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# PORTFOLIOS FOR LONG-TERM INVESTORS

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# Portfolios for long-term investors

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## Abstract

How should long-term investors form portfolios in our time-varying, multifactor and friction-filled world? Two conceptual frameworks may help: looking directly at the stream of payments that a portfolio and payout policy can produce, and including a general equilibrium view of the markets' economic purpose, and the nature of investors' differences. These perspectives can rationalize some of investors' behaviors, suggest substantial revisions to standard portfolio theory, and help us to apply portfolio theory in a way that is practically useful for investors.

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How should long-term investors form portfolios? How should they evaluate securities, portfolios and managers? In particular, how should they adapt to the fact that our world features all sorts of time-varying expected returns, volatilities, correlations, factors, and strategies?

## 1 Theory vs. practice

To academics, portfolio theory seems to be basically a solved problem. Merton (1969) gave the answer 50 years ago, building on Markowitz (1952) 70 years ago: Split the investment between the market portfolio, the risk free asset, and hedge portfolios for additional factors.

The hard work lies in naming the factors, calculating the hedge portfolios, calibrating and solving Merton models, and adapting them to the peculiarities of different investors. This remains an immense, productive and challenging enterprise. (Campbell and Viceira (2002) for example.) But the conceptual framework has not really changed. Even behaviorists view Merton's advice as the right normative thing to do, and behaviorally biased agents should learn to do Merton better. Indeed, one might see portfolio theory and advice as a bit of a backwater.

Yet Merton's theory has almost no impact on portfolio practice. With some exceptions, it does not well describe empirically the portfolios that households or institutions hold.

Institutions – endowments, family offices, insurance companies, pension funds, and the financial management and advice industry – formalize investment practice and advice in ways we can study and read, beyond just examining the outcome. Their portfolio practice and advice is quite uniform – and has little to do with Merton. Roughly, they start with intense attention to buckets, defined by names not betas and correlations: Debt and equity; subcategories mostly denoted by industry and other non-beta characteristics, but now also growth; value and other academic styles; domestic, foreign; “alternatives,” often just an alternative organizational forms which repackage the same securities; real estate, private equity, venture capital, and so forth. Having decided these buckets down to the last percentage point, they allocate each bucket to separate funds or managers, evaluated by rather short-term returns relative to a benchmark, and fre-

quently replaced based on short-term results. As market values change, they quickly rebalance to the benchmark allocations. As many private investors have moved to indexing, many institutions have moved in the opposite direction, preferring illiquid assets that one cannot mark to market and layers of high-fee management. Many such investors sell in downturns. University endowments are a case in point: Despite proclamations about being long-term investors that would ride out temporary price fluctuations, many universities sold equities and illiquid “alternatives” in a panic in 2008, during the buying opportunity of a generation.

Payout policy is an integral and overlooked part of portfolio theory. These investors often stick to fixed payout rules in downturns, even as in 2008 when liquidity crises, collateral for derivatives positions, debt-roll-over problems, and short-term revenue shortfalls loomed. Universities instituted budget cuts and hiring freezes in 2008 and again this year, freezing hiring during the best buy-side market in a generation. Endowments pay out an almost constant fraction of wealth each period, and do not provide any liability-stream hedging or buffer-stock value. (See Gilbert and Hrdlicka (2015) on endowment payout policies and internal investments.)

And this effort produces a pretty discouraging track record by standard evaluation, if you don't pick winners ex-post. For example, Ennis (2020) finds that “Public pension funds underperformed passive investment by approximately 1.0% a year for the 10 years ended June 30, 2018; the shortfall of educational endowments was 1.6% a year.” A colleague estimates that Stanford pays \$800 million a year in fees. Many wealthy individuals voluntarily pay more than the Sanders-Warren wealth taxes in fees, though an immense academic literature finds no skill in active management (for example, Fama and French (2010)). The wisdom of high-fee active management is another striking difference between academic empirical work and practice.

There is no visible mention of state variables or state-variable hedging in any of this process. Multifactor state-variable thinking is also absent from finance industry. Why do sophisticated quantitative hedge funds, whose reasons for being are time-varying returns volatilities, and correlations, multiple priced factors, and Merton state variables, whose clients' horizons are long-term, who have sophisticated portfolio optimizers, nonetheless form and evaluate their portfolios on one-period means and variances and similar return statistics?

Universities employ a large faculty of finance professors, who actively research asset markets. They teach undergraduates and MBAs, so become experts at communicating theory to practitioners, and merging academic and real world perspectives via case studies and connections to industry. Yet with a few exceptions, university endowments have essentially no connection to all this free high-priced talent. Indeed, endowments strongly resist input from their finance faculty. I can't help suspecting they know what we're going to say and don't want to hear the answers.

Standard fallacies abound. Hedge funds and other institutional investors lever, and think they can synthesize a free put option by selling as prices fall. "Carry," the business of essentially trying to synthesize the writing of put options, litters the finance industry. That's fine, so long as you don't evaluate it by alpha-beta statistics based on short samples and normal distributions. How many times have levered "arbitrageurs" forgotten that they may have to post additional cash collateral if the market gets more irrational before it gets better, or that "we'll sell on the way down" does not synthesize a free option?

One can read with a bit of hope the spread of "factor investing" as the provision and purchase of Mertonian hedge portfolios. But which factors should one buy? Just how are style portfolios Mertonian factors? Factor-investing funds may promise alpha relative to a factor benchmark, but which factor benchmarks should one buy? What do all these names mean? Their fund options look like a Chinese restaurant menu. In Chinese.

When theory is so persistently contrary to practice one of the two must be wrong. One might take the Chicago approach: What is the question to which these practices are the optimal answer? Or one might take the Cambridge approach: Let's deplore how dumb people are not to use the right answer, or find "nudges," or add to the mass of financial regulation to force better allocations.

I think the right answer drinks a bit of both salt and fresh water. Let us figure out why investors aren't implementing our theory, and try to make the theory more useful so they are more interested in using it. And let us keep our eyes open to aspects of the world in which their decisions make some sort of sense.

Cambridge is half right. Normative theory has improved practice. The wide spread of indexing, passive management, and alpha-beta evaluation stemmed from pure theory. It had nothing to do with practice in the 1960s, and was widely derided by practitioners at the time. The concept of value vs. growth, factor investing, and behavioral anomaly har-

vesting sprang from academic research. Chicago is half right too. As I survey seemingly irrational investment practice, below, I find a lot that actually does make sense as ways to implement Merton ideas in a world that is a lot less random walky than we thought 50 years ago.

Merton's theory also devilishly hard to implement, which surely accounts for its lack of success. The central problem: what are the expected returns, state variables, betas or correlations? These are really hard, nay impossible, to measure. What are the partial derivatives of the value function that guide state-variable investments? That's harder still. And portfolio maximization is very sensitive to inputs.

I offer two basic ideas to make progress on these questions. First, we should focus on the stream of dividends, or more generally payoffs, that an investment can produce, rather than focus on one-period returns. Second, we should take a general equilibrium perspective. An investor should ask, what is the economic function of markets, and what is my role in it? If I want to buy, who is selling and why? Answering this question can cut through knots of algebra and statistics, and avoid many fallacies. Putting both hats on and touring the new world of finance theories, I end up seeing, going forward, portfolio theories that are fundamentally different from Merton's vision, and ways of applying and marketing theories that are fundamentally different from the usual black box so rightly distrusted by real investors.

## 2 The Payoff Perspective

I am inspired by the brilliant Campbell and Viceira (2001) article on long-term bonds. The indexed perpetuity is the riskless asset for a long-run investor. Stop and savor that statement. I remember first seeing the paper when John gave it at Chicago. I instantly thought, "This is obvious." And then, I realized that neither I nor anyone in the room knew it, and I realized "this is brilliant."

Now, the statement is only obvious if you look at the *payoffs*. An indexed perpetuity gives a steady stream of income forever. It's the risk-free *payoff*. Duh. The statement is not at all obvious if you look at it Merton-style: When bond prices fall, bond expected returns rise. The indexed perpetuity's expected return happens to rise just enough when its price falls, that the dynamically-optimizing consumer can construct a riskless con-

sumption stream. Seeing that fact takes a lot of dynamic programming.

This verity is not at all obvious to the investing world. The idea that indexed perpetuities are the riskfree asset meets incredulity. How many MBA investment classes yet mention this obvious fact, instead calling money market funds the “riskless asset?” (Mine did, after 2001, but I wonder how many of my ex-students remember the proposition, or would ever say it out loud in their jobs.) In trying to sell my view that the Treasury should issue indexed perpetuities (Cochrane (2015)), I constantly hear “there is no demand” for the product. Indeed, the world is moving away from theory. Perpetuities were common in the early history of government debt, perhaps precisely because they never needed a lump sum roll-over. The Victorian U.K. funded an enormous government debt almost entirely with perpetuities, which under the gold standard were essentially indexed. Perpetuities were the go-to asset for aristocrats of 19th century novels to fund a leisurely lifestyle.

Well, let’s try to sell some indexed perpetuities. Bring along your infinitely risk-averse spouse who is not a finance professor, and let’s set up your retirement portfolio. We’ll start, of course, with enough indexed perpetuities to keep you and your altruistically-linked dynasty going at a minimum standard of living forever.

The next month, the statement comes in. Interest rates have gone up and the value of your investment has plummeted. “Hey, Mr. or Ms. brilliant finance professor,” your spouse intones, “you just lost a third of our retirement savings in your supposed risk-free asset!” Hmm. This is going to be a long conversation, as you delve into state variables, cross-derivatives of the value function, hedges for state-variable risk, and dynamic models of the term structure.

A much better explanation is, “Honey, look, this investment pays a steady coupon forever that we can live on. That’s the point. We’re not going to buy or sell those bonds, we’re going to live off the coupons. Just tear up the statements, and stop worrying, they’re meaningless.”

That bond returns are predictable from their prices or yields is the flip side of the fact that (nominal US government) bond cash flows are not predictable from their prices or yields. Fundamentally, I think the spouse is going to understand the latter better than the former. Rather than explain that a price decline is bad news, but a rise in expected returns means we’ll be ok in the end, let us look at the other side of the identity: When

government bond prices decline, that fact has no news whatsoever about the cashflows you will receive.

Now, many risky investments, including stocks, are a lot like bonds – lower prices correspond to high expected returns, a higher discount rate. Lower prices do not correspond to lower expected cashflows. (I use the term “discount rate” without, here, taking a stand on the root cause. Even if the root cause of temporary price fluctuation is behavioral or institutional, the *fact* is that higher prices mean lower returns not higher dividends, that stocks are a lot more like long-term bonds than claims to dividends discounted at a constant rate. That fact alone, which pervades empirical finance, matters for portfolio theory. “Correspond” is all we need. )

This fact means that stocks are not as risky for long-term investors as one might think. How can we express that fact? We might try again to say that stocks are natural hedges for their own intertemporal opportunity state variable risk. Low expected returns are bad news, but when that happens prices go up enough to fund the investment despite low returns. Price declines are bad news, but when that happens (are you still listening, honey?) expected returns going forward are better, so we’ll make up the losses. This is the fact beautifully expounded in Campbell and Viceira’s, and others, many papers, and Campbell and Viceira (2002).

Hmm, the spouse is looking dubious again. Do we want to rehash the time you shorted bitcoin?.

Why don’t we say that stocks pay a much steadier stream of dividends than the volatile prices might suggest? The flip side of return predictability is dividend non-predictability – the two facts are linked by an identity. But it might be more believable and understandable to say that low prices without a change in dividends, do not signal lower dividends ahead, so the strategy of just holding the stocks and eating the dividends is relatively safe.

