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DETERRENCE AND THE ADJUSTMENT OF SENTENCES DURING IMPRISONMENT

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Abstract: The prison time actually served by a convicted criminal depends to a significant degree on decisions made by the state during the course of imprisonment—on whether to grant parole or other forms of sentence reduction. In this article we study a model of the adjustment of sentences assuming that the state's objective is the optimal deterrence of crime. In the model, the state can lower or raise the sentence based on deterrence-relevant information that it obtains about a criminal during imprisonment. Our focus on sentence adjustment as a means of promoting deterrence stands in contrast to the usual emphasis in sentence adjustment policy on reducing recidivism.

Key words: Deterrence; imprisonment; sentence adjustment; sanctions; parole; recidivism

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1. Introduction

When an individual is convicted of a crime, the court's sentencing decision will not fully determine the length of his imprisonment. The time actually served by an offender will depend to a significant degree on decisions made by the state after he is incarcerated. Notably, prisoners are often granted parole[1] and can benefit from reductions of sentences through good time and earned time credits provided that they meet certain standards.[2] As a result of these adjustments, the sentences of individuals held in state prisons are estimated to be lowered by 49 percent.[3]

The contribution of this article is to study the general practice of altering sentences during imprisonment within a model of deterrence. Specifically, we examine how information about prisoners obtained during imprisonment can be employed to modify sentences so as to optimally deter crime.[4] Such information could concern, for example, whether the prisoner has a history of

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[1] In 2000, 24 percent of releases from state prisons were the result of discretionary decisions by parole boards. See "Reentry Trends in the U.S. Releases from State Prison," Bureau of Justice Statistics, Office of Justice Programs, U.S. Department of Justice (available at <https://www.bjs.gov/content/reentry/reentry_contents. cfm>; page last revised January 13, 2019). The year 2000 is the latest year discussed in this report.

[2] See generally National Conference of State Legislatures, "Good Time and Earned Time Policies for State Prison Inmates," updated January 2016 (available at <https://docs.legis.wisconsin.gov/misc/lc/study/ 2016/1495/030_august_31_2016_meeting_10_00_a_m_room_412_east_state_capitol/memono4g>). Although there is some overlap between good time and earned time, good time credits are typically awarded to a prisoner for obeying prison rules, while earned time credits are usually conferred for satisfactory participation in a self-improvement activity, such as vocational training or a drug treatment program. Whether a prisoner receives good time credits depends on an assessment of his behavior by a state agent while incarcerated. Likewise, whether a prisoner is granted earned time credits often depends on an evaluation of his involvement in a self-improvement program.

[3] See Durose and Langan (2004, p. 5, Table 4). This article from the Bureau of Justice Statistics reports the percentage of mean sentences served by individuals in state prisons who have been convicted of felonies (there is no subsequent Bureau of Justice Statistics publication that provides the percentage). Parole is currently unavailable to Federal prisoners, who constitute 13 percent of all prisoners, though they can benefit from good time credits, which lower their sentences on average by 12 percent. See Carson (2018, p. 3) and Motivans (2015, p. 39, Table 7.11).

[4] Although, to our knowledge, our article is the first to undertake this task, several authors address related issues. Miceli (1994), Garoupa (1996), and Polinsky (2015) examine how the modification of sentences can induce good conduct *within* prison. Pyne (2015) discusses a mechanism that leads prisoners to reveal their disutility from imprisonment, which can be used to foster deterrence. Also, Bernhardt, Mongrain, and Roberts (2012) and Kuziemko (2013), among many others, study the use of sentence adjustment to limit recidivism.

recalcitrance, suggesting that he might be more difficult to deter, or whether he is temperate in character and follows prison rules, suggesting that he is easier to deter.[5]

We begin in Section 2 by determining the optimal prison sentence in the standard model of crime, in which sentences are imposed at the time of conviction and are not adjusted afterwards.

In Sections 3 and 4 we analyze what we refer to as the *sentence-adjustment model*, in which the state might obtain information during imprisonment about the gain a prisoner derived from the crime he committed and may alter the initial sentence in light of it. (A prisoner's gain is a stand-in for any factor that affects his incentive to commit a crime, such as the cost of committing the crime or the disutility he suffers from being imprisoned.) In Section 3, we assume that the information about the gain is perfect. In that case, if the state learns a prisoner's gain, the sentence should be lowered to zero when the prisoner could not have been deterred and the sentence should be set at a level sufficient to accomplish deterrence when he can be deterred.[6] It also follows that social welfare in this version of the sentence-adjustment model is higher than in the standard model.

In Section 4 we consider the sentence-adjustment model when the information the state obtains about a prisoner's gain is imperfect. We first discuss an example in which what is learned about a prisoner is whether he is in an easy-to-deter group or a hard-to-deter-group of potential offenders and we explain why the optimal sentence rises when the information is that the prisoner is in the hard-to-deter group and declines when he is in the easy-to-deter-group. We

[5] For elaboration, see comment (c) in Section 5 below.

