k$  is different from  $v_\ell$ . Let  $x_1^+ \in \Delta(V)$  and  $x_m^+ \in \Delta(V)$  be the marginal distributions of  $u_1^i$  and  $u_m^i$  given  $x^+$ , respectively. Because the firm prefers recommending a product  $m$  at price  $v_k$  to recommending a product 1 at price  $v_\ell$ , I obtain

$$v_k \sum_{j=k}^K x_m^+(v_j) > v_\ell \sum_{j=\ell}^K x_1^+(v_j),$$

which implies

$$v_k \sum_{j=k}^K \alpha x_m^+(v_j) + (1 - \alpha)x_m^-(v_j) \geq v_k \sum_{j=k}^K \alpha x_m^+(v_j) > v_\ell \sum_{j=\ell}^K \alpha x_1^+(v_j) = v_\ell \sum_{j=\ell}^K \alpha x_1^+(v_j) + (1 - \alpha)x_1^-(v_j).$$

The last equality follows from  $x_1^-(v) = 0$  for any  $v \geq v_\ell$ . This contradicts that the firm prefers to recommend product 1 at price  $v_\ell$  at  $x$ .

Consider the following mapping  $\Phi : \mathcal{D} \rightarrow \mathcal{D}$ : given any disclosure policy  $\phi \in \mathcal{D}$ ,  $\Phi$  chooses a posterior belief  $x$  induced by  $\phi$  at which trade fails to occur with a positive probability. If there are more than one such belief,  $\Phi$  chooses the posterior belief corresponding to the lowest price and the smallest index  $k \in \mathcal{K}$ .<sup>22</sup>  $\Phi(\phi)$  is a disclosure policy which discloses whether the value for the recommended product is weakly greater than the price or not whenever posterior  $x$  realizes, in addition to the information disclosed by  $\phi$ .

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<sup>22</sup>If this does not pin down a posterior uniquely, then I define  $\Phi$  so that it first modifies  $\phi$  by merging multiple beliefs at which the same product is recommended at the same price.

To show that there exists a vertically efficient equilibrium, take any equilibrium disclosure policy  $\phi_0$ . Define  $\Phi^1(\phi_0) = \Phi(\phi_0)$  and  $\Phi^{n+1}(\phi_0) = \Phi(\Phi^n(\phi_0))$  for each  $n \geq 1$ . Because  $|V^K| < +\infty$ , there exists  $n^*$  such that  $\Phi^{n^*} = \Phi^{n^*+1}$ . Define  $\phi^* := \Phi^{n^*}(\phi_0)$ . By construction,  $\phi^*$  gives a weakly greater payoff to a consumer than  $\phi_0$ . Thus, it is an equilibrium under discriminatory pricing. Moreover, at each realized posterior, trade occurs with probability 1. Therefore,  $\phi^*$  is a vertically efficient equilibrium.

Next, I show that for a generic prior, any equilibrium is horizontally inefficient whenever  $x_0 \in X_{>v_1}$ . While the consumer has private type  $u$  drawn from  $x_0 \times \cdots \times x_0 \in \Delta(V^K)$ , for the ease of exposition, I interpret the model as having the total mass one of consumers with mass  $\prod_{k=1}^K x^*(u_k)$  having a valuation vector  $u = (u_1, \dots, u_K) \in V^K$ .

Let  $\mathcal{E} \subset \mathcal{D}$  denote the set of disclosure policies which lead to an efficient allocation for some best response of the firm. Take any disclosure policy  $\phi^E \in \mathcal{E}$ . Under  $\phi^E$ , if the firm prefers to recommend product  $k$ , then  $k \in \arg \max_{\ell \in \mathcal{K}} u_\ell$ . Thus, if both  $\phi^E$  and  $\hat{\phi}^E$  achieve an efficient allocation, they only differ in terms of which product is recommended to consumers who have the same valuation for more than one product. I show that without loss of generality, I can focus on disclosure policies that recommend each product in  $\arg \max u_k$  with equal probability whenever  $|\arg \max u_k| \geq 2$ .

To show this, take any  $(M, \phi) \in \mathcal{E}$ . Let  $P \subset \mathcal{K}^{\mathcal{K}}$  be the set of the permutations of  $\{1, \dots, K\}$ . Define  $\phi^E$  as the following disclosure policy. First,  $\phi^E$  publicly draws a permutation  $\tau \in P$  uniformly randomly. Second,  $\phi^E$  discloses information according to  $\phi(u_{\tau(1)}, \dots, u_{\tau(K)}) \in \Delta(M)$  for each realization  $(u_1, \dots, u_K)$ . Then, from the ex-ante perspective, the consumer is recommended a product  $k \in \arg \max u_j$  with probability  $\frac{1}{|\arg \max u_j|}$ .

I further modify  $\phi^E$  to obtain  $\phi^G \in \mathcal{E}$  which maximizes the consumer's payoff among the disclosure policies which lead to an efficient allocation. First,  $\phi^G$  decomposes the prior  $x^*$  into  $K$  segments so that  $x^* = x_1 + \cdots + x_K$ . (As a disclosure policy,  $\frac{1}{\sum_{n=1}^N x_k(v_n)} x_k \in \Delta(V^K)$  is a posterior belief that  $\phi^G$  draws) Each  $x_k$  consists of  $\frac{1}{|\arg \max u_j|} \cdot \prod_{j=1}^K x^*(u_j) \cdot 1_{\{k \in \arg \max u_j\}}$  mass of consumers with  $u \in V^K$ . Now, I apply the following procedure to each segment  $x_k$ . Without