Dividends are obviously driven by overall economic growth, not market gyrations. We all have a sense of how risky that is, and when that risk might kick in. If we’re just going to eat the dividends, we don’t have to worry about when stock prices might rise or fall.

Much of our accounting and finance practice and intuitions are built on the idea that prices behave like random walks, that expected returns and variances are constant



over time. But much of what people do, which we might have formerly disparaged as irrational, makes sense when risky assets are a lot like long term bonds. So, if looking at the cashflows that result from an initial investment instantly clarifies long-term bond investing, let us pursue the same perspective generally.

When evaluating bond managers, about the silliest practice I can imagine is to evaluate them by alpha and beta, even relative to a bond index, as if returns and alphas are independent over time. Like bonds themselves, any deviation of past return from the index is surely at least in large part a sign of positive returns ahead – and vice versa.

Why do so many institutions, like our endowments, prize private equity, venture capital and real estate with no clear market values? Well, perhaps they like those assets precisely because they're hard to mark to market, easy to just pay out 5% of a made-up value, not to sell in a panic and not fire the asset manager based on an irrelevant price – or rather a price that is only relevant when combined with the state variable, which accountants and oversight committees are no better at than our hypothetical spouse. (The chance to cut reported volatility in half with smoothed markings may also be a bit of a reason for this preference. See Stafford (2017). And while highlighting one, I do not mean to disparage all the other explanations for this preference.)

I used to try to help my mom with her small retirement investments. She would often complain that stocks don't pay much dividends anymore, so how was she to live? She was of a generation that believed one should never dip into "principal." I patiently explained that there is no difference between dividends and capital gains, and it's ok to sell some stock these days. But in a world of time-varying expected returns, my mom has a point. Dividend yields mean revert, so in the end the long-run return of a stock is the stream of dividends divided by the initial price. Living on the dividends is exactly the right strategy for the indexed perpetuity investor, and not a bad strategy for the long-term equity investor.

Why do stocks pay dividends? It's a longstanding puzzle. If you go back to the world my mom grew up in, of high transactions costs and long-term investors, when you had to hold physical shares in a safe deposit box, it makes sense to sell stocks to, say, my mom, that automatically include the amount they should take out and live off of. In turn, this observation may help us. In principle, dynamic trading can create many "dividend streams" out of an initial set of assets. But corporations and the finance industry have

every incentive to create and market the kinds of dividend streams people want. Why do firms smooth dividends? Because my mom liked a steady check!

In a sense the *definition* of “long-term investor” might be one who buys a security for the stream of cashflows, not for a hope of price appreciation.

Shiller (2014) advises people to buy stocks for the dividends. You can read his famous graph, updated there, of volatile stock price on top of very smooth plot of dividends as a plot of the very steady stream of consumption that stocks can support at any date by just consuming the dividends.

Our accounting systems are to some extent to blame. Under the early rational expectations efficient market i.i.d. returns view, accountants wisely started more and more marking to market. But often you can only mark the assets to market, not the liabilities, and no accountant can mark to market state variables that say “yes, we lost a lot on a mark to market basis, but these assets will gain back their value in time.” Accountants have resorted to tricks, like putting some assets in “hold to maturity” buckets to avoid marking to market. The desire for tricks reveals the problem.

Insurance companies are not required to mark to market their portfolios unless there is “impairment,” a permanent loss of value. They can hold assets at the booked purchase price through maturity, and they don’t need to flow into the income statement the fluctuations in their market value (other than impairment). They tend to care only about the income their portfolios produce, like my mom. They liquidate appreciated securities when they need to meet their income needs, But that is not done often.

When I pointed out that stock prices were high, so she could sell, my mom would reply, “Oh, those are only paper profits,” which sounds like another classic fallacy. I explained the conventional wisdom that the term is meaningless. Prices are prices, and you can sell at those prices! Once again, my mom might have been right. She would be exactly right for a long-term bond portfolio. If an indexed perpetuity price rises, that fact has zero implications for her, the long-term investor. If bond prices doubled and she sold bonds, she would get a high value, to be sure. But the present value of her consumption path doubled as well, so it would take exactly double the wealth to maintain the same consumption path. Regarding the price rise as “paper profits” guided my mom to the correct Merton decision.

Houses may be a good example halfway between stocks and bonds. House prices

vary a lot. Yet most of us do not market-time housing. Yes, the transactions costs are larger. But the main reason is, we have to live somewhere. The house you want to buy goes up just as much as the house you want to sell. Houses are perfect hedges for house-price investment-opportunity variation. Indeed one of the major benefits of buying a house is that you are protected from rent variation, and can live there as long as you want no matter how high rents go. A house is an indexed housing services perpetuity. (The absence of long-term rental contracts or rent insurance is interesting. One should be able to buy a rent swap, a financial contract that pays, say, the index of rents in a geographic area against a constant flow.)

This observation says much about wealth inequality by the way. To the extent that stocks are like long term bonds, a higher price/dividend ratio means nothing about future dividends and everything about lower expected returns. Then a higher price cannot fund any difference in permanent consumption stream. A huge amount of the increase in measured wealth inequality is just this - - higher prices for the same payoff streams, as the riskfree rate has plummeted at least 4 percentage points, and risk premiums have also compressed. It is then also just “paper wealth” inequality, as lifetime and dynastic consumption inequality does not change at all.

Merton’s theory is not just a portfolio theory. It is an integrated theory of portfolios and consumption, or portfolios and payout strategy. People follow its consumption advice even less than they follow its portfolio advice. In an i.i.d. world, Merton portfolio theory says to consume in proportion to wealth, which is the market portfolio. That means consumption growth should have a volatility of 20% per year not 1% per year. Estimates of the effect of higher stock values on consumption find very small numbers. (For example Lettau, Ludvigson, and Steindel (2002).) Indeed just about everybody ignores “paper profits” in their consumption planning. By not paying attention to this prediction of the theory, finance missed the equity premium puzzle for 20 years, merrily fitting portfolios with a risk aversion of two and not wondering why consumption volatility is less than two percent, not nearly 20 percent.

Perhaps people are right, and our usual calibration of the Merton model is wrong. Perhaps all increases in stock market values beyond what is reflected in consumption are indeed transitory, “paper profits” that cannot sustain additional consumption.

So, clearly, we should generalize Campbell and Viceira’s insight. If looking at the

stream of coupons and the initial price makes dynamic portfolio theory for the long-term bond investor crystal clear, so looking at the stream of payoffs, the stream of consumption that a portfolio can support, divided by the initial price should make dynamic Merton theory for the long-term risk-tolerant investor crystal clear.

I focus here on the first half of Campbell and Viceira's advice – stocks are relatively safe for long-term investors, stocks are hedge portfolios for their own investment opportunities. Campbell and Viceira also show that even long-run investors, whose desired duration and consumption path does not exactly match that of aggregate dividends, may wish to market-time, investing more when the market is low and less when the market is high.

However, the hedge portfolio advice strikes me as more interesting and robust than the market-timing advice at least for the aggregate market. Market-timing based on the aggregate dividend yield is a very slow-moving strategy, as the dividend yield takes decades to mean-revert. We don't really know where the average dividend yield is, so whether one is above or below average is questionable. Real time evaluations of the market-timing strategy, including the need to estimate the relationship using only past data, are not encouraging. Formalizing this observation with learning, Bayesian estimation, structural shifts, and so forth, makes market timing more dubious still. The hedge portfolio logic only requires us to think dividend yields are stationary, but not what the long-term mean is. It only requires us to think dividend yields do not predict dividend growth, but how much of that failure reflects return predictability and how much reflects dividend yield autocorrelation is irrelevant. Finally, we can't all market time, so it is advice that can only apply to half of the investors, with the other half taking the opposite advice. So, in my advice for long-term investors, I deemphasize this part of the strategy.

Now, the managed-portfolio theorem reminds us that firm dividends really do not matter to the portfolio problem, and there really is no difference between dividends and capital gains. The statistical character of returns and cashflows attaches to characteristics, not stock name and dividends. A dynamic strategy can synthesize many different cashflows out of portfolios of securities. For example, a fund could market-time, or invest in a dynamically-managed portfolio of stocks, pay out a 5% payout rate of total value, and that payout becomes the fund's "dividends." Of course, as before, firms may already do that. We might consider large corporations and their dividend payout policies already

as managed portfolios that provide a dividend tailored in some way to investor's desires. But funds can potentially improve on that effort, or tailor it to different investors.

But I think this too is an insight, an advantage, and a good way to do things. It forces us to look at the long-neglected payout policy part of Merton theory. Our endowments spend hundreds of millions on fees for portfolio managers, but then pay out a constant 5% of mark to model value, and freeze hiring in recessions. Let us look a lot harder at payouts, and their state-contingency!

In the end payout policy transfers consumption over time, just as portfolio choice transfers consumption across states of nature. If we look at time and state of nature symmetrically, we should consider payout policy and portfolio policy together. And we should evaluate the end result and match that result to investors.

There is a tendency to separate portfolio payout from consumption: Here's your 5% payout now you decide when to consume it. But anything not consumed today is put in a bank account or other investment to be consumed later. Put the bank account in the overall portfolio, and payout really is consumption.

Perhaps this observation reveals another tendency, to think only of certain assets as the "portfolio" and to ignore the others. Universities in 2008 discovered that their endowment portfolio was not distinct from their borrowing against construction projects. The need to roll over municipal debt and maintain a decent rating, or in some cases make good on swap contracts involving such debt, provoked some of the endowment and budget panic of that year. And in some sense my biggest advice will be to fold outside assets and liabilities, outside business income to a private investor and professors' salaries to a university, into the concept of the portfolio. Merton's advice applies to the whole thing, not one piece of it.

The Merton consumption, or payout advice, says to consume in proportion to the marginal value of wealth,

$$u'(c_t) = V_W(W_t, X_t)$$

where here  $X_t$  denotes the state variables. It is inconsistent to advocate state variable thinking in the portfolio, but to ignore it as if returns are i.i.d. in the payout policy, consume in proportion to wealth. (There are special cases that deliver this result, but they are special, and at least you have to think about it. More later.) For example, suppose interest rates fall, bond prices rise, and our indexed perpetuity investor starts selling bonds

to consume a constant fraction of the now higher market value. Sooner or later, he or she will have to reduce consumption before the pre-planned risk-free value. The consumption rule must react to state variables just as does the portfolio rule. In the example, the effect of wealth and the state variable, bond yield, exactly offset.

The managed portfolio theorem suggests a sensible dividing line between financial engineering, portfolio selection, and marketing and tailoring a combination of funds to the particular investor. Forming and managing the portfolio, calculating appropriate payouts, hedging state variables, and then cleanly characterizing the available payouts as a stream of “dividends” tailored to the individual might well belong under the hood of expert management, and worth a fee. Management companies now sell target date funds, annuities, and defined-benefit pensions. Let’s bring that practice into our current understanding of financial markets and heterogeneity!

Under the hood, one may well wish to take a dynamic programming, state-variable approach. A bond fund with a more complex portfolio than an indexed perpetuity, or a more interesting set of payouts such as annuities, should definitely use yields as a state variable, and model how changes in yields are likely to affect returns and payouts going forward. Likewise, a value fund, recognizing that book/market ratios forecast returns over time as well as (or more strongly than) the cross-section, should include the time-variation in book/market ratios as a state variable, and use that to forecast returns and cashflows, in order to determine its payout policy. And educated customers need to understand and believe in what the fund is doing. Modeling does have to be part of the business – figuring out if price declines, just how much returns will rise, or equivalently figuring out just how much dividends have fallen, and what will happen if those forecasts are off, is a crucial part of this cashflow-synthesis business. There is still a lot we can do to bring standard Merton theory to practice, and that includes bringing state variables in to the calculation. Still, I think it remains useful to communicate, and for the final investor to contemplate and evaluate the portfolio plus payout decision by the character of the stream of payoffs it can produce.

