

COMPRESSION AUCTIONS WITH
AN APPLICATION TO LIBOR-SOFR
SWAP CONVERSION

Darrell Duffie
Graduate School of Business, Stanford University

September, 2018
Working Paper No. 18-036

Compression Auctions

With an Application to LIBOR-SOFR Swap Conversion

Darrell Duffie*

Graduate School of Business, Stanford University

September 10, 2018

This note explains a new type of auction based on an existing derivatives risk-management technique known as “compression.” A compression auction can be used to convert centrally cleared contracts on an underlying benchmark, such as the London Interbank Offered Rate (LIBOR), to contracts on a different underlying benchmark, such as the Secured Overnight Financing Rate (SOFR).¹ I first proposed² compression-auctions for this purpose in October, 2017. This note adds some details,

*I am a Research Associate of the National Bureau of Economic Research and chaired the Market Participants Group on Reference Rate Reform for the Financial Stability Board. I am grateful for research assistance from David Yang and for conversations with Leif Andersen, Sam Antill, Imane Bakkar, Martin Bardenhewer, Ann Battle, Roman Baumann, Michael Cloherty, Lou Crandall, Sunil Cutinho, Mehtap Dinc, Piotr Dworzak, Martin Engblom, Joshua Frost, Jason Granet, Basil Guggenheim, Matthias Jüttner, Antoine Lallour, Dennis McLaughlin, Paul Milgrom, Agha Mirza, Dewet Moser, Henrik Nilsson, Romans Pans, Zoltan Pozsar, Raf Pritchard, Alex Roeber, Benedict Roth, Per Sjöberg, Sayee Srinivasan, Jeremy Stein, Larry Stromfeld, Fred Sturm, Reiko Tokukatsu, Sean Tully, Laura Yannuzzi, Josh Younger, Anthony Zhang, Haoxiang Zhu, and Marcel Zimmerman.

¹The selection of SOFR as the new benchmark reference rate for USD wholesale lending markets, and the process of converting the market from LIBOR to SOFR, has been managed by the Alternative Reference Rate Committee (ARRC). The [web site of the ARRC](#) is maintained by the Federal Reserve Bank of New York, and provides updates and details. SOFR is a broad measure of the cost of borrowing cash overnight collateralized by Treasury securities. The new CHF reference rate is SARON, the new GBP rate is SONIA, and the new JPY reference rate is TONA. See the [IBOR Global Benchmark Transition Report](#), issued jointly by International Swaps and Derivatives Association, Inc. (ISDA), the Association for Financial Markets in Europe (AFME), the International Capital Market Association (ICMA), the Securities Industry and Financial Markets Association (SIFMA) and SIFMA's Asset Management Group (SIFMA AMG) (Trade Associations), June 2018. The two most important euro benchmarks, EURIBOR and EONIA, are likely to be replaced by a [new overnight unsecured euro rate](#) to be produced by the European Central Bank.

² “[Converting LIBOR Contracts to New Benchmarks: An Auction-and-Protocol Approach](#),” GRI and IEO Financial Engineering Practitioners Seminar, October 24, 2017, Bloomberg, New York.

while still falling far short of a complete design.

The UK Financial Conduct Authority [announced in 2017](#) that it will end its supervision of LIBOR at the end of 2021.³ At that point, banks providing reports that determine LIBOR each day will be under less pressure to continue doing so. Banks are exposed to litigation and reputational risk when providing this information. It is far from assured that LIBOR will be available after 2021.

Because LIBOR may be disappearing, many market participants are likely to desire to convert their legacy LIBOR-linked contracts so that they become contractually linked instead to new benchmark reference rates. This may alleviate much of the costs and risks associated with a weakening or disappearance of LIBOR benchmarks. The liquidity costs associated with conversion will become more significant with eventual reductions in the depth of LIBOR-based markets.

Because, in each currency, LIBOR is a higher interest rate than the corresponding new reference rate, the receiver of LIBOR on a legacy contract will require compensation in order to agree to the conversion. This note addresses the determination of a fair rate of compensation, maturity by maturity, and how to obtain the agreement of contract holders. It will be some time before the normal ongoing over-the-counter market for long-term derivatives on the new reference rate (or basis swaps) is sufficiently active or deep to provide benchmark compensation rates for converting swaps. The long-term swap rates that could be estimated from daily transactions of new-rate derivatives may be noisy and exposed to manipulation. In the meantime, it may be better to have periodic auctions, for example monthly or quarterly, that improve the depth of the conversion market by focusing bids and offers at designated points in time. Similarly, many stock exchanges⁴ run an auction at the beginning of each trading day in order to set an opening price, because there is greater uncertainty at the open about the fair price. An auction concentrates market depth into a point in time.