loss of generality, I explain the procedure for  $x_1$ . I apply the “greedy algorithm” in [Bergemann, Brooks, and Morris \(2015\)](#) to  $x_1$  with respect to the value for product 1 so that I can decompose  $x_1$  into  $x_1 = \alpha_1 x_1^{S_1} + \dots + \alpha_{N_1} x_1^{S_{N_1}}$ . Here,  $S_1 = V$  and  $S_{n+1} \supset S_n$  for  $n = 1, \dots, N_1 - 1$ . Moreover, the marginal distribution of each  $x_1^{S_n}$  with respect to  $u_1$  is supported on  $S_n \subset V$ , and the firm is indifferent between charging any price for product 1 inside the set  $S_n$  if the value for product 1 is distributed according to  $x_1^{S_n}$ . In contrast to [Bergemann, Brooks, and Morris \(2015\)](#), the consumer’s type is  $K$ -dimensional. Thus, directly applying the algorithm does not pin down the valuation distribution for product  $k \neq 1$  in each segment  $x_1^{S_n}$ . To pin down the distribution of values for product  $k \neq 1$  in each  $x_1^{S_n}$ , I assume the following: whenever the algorithm picks consumers from  $x_1$  to construct  $x_1^{S_n}$ , it picks consumers whose value for product 2 is lower. If this does not uniquely pin down the valuation vector to pick, it picks consumers whose value for product 3 is lower, and so on. In this way, the algorithm pins down a unique segmentation.

Consumer surplus under  $\phi^G$  is weakly greater than under  $\phi^E$ . This is because the segmentation created by the greedy algorithm maximizes consumer surplus and that the valuation distribution of each recommended product is identical between  $\phi^E$  and  $\phi^G$ . Also, under  $\phi^G$ , the firm is willing to recommend product  $k$  to consumers in  $x_k^{S_n}$  because  $x_k^{S_n}$  only contains consumers such that  $u_k \geq u'_k$  for any  $k' \in \mathcal{K}$ .

Next, I show the following: there exists a set  $D \subset \Delta(V)$  satisfying the following:  $D$  has Lebesgue measure zero in  $\mathbb{R}^N$ , and for any prior  $x_0 \in \Delta(V) \setminus D$ , all consumers in  $x_1^{S_{N_1}}$  constructed by the last step of the algorithm have the same value for product  $k$ . The proof of this part consists of two steps.

In the first step, take any subsets of  $V$  as  $S_1 \supset S_2 \supset \dots \supset S_{N_1}$  such that  $|S_{N_1}| \geq 2$ . Then, define

$$Y(S_1, \dots, S_{N_1}) := \left\{ y \in \mathbb{R} : y = \sum_{n=1}^{N_1} \alpha_n x_1^{S_n}, \exists (\alpha_1, \dots, \alpha_{N_1}) \in \Delta^{N_1-1} \right\}$$

where  $\Delta^{N_1-1}$  is the  $(N_1 - 1)$ -dimensional unit simplex. Because  $|S_{N_1}| \geq 2$ ,  $N_1 \leq N - 1$ . Thus,  $Y(S_1, \dots, S_{N_1})$  is a subset of at most  $N - 1$  dimensional subspace, which has Lebesgue measure

zero in  $\Delta(V) \subset \mathbb{R}^N$ . Define  $\mathcal{S}$  as

$$\mathcal{S} = \{(S_1, \dots, S_{N_1}) : \exists N_1 \in \mathbb{N}, V \supset S_1 \supset S_2 \supset \dots \supset S_{N_1}, |S_{N_1}| \geq 2\}.$$

Let  $\mathcal{Q}$  be the set of  $x \in \Delta(V)$  such that consumers in  $x_k^{S_n}$  constructed in the last step of the algorithm have different values for product  $k$ . I can write it as  $\mathcal{Q} = \cup_{(S_1, \dots, S_{N'}) \in \mathcal{S}} Y(S_1, \dots, S_{N'})$ . Because  $|\mathcal{S}| < +\infty$  and each  $Y(S_1, \dots, S_{N'})$  has measure zero,  $\mathcal{Q}$  has Lebesgue measure zero as well.

In the second step, to show that there exists  $D$  with the desired property, consider a function  $\varphi$  which maps any prior  $x \in \Delta(V)$  to the valuation distribution of product  $k$  conditional on the event product  $k$  is recommended under  $\phi^E$ . Because the distribution does not depend on  $k$ , I consider  $k = 1$  without loss of generality.  $\varphi$  is written as follows.

$$\varphi(x) = K \cdot \begin{pmatrix} \frac{1}{K} x_1^K \\ x_2 \sum_{\ell=0}^{K-1} x_1^{K-1-\ell} x_1^\ell \cdot \frac{1}{\ell+1} \binom{K-1}{\ell} \\ x_3 \sum_{\ell=0}^{K-1} (x_1 + x_2)^{K-1-\ell} x_3^\ell \cdot \frac{1}{\ell+1} \binom{K-1}{\ell} \\ \vdots \\ x_N \sum_{\ell=0}^{K-1} (x_1 + \dots + x_{N-1})^{K-1-\ell} x_N^\ell \cdot \frac{1}{\ell+1} \binom{K-1}{\ell} \end{pmatrix}.$$

$\varphi$  is infinitely differentiable and its Jacobian matrix  $J_\varphi$  is a triangular matrix with the diagonal elements being positive as long as  $x_n > 0$  for each  $n = 1, \dots, N$ . Thus,  $J_\varphi(x)$  has full rank if  $x$  is *not* in a measure-zero set

$$\{(x_1, \dots, x_N) \in \Delta(V) : \exists n, x_n = 0\}. \quad (3)$$

By Theorem 1 of [Ponomarev \(1987\)](#),  $\varphi : \mathbb{R}^N \rightarrow \mathbb{R}^N$  has the “0-property”: the inverse image

of measure-zero set by  $\varphi$  has measure zero. In particular,  $D := \varphi^{-1}(\mathcal{Q})$  has measure zero. Thus, there exists a measure-zero set  $D$  such that for any  $x \in \Delta(V) \setminus D$ , all consumers in  $x_k^{S_n}$  constructed in the last step of the algorithm have the same value for product  $k$ .