Currently, funds including “factor beta” funds such as value characterize their performance for investors by alpha and beta relative to a market index such as the S&P500, and do not (or do not dare) to promise that low state variables today mean higher returns in the future. That characterization is next to useless for long-term investors. Alpha and

beta relative to the S&P 500 is only interesting if your portfolio, including your outside income and liability streams, consists only of the S&P500. Such alphas and betas are only interesting if your objective is one-period mean-variance optimization, i.e. if you care about the discount-rate betas, but the discount rates are not a state variable to you. They are only interesting if you are doing alpha-beta risk management and evaluating a whole portfolio on that basis. Our investors do not hold that portfolio and they do bucket-based not beta-based risk management.

Alpha and beta relative to a factor index such as a value index is a little more interesting. But only if that the deviation from the index,  $\alpha + \epsilon$  is uncorrelated over time and unforecastable, which it is likely not. Like bonds, if my fund's value stocks fell more than the index, they are likely deeper value than the index and thus even better bargains. The question for the long-term investor is, in any case, which of the dozens of factor indices to buy. Alphas relative to those indices are icing on the cake. First, let's figure out how to bake the cake.

To be concrete, let's take the hard job of a value fund manager in late 2020. It's a lot like our beleaguered finance professor trying to explain the indexed perpetuity to an incredulous spouse. If value stocks fall, and their dividends, earnings, etc. do not fall, value stocks are likely an even better deal than before. (Time-series value is stronger than cross-sectional value. Book/market variation over time predicts subsequent returns of the same portfolio more strongly than book/market variation across portfolios predicts returns. See Cochrane (2011) Table AIII.) The same is true of funds that operate on convergence trades or long-term arbitrage opportunities. But going to client meetings, explaining that you've just lost a ton of mark-to-market money, but the state variables just keep getting better and better, is awfully hard! "Paper losses" don't have quite that much ring in a modern client meeting. And not irrationally so. Any ambiguity or learning about the process puts weight on realized returns. But the same learning and statistical doubt would not believe that indexed perpetuity bond returns mean-revert as well.

If the conversation were about long-run payouts, the whole business might be a lot easier. It's still not easy – unlike a bond, one needs to model the long-run payouts. But it seems a better way to frame the conversation.

Looking much harder at the risks and risk absorbing capacity of the liability stream or planned consumption (and philanthropy) stream is a vital and overlooked component

of the portfolio process. Liability streams are not fixed, and often really a choice rather than a fixed set of payments. If an institution is not going to buy indexed perpetuities, then the supported income stream is risky, and will have to be cut back at times and states of nature dictated by the portfolio and payout strategy.

So, a university that invests in risky assets should be sure that the payment streams supported by the risky payouts can absorb the risk. Tenured professor salaries are, in the current institutional framework, hard to cut in bad times. Adapting to risk by cutting back investment in new faculty may not be the smartest way to absorb risk. Flashy new centers, staff, sexy initiatives, non-revenue teaching, outreach, “impact” programs, and related activities are easier to scale up in good times and scale back in bad times. Whatever one thinks of such activities, universities do a lot of them. If they can bear risk, they are better suited for funding from the risky payoff streams of an endowment invested in risky assets. A sovereign wealth fund might think likewise about which streams of payments can bear risk sensitive, and which can not. Many wealthy individuals invest, on the margin, to fund philanthropic ventures. Their preferences about such funding may well be quite risk tolerant. That fact leads to a strong positive beta in donation flow, another stream that university and nonprofit endowments need to think more about in their risk management. On the other hand, a defined benefit pension fund, an individual with a lot of debt, for example mortgages on houses that he or she really would be pained to sell, needs to find other consumption streams that can bear risk.

This remains vision, and it is all easier said than done. I tried to extend Campbell and Viceira (2001) in Cochrane (2014). If the indexed perpetuity is the riskless asset, then the consumption claim, that pays one unit of aggregate consumption, must be the risky claim and held by the average investor. People must mix their portfolios between these two claims, with more risk averse people taking more of the indexed perpetuity, and vice versa. And all of this must hold with an arbitrarily dynamic and multifactor view of the world.

Conceptually, the paper suggests we extend one-period asset pricing to multiple periods by looking at streams of payoffs, rather than make the one-period terminal utility more complex, as the value function of a dynamic program. We treat date and state symmetrically.

Let us write intertemporal dynamic portfolio choice as a choice of the final payoff



or consumption streams,

$$\max_{\{c(s_t)\}} \sum_{t=0}^{\infty} \sum_{s_t=1}^{S_t} \beta^t \pi(s_t) u[c(s_t)].$$

In a complete market, we can write the constraint

$$W = \sum_{t=0}^{\infty} \sum_{s_t=1}^{S_t} \beta^t \pi(s_t) m_t(s_t) c(s_t).$$

The optimal payoff is simply

$$u'[\hat{c}(s_t)] = \lambda m(s_t).$$

This condition fully describes the optimal payoff over time just as it does across states of nature, despite arbitrary dynamics, state variables, multiple factors, and so forth, Markowitz portfolio theory can describe the Merton environment, if you treat date and state symmetrically. This is well known, but less useful as we do not live in a complete market.

The challenge is to do the same sort of analysis in an *incomplete* market. Start with a set of basis payoffs  $x(s_t)$ , which can be expanded by dynamic trading and payout strategies. Then the constraint is

$$c(s_t) = \sum_i w_i x_i(s_t), \sum_i w_i p_i = W$$

Again, static Markowitz portfolio theory applies transparently to the dynamic Merton environment if we look at the payoff streams. I made it look even closer by defining “long run” moments that sum over date and state symmetrically,

$$\tilde{E}(x) \equiv (1 - \beta) \sum_{t=0}^{\infty} \sum_{s_t=1}^{S_t} \beta^t \pi(s_t) x(s_t)$$

Likewise we can define long-run variances, covariances, betas, and so forth.

Though the environment is arbitrarily dynamic, all the standard one-period characterizations obtain. We can define a long-run mean / long-run variance frontier, price assets from long-run betas, and so forth. But it all has a much different meaning. The long run mean, variance and covariances are of dividend streams divided by initial price. We look at asset prices, driven by the long-run covariance of dividend streams with the marginal utility stream, in place of one-period returns. Hedging outside income streams

is easy too, discussed below.

That long-horizon perspective can make a big difference. Consider the problem of betas, central to risk management and portfolio formation. Another great puzzle is how little we know about betas. In continuous-time diffusion theory, 10 seconds of millisecond data should be enough to measure betas with nearly infinite precision. In fact, betas are hard to measure and unstable over time.

Much of the reason, I think, is that most betas are discount-rate betas, not cash-flow betas. (Or they are sentiment betas, if you're in a behavioral mood.) Betas measure how one price moves with another price, while the dividend streams do not move at all. Most of this variation is irrelevant for our long-term use.

More generally, we are at a crossroads. Campbell and Viceira, and the related literature, have done a good job of adapting Merton theory to what we now understand about the evolution of the market, and perhaps a few styles like value and growth, over *time*. But applying Merton to our current understanding of the *cross section* remains *terra incognita*. Perhaps this payoff focused view will let us convincingly describe cross-sectional portfolio theory, overcoming the instability of  $\Sigma^{-1}\mu$ .

Alas, after (in my view) an inspiring introduction, my paper foundered a bit. The linear marginal utility or linear-quadratic approximation that one needs to generate two-fund separation in an incomplete market is a difficult fit to power utility and lognormal long-term returns. So the statement that investors who differ by risk aversion split their payoffs linearly between the indexed perpetuity and the aggregate consumption claim is not a great global approximation to the power-lognormal environment. For example, when investors have power utility, the complete-market sharing rule is

$$c_i^{-\gamma_i} = \lambda c_a^{-\gamma_a}$$

where  $i$  and  $a$  are individual and aggregate. Log consumption is a linear function of log aggregate consumption, but the level is nonlinear.

So, a linear two-fund payoff theorem may survive as useful conceptual benchmark. But you can see in this sharing rule that it may not be a good approximation far from the endpoints, indexed perpetuity and market dividend claim. I leave a better formal analysis an invitation for smarter people than I am to follow if, as I hope, I have persuaded you it's a good idea either for calculating or for marketing consumption-portfolio

advice and especially outside-income hedging where we do not have mark-to-market values. (Cochrane (2007) and Cochrane (2013) include many seemingly obvious paths that don't work. If you get inspired, see here for a list of things not to do.)

One may object that it takes a long time to evaluate whether the stream of dividends accruing to an investment is good or not, or to calculate the long-run mean, variance, and covariance of long-run cashflows. But it takes a lot of time to evaluate average returns too. The tyranny of  $\sigma/\sqrt{T}$  ensures that. Measuring and evaluating payouts will have to include some modeling, understanding how state variables like bond and dividend yield forecast future returns or cashflows. And prices never really go away for finite horizons and data. You can have a great stream for a while if you plunder the terminal value. But a less precise and model-based estimate of the right object, the cashflow stream, can be more useful than a direct and more precise estimate of something that doesn't matter, one-period return moments.

The long-run perspective also changes a good deal what is important or not. Translating returns to prices can make a big difference if return differences are persistent. For example, at a price/dividend ratio of 50,  $P/D = 1/(r - g)$  gives us  $r - g = 2\%$ . So a 20 bp change in return equals a 10% change in price. Many people don't get excited about a 20 bp change in average return. (Especially those paying 2%+ 20% fees!) But increasing lifetime consumption by 10% is something to write home about. On the other hand, things that loom large to the highly levered, high-speed trading and transient anomaly end of finance fade from interest, as they make little difference to the price of long-term cashflows.

At a simpler level, dividends are what matters for long-run returns. The price-dividend ratio mean reverts, so all of the eventual return comes from dividends divided by initial price, not from capital gains. All unexpected return comes from unexpected dividend growth. We can see this fact by rearranging the Campbell and Shiller (1988) decomposition, in logs

$$\sum_{j=1}^{\infty} \rho^{j-1} r_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} \Delta d_{t+j} - (p_t - d_t).$$

Even more simply, we can just write the definition of return,

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t} + \frac{D_{t+1}}{P_t}$$

In the long run, the price dividend ratio reverts to its mean,  $P/D$ , so, loosely (!) a very long-run return becomes

$$R_{t+1} = \frac{P/D}{P_t/D_t} \frac{D_{t+1}}{D_t} + \frac{D_{t+1}}{P_t} = \left(1 + \frac{P}{D}\right) \frac{D_{t+1}}{P_t} = R \frac{D_{t+1}}{D_t} \frac{D_t}{P_t}.$$

Dividend growth is what matters to long-run returns. Fama and French (2002) use this idea to measure the equity premium. The postwar period saw a large increase in the price/dividend ratio, which will not go on forever. One may also argue that even the postwar dividend growth was unexpected, representing decades of stronger economic growth than the world has ever seen or was expected in the aftermath of WWII.

In sum, the challenge of contemporary portfolio theory is to move from a one-period understanding to a multiperiod, dynamic view. This generalization is easy if the world is i.i.d. – just play the one period over and over. But our world is not i.i.d., so one-period tools don't work. Merton extends the one-period world to a dynamic world by dynamic programming, adding using state variables that capture changing conditional distributions to the terminal utility, which is the value function of the dynamic program. Now, dynamic programming often does capture common intuition and behavior and offers a digestible way to operationalize dynamic maximization. When someone says “I can't afford to go out to that fancy dinner,” they're trading  $u_c(c_t)$  for  $V_W(W_t)$ , marginal utility of wealth. Keeping a rough sense of the marginal value of money around is a much better heuristic for decision making than trying to trade off date and state contingent consumption – “I don't want to go out to a fancy dinner tonight because I might want to buy a better class of RV when I retire in 40 years.” But in the portfolio case, we don't know really what the state variables are, the derivatives of the value function don't have such an easy interpretation, and they depend on the environment as well as preferences. Viewing the problem as a selection among cashflow streams may be an easier way, at least to implement and market portfolio theory, if not directly to compute portfolios and recommended consumption.