In [an earlier technical note](#), I proposed an auction-and-protocol approach to conversion. An auction would be used to determine the fixed compensation rate r that will be given by LIBOR payers to LIBOR receivers, when converting their contracts from LIBOR to the new rate. For example, a 5-year swap promising 3-month LIBOR against some fixed rate F would be converted to a swap promising the new floating rate in exchange for $F + r$. Non-bidding market participants who have

³The Chief Executive of the FCA, Andrew Bailey, provided an [update](#) in July, 2018.

⁴For example, see [NYSE Arca Auctions](#), New York Stock Exchange Arca, 2014. According to NYSE Arca, “Our limit order opening auctions allows participants to participate in real-time price discovery and matches orders at the price that maximizes the amount of tradable stock.”

previously signed a protocol would convert their LIBOR contracts to the new reference rate at the auction-determined compensation rate.⁵

1 Floating rates for coupon periods longer than one day

When using a new overnight benchmark rate such as SOFR to settle floating-rate coupon payments at some longer tenor such as three months, one could in principle use the associated three-month overnight index swap (OIS) rate S . For a term of 90 days, for example, an OIS contract commits the fixed-side counterparty to pay $(S - R) \times 90/360$, per dollar of notional position, where R is the compounded overnight rate, defined by

$$1 + R \times \frac{90}{360} = \left(1 + \frac{r_1}{360}\right) \times \left(1 + \frac{r_2}{360}\right) \times \cdots \times \left(1 + \frac{r_{90}}{360}\right),$$

where r_n is the overnight benchmark rate on the n -th of the 90 days during the contract period. Standard adjustments apply to weekends and holidays.⁶ Thus, at the beginning of a quarter, the market value of receiving the OIS rate at the end of quarter is equal to the market value of receiving the compounded overnight rate R at the end of the quarter.

Unfortunately, there may not be enough depth in the three-month OIS market to fix S robustly from OIS transactions. If the OIS market is too thin, the fixing of S could be contaminated by noise, or even manipulated. This same concern is likely to apply for other typical contract periods, such as one month and six months.

An alternative is to settle coupon payments with the compounded daily rate R . For SOFR, the compounded daily rate is known as the Secured Average Financing Rate (SAFR), even though it is not literally an “average.” Figure 1 compares three-month LIBOR and estimates of three-month SAFR over the period 2014-2018. Although SAFR was not published until mid-2018, the Federal Reserve Bank of New York has provided estimates of what SAFR would have been during this period.⁷

⁵ For centrally cleared swaps, the protocol step is effectively a dark pool. The heavy side of the market is rationed pro rata, so that there is a match between the total volumes of payer and receiver swaps converted. The amount matched can be increased with the use of compression, which allows cross-maturity substitution.

⁶The rate applied over weekends and holidays is based on the standard money-market (actual/360) formula.

⁷The Federal Reserve Bank of New York has also provided a https://www.newyorkfed.org/markets/policy/operating_policy_180309 longer time series of average overnight repo rates going back to 1998, the volume-weighted mean rate of the primary dealers overnight Treasury general collateral repo borrowing activity.