Consider the algorithm applied to product  $k$ . Recall that  $x_k^{N_k}$  is the segment created at the last step. As I have shown, generically, all consumers in  $x_k^{N_k}$  have the same value for product  $k$ . Let  $v^*$  denote the value. In equilibrium, consumers in  $x_k^{N_k}$  obtain a payoff of zero given the firm's optimal price  $v^*$ . Moreover, if the optimal price at the prior is strictly greater than  $v_1$  (i.e.,  $p(x_0) \geq v_2$ ), then  $v^* > v_1$ . Indeed, if  $v^* = v_1$ , then  $v_1 \in S_n$  for  $n = 1, \dots, N_1$ . This implies that  $v_1$  is an optimal price for each  $x_1^{S_n}$  and thus for  $x_1 = \sum_{n=1}^{N_1} \alpha_n x_1^{S_n}$ , which is a contradiction. To sum up, except for a Lebesgue measure zero set of priors, if the optimal price is strictly greater than  $v_1$  under the prior, then consumers in  $x_k^{N_k}$  obtain a payoff of zero given the firm's optimal price strictly above  $v_1$ .

Now, I modify  $\phi^G$  to create a horizontally inefficient  $\phi^I$  that yields consumer surplus strictly greater than  $\phi^G$ , which completes the proof. To simplify the exposition, for any  $S \subset \mathcal{K}$ , let  $v_S^* \in V^K$  denote a vector whose coordinate for each  $k \in S$  is  $v^*$  and other coordinates are  $v_1$ . First, I replace  $\varepsilon$  mass of  $v_{\{2\}}^*$  in the segment  $x_2^{S_1}$  for product 2 created by the first step of the algorithm (applied for product 2) by the same probability mass of  $v_{\mathcal{K}}^*$  in the segment  $x_2^{S_{N_2}}$ . Now, this does not affect consumer surplus generated from product 2. However, I now have  $\varepsilon$  mass of  $v_{\{2\}}^*$  remaining. I pool this  $\varepsilon$  mass of  $v^*(2)$  with segment  $x_1^{S_1}$ . Let  $\hat{x}_1^1$  denote the segment created in this way. First, under  $\hat{x}_1^1$ , price  $v_1$  is uniquely optimal because I add a positive mass of consumers having value  $v_1$  to  $x_1^1$ , and price  $v_1$  is optimal for  $x_1^1$ . Second, the firm is willing to recommend product 1 for  $\hat{x}_1^1$ , as long as  $\varepsilon$  is small. This follows from the fact that the firm strictly prefers to set price  $v_1$  if the firm recommended product  $k \neq 1$  for  $x_1^1$ . Indeed, at  $x_1^{S_1}$ , the firm's optimal price is  $v_1$  no matter which product it recommends. While the firm is indifferent between recommending any prices of product 1, consumers who have value  $v_1$  for all products but product 1 reject any price strictly greater than  $v_1$ . (Note that such consumers must be in the segment in  $x_1^{S_1}$ .) Thus, for product 2, price  $v_1$  is uniquely optimal at  $x_1$ .

Because the firm strictly prefers to recommend product 1 at price  $v_1$  compared to any other choices, for some  $\delta > 0$ , I can bring mass  $\delta$  of  $v_{\mathcal{K}}^*$  from  $x_1^{S_{N_1}}$  who originally receives zero payoff. Let  $\tilde{x}_1^1$  denote the segment created in this way. As long as  $\delta$  is small, at  $\tilde{x}_1^1$ , the firm still recommends product 1 at price  $v_1$ . This strictly increases consumer surplus because consumers who obtain zero payoff at segment  $x_1^1$  now obtain a positive payoff at  $\tilde{x}_1^1$  without changing surplus accruing to other consumers. However, the resulting allocation is inefficient.

Therefore, for any disclosure policy which leads to an efficient allocation, there exists a horizontally inefficient disclosure policy which gives a strictly greater consumer surplus. This completes the proof.

## F Proof of Proposition 3 and Proposition 4

*Proof of Proposition 3.* For each  $K$ , the consumer chooses  $\delta = 1$  in the unique symmetric equilibrium under nondiscriminatory pricing because disclosure does not affect prices and increases his payoff through a better recommendation. Let  $F$  denote the CDF of the value for each product. Take any  $\varepsilon > 0$ . Suppose that the firm sets a nondiscriminatory price of  $b - \varepsilon/2$  for each product. For a sufficiently large  $K$ , the probability  $1 - F(p)^K$  that the consumer buys the recommended product goes to 1. Thus, there is  $\underline{K}$  such that the firm's revenue is at least  $b - \varepsilon$  if  $K \geq \underline{K}$ . This implies that the consumer's payoff is at most  $\varepsilon$  for any such  $K$ . This completes the proof of the first part.

To see that the consumer can always guarantee some positive payoff  $\underline{u}$  under discriminatory pricing, observe that the consumer can choose to disclose no information and obtain a payoff of  $\int_{p_0}^b v - p_0 dF(v)$  where  $p_0 < b$  is the optimal price given no disclosure, which is independent of  $K$ .  $\square$

*Proof of Proposition 4.* First, the result under nondiscriminatory pricing follows from the previous result, as total surplus is greater than the firm's revenue.

Second, I show that total surplus under discriminatory pricing is uniformly bounded away from  $b$ . Suppose to the contrary that for any  $n \in \mathbb{N}$ , there exists  $K_n$  such that when the firm sells  $K_n$

products, some equilibrium under discriminatory pricing achieves total surplus of at least  $b - \frac{1}{n}$ . Then, I can take a subsequence  $(K_{n_\ell})_\ell$  such that  $K_{n_\ell} < K_{n_{\ell+1}}$  for any  $\ell \in \mathbb{N}$ . Next, I show that for any  $p < b$  and  $\varepsilon < 1$ , there exists  $\ell^* \in \mathbb{N}$  such that for any  $\ell \geq \ell^*$ ,

$$\mathbf{P}_\ell(\text{the consumer's value for the recommended product} \geq p) \geq \varepsilon. \quad (4)$$

where  $\mathbf{P}_\ell(\cdot)$  is the probability measure on the consumer's value for the recommended product in equilibrium of  $K_{n_\ell}$ -product model. To show [inequality 4](#), suppose to the contrary that there is some  $(p, \varepsilon)$  and a subsequence  $(K'_m)_m$  of  $(K_{n_\ell})_\ell$  such that the inequality is violated. Then, given any  $K'_m$  in this subsequence, the total surplus is at most  $p\varepsilon + b(1 - \varepsilon) < b$ . This contradicts that the equilibrium total surplus converges to  $b$  as  $K'_m \rightarrow +\infty$ .