[6] Of course, since this policy will result in individuals who can be deterred choosing not to commit a crime, they will not be sentenced in fact.

then show that although imperfect information about prisoners' gains often has social value, as in the example, it is possible that such information has no social value. We also supply two general sufficient conditions for imperfect information to have social value.

In Section 5 we conclude with several comments on the interpretation of the model. We also remark on the failure of prison authorities and scholars to consider the role of deterrence in sentence adjustment policy—including that they ignore the possibility that raising sentences may be desirable and that lowering sentences dilutes deterrence. Instead, their virtually exclusive concern is with how the early release of prisoners affects recidivism.

2. The Standard Model

We first consider the standard model of deterrence, in which criminals are sentenced before imprisonment and sentences are not adjusted afterwards.[7] In this model individuals choose whether to commit a harmful act and differ in the gains that they would obtain if they do so. Let

h = harm caused by the act; $h > 0$;

g = gain that an individual would obtain from the act; and

$f(g)$ = probability density of g in $[0, \bar{g}]$; $f(g) > 0$; $\bar{g} > 0$,

where F is the cumulative distribution function of f.[8] We assume that $\bar{g} < h$ in order to rule out cases in which acts are socially desirable to commit. We also assume that the state cannot observe g but knows its probability density.

[7] See Becker (1968) and Polinsky and Shavell (2000).

[8] The function f should be interpreted as the distribution of gains conditional on the information available to the state at the time of sentencing.

Individuals who commit the act are caught with a probability and sentenced to a term of imprisonment. Imprisonment imposes disutility on the offender and results in the state bearing costs associated with the operation of prisons. Let

p = probability that an individual who commits the harmful act is caught;

$p > 0$;

s = prison sentence; s is in $[0, \bar{s}]$; $\bar{s} > 0$; and

k = cost to the state per unit of imprisonment; $k > 0$.

We treat p as fixed in order to focus on optimal sentencing policy. The sentence s is assumed to be equivalent to its disutility to a criminal. Hence, an individual will commit the harmful act if $g > ps$ and otherwise will be deterred.[9] We assume that $\bar{s} < \bar{g}$ to guarantee that it is not possible to deter all individuals from committing the harmful act; the case of full deterrence would not be of interest.

Social welfare is the sum of individuals' gains from committing the act, less the harm caused, less the disutility of imprisonment, and less the cost to the state of imprisonment:[10]

$$W(s) = \int_{ps}^{\bar{g}} [g - h - p(1 + k)s] f(g) dg. \tag{1}$$

In other words, each individual who commits the act changes social welfare by the amount $g - h - ps - psk = g - h - p(1 + k)s$. The state's problem is to choose the sentence s to maximize social welfare (1). The solution, s^*, can be characterized as follows.

[9] We assume for convenience that the individual is deterred if $g = ps$.

[10] This measure of social welfare does not reflect the cost to the state of maintaining the probability of detection p. Were we to include that cost, it would be a constant term in social welfare.

Proposition 1. In the standard model of deterrence,

(a) the optimal sentence s^* may be at either endpoint or in the interior of $[0, \bar{s}]$;

(b) if $0 < s^* < \bar{s}$, then s^* must satisfy condition (5); and

(c) if $s^* > 0$, then a positive number of individuals will be imprisoned.

Note. The optimal sentence s^* reflects a tradeoff between the benefit of deterring more individuals as s is raised and the social cost of longer sentences borne by individuals who are not deterred. The optimal sentence might be zero because the social costs of imprisonment might always outweigh the social benefits of deterrence, or maximal for the opposite reason. When s^* is positive, some individuals will always be imprisoned because a positive fraction of individuals are undeterrable (those in $[\bar{s}, \bar{g}]$)

Proof. Part (a): The derivative of social welfare is

$$W'(s) = -p[ps - h - p(1 + k)s]f(ps) - \int_{ps}^{\bar{g}} p(1 + k)f(g)dg \tag{2}$$

$$= p[(h + pks)f(ps) - (1 - F(ps))(1 + k)],$$

where the first term is the marginal benefit from deterring individuals of type ps and the second term is the marginal cost from sentencing undeterred individuals for an additional unit of time. We first show that s^* can be 0 by demonstrating that $W'(s)$ can be negative for all s in $[0, \bar{s}]$. We do this by providing an example in which there is an upper bound on the first term in brackets in the second line of (2) that is strictly less than a lower bound of the second term in brackets. In particular, suppose that $f(g)$ is uniform at height $\hat{f}$ between 0 and $p\bar{s}$. Then $(h + pk\bar{s})\hat{f}$ is an upper bound on the first term. Clearly, $(1 - F(p\bar{s}))(1 + k)$ is a lower bound for the second term. Thus, $W'(s)$ will be negative for s in $[0, \bar{s}]$ if

$$(h + pk\bar{s})\hat{f} < (1 - F(p\bar{s}))(1 + k), \tag{3}$$

which will hold for a sufficiently low $\hat{f}$.