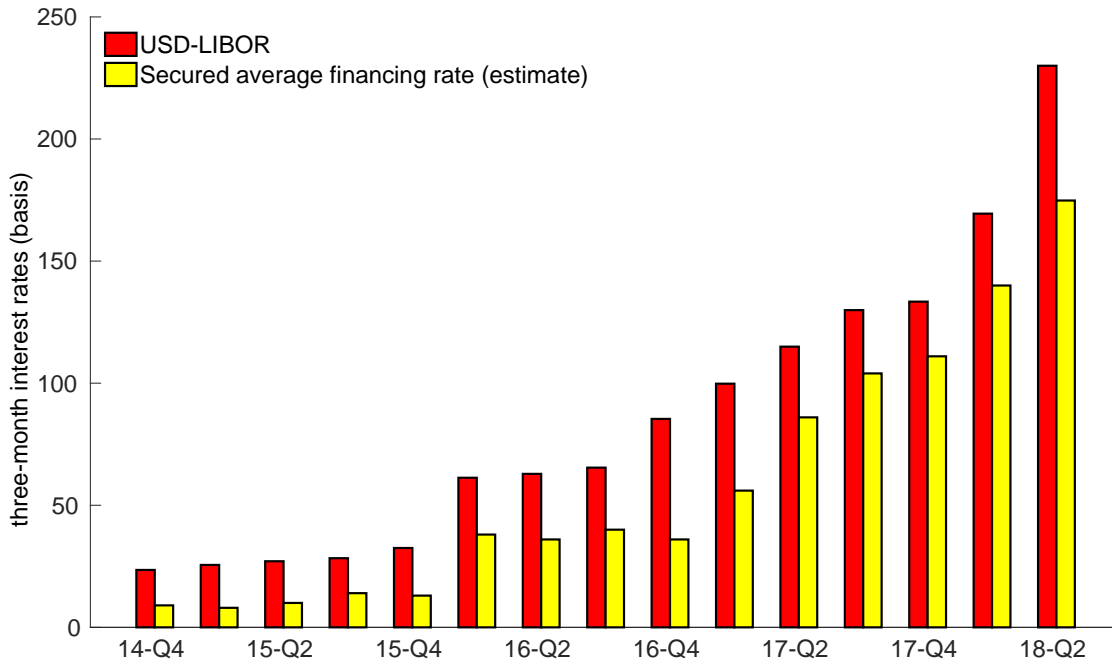


Figure 1: Three-month LIBOR and estimated three-month secured average financing rate (SAFR), which is the rate for a given quarter implied by compounding SOFR for all days of that quarter. The underlying SOFR rates (or estimated SOFR rates, for the period preceding official SOFR publication) were obtained from the web site of the Federal Reserve Bank of New York. The mean difference between LIBOR and SAFR for the illustrated period is 27.6 basis points. The sample standard deviation of the differences is 12.1 basis points.

The mean difference between three-month LIBOR and estimated three-month SAFR for the illustrated period is 27.6 basis points. The standard deviation of the difference in these two rates is 12.1 basis points, representing a substantial amount of fluctuation over time in the compensation one should expect for converting a LIBOR swap to a SAFR swap. The auction approach that I will describe can be used to convert swaps on any underlying index to swaps on any other underlying index, and does not depend on using SAFR as the new floating rate.

2 Why focus on centrally cleared swaps?

Figure 2 illustrates central clearing in a swap market with participation by major dealers who are the clearing members of a central counterparty (CCP), and by the clients of dealers who participate in clearing through dealers. Centrally cleared swaps are ideal for the purpose of conversion because they are fungible across all firms participating in the CCP, thus avoiding the need to obtain pair-wise

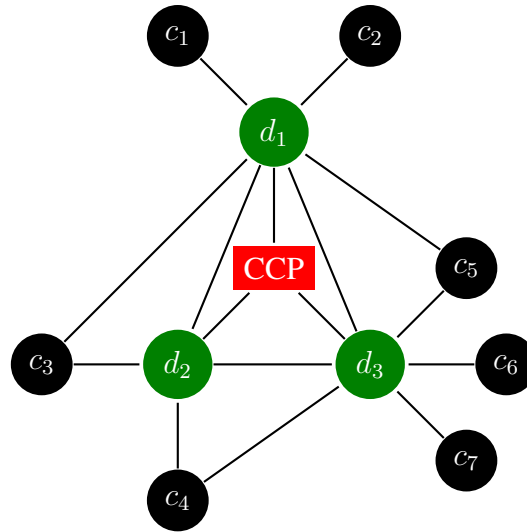


Figure 2: Schematic of a centrally cleared swap market. Dealers and other clearing members, marked in green, novate their trades to a central counterparty (CCP), which becomes the buyer to each seller and the seller to each buyer. Centrally cleared contracts of a given type with different clearing members are fungible, in that they are all obligations of the same type to the same counterparty, the CCP. This fungibility allows submissions of contracts from many long and short position holders to be netted, which is not possible with bilateral contracts.

conversion agreements. Swaps that are “bilateral,” meaning not centrally cleared, can be converted only with the agreement of both counterparties. For example, if I wish to convert my swap but my bilateral counterparty does not wish to convert at any compensation rate that I am willing to accept, then my swap will remain unconverted.