Now, I use [inequality 4](#) to show that the firm's equilibrium revenue converges to  $b$  along  $(K_{n_\ell})_\ell$ . Take any  $r < b$ . If the firm sets price  $\frac{r+b}{2}$ , then for a sufficiently large  $\ell$ , the consumer accepts the price with probability greater than  $\frac{2r}{r+b} < 1$ . That is, for a large  $\ell$ , the firm's expected revenue exceeds  $r$ . Since this holds for any  $r < b$ , the firm's revenue converges to  $b$  as  $\ell \rightarrow +\infty$ . This contradicts that the consumer's payoff is bounded from below by a positive number independent of  $K$ , which is shown [Proposition 3](#).  $\square$

## G Does Nondiscriminatory Pricing Increase Total Welfare?: Case of Restricted Information Disclosure

**Example 2.** Nondiscriminatory pricing can be welfare-detrimental. Consider the prior  $x_0$  and the resulting probability mass function  $f_1$  of  $\max(u_1, u_2)$  in [Table VII](#).

If the firm commits to nondiscriminatory pricing, then in equilibrium, the consumer chooses disclosure level 1 and the value of the recommended product is drawn from  $f_1(F_1)$ . In this case, the monopoly price is 3, which gives a total surplus of  $9/4$ . If the consumer discloses no information, the value is drawn from  $x^*$ . In this case, the monopoly price is 2, which gives a total surplus of  $5/2 > 9/4$ . At equilibrium of the discriminatory pricing regime, the consumer chooses the highest

$\delta$  at which the firm is willing to set price 2. This gives the total surplus greater than no disclosure because both the consumer and the firm obtain greater payoffs. Thus, the equilibrium total surplus is higher under discriminatory pricing in which the consumer discloses less information than under nondiscriminatory pricing.

**Example 3.** Conversely, nondiscriminatory pricing can be welfare-enhancing. Consider the prior  $x^*$  and the resulting probability mass function  $f_1$  of  $\max(u_1, u_2)$  in [Table VIII](#).

If the firm commits to nondiscriminatory pricing, then in equilibrium, the consumer chooses disclosure level 1 and the value of the recommended product is drawn from  $f_1$ . In this case, the monopoly price is 3, which gives total surplus of  $9/4$ . Under discriminatory pricing, the consumer discloses no information. Indeed, the prior  $x^*$  is such that the firm is indifferent between prices 2 and 3, which it strictly prefers to price 1. Thus, any disclosure level  $\delta > 1/2$  makes the firm prefer to set price 3. Given no disclosure, the monopoly price is 2, which gives total surplus of  $2 < 9/4$ . Thus, the equilibrium total surplus is higher under nondiscriminatory pricing in which the consumer discloses more information. The result is not a knife-edge case in the sense that qualitatively the same result holds if the prior is stochastically smaller but close to  $x^*$ .

## H Disclosure Policy for Case 3 of [Section 4.4](#)

I consider the third case:  $x^{OPT}(H) < x_0(H)$ . One representation of an equilibrium disclosure policy is shown in [Table IX](#).

$\beta$  and  $\gamma$  are determined as follows. Let  $x'(H)$  denote a threshold value of  $x_0(H)$  such that, if  $x_0(H) = x'(H)$ , then the firm is indifferent between charging prices  $L$  and  $H$  given message *Low* with  $(\beta, \gamma) = (0, 1)$ . Note that  $x^{OPT}(H) < x'(H)$ . If  $x_0(H) \leq x'(H)$ , the optimal disclosure sets  $\gamma = 1$ , and  $\beta < 1$  is such that the firm is indifferent between prices  $L$  and  $H$  conditional on message *Low*. If  $x_0(H) > x'(H)$ , then  $\beta = 0$  and  $\gamma$  is such that the firm is indifferent between prices  $L$  and  $H$  conditional on message *Low*. This is an equilibrium because the consumer obtains the first-best payoff  $(H - L)x_H^{OPT}$  with the maximum probability. The following is the formal proof.

*Proof.* Take any disclosure policy  $\phi$  that maximizes consumer surplus. I show that  $\phi$  is modified to be the form of [Table IX](#) without decreasing consumer surplus. By Proposition 1 of [Kamenica and Gentzkow \(2011\)](#), without loss of generality, I assume that  $\phi$  sends at most four messages  $1L$ ,  $1H$ ,  $2L$ , and  $2H$ , where a firm recommends product  $k \in \{1, 2\}$  at price  $j \in \{L, H\}$  conditional on message  $kj$ . Let  $x_{kj}$  be the segment that corresponds to  $kj$ . Namely,  $x_{kj}$  specifies the ex-ante probability mass of each type  $(u_1, u_2)$  which sends message  $kj$ . Thus,  $x_{1L} + x_{1H} + x_{2L} + x_{2H} = x^*$ . Now, without loss of generality, I can assume that  $x_{kH}$  puts no weight on  $(u_1, u_2) = (L, L)$  and that  $x_{1L}$  and  $x_{2L}$  contain equal mass of  $(L, L)$ . Next, the firm must be indifferent between prices  $L$  and  $H$  at any  $x_{kL}$ . Indeed, if the firm strictly prefers price  $L$ , say at  $x_{1L}$ , then I can take a positive mass of any of  $(H, H)$  and  $(H, L)$  from  $x_{1H} + x_{2H}$  and pool it to  $x_{1L}$  to strictly increase consumer surplus. Note that  $x_{1H} + x_{2H}$  must contain  $(H, H)$  or  $(H, L)$ : if they do not, which means  $x_{1L} + x_{2L}$  contains all the mass of  $(H, H)$  and  $(H, L)$ , it contradicts that  $x^{OPT}(H) < x_0(H)$  and that the firm is willing to set price  $L$  at the original  $x_{1L}$  and  $x_{2L}$ .