### 3 A general equilibrium approach

The second conceptual aid I want to offer is that researchers, investors, and advisers should routinely state a view of the economic function of financial markets, their place in markets, what activity or risk bearing generates a reward, what justifies doing something different from everybody else. In our language, portfolio theory and application should describe a general equilibrium, and how investors are heterogenous.

Why, you may ask? When you buy tomatoes at the grocery store, you don't need to think about general equilibrium. If tomatoes are expensive, buy less of them. Who cares if they are expensive because of a frost in the tomato farms or a behavioral bubble in the tomato futures market? Consult your preferences, look at the price and buy accordingly. This is exactly the Merton approach in financial markets. Whether expected returns are low because of a behavioral bubble or because everyone else is less risk averse or because of a savings glut in China makes absolutely no difference to your portfolio calculation. Whether markets are efficient, behavioral or institutional makes no difference.

But, asset markets are different from grocery stores in one crucial respect: *We don't know the prices.* We don't know the expected returns and betas that appear in Merton portfolio theory. Expected returns take decades to learn, thanks to  $\sigma/\sqrt{T}$ , and then they change. The factor structure of returns is devilishly hard to measure, and also varies over time. (Again, I suspect that is because so much of it represents discount-rate betas. I hope that long-run cashflow betas are more stable and sensible. But nothing in finance turns out the way you think before you look.)

Maybe a better analogy: We know asset prices, but we don't know what we get for that price. It's as if the store says tomatoes are \$5.00, but you have to take them home and wait 20 years to find out how many tomatoes you actually got for your \$5.00.

Now if you can't see the prices and have to do a lot of statistics to guess what they are, thinking about general equilibrium can help to make a shopping decision. If you know that tomatoes grow on farms, that they are ripe in August, and these facts drive the seasonal behavior of tomato prices, then you know buying a lot of tomatoes in August when the posted price is low is likely to be a good idea.

General equilibrium thinking starts with a deep, powerful and frequently overlooked theorem:

- *The average investor must hold the market portfolio.*

The first implication: If you are not identifiably different than average, then you should hold the market portfolio. You should not even rebalance. If stock prices go up, from 60/40 of total assets, to 80/20, and you rebalance to 60/40, someone else has to overweight equities. Rebalancing is a market-timing strategy.

The average-investor theorem is powerful. Portfolio theory is hard. You have to estimate time-varying means, covariances of asset returns, estimate alphas and betas, and make a difficult computation, and do a lot of massaging to keep it from blowing up. But if you know that you're no different than the average – or if you don't really know you are different and how – you're done, you know the answer. Off to the total market portfolio with you.

Indeed, if you have ever tried to do a portfolio optimization, even at the level of MBA class problem set, you know how essentially impossible it is. The optimization is sensitive to small variation in mean returns and covariance matrix. If you want the maximization to come up with anything like the market portfolio, you have to reverse-engineer that result by forcing expected returns to equal beta times the market return. Starting with the market portfolio, and then isolating respects in which you are different from average is a much more promising way to construct a portfolio.

A lemma:

- *Any deviation from the market portfolio is a financial zero-sum game.*

For everyone who gains, or earns an alpha, someone else must lose. I add the “financial” qualifier because, the whole point is to find deviations that make sense for everybody in a marginal utility sense. But that only happens if we are different from each other in some important sense.

- *Portfolio theory must be all about heterogeneity.*

So how are you different than average? Everyone thinks they're smarter than average, or less behavioral, or better informed. Half are, by definition, wrong about that.

Well, just hire a smart person. Sorry, the finance profession has free entry of charlatans at all levels. (This is now a technical term, see Berk and van Binsbergen (2020) for an equilibrium model of charlatans.) You have to be better than average about picking

managers. Half of the people are wrong about that too. Maybe you think you deserve alpha because you golf with this brilliant options trader named Bernie. Your social connections are better than average. Half the people are wrong about that too.

In turn, a good test of a portfolio theory, a portfolio maximization program, or portfolio adviser, is that there must be a question to which the answer is "do nothing" – hold the market portfolio without rebalancing. This is a vital placebo test.

Moreover, if the program or adviser says "you should buy value and short growth," then ask, "and who are you telling to short value and buy growth?" If the answer is "nobody," at a minimum the portfolio view is not fleshed out. If it's honest, then it's useless for almost all investors – the vast majority of people are solving some other portfolio problem, which gives the "buy growth" answer. Are you sure you're not one of them? If it's not useless, then it's back in the land where all the children are smarter than average. At best, when they take the advice and wise up, the phenomenon will disappear. "Buy value" is clearly a much more stable portfolio advice if we know the question whose answer is "buy growth," and the question whose answer is "hold the market," and we can check that this is not our question.

The average investor theorem offers one excellent piece of advice. At least by just indexing you can protect yourself from being below average! The dismal literature on active management, constantly reminding us that there is essentially no performance better than indexing, is a powerful incentive to remember the average investor theorem. At least avoid being the "liquidity" trader who the "informed" traders and "market makers" pick off.

In sum, the average investor theorem suggests a few related tests for portfolio formation:

- *Apply the placebo test.*
- *Apply the look in the mirror test.*
- *Apply the dinner with lions / look around the table test.*

The placebo test is, again, the test that there must be a well defined question to which the portfolio advice spits out, hold the market. If you're buying someone else must be selling, and he or she has a reason to do so. Maybe they're more behavioral than you, or less informed, or dumber. Maybe they have some exposure to state variables you don't.

Maybe they have a “liquidity need.” But first, look in the mirror. Are you sure you’re not the one who should be selling? When having dinner with lions, make sure you’re at the table not on the menu. Someone is. When sitting down to play poker, find the mark. Make sure it’s not you.

Cross sectional portfolio theory is hard and  $\Sigma^{-1}\mu$  unstable, so if one deviates from the market portfolio, thinking, “What are the risks of this security, why are others unwilling to hold that risk, and why am I the right person to do so?” is the heart of the cross-sectional advice.

General equilibrium, or at least some economics, should be useful also in guiding intertemporal thinking. I have simply asserted that stocks are a lot like bonds, that expected returns vary over time and higher, say, price-dividend ratios or market/book ratios correspond to lower expected returns. But time-series predictability and the states of nature when it goes wrong are if anything more statistically fraught than characterization of cross-sectional means and the risks those pose to investors. Out of sample prediction doesn’t fare too well. Even without the siren song of market timing, faith in state-variable hedging, or will to mark future returns to market and produce a rosy estimate of the dividend stream that a long-term strategy provide, might be excusably weak. Will a low stock price really mean revert, as a low bond price does? Equivalently, are we really sure that a low stock price does not presage lower dividends? If one states a general equilibrium view of the causes of discount rate variation, that can give one more confidence in state variables. For example, in the fall of 2008, one can clearly see many other investors dumping securities, as they get nearer to balance sheet capacity or default boundaries. The smell of time-varying risk aversion was in the air. Such thinking gives an investor who does not have such constraints, or reason to become more risk averse, confidence that this is a time of higher time-varying expected return. And such thinking invites a look in the mirror to make sure our investor really does have more risk-bearing capacity than the average.

### 3.1 Portfolio theory and asset pricing

The last 50 years have seen immense progress in asset pricing, both theory and empirical work. Much of work this still awaits integration with portfolio theory. Simply throwing every anomaly in the soup and sending it off to a Merton portfolio optimizer is obviously



a bad idea for all the above reasons.

Why has so much asset pricing and empirical work since the ICAPM not translated into parallel portfolio theory and application? In part, I think because we simply have not asked them to do so. We have mostly used new theories, including new preferences and technology specifications, to ask the opposite question – what asset price variation do we reverse-engineer so the portfolio theory answer is just to hold the market portfolio?

That is already interesting. It reminds us that perhaps we too are not the Merton investor, and have the same novel preferences or face the same opportunities as everyone else, so what look like delicious trading opportunities are in fact prices that should guide us to hold the market portfolio just like everyone else.

But asset pricing per se does not give any portfolio advice other than that the average investor should hold the market, no matter how many the asset prices are. Here, I think the brilliant insight of Lucas (1978) may have taken us too far. Before Lucas, we thought of asset prices with the traditional general equilibrium framework: write down supply and demand (portfolio) curves, then intersect them to solve for asset prices. The CAPM and mean-variance theory came hand in hand; the ICAPM and Merton theory came hand in hand. Lucas taught us to skip portfolio theory altogether, and just read asset prices from first-order conditions in equilibrium – when the investor holds the market portfolio. That was a tremendous shortcut for asset pricing, but it put portfolio theory a bit in the rear-view mirror.

So, let us survey quickly some of the new asset pricing theories, or views of the function of markets, buttressed by facts. Let us think about how to introduce relevant and interesting measures of heterogeneity. Let us think about how to complete them – many theories don't describe a full set of agents, or supply conditions. And then we can speculate how the new understanding of asset pricing might change portfolio theory, focusing here on long-run investors.

### **3.2 The classic view and updates**

I start with the classic view, completing Merton portfolio theory with CAPM or ICAPM general equilibrium, to get this way of thinking going. Here, the purpose of financial markets is to intermediate between investors managing portfolios of wealth, with time separable expected utility, no substantial outside income, and firms that need to issue

equity to finance investment. In the classic i.i.d. case, investors differ by risk aversion and hold more or less of the market portfolio and riskfree asset accordingly.

This general equilibrium view is already useful for practical portfolio advice. If investors are to answer “What’s my risk aversion?” it’s awfully hard to come up with a number. Various surveys and questionnaires to elicit such a number are only fodder for behavioral finance in their inconsistency. Like rats in a maze, we are as-if maximizers not self-aware calculating automatons. But if investors have only to answer, “Are you more risk averse than the average investor?” perhaps we can make progress. Of course, beware surveys. Not everyone can be less risk averse than average either! And people who state low risk aversion in good times have a remarkable habit of changing their minds in the tough after-the-big-loss meetings with their managers. But that fact only suggests time-varying risk aversion which we’ll get to later.

Moreover, as before, actual portfolio maximization using average return and covariance matrix estimates is devilishly unstable. By posing the question in this general equilibrium form, we reduce the problem to a simple two-fund question. That is the greatest simplification.

However, this model fails as an empirically relevant general equilibrium model. If everyone were Merton (power utility, no outside income, common information) investors, it’s awfully hard to have a market that produces the kind of multifactor time-varying opportunities we see. Why are all these opportunities there? Why didn’t everyone else take them? Whose portfolio demands are causing these apparent opportunities to emerge in the first place? If you can’t answer these questions, then you should really wonder whether the advice is right.

Substantial progress has been made applying and extending the basic Merton framework, recognizing heterogeneity beyond risk aversion, and some of this progress has made its way to practice. I’ll again let Campbell and Viceira (2002) be the exemplar of a huge literature. They largely implement Merton-style theory, though with recursive utility, in a statistical environment that captures broad newly-understood dynamic features of asset returns and avoiding the nettlesome  $\Sigma^{-1}\mu$  step.

In the classic i.i.d. world, portfolio allocation is independent of horizon, and we disparaged conventional wisdom to the contrary. Hedging demand and calibrated return predictability from valuation ratios rescues another piece of disparaged practical

world advice. Long-term investors who can “wait out market downturns” should invest in stocks, just as their risk-averse long-term investor should invest in indexed perpetuities. Stocks are like bonds, a good hedge for their own expected-return risk. Campbell and Viceira quantify this effect.