To demonstrate that s^* can be an interior point, we claim that $W'(0) > 0$ and $W'(\bar{s}) < 0$ can hold simultaneously. We know that $W'(0) = p[hf(0) - (1 + k)]$, which will be positive if $f(0)$ is sufficiently high. Similarly, $W'(\bar{s}) = p\{[h + pk\bar{s}]f(p\bar{s}) - (1 - F(p\bar{s}))(1 + k)\}$, which will be negative if $f(p\bar{s})$ is sufficiently low.

To prove that s^* can be $\bar{s}$, we show that $W'(s)$ can be positive for all s in $[0, \bar{s}]$. Let $f(g)$ be uniform over $[0, \bar{g}]$, and thus $f(g) = 1/\bar{g}$. Then $h(1/\bar{g})$ is a lower bound for the first term in brackets of the second line of (2). Clearly, $(1 + k)$ is an upper bound for the second term in brackets. Thus, $W'(s)$ will be positive for s in $[0, \bar{s}]$ if

$$h(1/\bar{g}) > (1 + k), \tag{4}$$

which will hold for a sufficiently high h.

Part (b): If s^* is an interior solution, it must satisfy the first-order condition from (2), namely

$$[h + pks]f(ps) = (1 - F(\mathrm{ps}))(1 + k). \tag{5}$$

Part (c): Individuals who will not be deterred include those for whom g is in $(p\bar{s}, \bar{g}]$. This group has positive mass since $p\bar{s} < \bar{g}$. The fraction p of these individuals will be imprisoned for the positive length of time s^*. □

3. The Sentence-Adjustment Model with Perfect Information

We now modify the standard model by assuming that after an offender is initially sentenced and imprisoned, the state will learn his gain with some probability. In that event, the state can change the offender's sentence. Let

s_I = initial sentence; s_I is in $[0, \bar{s}]$;

q = probability that the state learns g when a person is in prison; $0 < q < 1$;[11] and

$s(g)$ = adjusted sentence if the state learns g; $s(g)$ is in $[0, \bar{s}]$.

A person of type g will commit the harmful act when

$$g > p[(1-q)s_I + qs(g)], \tag{6}$$

for if the person is caught and sentenced to s_I, the probability is $(1 - q)$ that the state will not observe g and thus that the sentence will remain s_I, and the probability is q that the state will observe g and alter the sentence to $s(g)$.

The state's problem is to choose s_I and the function $s(g)$ to maximize social welfare, which is

$$W(s_I, s(g)) = \int_0^{\bar{g}} \lambda(g)[g - h - p[(1-q)s_I + qs(g)](1+k)]f(g)dg, \tag{7}$$

where $\lambda(g)$ is 1 if (6) is satisfied and 0 otherwise.[12] The solution is as follows.

Proposition 2. In the sentence-adjustment model of deterrence with perfect information,

(a) the optimal initial sentence s_I^* may be at either endpoint or in the interior of $[0, \bar{s}]$;

(b) if $0 < s_I^* < \bar{s}$, then s_I^* must satisfy condition (12);

(c) the optimal adjusted sentence $s^*(g)$ accomplishes deterrence for all types g who can be deterred, and is zero for all types g who cannot be deterred: $s^*(g) = 0$ for g in $[0, p(1-q)s_I]$;

[11] For simplicity, we assume that q does not depend on the length of the initial sentence. If we were to assume that q rises with s_I and that $q = 0$ if $s_I = 0$, the essential character of our results would not be affected.

[12] This problem can be interpreted as that facing the state with respect to information it might obtain during trial rather than during imprisonment. Then s_I would be the sentence the state would impose if it only knew the offense that was committed, g would be the possible information obtained about the offender during trial, and $s(g)$ would be the sentence determined at trial given g.

$s^*(g) = [(g/p) - (1 - q)s_I]/q$ and rises from 0 to $\overline{s}$ for g in $(p(1 - q)s_I, p[(1 - q)s_I + q\overline{s})]]$; and $s^*(g) = 0$ for g in $(p[(1 - q)s_I + q\overline{s})], \overline{g}]$;

(d) if $s_I^* > 0$, then a positive number of individuals will be imprisoned; and

(e) some individuals will be deterred in the optimal solution.

Notes. (i) It is clear that if $s(g)$ can be chosen so as to deter individuals of type g, it is desirable to do so because the harmful act is socially undesirable. (For low values of g, this does not require $s(g)$ to be positive since the initial sentence s_I is imposed with a positive probability.) Conversely, if deterrence is not possible, $s(g)$ should be zero since nothing is accomplished by imposing costly sentences.[13] This explains part (c). It also explains part (e) because it will always be possible to deter some individuals with a positive $s^*(g)$.