Furthermore,  $x_{1L}$  contains a weakly greater mass of  $(H, L)$  than that of  $(L, H)$ . If any of  $x_{1H}$  and  $x_{2H}$  contains a positive mass of  $(L, H)$ , then I can put this  $(L, H)$  into  $x_{1L}$ . I consider three cases:

(Case I) Suppose that after putting all the mass of  $(L, H)$  in  $x_{1H} + x_{2H}$  into  $x_{1L}$ , the new  $x_{1L}$  still contains a strictly greater mass of  $(H, L)$  than that of  $(L, H)$  and the firm is willing to set price  $L$  at the new  $x_{1L}$ . This implies that I can also put the remaining mass of  $(H, L)$  and  $(L, H)$  into  $x_{2L}$ . At the resulting  $x_{1L}$  and  $x_{2L}$ , segment  $x_{2L}$  contains a strictly greater mass of  $(L, H)$  than that of  $(H, L)$  because the total probability masses of  $(H, L)$  and  $(L, H)$  are equal. I show that this leads to a contradiction. To show this, I can exchange mass  $\varepsilon/2$  of  $(H, L)$  in  $x_{1L}$  and mass  $\varepsilon/2$  of  $(L, H)$  in  $x_{2L}$ , where  $\varepsilon$  is the difference between masses of  $(H, L)$  and  $(L, H)$  in  $x_{1L}$ . Then, I put a probability mass slightly greater than  $\varepsilon/2$  of  $(H, H)$  from  $x_{1H} + x_{2H}$  to each of  $x_{1L}$  and  $x_{2L}$ . Note that  $x_{1H} + x_{2H}$  must be containing a mass strictly greater than  $\varepsilon$  of  $(H, H)$ . Indeed, if the total mass of  $(H, H)$  in  $x_{1H} + x_{2H}$  is  $\varepsilon' \leq \varepsilon$ , then this implies that the firm is willing to set price  $L$  at  $x_{1L}$  and  $x_{2L}$  after we take all mass  $\varepsilon'$  of  $(H, H)$  from  $x_{1H} + x_{2H}$  to each of  $x_{1L}$  and  $x_{2L}$ . However,

this means that the firm prefers price  $H$  at  $x_{1L}$  because it prefers price  $H$  at the prior. This is a contradiction. This exchange strictly increases consumer surplus, which is a contradiction.

(Case II) Suppose that before putting all the mass of  $(L, H)$  in  $x_{1H} + x_{2H}$  into  $x_{1L}$ , the new  $x_{1L}$  contains an equal mass of  $(H, L)$  and  $(L, H)$ . Then, I apply the same procedure to  $x_{2L}$ . After putting all the mass of  $(L, H)$  in  $x_{1H} + x_{2H}$  into  $x_{2L}$ , if the new  $x_{2L}$  still contains a strictly greater mass of  $(H, L)$  than that of  $(L, H)$ , then this is a contradiction, because this implies that the total mass of  $(L, H)$  is strictly greater than that of  $(H, L)$ .

(Case III) Suppose before putting all the mass of  $(L, H)$  in  $x_{1H} + x_{2H}$  into  $x_{1L}$ , the new  $x_{1L}$  contains an equal mass of  $(H, L)$  and  $(L, H)$ . Then, I apply the same procedure to  $x_{2L}$ . Suppose that before all the mass of  $(L, H)$  in  $x_{1H} + x_{2H}$  into  $x_{2L}$ , the new  $x_{2L}$  also contains an equal mass of  $(H, L)$  and  $(L, H)$ . Because a consumers and the firm are now indifferent between two products at  $x_{1L}$  and  $x_{2L}$ , I can combine them to create a new segment *Low*.

If  $x_0(H) \leq x'(H)$ , then I can take all the remaining masses of  $(H, L)$  and  $(L, H)$  from  $x_{1H} + x_{2H}$  to *Low*. If the firm strictly prefers price  $L$  after this procedure, I take a positive mass of  $(H, H)$  from  $x_{1H} + x_{2H}$  to make the firm indifferent between prices  $L$  and  $H$ . This corresponds to [Table IX](#) with  $\gamma = 1$ .

In contrast, if  $x_0(H) > x'(H)$ , then, I take equal masses of  $(H, L)$  and  $(L, H)$  from  $x_{1H} + x_{2H}$  to *Low* up to the point that the firm is indifferent between prices  $L$  and  $H$ . This corresponds to [Table IX](#) with  $\gamma < 1$  and  $\beta = 0$ . □

## I Proof of [Lemma 4](#)

*Proof.* First, I show that there is an equilibrium in which each firm recommends product  $k$  conditional on message  $k$ . Because an identical argument applies to both pricing regimes, I consider discriminatory pricing. Take any equilibrium and consumer  $i \in I$  who has chosen a disclosure level  $\delta$ . Let CDF  $G$  denote the equilibrium price distribution of Firm  $B$ . Suppose that Firm  $B$  recommends product  $k$  with probability  $\beta$  conditional on message  $k$ . I calculate the payoff of Firm  $A$  conditional on the event that the consumer prefers a product 1.