3 How does a conversion auction work?

We first consider an auction for the conversion of legacy LIBOR swaps of a given maturity, say 10 years, to new-rate swaps. Later, we explore the advantage of using a single auction for simultaneously converting swaps of multiple different maturities.

In the most basic form of a conversion auction, a *bid* is a pair (r, q) consisting of the maximum compensation rate r the bidder is willing to pay (annualized over the life of the contract) to convert legacy pay-LIBOR contracts of up to the notional quantity q to a new contract that pays the new rate NR. Conversely, an *offer* (r, q) specifies the minimum compensation rate r that would be accepted to convert up to q units (notional) of legacy receive-LIBOR contracts into new contracts that will receive the new rate, NR. If a bid or offer is accepted, any converted contract will pay, in lieu of LIBOR, interest payments at the rate $NR + r^*$, where r^* is the rate at which the total quantity of bids to pay

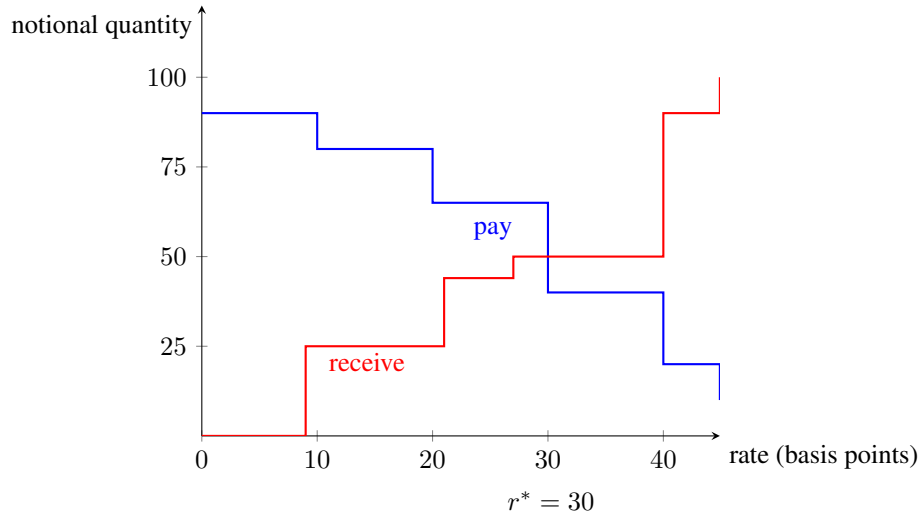


Figure 3: The total demand and supply schedules constructed from bids and offers. The market-clearing compensation rate is $r^* = 30$ basis points.

compensation at rates at or above r^* is equal to the quantity of offers to receive compensation at rates no higher than r^* . As usual, market clearing can be assured by pro-rata allocations of quantities bid at r^* .

Figure 3 illustrates the total demand and supply schedules that were realized in a hypothetical auction. For example, as illustrated, a firm with legacy swaps that receive LIBOR was willing to convert up to $q = \$19$ billion notional if compensated at a rate of at least $r = 21$ basis points (bps) running. This firm will be awarded all \$19 billion and be compensated at the auction clearing rate $r^* = 30$ bps. Another firm with legacy swaps that pay LIBOR, was willing to convert up to $q = \$15$ billion notional if required to pay in compensation no more than $r = 20$ basis points running. None of this bid was executed, because $20 \text{ bps} < r^*$.

An auction participant would be permitted to place as many different such (r, q) limit orders as desired. Many sorts of rules could be imposed on the auction design. For example, an auction participant could be permitted to place both buy and sell orders, subject to not expanding that bidder's total absolute amount of LIBOR swaps.

4 What is compression?

Compression is a family of methodologies that reduce the gross notional amounts of the swaps of participating firms while maintaining approximately the same market exposures for each firm. Com-

pression benefits participating firms by reducing their counterparty risk, and thus their collateral and capital requirements. Well over one quadrillion USD notional equivalent of interest rate swap counterparty exposures have been eliminated using compression methodologies supplied by just one of the available technology providers, [TriOptima](#).

Compression can be applied to the non-cleared bilateral swap positions of multiple pairs of participating firms, or alternatively can be applied to swaps that have been centrally cleared at a particular CCP. We focus on the latter case, called “multilateral compression.”