(ii) The explanations of parts (a), (b), and (d) regarding s_I^* parallel the explanation of the results regarding s^* in Proposition 1.

Proof. Part (c): If an individual is deterred, there will be no change in social welfare, whereas if he is not deterred, the change in social welfare will be $g - h - p(1 + k)[(1 - q)s_I + qs(g)]$, which is negative. Therefore, it is desirable to deter any individual who can be deterred. It is clear from (6) that a person can be deterred if and only if $g \leq p[(1 - q)s_I + q\overline{s}]$. For individuals with g in $[0, p(1 - q)s_I]$, deterrence will be achieved if $s(g) = 0$. For individuals with g in $(p(1 - q)s_I, p[(1 - q)s_I + q\overline{s}]]$, it can be verified from (6) that deterrence will just be achieved if $s(g) = [(g/p) - (1 - q)s_I]/q$; in this interval, $s(g)$ rises from 0 to $\overline{s}$. We will use $[(g/p) - (1 - q)s_I]/q$ for $s^*(g)$ when g is in $[0, p[(1 - q)s_I + q\overline{s}]$ even though $s^*(g)$ in this region is not unique because a higher $s(g)$ would also deter. For individuals with g in $(p[(1 - q)s_I + q\overline{s}], \overline{g}]$, who cannot be

[13] We abstract from the fact that the prisoner has served some of his sentence before the state learns his g.

deterred, the change in social welfare is $g - h - p(1 + k)[(1 - q)s_I + qs(g)]$, which is maximized when $s(g) = 0$.

Part (e): Some individuals can be deterred, those for whom $g \le p[(1 - q)s_I + q\bar{s}]$, and from part (c) we know that they will be deterred.

Part (a): Given part (c), social welfare (7) can be written as

$$W(s_I) = \int_{p[(1-q)s_I + q\bar{s}]}^{\bar{g}} [g - h - p(1 - q)(1 + k)s_I] f(g)dg, \tag{8}$$

so that

$$W'(s_I) = -p(1 - q)\{p[(1 - q)s_I + q\bar{s}] - h - p(1 - q)(1 + k)s_I\} f(p[(1 - q)s_I + q\bar{s}]) \tag{9}$$

$$- \int_{p[(1-q)s_I + q\bar{s}]}^{\bar{g}} p(1 - q)(1 + k) f(g)dg$$

$$= p(1 - q)[(h - pq\bar{s} + p(1 - q)ks_I) f(p[(1 - q)s_I + q\bar{s}]) - (1 - F(p[(1 - q)s_I + q\bar{s}]))(1 + k)].$$

We first show that s_I^* can be 0 by demonstrating that $W'(s_I)$ can be negative for all s in $[0, \bar{s}]$. The proof parallels that of the corresponding result in Proposition 1. Again, assume that $f(g)$ is uniform at height $\hat{f}$ between 0 and $p\bar{s}$. Then $(h - pq\bar{s} + p(1 - q)k\bar{s})\hat{f}$ is an upper bound on the first term in brackets in the last line of (9) and $(1 - F(p\bar{s}))(1 + k)$ is a lower bound for the second term. Thus, $W'(s_I)$ will be negative for s_I in $[0, \bar{s}]$ if

$$(h - pq\bar{s} + p(1 - q)k\bar{s})\hat{f} < (1 - F(p\bar{s}))(1 + k), \tag{10}$$

which will hold for a sufficiently low $\hat{f}$.

To demonstrate that s_I^* can be an interior point, we claim that $W'(0) > 0$ and $W'(\bar{s}) < 0$ can hold simultaneously. We know that $W'(0) = p(1 - q)[(h - pq\bar{s})f(q\bar{s}) - (1 - F(q\bar{s}))(1 + k)]$, which will be positive if $f(q\bar{s})$ is sufficiently high. Similarly, $W'(\bar{s}) = p(1 - q)[(h - pq\bar{s} + p(1 - q)k\bar{s})f(p\bar{s}) - (1 - F(p\bar{s}))(1 + k)]$, which will be negative if $f(p\bar{s})$ is sufficiently low.

To prove that s_I^* can be $\bar{s}$, we show that $W'(s_I)$ can be positive for all s_I in $[0, \bar{s}]$. Let $f(g)$ be uniform over $[0, \bar{g}]$, and thus $f(g) = 1/\bar{g}$. Then $(h - pq\bar{s})(1/\bar{g})$ is a lower bound for the first term in brackets of the second line of (9) and $(1 + k)$ is an upper bound for the second term in brackets. Thus, $W'(s_I)$ will be positive for s_I in $[0, \bar{s}]$ if

$$(h - pq\bar{s})(1/\bar{g}) > (1 + k), \tag{11}$$

which will hold for a sufficiently high h.