But people and institutions may now differ in terms of investment horizon, not just risk aversion. A one-period investor does not care about state variables. State variables and hedging demand are only important for investors with longer horizons. And now we have an additional dimension of heterogeneity to violate the average-investor theorem.

The investment industry has responded to academic advice and horizon heterogeneity on top of risk-aversion heterogeneity by implementing target-date funds, that invest more in equity when people are younger, and then gradually move to an all-bond investment when people retire. However, in the meantime academics have become less sure that is the right advice, or why it is the right advice. Yes, intertemporal hedging suggests some of this behavior. But why should people in very long retirements not wish any equity? Even with short horizons they are classic Merton investors who should take some risk for its return. Most people with any money to invest are also planning to make bequests or charitable contributions.

More deeply, much of the target-date strategy in industry is motivated by the idea that wage income is bond-like, so people with wage income should tilt more to stocks, and people who retire should replace the bond-like wage with bond-like investments. But it is no longer clear that wage income is a bond-like investment (more below). Such funds’ aggressive rebalancing also runs afoul of the average investor theorem, and according to Parker, Schoar, and Sun (2020) such rebalancing drives supply shocks of the institutional finance sort in equity prices.

### **3.3 Outside income risk, a giant insurance market**

Consider a different vision of markets and heterogeneity: Investors also differ by outside income risk or liability streams. The economic function of markets is a giant insurance mechanism for spreading risk around.

Most individual investors, especially weighted by wealth and thus market impact, own substantial non-marketed businesses or real estate. Our universities invest to fund

stochastic liability streams such as the difference between tuition, gifts, and government subsidies less our and burgeoning staff salaries. Pension funds and insurance companies invest to support liability streams. And human capital, the stream of wage income is perhaps the largest of all sources of non-marketed income.

Before chasing alpha, investors should hedge these outside income streams. That hedging demand moves the typical investor away from the market portfolio. It can move prices and expected returns away from the i.i.d. CAPM, and thus generate the multifactor and time-varying premiums that we see in asset returns. The few lucky pure wealth investors then can earn superior returns by selling the hedge portfolios, writing insurance to the rest. More importantly, those who are less exposed to a common factor in outside income streams sell insurance to those who are positively exposed.

Now, just *how* do you hedge an outside income stream? The advice to do it has been given many times. People and institutions don't do it. We need to think a lot harder about *how* to do it. Academic attempts to calculate such hedge portfolios have an awfully hard time doing it. (The literature review in Cochrane (2014) contains many valiant examples to calculate portfolios with outside income in incomplete markets.)

Here too, I think this goal is more easily thought of and implemented in terms of streams of cashflows rather than the properties of one-period returns.

The central problem in hedging an income stream is marking it to market. Most asset return variation is, again, due to discount-rate betas not cashflow betas. You don't see any change in the cashflows yet all of a sudden you're supposed to cut its value in half, and look for securities that also fall by half at the same time?

Suppose we had only the dividend stream of the S&P 500 in hand. None of us could come within a factor of two of the S&P 500 market price, and less so of changes in that price. So if your labor income stream, or your private business' profit stream were *exactly* equal to the S&P500 dividend stream, and you set out to mark to model, then search for securities correlated with the resulting monthly, quarterly, or annual returns, it would be very unlikely that you would figure out to hedge these streams by reducing your market allocation. In evaluating the portfolio strategy, there is little chance that you and your accountants would see this as a successful hedge a year later when the market rises 50% and you missed it.

Well, let's just look at the S&P500 *dividends*, or the cashflows from managed strate-

gies, and find those dividend streams that best match the labor income stream directly, ignoring one-period returns.

This method might produce a pretty thorough revision of the standard picture. The income streams of well-paid people or business owners are likely a lot like S&P500 dividends, and if anything more cyclical, and prone to disaster shocks in bad times, and correlated with the dividend streams of their firms or industries.

Where did we get the idea that wages are a bond-like investment? Well, because we are not very good at marking wages to market with the nebulous but highly time-varying discount rates that apply to marketed dividend streams! If you marked S&P500 dividends to market using a constant discount rate, they would look pretty bond-like too.

(Ideas to better complete markets by selling insurance for idiosyncratic wage, house price, rent, and business income complement this vision. But here let's think just about portfolio theory in existing markets, or at best how intermediaries in such an effort could hedge the aggregate risks in their portfolios.)

The long-run mean concept makes this idea transparent. Let the investor have an outside income stream  $h$ . Then the optimal portfolio  $x$  comes from solving

$$\max \tilde{E}[u(c)] \quad \text{s.t.} \quad W = \tilde{E}(mx); c = h + x.$$

In a complete market the optimal portfolio is

$$u'(\hat{x} + h) = \lambda m$$

which we can solve for

$$\hat{x} = u'^{-1}(\lambda m) - h$$

Sell off the hedge portfolio for outside income, then invest as before. But the purchased portfolio  $\hat{x}$  contains both components.

Cochrane (2014) takes up this idea in *incomplete* markets. You can easily construct a hedge portfolio for income  $h$  by running a long-run regression of the stream  $h$  onto the set of asset payoffs. The hedge portfolio is the fitted value of that regression. I also show how market prices are affected when a mass of investors has such outside income and adds up to a hedging demand.

This view is a coherent and plausible general equilibrium view. It explains why opportunities might be there for the lonely mean-variance investor: he or she is selling insurance to a mass of others whose incomes vary with a state variable, such as recessions, with a component orthogonal to the dividend stream of the market index. Fama and French (1996) offer this view as one cohesive explanation of the value effect and factor.

Given this view, however, it is a puzzle that finance theory and industry are so exclusively devoted to the small number of investors who have no outside streams, can profit by selling insurance to the others, and have not yet done so. If this is the view of the world, the majority of investors should want to buy insurance of one sort or another. At a minimum, for every investor we help to guide to value because his or her outside income is not correlated with value payoff streams, there should be another investor we help to short value, despite its attractive return (dividends/initial price), because that loss of return is a small premium paid for valuable insurance. The absence of the latter ought to be unsettling. But maybe that is only a failure of marketing, or a historical relic of the era before mass investing and institutional investing.

Moreover, in this view of the world, it is a puzzle that portfolio theory and practice does not focus on the *unpriced* factors, and that the pages of our journals are so completely devoted to finding priced factors. The most interesting portfolios are not the “priced” ones useful to the last mean-variance investor, but portfolios in which there is a lot of *heterogeneity* in risk exposure, and strong hedgeability, high correlations of the portfolio stream with the income streams of a large mass of potential customers. Whether such portfolios have S&P500 alpha is irrelevant. Indeed zero-alpha portfolios are better as they offer actuarially fair insurance.

Industry portfolios are a prime example. These portfolios are well priced by standard factor models, and not even badly priced by the CAPM. Helping people to short their own industry should offer valuable utility improvement, worth a nice fee. It should be the cornerstone of every 401(k) retirement plan, though it has no alpha to the few lucky investors who do not have a job.

To date, this general equilibrium view of the world has not fared that well. Historically value did not do that badly in recessions. The disastrous results of value funds in the last two recessions may restore the theory, if not the fortunes of value funds in a

world still driven by short-term alphas and ignorant of state variables! And this empirical evaluation is still based on one-period returns, not on the cashflows. If we look at value patterns in dividends – which is exactly the general pitch for value stocks; buy dividends at low prices – do we see that stream of dividends more closely correlated to the streams of incomes of many investors who shun value stocks? But these are future possibilities, not a summary of current understanding.

Much of value investing consists of deliberately not looking under the hood. When investors look at value stock names, companies that have fallen, companies with no news or glamor, companies on the edge of failure that statistically come back more frequently than not, companies just offering steady streams of dividends at low prices, those investors run away. Now, perhaps by forcing a quantitative selection over behavioral bias, value strategies allow people to buy what they would not buy if they looked under the hood, and all investors to buy stocks that others irrationally shun. But perhaps investors who shun value stocks when they see the names are seeing in those names some commonality with their own fortunes. But if it is wise not to look under the hood at company names, perhaps not looking under the hood at high frequency returns, at least without really understanding state variables and cross-derivatives of the value function, is also valuable discipline.

It helps, of course, to name the state variable and convince one's self that a low price really does not mean lower dividends, and does mean higher returns. But even Fama and French (2020), evaluating value's performance and thereby the chances of its future success, are strangely unwilling to do this. Fama and French evaluate value going forward by average past returns. They do not assess value going forward by the obvious question: Since value has done so badly recently, does lower market to book ratios over time within the value category mean higher returns in the future?

We move on, but I first emphasize the general point, more general than my two frameworks (streams and general equilibrium). Most income is not marketed, and most investors have outside income or outside liability streams. These streams are risky and heterogenous. It is likely that many such streams are hedgeable in markets. The second most important thing a portfolio strategy should do is to hedge that risk. (The first is not to pay taxes needlessly.) Such hedging does not depend being smarter than average, knowing something others don't know, harvesting alphas, and so forth. Yet hedging

outside streams is painfully absent in professional asset management. If anything many individual and institutional portfolios take on more of the risk they are already exposed to. At a minimum, 401(k) options or defined benefit plans can avoid their own company stock, and then their own industry! Universities with medical centers should not hold hospital stocks!

There is a great desire in the portfolio industry to sell individually tailored portfolios, and charge a fee for that tailoring service. There is some hedging, in the form of rate of return swaps or option positions for extremely wealthy individuals with concentrated ownership of companies they started. But their tailoring attempts for the rest of us are often so ineffective as to be humorous. For example, the Fidelity “change in plans” ad<sup>1</sup> featuring an annoying<sup>2</sup> couple who run back to their portfolio adviser because one of them wants to sell hobby art shows as much sophistication in finance as it does in the art market. Just what change in portfolio strategy is Fidelity going to offer in response to this news?

The heart of portfolio theory is to identify relevant heterogeneity. Yes, the news in this ad is silly. But more seriously, when I read investment companies’ required survey questions about my investment goals – “capital preservation,” “growth,” “income” and so forth – I, with some background in finance, have no idea what they’re talking about. I think academic portfolio theory can help to make this a serious enterprise.

### 3.4 Cashflow betas, factors, and state variables

The long-run, cashflow perspective has also begun to change our picture of asset pricing. So far this research has not led to much explicit portfolio theory, but it is clearly suggestive of new directions for portfolios. This effort nicely coincides with an effort to understand the value-growth premium in terms of the different character of their cashflows.

Following Campbell and Vuolteenaho (2004), the separation of beta into cashflow betas and discount rate betas, has become more and more widespread. Sensibly they find that cashflow news is more persistent and cashflow or “bad” beta generates a larger premium,

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<sup>1</sup><https://www.ispot.tv/ad/ZqHs/fidelity-investments-grandparents-song-by-tears-for-fears>

<sup>2</sup>[https://www.reddit.com/r/CommercialsIHate/comments/ja829m/the\\_obnoxious\\_fidelity\\_change\\_in\\_plans\\_couple/](https://www.reddit.com/r/CommercialsIHate/comments/ja829m/the_obnoxious_fidelity_change_in_plans_couple/)



Cho and Polk (2020) find that a long-run CAPM focusing on cashflows describes the cross section of prices much better than the cross-section of returns, and an interaction of book to market and quality provides a strong second factor.

Campbell, Polk, and Vuolteenaho (2009) find that cash flows of growth stocks are sensitive to market discount rate changes, while cashflows of value stocks are driven by market cashflow changes. Thus, growth stocks are good hedges against expected *market* equity returns. When the market expected return declines, that's bad state variable news. Market prices go up, and future growth-stock cashflows go up. So the market is a good hedge for its own state-variable risk, and growth stocks are an especially good hedge for that risk. This finding suggests that growth stocks may not be such bad investments for long-horizon investors even without outside income or other sources of heterogeneity. And therefore, when we add heterogeneity driven by horizon, long-term investors may want to hold hold growth stocks while short-term investors hold value stocks.