Figure 4 provides a highly simplified illustration of how a firm can reduce the gross notional amount of its positions at a given CCP by compressing its receiver and payer positions into a “smaller” portfolio of new swap positions that has the original total duration-adjusted exposure to interest rate changes. In practice, a participating firm can submit into a single compression event any of its positions from across the entire range of maturities handled by the CCP, while imposing “risk tolerances” on the extents to which its positions can be substituted with positions at other maturities.⁸ A participating firm also receives or pays monetary compensation associated with compression-induced changes in its positions. Each participant specifies minimum monetary compensation tolerances by, in effect, submitting either a bid swap rate or an offered swap rate, maturity by maturity. In effect, then, a compression is a form of multidimensional auction.

In the illustrated example, the firms’s original payer swaps (pay fixed, receive LIBOR) have magnitudes (adjusted to a 10-year duration equivalent) of 50, 40, and 20 billion USD at maturities of 8, 9, and 10 years, respectively. The receiver swaps have duration-adjusted notional sizes of 20 billion USD at maturities of 8.5 years and 9.5 years. Compression retains the original net duration-adjusted position of \$70 billion while reducing the gross total position from \$110 billion to \$70 billion.

The compression manager uses a mathematically sophisticated algorithm to search for the adjustments to the swap positions of all participating firms that minimizes the post-compression aggregate total gross notional positions of all the firms, subject to meeting certain constraints. First, the CCP must maintain a net position of zero at each maturity. So, in the illustrated example, the compression algorithm found a list of adjustments to the compressible positions of the other clearing members that total, in billions, -30 at 8 years, $+20$ at 8.5 years, -20 at 9 years, $+20$ at 9.5 years, and $+10$ at 10 years. Second, for each participating firm, the adjustments must reduce that firm’s gross notional

⁸See, for instance, “[Interest Rate Swaps Compression: A Progress Report](#),” ISDA, February, 2012.