Suppose that Firm  $A$ 's recommends product  $k$  with probability  $\alpha_k$  conditional on message  $k$ . Note that conditional on the consumer's preferring product 1, the probability that Firm  $B$  recommends each product does not depend on  $(\alpha_1, \alpha_2)$ , because Firms  $A$  and  $B$  receive stochastically independent messages. With probability  $b := \delta\beta + (1 - \delta)(1 - \beta)$ , Firm  $B$  recommends a product 1. In this case, the consumer buys product 1 from Firm  $A$  as long as his value is above  $p$  and Firm  $B$  sets a price above  $p$  (which occurs with probability  $[1 - G(p)]$ ), and Firm  $A$  recommends product 1. With the remaining probability  $1 - b$ , Firm  $B$  recommends product 2. In this case, the consumer buys product 1 from Firm  $A$  as long as his value is above  $p$  and Firm  $A$  recommends product 1, which happens with probability  $\delta\alpha_1 + (1 - \delta)(1 - \alpha_2)$ .

To sum up, Firm  $A$ 's payoff is

$$[\delta\alpha_1 + (1 - \delta)(1 - \alpha_2)] \cdot \{(1 - b)\Pi(p) + b[1 - G(p)]\Pi(p)\}.$$

Applying the same calculation for the event that the consumer prefers product 2, I obtain the payoff

$$[\delta\alpha_2 + (1 - \delta)(1 - \alpha_1)] \cdot \{(1 - b)\Pi(p) + b[1 - G(p)]\Pi(p)\}.$$

Thus, the ex-ante expected payoff is

$$\begin{aligned} \Pi_A = & \frac{1}{2} [\delta\alpha_1 + (1 - \delta)(1 - \alpha_2)] \cdot \{(1 - b)\Pi(p) + b[1 - G(p)]\Pi(p)\} \\ & + \frac{1}{2} [\delta\alpha_2 + (1 - \delta)(1 - \alpha_1)] \cdot \{(1 - b)\Pi(p) + b[1 - G(p)]\Pi(p)\}. \end{aligned}$$

This is maximized at  $\alpha_1 = \alpha_2 = 1$ .

Next, I show that  $(G_\delta^*, G_\delta^*)$  is an equilibrium given the above recommendation strategy. First, I show that  $(G_\delta^*, G_\delta^*)$  is an equilibrium under both pricing regime. First, at price  $p^M$ , consumer  $i$  purchase from a firm only if the other firm recommends the products with value zero. Thus, a firm obtains expected payoff of  $(1 - \delta)\Pi^M$  from price  $p^M$  conditional on the event that the firm recommends the product that the consumer values. The indifference condition of Firm  $j$  requires

that any price  $p \in [p^*(\delta), p^M]$  gives a payoff of  $(1 - \delta)\Pi^M$ .

The indifference condition is as follows:

$$(1 - \delta)\Pi(p) + \delta[1 - G_\delta^*(p)]\Pi(p) = (1 - \delta)\Pi^M, \quad \forall p \in [p^*(\delta), p^M].$$

The direct substitution shows that  $G_\delta^*$  satisfies these equations. Finally, any price  $p > p^M$  cannot be a profitable deviation because it gives a firm a payoff of  $(1 - \delta)\Pi(p) \leq (1 - \delta)\Pi^M$ . Any price  $p < p^*(\delta)$  cannot be a profitable deviation either, because a firm obtains a revenue strictly lower than  $(1 - \delta)\Pi^M$ . Thus,  $(G_\delta^*, G_\delta^*)$  consists of an equilibrium.

Next, I show the uniqueness of the equilibrium. Take any equilibrium price distributions  $(G_A, G_B)$ . First,  $G_A$  and  $G_B^*$  do not have a positive mass at the same price: otherwise, one firm can profitably undercut the price of the other. Second, let  $\Gamma_j$  denote the support of  $G_j$ . Define  $\bar{\Gamma}_j = \sup \Gamma_j$  for each  $j \in \{1, 2\}$ . Then,  $\bar{\Gamma}_1 = \bar{\Gamma}_2 = p^M$  must hold. Suppose to the contrary that at least one of  $\bar{\Gamma}_1 < p^M$  and  $\bar{\Gamma}_2 < p^M$  holds. First, suppose that  $\bar{\Gamma}_j > \bar{\Gamma}_{-j}$ . Then, Firm  $j$  uses  $p^M$  instead of prices in  $(\bar{\Gamma}_j, \bar{\Gamma}_{-j})$  in the equilibrium. Then, Firm  $-j$  prefers to set prices arbitrarily close to  $p^M$  instead of those around  $\bar{\Gamma}_{-j}$ . This contradicts  $\bar{\Gamma}_j > \bar{\Gamma}_{-j}$ . Second, suppose that  $\bar{\Gamma}_j = \bar{\Gamma}_{-j} < p^M$ . Then, if one firm, say  $j$ , does not have a mass at  $\bar{\Gamma}_j$ , then the other firm prefers to deviate and set  $p^M$ , which contradicts  $\bar{\Gamma}_{-j} < p^M$ .

Third,  $\underline{\Gamma}_1 := \inf \Gamma_1 = \inf \Gamma_2 =: \underline{\Gamma}_2$  hold. Otherwise, firm  $j$  with  $\underline{\Gamma}_j < \underline{\Gamma}_{-j}$  has no incentive to charge below  $\underline{\Gamma}_{-j}$ . Now,  $\underline{\Gamma}_1 \geq (1 - \delta)\Pi^M$  has to hold, because Firm  $B$  can always guarantee at least  $(1 - \delta)\Pi^M$  whenever it recommends correct products at price  $p^M$ . Thus,  $\underline{\Gamma}_2 \geq (1 - \delta)\Pi^M$ . This implies that Firm  $A$  can get a profit arbitrarily close to  $(1 - \delta)\Pi^M$ , which implies  $\underline{\Gamma}_1 \geq (1 - \delta)\Pi^M$ .