Growth stocks have growing, and thus delayed cashflows. Value stocks have steadier or declining cashflows with shorter duration. A big literature accounts for the value-growth premium in this way, for example Gormsen and Lazarus (2019). Again, premiums for near-term or delayed cashflows suggest a portfolio theory in which horizons should also matter for the cross section.

Betermier, Calvet, and Sodini (2017) find that individuals do obey many of the suggestions we have sniffed out so far for long-run portfolios, an unusual example contradicting my negative assessment of the theory's empirical performance. In a large panel of Swedish residents, they find that "households progressively shift from growth to value as they become older and their balance sheets improve. Furthermore, investors with high human capital and high exposure to macroeconomic risk tilt their portfolios away from value," which is "consistent with the portfolio implications of risk-based theories of the value premium."

### **3.5 Macro-finance, new preferences, state variables, rare disasters**

Merton portfolio theory is based on power utility. If investors were all like this, we wouldn't see the asset market factors and dynamics that we see. In the 50 years since Merton, lots of different preferences have been brought to finance to account for those factors and dynamics. Recursive utility (Epstein and Zin (1989)), habits (Campbell and Cochrane

(1999), and precursors cited there), ambiguity aversion (Hansen and Sargent (2001) is the first of a long series), aggregation theory bringing in cross-sectional risk to asset prices (Constantinides and Duffie (1996)), or cross-sectional preference variation (Garleanu and Panageas (2015)), and individual and economy-wide rare disasters (Reitz (1988), Barro (2006)) are just a few. Technology has gotten more interesting than endowments or linear production technologies that look like rates of return. Gomes, Kogan, and Zhang (2003) is an early influential example of a large literature.

It seems that fundamentally changing preferences or technology should change portfolio theory. Yet these advances have not yet made a large inroad to Merton portfolio theory. For example, Campbell and Viceira (2002) include recursive utility, yet this change per se has not yet substantially changed Merton's basic advice. Most of Campbell and Viceira's novelty comes from calibrating the input parameters to Merton calculations, finding ways to easily calculate the portfolios, and making clear sense of the results.

With log utility – both risk aversion and intertemporal elasticity of substitution equal to one – the investment is myopic, ignoring state variables, and the payout policy gives consumption proportional to wealth. A risk aversion substantially above one while keeping the intertemporal elasticity near one generates Mertonian portfolio advice while keeping the constant payout rule. Since institutions pay out a constant fraction, that may seem a realistic parameter configuration.

But it does not seem that institutions are doing much Mertonian state-variable hedging. They don't describe portfolio allocation in such terms. They report one-year returns. They do not report state variables or expected returns going forward based on changes in state variables. If we take their current behavior as optimal, they look pretty myopic, i.e. risk aversion near one. Now, low risk aversion implies a counterfactually large equity allocation. But perhaps considering their large explicit debt – tax-free debt issued against construction – implicit debt – tenured faculty and hard-to-fire staff salaries – and the high beta of donations, perhaps they are fairly exposed overall. But then they should be holding the market portfolio, which they manifestly do not do.

As before, wealthy investors do not consume a fixed fraction of wealth, but seem largely to ignore discount-rate related “paper profits.” Finally, even a desire to consume a fixed fraction of mark-to-market wealth does not make sense of university decisions to

ignore outside income streams. This year most universities continued to pay a fixed amount out of endowment, while adapting to revenue losses with budget cuts and eliminating investment, especially in new faculty. So overall, recursive utility, while an advance, is not by itself the answer.

Mostly, I think, these insights have not made inroads in portfolio theory because we simply have not asked them to do so. We have mostly used these preferences and technologies to ask the opposite, asset-pricing, equilibrium question – what asset price variation do we reverse-engineer so the portfolio theory answer is just to hold the market portfolio?

That is already interesting. It reminds us that perhaps we too are not the lone Merton investor living in a multifactor dynamic world, and maybe we have the same novel preferences or face the same opportunities as everyone else, so what look like delicious trading opportunities are in fact prices that should guide us to hold the market portfolio just like everyone else.

Habits, for example, are a parable for time-varying risk aversion. They generate a market in which prices have a strong temporary component and expected returns vary over time. The representative agent sees a delicious low-price opportunity, but just before jumping on it, notices that his or her risk aversion and precautionary saving motive has risen so much that, despite the opportunity, he or she will just end up sitting on the market after all.

To be useful for portfolio theory, then, we need to include interesting and empirically plausible dimensions of heterogeneity in equilibrium models and informal equilibrium thinking. Does my risk aversion vary over time less than yours? Why? Habits look a lot like debt boundaries. Are you less indebted than average? Even if true habits, in the model John and I wrote down, people arguably differ. Most academics seem to live well below their means. Others are the highly-leveraged high-beta wealthy, Queens of Versailles who buy multiple highly mortgaged enormous houses. If my “worst-case” scenarios are different from yours, we will obviously have different portfolios. If my idiosyncratic risk exposure varies with aggregates differently from yours, we have a reason to trade.

The behavior of our endowments and university budgets in 2008 and this year are worth thinking about in this context. Why did they sell in a panic in 2008? Why are

there budget cuts and hiring freezes then and this year, each university missing a golden chance to snap up talent the others are ignoring?

In 2008, universities discovered that they had liability streams and leverage. Their rise in risk aversion looks a lot like time-varying risk aversion induced by getting closer to a boundary, with tenured faculty salaries and municipal bonds serving as the reference point that induces a spike in risk aversion. Surely, however, knowing this fact, they should have set up portfolios differently, at least for the next crisis?

That the same budget crisis is happening only 12 years later is more puzzling. OK, in March 2020, one could see a second financial crisis on the horizon. But by June that risk was clearly over. A Powell version of Draghi's "do what it takes" did what it took. Well, university cash flows are clearly disrupted by the pandemic. For many schools there is a good question just how long they will remain financially viable. That is not an issue for large-endowment schools, but they still faced disruptions to their cashflow streams. Stanford, for example<sup>3</sup> saw a greater need for financial aid, to support PhD students in a disastrous job market, and a drop in revenue. Still, this is clearly temporary. \$35 billion in endowment should not lead to a strong sensitivity of investment (new faculty) to temporary cashflows! So, it seems our universities experienced a big increase in risk aversion and precautionary savings desires. But why? And how can we better structure portfolio and payout rules to meet the possibility of such changes in the future?

Rare disasters offer a different and deep challenge to portfolio theory as well as asset pricing. Both the economy and financial markets display a non-normal distribution of shocks, with occasional crises and then quiet good times. Rare disasters, of quantities or of marginal utility, mean, however, that the covariances with marginal utility  $cov(x_t, u'(c_t))$  that drive both one-period risk premiums and long-run payoff covariances and asset prices may have nothing to do with the covariances we measure in normal times. All that matters is, how does the asset do in the crash state?

To a long-run investor, the rare disaster view focuses all attention on the rare disaster. How does the payoff, which includes portfolio and payout policies, behave in a state with very high marginal utility of consumption? Is the rare disaster in outside cashflow hedged on this, the only really important day? Not only do everyday discount rate betas not matter, everyday cashflow betas don't matter either! All that matters is the portfolio

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<sup>3</sup><https://news.stanford.edu/2020/06/12/trustees-approve-2020-21-budget-set-endowment-payout-light-covid-19/>

and payout strategy in the rare disaster – the latter meaning that discount rate betas in the rare disaster likely are important – and the behavior of outside income and liabilities in the rare disaster. Robust control or ambiguity averse preferences give much the same feeling.

That's right, but discouraging. Quantitative portfolio management and risk analysis are hard enough when monthly betas matter. If they are irrelevant, the problem becomes even more speculative. If rare disasters are important, to institutional survival if nothing else, than most of what standard portfolio theory does, evaluating annual returns and hiring and firing managers on that basis, is fairly irrelevant. On the other hand, if rare disasters are important, then thinking through how markets work, what one's place is in them, and how they are likely to behave in a rare disaster state – the general equilibrium + heterogeneity approach – is even more important relative to statistical analysis and portfolio maximization.

Disasters raise a principle that perhaps I should have stated at the beginning:

- *Portfolio formation begins with risk management*

Think about the worst states. What are the states of the world in which you, or your institution are in deep trouble? How does the portfolio and payout policy hedge those states? Only then start getting greedy about paying a low price.

The Fed's stress tests for banks are an interesting example of this approach, mixing some robust control and general equilibrium thinking in risk management. Bank's risk should be well captured by value-at-risk analysis which computes standard betas to sophisticated statistical models of the evolution of asset returns and their correlations. It's interesting that Fed does not trust this, and instead wants to start with some simple scenarios that don't even have probabilities attached to them. Perhaps the question to which stress tests are an answer is one more portfolio managers should ask.

Heterogeneity is exploding in macroeconomics and finance, in part due to technical improvements in our ability to solve models with heterogeneity, and in part due to realizing it matters. Heterogeneity is ideal for portfolio theory, because portfolio theory is all about heterogeneity! In early models like Constantinides and Duffie (1996) people just face idiosyncratic risks, so there is not much immediate portfolio advice. Their paper largely reminds us that the representative investor preferences can include measures of heterogeneity of the individuals. Garleanu and Panageas (2015) show that when

people differ by risk aversion, the market develops time-varying expected returns. Less risk averse people have larger equity positions, so in downturns the average investor becomes more risk averse. That has a direct implication for time-varying portfolios. Recent models with more kinds of heterogeneity interacted with portfolio constraints offer portfolio advice directly. See Garleanu and Panageas (2019) for a survey.

### 3.6 Trading and liquidity

Perhaps however the central function of asset markets is neither saving and investment, nor a giant insurance market, but to facilitate trading on information. If so, then long-term investors are to some extent a side-show. But their investment strategies should take account of this fundamental purpose of markets.

Trading and volume are, I think, the great unsolved finance question of our time. Why is there so much volume? The standard models we have thought about so far do not produce a hundredth of the observed trading volume.

Of course, we sort of know the answer: trading volume is high because people trade on information – or what they think is information. But standard theory runs smack into the no-trade theorem, part of the average investor theorem, and the dinner with lions lemma. If someone wants to sell you something, he or she knows something you don't know. All deviation from the market index must be zero sum. We can't all be more informed than average. If uninformed traders refuse to budge off the market portfolio, then efforts to trade simply move prices to reflect information with no volume.

Yes, we have an immense theory of information trading to try break that logic. We write models with a signal about a terminating dividend, but most price movement is discount-rate movement, i.e. disconnected from dividends. Trading models require exogenous “liquidity traders” or overconfident agents to surmount the no-trade theorem. They are the suckers who do worse than average. Who are they? Are you sure that's not you?

Our exchanges are *built* to facilitate “information” trading. That is their most obvious function. Share issues, retirement saving, even outside-income hedging, do not need exchanges and instant trading. Exchanges were obviously not built to accommodate those trading demands. Those demands could all be handled in retail markets, like houses, cars, and insurance. Is the *existence* of exchanges, going back half a millenium,

really just testament to human folly? Would they not exist if we all processed information better? I think not. But that means we really don't have a complete theory of what trading is about.

The standard trading models clearly can be improved. Why does it take a hundred trades for information to make its way into the market, or really for traders to merge their actual information and to develop a consensus view? (For example, Santosh (2014).) The world is filled with thousands of confusing signals, not the tight structure of our models. Information is something like internet rumors that snowball, merge, and refine to success, or die out, each tweet a trade, not a card game with exactly 52 cards and we know which ones. Perhaps the next generation will find a mathematical structure to model this process

I digress a bit, because this is I think vital direction for asset pricing in the future. But we are here to think about portfolio theory, using what we know today. In advance of such deeper understanding, how does the evident fact that markets are built to facilitate a lot of high frequency "information" trading affect our long-term investor?