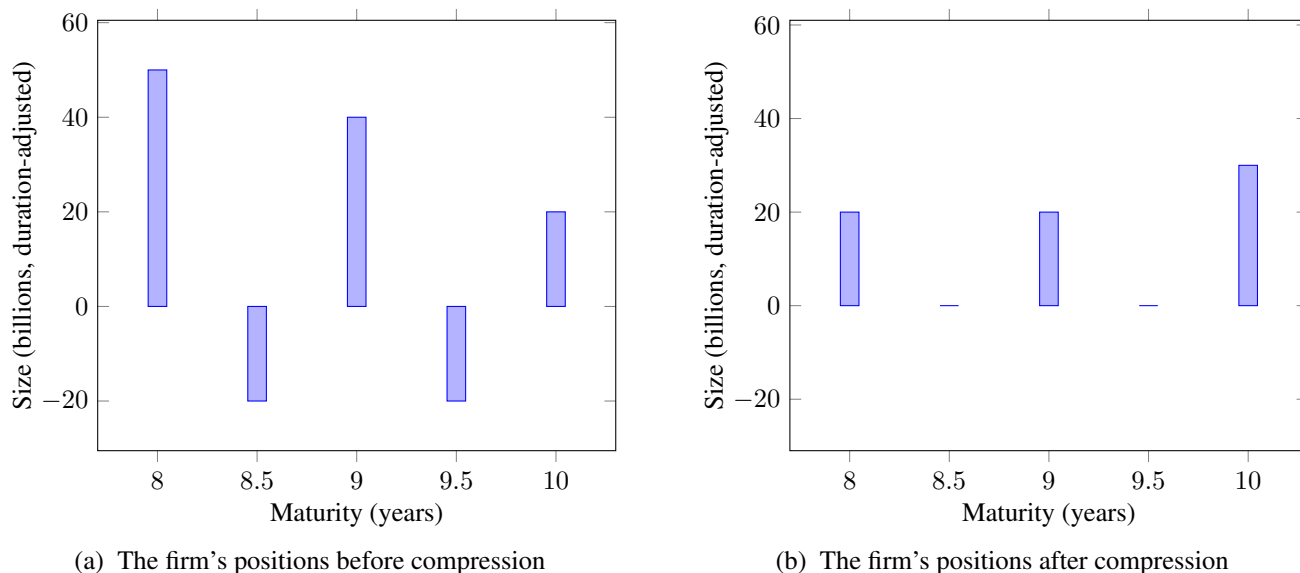


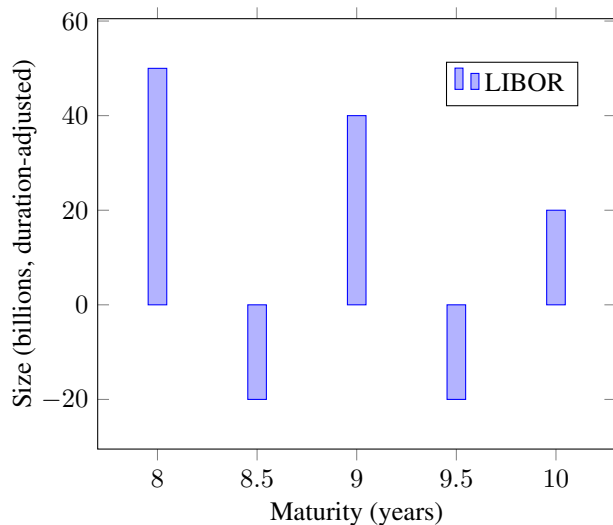
Figure 4: Figure 4(a) shows the duration-adjusted payer (pay fixed, receive LIBOR) positions of a given clearing member before compression, which total to a net long (payer) position of 70 billion USD notional equivalent. The clearing member submits all of these positions for compression. In the hypothetical outcome shown in Figure 4(b) for a compression event, the firm retains its net long position of 70 billion USD notional equivalent. The CCP must keep a net position of zero at each maturity, so has found a list of adjustments to the compressible positions of the other clearing members that retain the original duration-adjusted net positions of each the other clearing members, while changing the total positions of the other clearing members by -30 at 8 years, $+20$ at 8.5 years, -20 at 9 years, $+20$ at 9.5 years, and $+10$ at 10 years.

position and also satisfy the risk tolerances and compensation requirements of the firm.

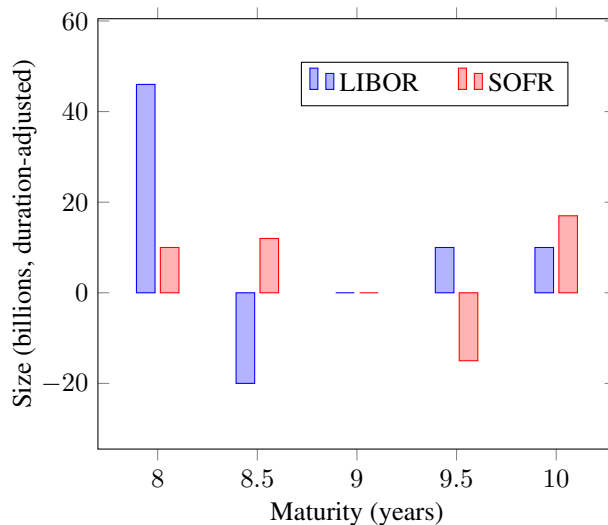
In the next section, we will see how to extend the compression approach into a new form of auction for converting legacy LIBOR contracts into new-rate contracts.

5 Compression auctions

When converting LIBOR swaps to new-rate swaps, running a separate auction for each maturity is not efficient because of the associated loss in potential netting of long and short positions across different maturities. For example, suppose that the demand to convert 9.5-year LIBOR receiver swaps is high relative to that for 9.5-year LIBOR payer swaps, whereas the demand to convert 10-year LIBOR receiver swaps is low relative to that for 10-year LIBOR payer swaps. In this case, the total amount of conversion obtained in two separate auctions would be much smaller than that for an auction in which there is substitutability between 9.5-year and 10-year swaps. To obtain substitutability, the CCP could adapt an existing form of cross-maturity multilateral compression, allowing a significant increase in



(a) The firm's swap positions before the auction.



(b) The firm's swap positions after the auction.