Part (b): If s_I^* is an interior solution, it must satisfy the first-order condition from (9), namely

$$[(h - pq\bar{s} + p(1 - q)ks_I)]f(p[(1 - q)s_I + q\bar{s}]) = (1 - F(p[(1 - q)s_I + q\bar{s}]))(1 + k)]. \tag{12}$$

Part (d): Individuals who will not be deterred include those for whom g is in $(p\bar{s}, \bar{g}]$. This group has positive mass since $p\bar{s} < \bar{g}$. The fraction pq of these individuals will be imprisoned for the positive length of time s_I^*. □

The next result concerns social welfare under the sentence-adjustment model and compares that model to the standard model.

Proposition 3. Under the optimal solution to the sentence-adjustment model,

(a) social welfare is higher than under the optimal solution to the standard model;

(b) social welfare is increasing in the probability q that the state will observe the prisoner's gain during imprisonment; and

(c) the use of sentences is always desirable, whereas in the standard model the use of sentences might not be desirable.

Notes. (i) There are two welfare advantages of the sentence-adjustment model over the standard model. First, sentencing costs can be lowered. To see this, suppose that sentences in the sentence-adjustment model are initially chosen so as to duplicate the sentence in the standard model—that is, $s_I = s(g) = s^*$—in which case the same individuals will be deterred in both models, those for whom $g \leq ps^*$. However, sentencing costs can be reduced by lowering $s(g)$ to 0 for all individuals for whom $g > ps^*$, that is, for those who are not deterred. The second advantage of the sentence-adjustment model is that, if $s^* < \overline{s}$, greater deterrence can be accomplished without incurring additional sentencing costs. To see this, suppose again that the sentence-adjustment model duplicates the standard model. Consider individuals whose g is slightly greater than ps^* and who therefore would not be deterred in the standard model. These individuals can be deterred in the sentence-adjustment model by raising $s(g)$ above s^*.

(ii) The two welfare advantages of the sentence-adjustment model just described depend on the observation of an offender's gain g. Therefore, if the probability q of observing g rises, welfare will rise.

(iii) In the standard model the use of sentences might not be desirable because the deterrence benefits of any positive sentence might be exceeded by its sentencing costs. In the sentence-adjustment model, however, it is always possible to use positive sentences to accomplish deterrence without incurring sentencing costs. For example, set $s_I = 0$, $s(g) = \overline{s}$ for g in $[0, pq\overline{s}]$, and $s(g) = 0$ for g in $(pq\overline{s}, \overline{g}]$; then individuals with g in the first interval will be deterred and everyone else will not face sentences.

Proof. Part (a): The optimal solution to the standard model, s^*, can be duplicated in the sentence-adjustment model by setting $s_I = s^*$ and $s(g) = s^*$ for all g. But Proposition 2(c) shows that, if $s^* = s_I = 0$, $s^*(g)$ is positive in the interval $[0, q\overline{s}]$, implying that social welfare must be higher in the sentence-adjustment model. Similarly, if $s^* = s_I > 0$, $s^*(g)$ is 0 in the interval $(p[(1 - q)s_I + q\overline{s})], \overline{g}]$, again implying that social welfare must be higher in the sentence-adjustment model.

Part (b): We prove this result by showing that social welfare (8) rises with q holding s_I constant. (Social welfare would be at least as high if s_I were chosen optimally as a function of q.) When q rises, the integrand rises, raising social welfare. The lower limit of integration remains the same if $s_I = \overline{s}$ and rises if $s < \overline{s}$. Thus, since the integrand is negative, social welfare must rise.

Part (c): We know from Proposition 2(c) that in the sentence-adjustment model $s^*(g)$ is positive over some positive interval of g. And we know from Proposition 1(a) that in the standard model $s^* = 0$ is possible. □

4. The Sentence-Adjustment Model with Imperfect Information

We next consider the sentence-adjustment model when there exist imperfectly informative signals about prisoners' gains. In particular, suppose that a signal identifies only a subset of individuals to which a prisoner belongs, where the possible subsets are mutually exclusive and exhaustive and are denoted $Z(1), Z(2), \ldots, Z(n)$, with $n \geq 2$. Although each individual in the population is a member of a single subset, we assume that different individuals with the same g may be members of different subsets. Let $f(g, j)$ be the conditional density of g over subset $Z(j)$ and let $r(j) > 0$ be the probability of that subset. We assume that signals are informative in the sense that at least two of the subsets have different conditional densities. The unconditional density $f(g)$ is the sum of $r(j)f(g, j)$ over j. Each individual in the population is assumed to know his $Z(j)$.

In this section the adjusted sentence must be a function of the subset $Z(j)$ to which a prisoner belongs. Let

$s(j)$ = adjusted sentence if the state learns $Z(j)$,

so that an individual who is a member of $Z(j)$ will commit the harmful act when

$$g > p[(1-q)s_I + qs(j)] \tag{13}$$

and will be deterred otherwise.