Fourth, I show  $\underline{\Gamma}_1 = \underline{\Gamma}_2 = (1 - \delta)\Pi^M$  for each  $j$ . Suppose to the contrary that  $\underline{\Gamma}_j > (1 - \delta)\Pi^M$  for  $j = 1, 2$ . Then, Firm  $B$  has to win by setting  $p^M$  when Firm  $A$  recommends correct products at the same price. Otherwise, Firm  $B$  obtains a strictly lower payoff from  $\underline{\Gamma}_2$  than from  $p^M$ , which contradicts that  $p^M$  is in the support of the equilibrium strategy. For Firm  $B$  to win with positive probability at price  $p^M$ ,  $G_A$  must have a mass at  $p^M$ . If  $G_A$  has a mass at  $p^M$ , Firm  $B$

undercuts  $p^M$  unless consumers accept the recommendations of Firm  $B$  with probability 1 when both firms recommend correct products at  $p^M$ . However, if Firm  $B$  wins for sure whenever both firms recommend correct products at price  $p^M$ , then Firm  $A$  obtains a payoff strictly lower than  $(1 - \delta)\Pi^M$ , which is a contradiction.

Next, I show that the supports of  $\Gamma_1$  and  $\Gamma_2$  are convex. Suppose to the contrary that  $\Gamma_j$  has a hole  $(a, b) \subset (\underline{\Gamma}_j, \bar{\Gamma}_j)$ . Then, Firm  $-j$  does not set prices in  $(a, b)$  either. Define  $p'_j := \inf \{p \in \Gamma_j : p \leq a\}$ . Firm  $j$  with  $p'_j > p'_{-j}$  has an incentive to set a price  $b - \varepsilon$  instead of setting prices around  $p'_j$ . If  $p'_j = p'_{-j}$ , then a firm who has no mass at this price has an incentive to deviate in the same way.

Finally, I show that neither  $G_A$  nor  $G_B$  has a probability mass. Indeed, if  $G_j$  has a mass at  $x$  in  $\Gamma_j$ , then Firm  $-j$  prefers to set a price  $x - \varepsilon$  instead of prices in  $(x, x + \varepsilon)$  for some  $\varepsilon > 0$ . This contradicts that  $\Gamma_1$  and  $\Gamma_2$  are convex.

To sum up,  $G_A$  and  $G_B$  are supported on  $[p^*(\delta), p^M]$ , and neither  $G_A$  nor  $G_B^*$  has a probability mass. The indifference conditions uniquely determine the price distributions.  $\square$

## J Proof of Proposition 6

*Proof.* Take any equilibrium under nondiscriminatory pricing. Suppose that the consumer chooses a disclosure level  $\delta$ . Then, each firm sets a price according to  $G_\delta^*$  in Lemma 4. Let  $p_A(\delta)$  and  $p_B(\delta)$  be independent random variables drawn from CDF  $G_\delta^*$ .

If the consumer unilaterally deviates to  $\delta_i$ , the expected payoff of the consumer  $X(\delta_i, \delta)$  is as follows.

$$X(\delta_i, \delta) := \mathbb{E} \left[ \delta_i^2 (u(1, \min \{p_A(\delta), p_B(\delta)\})) + 2\delta_i(1 - \delta_i)u(1, p_A(\delta)) \right] - c(\delta_i). \quad (5)$$

Here, the expectation is taken with respect to the value of a recommended product and the realizations of prices  $(p_A(\delta), p_B(\delta))$ . The first and the second terms correspond to the events that the consumer is recommended a product he prefers from both firms and only one firm, respectively.

I show that  $X(\delta_i, \delta)$  is strictly concave in  $\delta_i$ . Define  $X_1 := \frac{\partial X}{\partial \delta_i}$ . It holds that

$$\frac{X_1}{2} = \mathbb{E} [\delta_i (u(1, \min \{p_A(\delta), p_B(\delta)\})) + (1 - 2\delta_i)u(1, p_A(\delta))] - c'(\delta_i). \quad (6)$$

$X_1$  is decreasing in  $\delta_i$ . In particular,  $\mathbb{E} [\delta_i (u(1, \min \{p_A(\delta), p_B(\delta)\})) + (1 - 2\delta_i)u(1, p_A(\delta))]$  is decreasing in  $\delta_i$  because differentiating it in  $\delta_i$  gives

$$\mathbb{E} [(u(1, \min \{p_1(\delta), p_B(\delta)\})) - 2u(1, p_A(\delta))] \leq 0,$$

where the inequality follows from

$$\begin{aligned} & \mathbb{E} [(u(1, \min \{p_A(\delta), p_B(\delta)\})) - 2u(1, p_A(\delta))] \\ &= \mathbb{E} [\max \{v - \min \{p_A(\delta), p_B(\delta)\}, 0\} - \max \{v - p_A(\delta), 0\} - \max \{v - p_B(\delta), 0\}] \leq 0. \end{aligned}$$

For each  $\delta$ , define the correspondence  $BR(\delta) := \arg \max_{\delta_i \in [1/2, 1]} X(\delta_i, \delta) - c(\delta_i)$ .  $\delta^*$  consists of a symmetric equilibrium if and only if  $\delta^* \in BR(\delta^*)$ . The compactness of  $[1/2, 1]$  and the continuity of  $X(\delta_i, \delta)$  in  $(\delta_i, \delta)$  guarantees that  $BR_i(\cdot)$  is non-empty valued and has a closed graph. Because the consumer's payoff is concave in  $\delta_i$ ,  $BR(\cdot)$  is convex-valued. By Kakutani's fixed point theorem, there exists  $\delta^*$  such that  $\delta^* \in BR(\delta^*)$ . Now,  $1 \notin BR_i(1)$ , because  $X_1(1, 1) - c'(1) = -c'(1) < 0$ . Thus, there is no equilibrium in which the consumer choose disclosure level 1. Also,  $1/2 \notin BR_i(1/2)$ , because  $X_1(1/2, 1/2) - c'(1/2) \geq X_1(1/2, 1/2) > 0$ .