I'll stick to long-run investors who do not want to participate in information trading, or even to shade their portfolios based on hunches that this or that security or industry is the wave of the future, investors who do not wish to be the short end of the average investor theorem. You might immediately say "index," but that is not necessarily true. If trading demands push prices around, then an uninformed investor can profit by underweighting, or even shorting, assets that trading has pushed up in value, just as the unexposed investor can profit from by selling insurance if hedging demands are pushing prices around.

In the first blush of efficient markets, we thought we could separate information trading from portfolio theory, on an empirical understanding that trading activity had no effect on the level of prices. Since then, however, we have more and more evidence that trading activity moves the level of prices.

I start with one simple phenomenon: Highly traded securities have higher prices, lower average returns, due to the convenience yield that their shares give to information traders. The conclusion is natural: long-term investors should avoid them.

At a most basic level, money pays less than bonds (or at least it used to do so). Well, money turns over more, and is used for trading. The implication for long-term investors

is obvious: hold as little money as possible and do your investing in bonds. Don't stuff your mattress with money.

We now recognize sizable liquidity and trading-related spreads in government bonds. On-the-run vs. off-the-run spreads exist. Longstaff (2004) is particularly clean, comparing treasury yields with Refcorp bond yields, which had explicit Treasury credit guarantees, and finds 10-15% price discounts. Krishnamurthy and Vissing-Jorgensen (2012) document a sizeable liquidity premium in all US Treasury debt. That premium declines when the supply is larger, just as a "money demand" might do, and is traceable to the value of Treasury debt in financial transactions and the immense daily turnover of Treasuries.

As I write, the Moody's AAA yield is 2.6%. (Dec 2020.) These are bonds of more than 20 years maturity, so let's compare them to the 30 year constant-maturity Treasury rate of 1.7%. A measly 0.9 percentage point yield spread does not seem to be much to get excited about. But look at the flows. For each \$100 invested, you can get a steady \$2.6 per year coupon in AAA rather than \$1.70 per year in Treasuries. AAA bonds are not a 0.9 percentage point better return. They are a 53% better return; a 53% better cashflow for the same price! (Yes, this is a bit of an overstatement as there is presumably more default risk in AAA. A better calculation would cover that risk with CDS. But the probability of default times recovery is certainly not this large in the index of AAA bonds.) The long-term investor should buy AAA bonds not on-the-run Treasuries.

What if the AAA bonds lose value, if liquidity spreads spike as in 2008? The long-term investor should not care. If you ask the question, look hard in the mirror.

If a security has a high value due to its liquidity, or transactions value, and you don't care about liquidity or transactions value, don't buy it. This is a consistent general equilibrium view. Another class of investors, evidently, does care about the liquidity and transactions value of Treasuries, and is willing to hold them despite low returns. Again, look hard in the mirror – our endowments said they were the long run investor who does not panic in a crash, holds for the long term, harvests liquidity premiums. And then tried to sell in a panic. It turns out they were the ones who should have been overpaying for liquidity all along!

An enormous amount is written about "safe asset demands," "safe asset shortage," and so forth. Much of this has a behavioral, or yet-to-be-worked-out general equilibrium



basis, as it is not clear just why there should be so much demand for nominally default-free assets. Still, on these stories too, if it passes the look in the mirror test, the long term investor should not buy safe assets that other investors are driving up in value.

I advanced this view of asset prices in Cochrane (2003). In stocks also, we see high prices and low expected returns where turnover is high, share supply is low, and a large amount of “information” trading (speculation if you wish) is going on. The pattern is particularly clear when it rises to the point of apparent arbitrage opportunities. I looked at the 3Com/Palm arbitrage brilliantly analyzed by Lamont and Thaler (2003). A small number of Palm shares had been carved out, and 3Com shareholders would receive the rest in 6 months. The direct Palm shares traded at a huge price premium over the Palm shares embedded in 3Com. The overpriced end of the arbitrage (Palm) had much higher turnover, higher bid/ask spreads, strong short sales constraint, all indications of a strong demand to trade on Palm “information.” Who are the behavioral dummies who hold Palm rather than the Cheaper 3Com? Nobody – the average Palm holding was two weeks. Palm overpricing is a small cost of doing business if you think you have information about Palm, less than bid-ask spreads. Even to short Palm you have to own the shares for a little while. Share supply constrains information trading, and a demand for information trading can drive prices up and required returns down.

If you are a long run investor, who wants to hold Palm for the dividends, the strategy is simple: buy 3 Com and wait 6 months for the distribution of Palm shares.

The pattern is widespread, suggesting to me that a great deal of security value may lie in the value of shares for information trading – and that there are broad categories of stocks that long-term investors should avoid, substantially shading their portfolio away from the market-weighted index.

Figure 1 shows price and trading volumes through the first internet boom and bust. The Nasdaq tech trading volume exploded, and the prices went up with them, just as one might expect of a money-like security that offers a convenience yield, in that it facilitates trading on information about this dramatic new technical possibility. And when the trading boom vanished, so did the extra value. (If there is a reverse causality story, that would be just as interesting, but I cannot think of one. )

Table 1 shows the correlation extends across all individual securities. There is a strong (Fama MacBeth t statistic over 7) cross-sectional correlation between volume and

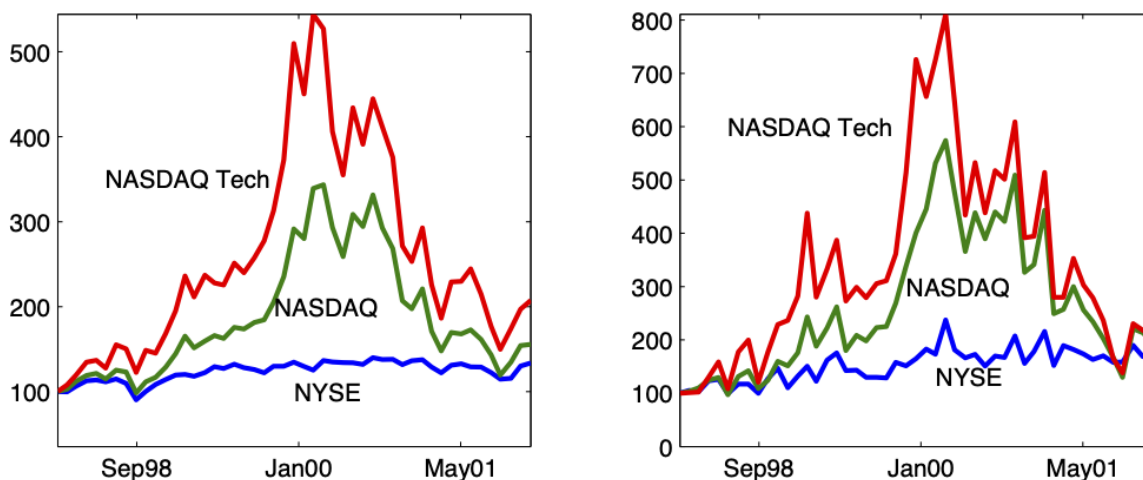


Figure 1: Index value and dollar volume.

$$\ln \left( \frac{ME}{BE} \right)_i = a + b \ln \left( \frac{\text{share volume}}{\text{shares outstanding}} \right)_i + \varepsilon_i.$$

Sample	a	b	OLS t	FM t	$R^2$	$\rho$
All CRSP Dec 1999	0.85	0.33	27		0.12	0.34
NASDAQ Dec 1999	0.89	0.38	23		0.13	0.37
All CRSP 1996-2000 (averages)	0.83	0.21	18	7.5	0.06	0.24
NASDAQ 1996-2000 (averages)	0.85	0.23	16	7.2	0.07	0.27

Table 1: Cross sectional regressions of market value / book value on share turnover. OLS t gives the OLS cross-sectional regression t statistic. FM t presents the Fama MacBeth t statistic, calculated from the time-series standard deviation of the cross sectional regression coefficients  $b_i$  divided by  $\sqrt{5}$ .  $\rho$  is the correlation between log market/book and log turnover. Book values follow Fama and French (2002). Source: Cochrane (2003).

market to book ratio. The “growth” stocks are where all the trading action is, where all the information is, where all the fun is. And, as we know, growth is where the prices are highest and returns are lowest.

A similar and strong correlation of market value with volume occurred in in the radio, phonograph, washing machine and automobile boom and bust of the 1920s. High valuations coincided with high trading volumes, and the market decline correlates beautifully with a decline in volume. House price (price/rent ratios) are high when trading volume is high and vice versa.

The small growth overpricing anomaly is one of the largest around, and strangely ignored. In Fama and French (1996), Table 1, for example, the small growth portfolio has the distinction of a -45bp/month three-factor alpha, with a 4.19 t statistic, twice as large as any other portfolio. The small growth portfolio is where all the information trading action is. For example, Davis (2001) examines fund performance by manager style. In the usual story of rampant underperformance, Davis finds one ray of hope for active management, a positive 17bp/month alpha for the deepest growth funds. Active managers, who supposedly trade on information, are able to beat the index of the most overpriced stocks around.

The implication of all this for a long-term investor is clear: Don't hold stocks with high prices relative to book, dividends, earnings, etc, when those high prices are correlated with high trading volume, low share supply, and lots of buzz in social media, lots of information traders. To be clear, we're not (just) worrying about being on the wrong side of asymmetric information here. We're worrying about buying stocks whose price has been driven up by the combined demands of traders, for whom the shares are an entry ticket to betting on information. If you're investing to get long-term relatively risk free dividends at a low price today, stay out of Tesla, Gamestop, and bitcoin.

The resulting portfolio is likely to differ substantially from the market index. The value-weighted market index includes all these high-flying stocks, and in larger proportions precisely because of their inflated prices!

Avoiding stocks with high information trading is again something professional management can help with, for a fee. For who owns the overpriced shares? Who owns the short end of an arbitrage? Who owns Palm rather than 3Com? Who owns the more expensive of two identical classes of payoff-identical HSBC shares? (Maymin (2011).) Who

owns Royal Dutch rather than Shell when the latter is more expensive? (Rosenthal and Young (1990).) Who owns shares in the all-time prize for negative alpha – companies that are having a battle between short-sellers and management? (See the delightful Lamont (2012).) Who owns Gamestop and Tesla? Well, not just the information traders. Index holders and quantitative factor-beta funds hold all of these, and the more overpriced, the more they hold. If you buy one of everything, you will not notice even an arbitrage opportunity!

I have emphasized one story for these anomalies, the convenience yield for information-based trading. There are many anomalies, many “overpriced” securities, and many more stories to explain them. Lamont and Thaler chalk up the 3Com / Palm arbitrage opportunity to (p. 231) investors who are “irrational, woefully uninformed, endowed with strange preferences, or for some other reason willing to hold over-priced assets.” Such an interpretation has the same implications for long-term investors. However, it invites a different look-in-the-mirror test. We are all human, and everyone thinks they’re less behavioral than the others. I prefer the information-trading guide because it is more restricted. You have to look for high trading volume as well as “overpriced” securities. It also gives a general equilibrium story and a description of heterogeneity in which everyone is almost rational, up to the small problem of the no trade theorem which I think we may solve someday.

Likewise, a liquidity premium is a different object. Palm shares had huge turnover *despite* large bid/ask spreads, reflecting a large demand for informed trading. Treasury bonds have huge turnover *because of* low bid/ask spread, reflecting an easy supply of liquidity for information-insensitive securities. (There is always a supply and demand of everything, including liquidity!)