The state's problem is to choose s_I and the function $s(j)$ to maximize social welfare, which is

$$W(s_I, s(j)) = \sum_{j=1}^{n} \{\int_0^{\bar{g}} \lambda(g, j)[g - h - p[(1-q)s_I + qs(j)](1+k)]\, r(j) f(g, j) dg\}, \tag{14}$$

where $\lambda(g, j)$ is 1 if (13) is satisfied and 0 otherwise. To explain (14), consider a person who is a member of $Z(j)$. That person will commit the harmful act when (13) is satisfied, in which case $\lambda(g, j)$ will be 1. The term in brackets then is his contribution to social welfare, and $r(j)f(g, j)$ is the unconditional density of g in $Z(j)$.

It will be useful to define the conventionally optimal sentence s_j* for subgroup $Z(j)$ to be the optimal sentence in the standard model if the distribution of gains g were given by the conditional distribution $f(g, j)$.

We now illustrate the sentence-adjustment model with imperfect information. Suppose that the maximum gain $\bar{g}$ of individuals is 100; the harm h is 150; the probability p of catching offenders is 0.5; the maximum sentence $\bar{s}$ is 160; the probability q of a signal is 0.3; and the public cost k of sentences is 1 per unit time. Note that the highest expected sentence is 80, so that any individual with a gain exceeding 80 is undeterrable.

Suppose also that the population is divided into two subsets of equal size, so that $r(1) = r(2) = 0.5$. The density of gains for each subset is a triangle over a lower region that consists of deterrable individuals and is uniform over a higher region that consists of undeterrable individuals. In particular, let the conditional density of subset $Z(1)$ be 0 between 0 and 10; rise linearly between 10 and 30 to a peak at height 1/30; decline linearly between 30 and 50 to 0; remain at 0 until 80; and then be constant at 1/60 between 80 and 100. Similarly, let the conditional density of subset $Z(2)$ be 0 up to 30; rise linearly between 30 and 50 to the same peak and then decline linearly to 0 at 70; remain at 0 until 80; and be constant at 1/60 between 80 and 100. We will refer to the first group as easy-to-deter and the second group as hard-to-deter because the triangle of deterrable individuals in the first group is to the left of the triangle of deterrable individuals in the second group.

When this model is solved (which was done numerically), the optimal initial sentence s_I^* is 130; the optimal adjusted sentence for the easy-to-deter group $s(1)^*$ is 10; and the optimal adjusted sentence for the hard-to-deter group $s(2)^*$ is 150.[14] Consequently, the optimal expected sentence for individuals in the easy-to-deter group is 47 (= 0.5[0.7(130) + 0.3(10)]) and the optimal expected sentence for individuals in the hard-to-deter group is 68 (= 0.5[0.7(130) + 0.3(150)]).

To understand these results, it is useful to compare them to the conventionally optimal sentences for the two groups. For the easy-to-deter group, this sentence s_1^* is 94, which results in an expected sentence ps_1^* of 47. The conventionally optimal sentence s_1^* reflects, for this group, a balancing of the benefits of greater deterrence as the sentence is raised and the increased costs of sentencing associated with individuals who are not deterred, some of whom are imprisoned. As seen in the previous paragraph, the optimal solution to the sentence-adjustment model achieves the same expected sentence for individuals in the easy-to-deter group—by lowering the initial sentence $s_I^* = 130$ to the adjusted sentence $s(1)^* = 10$—and therefore accomplishes the same balancing of deterrence benefits and sentencing costs for individuals in this group. A parallel explanation applies to the hard-to-deter group, for which the conventionally optimal sentence s_2^* is 136 and ps_2^* is 68. This outcome, too, can be duplicated in the sentence-

[14] The solution is not unique although optimal expected sentences are unique.

adjustment model by raising the initial sentence s_1* = 130 to the adjusted sentence $s(2)$* = 150 when a signal identifies a prisoner as a member of the hard-to-deter group.[15]

If there were no chance of a signal identifying the group to which a prisoner belonged, a single sentence would have to be used for everyone—this is the standard model. Then the optimal sentence s* would be 132 and the expected sentence ps* would be 66 for all individuals. Consequently, the resulting tradeoff between deterrence benefits and sentencing costs would be suboptimal for both groups—excessive deterrence for the easy-to-deter group and insufficient deterrence for the hard-to-deter group. The ability to adjust sentences on the basis of signals raises social welfare by permitting different expected sentences to be employed to deter individuals in the two groups, achieving a better tradeoff between deterrence benefits and sentencing costs for both groups.

A natural question is whether an informative signal about individuals' gains from crime necessarily leads to an increase in social welfare. In some incentive contexts, informative signals about individuals always can be employed to enhance welfare. Notably, this is true of signals about individuals' effort in principal-agent models.[16] Here, however, informative signals about individuals' gains might not raise welfare, as the next proposition demonstrates.

Proposition 4. Information about individuals' types may not have social value; that is, the optimal solution to the sentence-adjustment model may be equivalent to that of the standard model.