Take any equilibrium disclosure level  $\delta^*$  under nondiscriminatory pricing. By the previous argument,  $1/2 < \delta^* < 1$ . Because  $X_1(\delta^*, \delta^*) = c'(\delta^*)$ ,  $X_1(\delta^*, \delta^*) + X_2(\delta^*, \delta^*) > c'(\delta^*)$ . Thus, there is  $\varepsilon > 0$  such that, if the consumer pre-commit to  $\delta^* + \varepsilon$ , he obtains a strictly greater payoff. This is equivalent to saying that the consumer obtains a strictly greater payoff if he was in discriminatory pricing and chooses  $\delta^* + \varepsilon$ .  $\square$

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Table I: Disclosure policy  $\phi$  revealing ordinal ranking

	$\phi(1 u_1, u_2)$	$\phi(2 u_1, u_2)$
(2, 2)	1/2	1/2
(2, 1)	1	0
(1, 2)	0	1
(1, 1)	1/2	1/2



Table II: Efficient disclosure policy  $\phi^G$ .

	$\phi^G(11 u_1, u_2)$	$\phi^G(12 u_1, u_2)$	$\phi^G(21 u_1, u_2)$	$\phi^G(22 u_1, u_2)$
(2, 2)	0	1/2	0	1/2
(2, 1)	1/4	3/4	0	0
(1, 2)	0	0	1/4	3/4
(1, 1)	1/2	0	1/2	0

Table III: Horizontally inefficient disclosure policy  $\phi^I$ .

	$\phi^G(11 u_1, u_2)$	$\phi^G(12 u_1, u_2)$	$\phi^G(21 u_1, u_2)$	$\phi^G(22 u_1, u_2)$
(2, 2)	0	1/2	$\varepsilon'$	1/2 - $\varepsilon'$
(2, 1)	1/4	3/4 - $\varepsilon$	$\varepsilon$	0
(1, 2)	0	0	1/4	3/4
(1, 1)	1/2	0	1/2	0

Table IV: Asymmetric equilibrium

	$\phi^A(1 u_1, u_2)$	$\phi^A(2 u_1, u_2)$
$(H, H)$	1	0
$(H, L)$	1	0
$(L, H)$	0	1
$(L, L)$	1	0

Table V: Symmetric equilibrium

	$\phi^S(1 u_1, u_2)$	$\phi^S(2 u_1, u_2)$
$(H, H)$	1/2	1/2
$(H, L)$	1	0
$(L, H)$	0	1
$(L, L)$	1/2	1/2

Table VI: Equilibrium disclosure for  $x^{MAX}(H) \leq x^{OPT}(H)$ .

	$\phi^*(1 u_1, u_2)$	$\phi^*(2 u_1, u_2)$
$(H, H)$	$1/2$	$1/2$
$(H, L)$	$\alpha$	$1 - \alpha$
$(L, H)$	$1 - \alpha$	$\alpha$
$(L, L)$	$1/2$	$1/2$

Table VII: Nondiscriminatory pricing reduces welfare

	$v = 2$	$v = 3$
$x_0$	$1/2$	$1/2$
$f_1$	$1/4$	$3/4$

Table VIII: Nondiscriminatory pricing increases welfare

	$v = 1$	$v = 2$	$v = 3$
$x^*$	$1/4$	$1/4$	$1/2$
$f_1$	$1/16$	$3/16$	$3/4$

Table IX: Equilibrium disclosure for  $x_H^* > x^{OPT}(H)$ .

	$\phi^*(Low u_1, u_2)$	$\phi^*(High u_1, u_2)$
$(H, H)$	$\beta$	$1 - \beta$
$(H, L)$	$\gamma$	$1 - \gamma$
$(L, H)$	$\gamma$	$1 - \gamma$
$(L, L)$	1	0



Figure I: Timing of moves under each pricing regime

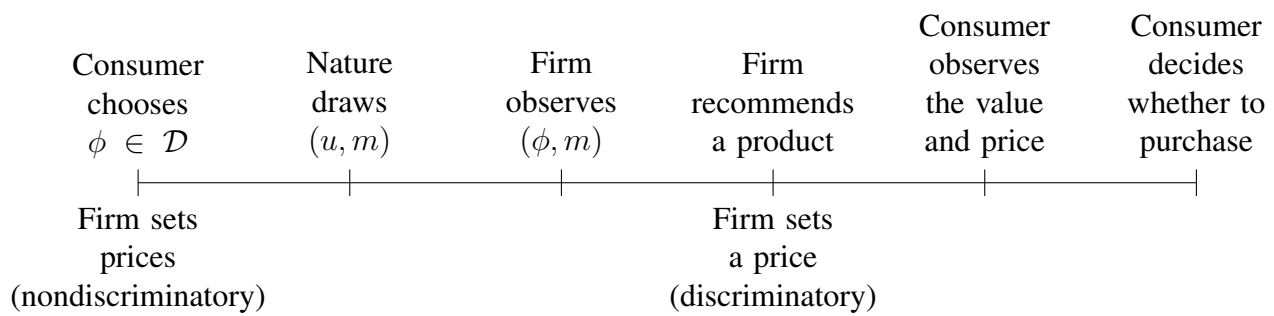


Figure II: Disclosure policy for  $\delta \in [1/2, 1]$

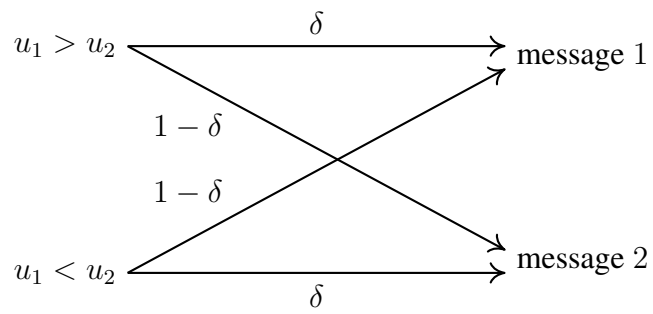


Figure III: Timing of Moves: Competition