Long-term investors should generally avoid securities whose values are boosted by “liquidity” as well. Endowments offer this reason for investing in unmarked, illiquid securities including private equity. Everyone wants to be the next David Swensen. This view is dicier because it still lacks a complete general equilibrium foundation. Who are the buyers of liquidity? Why do they not arrange their affairs so they don’t need to buy and sell so much so fast? We put “liquidity traders” in models blithely. OK, they are there, but to pass the look in the mirror test we need to understand them and make sure they are not us.

### **3.7 Institutional finance, intermediary asset pricing, slow-moving capital, supply and demand.**

Recently, several novel views of financial markets have emerged under the banners “institutional finance” (Brunnermeier, Eisenbach, and Sannikov (2013)), “intermediary asset pricing” (He and Krishnamurthy (2013)) “slow-moving capital,” ( Mitchell, Pedersen, and Pulvino (2007)) “supply and demand” ( Gabaix and Koijen (2020), Koijen and Yogo (2020).) When we can sign volume – when we can tell who initiated an order – prices rise when buyers initiate and sell when sellers do so. (Evans and Lyons (2002). However, see Brandt and Kavajecz (2004) for a cautionary note on “price pressure” vs. “price discovery.”) Each of these offers novel views of the function and operation of financial markets, opening many expected-return spreads. Since much of what the institutions of institutional finance do is high-frequency “information” trading, the visions are related. However, authors in this literature express price distortions in different economic terms, so it’s worth thinking about them separately.

Really the whole debate within asset pricing has shifted from macro-finance vs. behavioral to macro-finance vs. institutional. And the latter debate is only about the dividing line: how frequent, pervasive, and quantitatively important institutional frictions are, and how much a background of time-varying investment opportunities, preferences, outside streams, or even macro and monetary policies account for asset price fluctuations.

I emphasize theoretical interpretations here rather than the huge list of anomalies, consistent with the theme of this section: statistical expected return and covariance estimates are poor inputs to portfolio optimization. Instead we are exploring plausible general-equilibrium theories with heterogeneity to guide a portfolio view.

We must be careful however. In many of these stories, making even conventional alpha and beta appears to be easy. We have to constrain the models to the abject failure of active management to earn conventional alpha. At a minimum that warning means we should calibrate our view of the models to generate persistent price differences that do not amount to much for one-period expected returns. But such price discounts are exactly what should interest our long-term investor.

In the institutional and intermediary finance view, markets are dominated by traders who are highly leveraged and have little capital. They try to sell on the way down, but in

doing so they create “fire sales,” especially as balance-sheet constraints become closer to binding. (This view requires some reason why a firm nearing bankruptcy becomes more risk averse not less, exploiting the call option inherent in bankruptcy.) They even leave arbitrage opportunities like uncovered interest parity sitting like proverbial \$20 bills on sidewalks because they don’t have balance-sheet capacity to trade them.

If this view is correct, then our long-run investors, deploying pools of unleveraged long-only capital ought to be able to improve their investment performance – to buy a larger stream of payoffs at a lower price – by stepping in. They should provide capital to capital-constrained trading activities, persistent arbitrages like covered interest parity, and buy during the fire sales. To “slow-moving capital,” institutions should respond by investing in large multi-strategy hedge funds that have the expertise to quickly deploy across markets. These activities take expertise, but the long-only investors should fund managers who can deploy these strategies. The spread of securities lending by long-term investors is a small but practical and successful deployment of these views.

This funding does not take any special new insights, and it should trouble us that it does not happen. If you read the pages of our academic journals, you see empirical anomaly after anomaly and theoretical friction after friction. Based on this reading, you would expect rivers of alpha to flow from Greenwich, if not Wall Street, and from transparent organizations who fund easily identifiable arbitrage strategies not secret-sauce trading, and generate easy-to-measure alpha. Yet the striking difference between the alphas reported in theory and empirical papers and the alphas reported by studies of fund performance remains as true of institutional or intermediary finance as it is of behavioral or other anomalies.

Perhaps the frictions are so deep nobody can get around them, though that case is hard to make. Perhaps the opportunities are small and fleeting. As one banker remarked a while ago, commenting on the arbitrage opportunity of repo spikes, “sure, it’s a \$20 bill on the sidewalk, but it’s in Chicago and it’s not worth flying there to pick it up.” That is a deeper challenge to the friction-finance view for another day: Constraints bind in moments of crisis and stress, but inequality constraints don’t always bind, and if they do there is a strong force for institutional development to make them unbind. To what extent is this a theory of rare crises, and to what extent is this a theory of every day?

Perhaps, however, the alphas to be so gained are small but persistent, and sub-

ject to mark-to-market drawdowns. Then they are not visible in conventional return accounting, but looking at the issue as long-run payoffs would allow long-run investors to see and fund the opportunity.

Nonetheless, there is a deep and robust opportunity that someone should be taking – at a minimum family offices or wealthy individuals who do not have debt or other liability streams. Whether we call the phenomena time-varying risk aversion (habits), share demand due to trading frenzies, slow moving capital, institutional finance, or something else, there is a robust call for someone to take the other side.

Deeply, the proposition in finance that demand curves are flat, so price equals present value of payoffs, does not stem from arbitrage. It stems from risk-bearing capacity, from the willingness of a pool of investors to take risks that others are, temporarily unwilling to take. If someone sells a million dollars of Apple stock, the rest of the market has to collectively over-weight Apple, and thereby take risk. The price goes down a little bit to generate a reward. Since this requires risk-bearing it cannot be done by leveraged intermediaries. Big pots of money are there to do it. But by and large they do not.

Our endowments along with most large institutional investors behave exactly in the opposite way. In 2008, they sold during a once-in-decades buying opportunity. Perhaps this is all the result of “agency frictions,” focusing on short-term results, or behavioral biases of panicking when markets decline. If so, perhaps looking at payouts and tearing up the statements may help to overcome the agency friction or bias. But perhaps the Chicago view is right, and we have misunderstood the question these large investors and institutions are asking. And so have they, at least in their pronouncements about being long-term investors.

These views are also not yet fully fleshed out to describe heterogeneity, which is the key to turning an equilibrium insight into a portfolio theory. Model that rely on external liquidity traders, irrational agents, or artificial financial frictions – *only* leveraged intermediaries buy stocks – make a sensible shortcut for studying the friction at hand, but they beg the look-in-the-mirror portfolio theory question. That’s an important step.

## 4 Concluding comments

In sum, I opine that portfolio theory and practice for long-run investors, is like all asset pricing, ripe for important changes. Portfolio theory is perceived as somewhat of a dead end: a 50 year old model that is so hard to calculate in practice that nobody uses it in the investment world, or a benchmark from which it is easy to accuse even the most sophisticated people and institutions of being behavioral morons.

Finance has changed a lot however, and the progress opens the door to more fruitful portfolio theory, and also for more fruitful marketing to bringing theory to the realities of practice. I emphasize two conceptual steps which are part of both efforts: looking at the world as prices and payoffs, the ultimate object of interest for the long-term investor, and looking at the world informed by a view of the general equilibrium of financial markets and one's place in it. Both views offer heuristics for avoiding common investment mistakes, such as the average investor theorem. Both views also make a lot of sense of much apparently puzzling portfolio practice, and even apparent fallacies. In this way they bridge the Cambridge-Chicago approaches to behavior that persistently deviates from economic theory. The explosions in our general equilibrium view of the market since the days of CAPM and ICAPM with poorly defined factors, the explosion in our understanding of the facts, each have portfolio implications for long-run investors that are not really yet explored.

There is, I admit, a sense that I've arrived at many different questions to which value, or some sort of enhanced value seems to be the answer – after hedging outside income, buy a stream of dividends at a low price – along with heuristics to stay the course. But just what constitutes value, and who this advice applies to is more subtle than it appears. My different stories - heterogeneous outside income vs. avoiding securities caught up in a trading frenzy - isolate different securities, and for different investors.

The need to properly hedge outside income or liability streams looms large, and I think it is something done poorly by our current portfolio theory and practice. That involves thinking about payout policies as much as portfolio policies.

Risk management – describe what the bad states of the world are to you the investor, and make sure your portfolio isn't bad at just that time – should be the core of investing, portfolio management and evaluation, not a small afterthought. If this century has taught us anything, it ought to be that unexpected things will happen, they will



be financially painful, and resilience is more important than a few basis points of alpha.

There is also a defense in here of the finance industry and modest fees, if not for what they do now for what they could do in the future. Some of the disdain for active management rings of the advice a colleague gave me back when personal computers were new: Why buy a computer from Dell, when you can go down to Fry's and pick up a motherboard, power supply, hard drive, monitor, and case and put it together yourself for 20% less? Indeed, financial management consists of buying generic ingredients, putting them together in an attractive package, branding and marketing them. Perhaps we should give over the training of MBAs for asset management jobs to the marketing departments of our business schools. But in all the visions I've outlined so far, constructing the right payoff for a long-run investor, matching it to that investor's liability stream, then managing the portfolio, is not easy. It demand skill, and is worth paying fees for, if not quite the astounding level of fees many people and institutions pay now. Such fees survive the average investor theorem, as their value comes from better matching products to individual circumstances. I long felt some guilt working for an institution that charged a lot of tuition to teach MBAs that the industry to which we sent them was pointless. Now, perhaps, we can send them off with a better answer, and a bit of a business plan.

Why don't more institutions take our advice? Are there agency frictions or other barriers? I think first of all we must look to ourselves. We have a beautiful theory, but we have done little to make it practically useful. The people I have met at endowments, asset managers, sovereign wealth funds, and hedge funds are really smart. They know portfolio theory inside and out. Many are PhDs or ex academics after all. They would like dearly, for example, to use betas not buckets for risk analysis and diversification. But they face on a daily basis the fact that  $\Sigma^{-1}\mu$  simply does not work, that betas are unstable, and so forth. They understand hedge portfolio logic, but they also know that the underlying statistical models are uncertain. And they have to explain what they're doing in simple understandable terms. Much of what I offer here are first glimpses of how to make theory more useful, and more easily explained. We do a better job.

Second, there are as usual legal and regulatory barriers. Endowments are not free to adjust payouts as they wish or to use the endowment as a piggy bank. IRS rules put minimums and maximums on payouts. The terms of many gifts specify payout rates and tie the payout to specific expenditure streams, something that can be changed but would

take a century to do.

Third, there surely are agency frictions in many institutional investment contexts. University endowments are supervised by an upper level management that may make promises, but ultimately does hire and fire based on relatively short term (reported) reports. The obscurity of endowment performance is a natural result. The immense multi-layered fees paid by such institutions is a natural source of agency friction. Endowment investment is often exceptionally opaque. They do not make it easy to know how much is invested where, how many fees were paid, and how results are calculated, even for faculty on oversight committees. Opacity is a symptom of agency conflicts. The incentives of university top management, or those of similar philanthropies, may be likewise distorted. University and nonprofit boards of trustees, are typically wealthy people (alumni) who by selection bias are often lucky investors who think themselves skillful, or the recipients of large fees unlikely to object to the practice. At the bottom of it all, non-profit institutions are insulated from the market for corporate control. If a university or philanthropy (or hospital) is badly run, you can't buy up the stock, fire the management and clean it up. Like a monopoly, the first reward of a profitable non-profit is an easy life, iron rice bowls, and gravy trains. But agency frictions and dysfunctional institutional structures are not carved in stone either.

Doubting that view, many other institutions follow similar practices, including family offices, wealth managers, and so forth, who don't have similar frictions – just the usual caveat emptor we all face when dealing with technically complex services. That thought leaves me emphasizing the first point – we should do a better job, before pointing too many agency-frictions fingers at the world for not doing what we recommend.

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