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[15] The optimal expected sentences in the sentence-adjustment model will equal the conventionally optimal expected sentences for all groups only if the signal probability q is sufficiently high (the intuition is that if $q = 0$, the expected sentence has to be the same for all groups, while if $q = 1$, the expected sentence can be perfectly tailored for each group).

[16] This statement presumes that agents are risk averse. See Hölmstrom (1979) and Shavell (1979).

Note. We will show that if signals convey information only about individuals who would be deterred in the standard model, there may be no advantage in having that information.

Proof. We employ a version of the above example in which the density of gains for each subset is a modified triangle over a region between 20 and 60. For both groups, $Z(1)$ and $Z(2)$, the conditional density is 0 up to 20 and between 60 and 80; and then constant at 1/60 between 80 and 100. For $Z(1)$, we modify the triangle between 20 and 60, which has a height of 1/30 at 40, in the following way. Let the density between 25 and 35 rise by 1/200. For $Z(2)$, we employ the same triangle but lower the density between 25 and 35 by 1/200.

Observe that the unconditional density of gains is 0 between 0 and 20; rises linearly between 20 and 40 to a peak at height 1/30; declines linearly between 40 and 60 to 0; remains at 0 until 80; and then is constant at 1/60 between 80 and 100. Given this density, the optimal solution of the standard model is $s^* = 114$, with an expected sentence of 57 for everyone.

Note that $Z(1)$ and $Z(2)$ convey information about gains below 57, and thus about individuals who are deterred in the standard model. An optimal solution to the sentence-adjustment model is $s_I^* = s(1)^* = s(2)^* = 114$, also resulting in each individual facing an expected sentence of 57. Thus, social welfare in the two models is the same. □

Although informative signals are not always socially valuable, we now provide two sufficient conditions for them to have value.

Proposition 5. A sufficient condition for information about individuals' types to have social value is that the optimal sentence s^* in the standard model is positive and that there exists a type whose support lies above ps^*; that is, the support of one of the subsets $Z(j)$ lies above ps^*.

Notes. (i) This condition should ordinarily apply because it will commonly be true that there are *some* individuals who can be recognized as undeterrable given p and s^*.

(ii) The reason that this is a sufficient condition is that the sentence for individuals of this type can be lowered without affecting the deterrence of anyone else.

Proof. First set $s_1 = s^*$ and $s(j) = s^*$ for all j, so that the outcome in the sentence-adjustment model duplicates the optimal outcome in the standard model. Let group $Z(1)$ be a subset of individuals whose support lies above ps^*. In both the standard model and the sentence-adjustment model under our present assumptions, these individuals will commit the harmful act. Now change $s(1)$ to any $s < s^*$. Individuals in subset $Z(1)$ will continue to commit the harmful act, but sentencing costs with respect to them will decline from $ps^*(1 + k)$ to $p[(1 - q)s^* + qs](1 + k)$. Their contribution to social welfare will thus rise, while social welfare with respect to everyone else will be unaffected. □

Proposition 6. A sufficient condition for information about individuals' types to have social value is that there is at least one subset $Z(j)$ for which the conventionally optimal sentence s_j^* differs from s^* and that social welfare is concave in s_j.

Note. The reason that this condition is sufficient for information to have social value is that it permits a sentence adjustment to be made that moves the expected sentence in the direction of the ideal sentence for the specified group.

Proof. First set $s_1 = s^*$ and $s(j) = s^*$ for all j, so that the outcome in the sentence-adjustment model duplicates the optimal outcome in the standard model. Let group $Z(1)$ be a subset of individuals for whom s_1^* differs from s^*. If $s_1^* > s^*$, raise $s(1)$ by a small amount γ from s^* to $s^* + \gamma < s_1^*$. The expected sentence for individuals in group $Z(1)$ will thus increase from ps^* to $p[(1 - q)s^* + q(s^* + \gamma)]$. Because this expected sentence is between ps^* and ps_1^*, the assumption of concavity implies that the contribution to social welfare of individuals in $Z(1)$ will also increase. Since social welfare with respect to everyone else is unaffected, social welfare rises. If $s_1^* < s^*$, an analogous argument (with $s(1)$ declining by a small amount γ) applies. □

5. Concluding Comments

(a) *Determinate versus indeterminate sentencing*. The sentencing decision of a court may be determinate—name a particular term—or indeterminate—specify lower and upper bounds on the term, such as five to ten years. In the United States, approximately two-thirds of the states employ indeterminate sentencing, with parole boards deciding on the length of time that prisoners serve within the sentence's limits, together with adjustments for good time and earned time.[17] In other countries, indeterminate sentencing is even more dominant as the form of sentencing.[18] Although we assumed for simplicity that sentencing was determinate in our model, it is clear that the thrust of our conclusions would not be altered if we studied indeterminate sentencing. It would still be true that deterrence-relevant information acquired during imprisonment would be socially desirable to employ in deciding on the length of incarceration.