Figure 5: Figure 5(a) shows the duration-adjusted legacy payer (pay-fixed, receive LIBOR) swap positions comprising one of the packages of swaps a firm submitted for conversion to new-rate swaps. At the respective maturities, these 10-year-duration-adjusted LIBOR payer positions are of sizes 50, -20, 40, 20, and 20, in USD billions. (A negative payer swap is a receiver swap.) In a hypothetical outcome for a compression auction, the submitted package of LIBOR swaps is converted to the new package of LIBOR and SAFR swaps shown in Figure 5(b). The new LIBOR positions at the respective maturities have duration-adjusted notional sizes of 46, -20, 0, 10, and 10 billion USD. The new SAFR positions at the respective maturities have duration-adjusted notional sizes of 10, 12, 0, -15, and 17 billion USD.

conversion volumes.

In this section, I outline one possible design for compression auctions, leaving out some details. My objective is to illustrate some of the key concepts. Practitioners would likely choose some variation of this design.

Figure 5 illustrates a simple example of a given firm's participation in a compression auction. The firm could be a clearing member of the CCP, or could be another firm whose swaps are cleared for it by a clearing member. Figure 5(a) shows the duration-adjusted legacy payer (pay-fixed, receive LIBOR) swap positions comprising one of the packages of swaps submitted by this firm for potential conversion to SAFR swaps. The same firm is permitted to submit additional packages of swaps for conversion, with different compensation requirements, just as a bidder in a treasury securities auction may submit multiple bids at different yields for the same treasury note. At the respective maturities, the submitted 10-year-duration-adjusted LIBOR payer positions are of sizes 50, -20, 40, -20, and 20, in USD billions. (I treat a receiver position as a negative payer position.)

Table 1: Hypothetical bid and offer swap rates of given firm in basis points, for both LIBOR and SAFR swaps. For example, the firm is willing to enter new 8-year receive-fixed pay-LIBOR swaps at a swap rate no lower than 310.1 basis points and is willing to enter new 8-year pay-fixed receive-LIBOR swaps at a swap rate no higher than 310.0 basis points. More generally, the firm is willing to enter any combination of new LIBOR and SAFR swaps that satisfies the firm’s stipulated risk tolerances and is consistent with the tabulated bid and offer rates.

	LIBOR offer	LIBOR bid	SAFR offer	SAFR bid
8 years	310.1	310.0	290.1	289.9
8.5 years	312.5	312.3	292.2	292.0
9 years	315.4	315.2	295.5	295.3
9.5 years	324.3	324.1	305.0	304.8
10 years	330.1	329.9	311.8	311.6

Table 1 shows the firm’s required rates for converting any package of LIBOR and SOFR swaps to any other package of LIBOR and SOFR swaps that meet the firm’s risk tolerances. For instance, the firm is willing to add 8-year LIBOR receiver swaps at any swap rate higher than 310.1 basis points and to add 8-year LIBOR payer swaps at any swap rate lower than 310.0 basis points. Similarly, the firm is willing to add 8-year SAFR receiver swaps at any swap rate higher than 290.1 basis points and is willing to add 8-year SAFR payer swaps at any rate lower than 289.9 basis points. This implies that if the compression auction merely converts this firm’s 8-year LIBOR payers into 8-year SAFR payers, with no other changes in this firm’s portfolio, then the firm must receive a net compensation rate of at least $310.1 - 290.1 = 20$ basis points. The compensation could be received in some combination of basis points running and up-front payment that has the implied total present market value. (I am not aware of how this split would likely work in practice.) This firm might alternatively be allocated a reduction of 8-year LIBOR payer swaps in exchange for 9-year SAFR swaps, or could be allocated any other combination of LIBOR and SAFR swaps that reduces its gross notional outstanding LIBOR swap position, duration-adjusted, while meeting its risk tolerances and compensation tolerances.

In this example, the firm retains its original duration-adjusted net long position of 70 billion USD notional. The compression algorithm found offsetting adjustments to the compressible positions of the other firms that retain the original duration-adjusted net positions of each of the other firms (within their stipulated risk and compensation tolerances). For the illustrated case, the total of the offsetting changes in the positions of other firm’s LIBOR payer swap positions at the respective maturities are $-4, 0, -40, +30,$ and -10 billion USD, duration adjusted. The total of the offsetting changes in the

SAFR payer swap positions of the other firms at the respective maturities are -10 , -12 , 0 , $+15$, and -17 billion USD, duration adjusted.

In addition to respecting the risk tolerances and compensation requirements stipulated by each participating firm, the total of the notional sizes of each of the type of swaps allocated to the participants must sum to zero, so that the CCP maintains a balanced swap position.