(b) *Do potential offenders take sentence adjustment into account?* Substantial evidence exists that individuals are responsive to the threat of criminal sanctions.[19] This could only be the case if individuals view the magnitude of possible sanctions as serious. Given that the level of sentences is significantly altered during imprisonment—we noted in the introduction that sentences are reduced on average by about half—it is reasonable to assume that potential

[17] See Lawrence (2015, p. 4) and note 2 above.

[18] See Aharonson (2013, pp. 164-175). He views the United States as an outlier due to its increasing reliance in recent decades on determinate sentencing.

[19] See, for example, Andenaes (1966, pp. 960-973), Levitt and Miles (2007, pp. 466-474), Durlauf and Nagin (2012, pp. 47-71), and Chalfin and McCrary (2017, pp. 13-32).

offenders would factor into their calculation of expected sentences the general effect of these adjustments. This belief is reinforced, moreover, by the fact that the institution of parole is part of our popular culture, as reflected in books, television and movies, and the news.

(c) *Can the state acquire information about prisoners that bears on deterrence?* It is plausible that the state can obtain information during imprisonment that is pertinent to deterrence. Suppose, for example, that a parole board ascertains that a prisoner has generally followed prison rules, has made an effort to obtain job training, and is remorseful for his crime. This kind of record might indicate that a shorter sentence than would otherwise apply would be appropriate on grounds of deterrence.[20] Or suppose that prison authorities find that a prisoner is mentally ill. Such a discovery might suggest that he would not be very responsive to the possibility of being caught and punished for criminal acts, again implying that a shorter sentence would be desirable from the perspective of deterrence.[21]

Conversely, suppose that it becomes apparent to a parole board that a prisoner has a history of insubordination, altercations with other prisoners, and the like. Behavior of this sort could suggest the need for a lengthier sentence to properly deter. Likewise, suppose that it is learned that the prisoner is a gang member. This too could indicate that a longer sentence would be appropriate, for gang members might derive status benefits from committing crimes and also

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[20] The personality type of such a prisoner suggests that his criminal act may have been motivated by unusual circumstances and that ordinarily individuals with his set of characteristics could be deterred by a shorter sentence. The reason for lowering the sentence is not that the particular individual in prison would have been deterred with a shorter sentence, for obviously he would not have. Rather, it is that a shorter sentence is more desirable in expectation for individuals of this type. The rationale for lowering the sentence in the example in Section 4 when a signal identifies a prisoner as a member of the easy-to-deter group is of this character.

[21] The basis for this conclusion is essentially that given in Section 3 for the result that, if an individual's gain g can be observed and his gain is so high that he cannot be deterred, the optimal sentence is zero. See Proposition 2(c) and note (i) following the proposition. Here the reason for the inability to deter an individual is his mental illness rather than his high gain.

view imprisonment as having less disutility because they will be protected from violence within the prison or receive favorable treatment from guards.

(d) *The failure of parole boards and prison authorities as well as scholars to consider the effects of sentence adjustment on deterrence.* As we observed in the introduction, sentences are reduced in various ways, through parole, good time, and earned time. But none of these reductions are motivated by considerations of deterrence. When parole boards decide on sentence reduction, they focus on whether a prisoner is likely to commit another crime if he is released, that is, whether he will become a recidivist.[22] Good time credits are employed by prison authorities mainly as a reward for good behavior within prison; and earned time credits are used to encourage prisoners to participate in self-improvement activities.[23] Moreover, discussions of sentence adjustment by scholars focus on issues of recidivism and encouraging good behavior of prisoners.[24]

This lack of attention to the effects of sentence adjustment on deterrence can lead to undesirable social policy. First, the practice of lowering sentences during imprisonment can obviously undermine deterrence. Suppose a person is sentenced to a twenty-year prison term for murder but is released after ten years because a parole board concludes that, given his exemplary behavior in prison, he would no longer be a risk to society.[25] Clearly, other individuals

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[22] See generally Cohen (2018, § 4:30), who observes that "the most basic criteria for release on parole" are "whether there is a reasonable probability that a prison inmate, if placed on parole, will be able to live and conduct himself or herself as a respectable, law-abiding person, and whether release will be compatible with the offender's own welfare and the welfare and safety of society."

[23] See note 2 above.

[24] For example, Bernhardt, Mongrain, and Roberts (2012) and Kuziemko (2013) focus on the effect of early release of prisoners on recidivism, not on deterrence.

[25] According to Durose and Langan (2004, p. 5, Table 4), the mean prison sentence for murder (including nonnegligent manslaughter) is 225 months (18.8 years), whereas prisoners serve on average 142 months (11.8 years).

contemplating committing murder could believe that they too would serve a sentence much shorter than twenty years if they behaved well in prison. Second, it may obviously be desirable to raise sentences if information indicates that doing so would promote deterrence (see comment (c)), yet sentences are never raised in practice.

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