For a uniform-price auction, the compression algorithm will choose a unique fixed rate for each type of swap (LIBOR or SAFR, maturity by maturity), and assign this same fixed rate to all of the newly allocated swaps. With a uniform-price auction, the cash flows of the CCP are automatically balanced.

For the alternative of a discriminatory-price auction, different firms may be assigned different swap rates for the same type of swap, subject to satisfying their bids and offers. In this case, the compression algorithm must choose swap rates and notional sizes for each firm's new swaps with the property that, for each type of swap, the size-weighted average rate of the added payer swaps is equal to the size-weighted average rate of the added receiver swaps, possibly after compensating up-front payments that sum to zero across firms. With this restriction, the CCP again has balanced cash flows.

The most important difference between uniform-price and discriminatory-price auction designs is the incentive that is created for firms to bid aggressively. While there has been some related research,⁹ I am not aware of any analysis of this multi-dimensional setting.

Additional requirements on the allocation may be imposed by individual firms. For example, an individual firm may or may not require that the magnitude of its total gross notional LIBOR position is reduced. An individual firm whose total gross LIBOR position is increased by a compression event could, in essence, be making a market in LIBOR conversion, facilitating a reduction in the aggregate gross outstanding amount of LIBOR swaps that might not otherwise occur.

A key overall objective is a reduction in the duration-adjusted total gross notional amount of LIBOR swaps outstanding. This reduction is some amount that we denote by X . Simply maximizing X might increase the aggregate post-compression gross notional amount of SAFR swaps outstanding by some amount Y that is inefficiently large. In the example shown in Figure 5, the submitted package of \$110 billion gross notional of LIBOR swaps was converted into a package of \$86 billion gross

⁹See for example “[Demand Reduction and Inefficiency in Multi-Unit Auctions](#)” by Lawrence Ausubel, Peter Cramton, Marek Pycia, Marzena Rostek, and Marek Weretka, *The Review of Economic Studies*, Volume 81 (2014), pages 1366-1400.

notional of LIBOR swaps and \$54 billion gross notional of SAFR swaps. There is a tradeoff between (i) the objective of reducing the aggregate gross amount LIBOR positions, because of the weak quality of LIBOR as a benchmark; and (ii) the objective of reducing the total of the aggregate gross amounts of swaps referencing LIBOR and SAFR, because swap positions of any type involve costs associated with counterparty risk, collateral requirements, and capital requirements.

In practice, the compression manager might maximize $X - cY$, for some tradeoff coefficient c between 0 and 1. Some experimentation with c may result in a reasonable tradeoff. Other tradeoff approaches could be considered. A smaller choice for c implies that gross LIBOR positions are viewed as less desirable, in light of the risk of a potential discontinuation or weakening of LIBOR as a benchmark. The discontinuation of LIBOR could lead to costly contract frustration¹⁰ or reliance on undesirable fallbacks.¹¹ A weakening of LIBOR as a benchmark could lead to lower market depth and thus higher transactions costs.

At the earlier stages of market transition from LIBOR to SAFR, one should expect relatively poor efficiency in terms of the aggregate gross outstanding notional amounts of swaps. That is, the total of the gross amounts of LIBOR and SAFR swaps outstanding is likely to go up more rapidly when there is relatively less opportunity to net the newly created SAFR swaps against previously outstanding SAFR swaps. This is the situation reflected in Figure 5. This may cause some increases in margin and capital requirements, which perhaps can be mitigated by some rule changes that allow significant recognition of netting between LIBOR and SAFR swaps.

Once SAFR swap markets develop and market participants have built non-trivial outstanding SAFR swap positions, efficiency will improve. Compression participants will likely submit for compression more and more of their outstanding LIBOR and SAFR swaps at a given CCP. As the liquidity improves in the market for SAFR swaps, conversion efficiency will also improve through a narrowing of the bid-offer spreads submitted by compression participants.

As a part of their preparation for the upcoming transition from IBOR to new reference rates, market participants and infrastructure providers may now wish to begin designing and testing alternative compression-auction methods.

¹⁰See Chapter 6, “Legal Analysis,” of [“Market Participants Group on Reforming Interest Rate Benchmarks, Final Report,”](#) Financial Stability Board, March 2014.

¹¹See [ISDA Publishes Consultation on Benchmark Fallbacks](#), ISDA, March, 2018.