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STANFORD INSTITUTE FOR ECONOMIC POLICY RESEARCH

SIEPR Discussion Paper No. 13-013

The Floor-Leverage Rule for Retirement

By

Jason S. Scott and John G. Watson

Stanford Institute for Economic Policy Research
Stanford University
Stanford, CA 94305
(650) 725-1874

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Jason S. Scott[a], John G. Watson[b,c]

[a] Managing Director, Retiree Research Center
Financial Engines, Inc.,1050 Enterprise Way, Sunnyvale, CA 94089
[b] Financial Engines, Inc., 1050 Enterprise Way, Sunnyvale, CA 94089
[c] Stanford University Graduate School of Business, 655 Knight Way, Stanford, CA 94305

Abstract

The Floor-Leverage Rule is a spending and investment strategy designed for retirees that can tolerate investment risk, but insist on sustainable spending. The rule calls for purchasing a spending guarantee with 85% of wealth and investing the remaining 15% in equities with 3x leverage. Surprisingly, this leverage is a tool for managing risk. We compare our rule to some popular strategies, illustrate it for a variety of retiree preferences, and evaluate its historical performance.

Keywords: Retirement, consumption, investment, floor, leverage

1. Introduction

Over the past 40 years, the responsibility for making retirement spending and investment decisions has shifted away from governments and corporations and toward individuals. Traditional defined benefit plans, where an individual receives a steady stream of lifetime pension payments, have been increasingly replaced by defined contribution plans, where an individual contributes to a retirement account, chooses the assets to purchase, and withdraws funds to cover retirement expenses. In the United States, the Investment Company Institute [21] estimates that $8.5 trillion in assets are currently invested in individual retirement accounts, versus the $2.2 trillion held in traditional pension plans. In Chile, the national defined contribution plan, which began in 1981, currently holds assets that exceed 60% of Chile's annual GDP [28]. This global shift is expected to accelerate as more countries follow suit.[1] In addition to assets invested in employer and government sponsored retirement accounts, many retirees have private savings and brokerage accounts earmarked for retirement. Hence, for millions of current and future retirees, the bulk of their retirement savings will be in self-directed accounts. These retirees need strategies to manage their assets and budget their drawdowns. Here, we motivate, describe, and explore a simple spending and investing strategy for retirement, the Floor-Leverage Rule.

Many financial planners specialize in retirement planning, and successful ones take pains to understand a retiree's spending preferences, or risk losing the client. A review of this literature reveals that retirees have a clear preference for sustainable, non-decreasing, spending. For example, William Bengen [2], the author of the popular 4% Rule, described his goal as helping clients "spend as much as possible each year from their retirement accounts, while maintaining a consistent lifestyle throughout retirement." Though many authors suggest refinements to his basic strategy, no one challenges his premise that clients prefer sustainable spending.

A preference for sustainable spending does not mean that retirees shun risk. In fact, a look at their investments shows that most retirees have an appetite for risk. In the US, a retiree with no tolerance for risk can virtually guarantee constant nominal spending by purchasing a bond ladder of Treasuries or real spending by purchasing a ladder of Treasury Inflation Protected Securities (TIPS). Alternatively, a retiree can purchase an immediate annuity, with or without a cost of living rider, and ensure a constant, sustained spending level for life. Similar securities exist for retirees in other countries. Most retirees eschew these conservative strategies, suggesting a willingness to invest in equities and a belief that investment gains will support increased future spending. Indeed, a significant challenge of retirement planning is reconciling simultaneous desires for sustainable spending and equity investments.

Researchers, beginning with Dybvig [14, 15] and later with Watson and Scott [39], have used an expected utility maximization framework to analyze optimal spending and investment decisions when preferences include a requirement for sustainable spending. Unfortunately, the suggested optimal strategy for a retiree with these preferences requires an implausibly complex and costly trading strategy. This paper bridges the gap between theoretical and practical by suggesting a relatively simple retirement rule, the Floor-Leverage Rule, which is near-efficient in terms of expected utility maximization, but practical to implement.

☆We gratefully acknowledge the many helpful comments of our colleagues Geert Bekaert, Wei-Yin Hu, and Robert Young. Further, we are indebted to our FAJ referees, whose suggestions added much to this papers applicability and readability.

[1] Canada has registered retirement savings plans (RRSPs), the United Kingdom has National Employment Savings Trusts (NESTs), and Australia has Superannuation (Super) funds.

2. The Floor-Leverage Rule

The classic utility-based model of Merton [26] and Samuelson [31] recommends that retirees spend a constant fraction of wealth and invest in a constant risk portfolio (e.g., a 60/40 stock/bond mix for retirees with average risk tolerance). This is a contrarian investment philosophy, i.e., a decline in the stock market triggers bond sales and stock purchases. If we add a strong preference for sustainable spending to the classic model, then the optimal investment strategy must change drastically. Otherwise, during a prolonged stock market collapse, sales of bonds, losses in stocks, and sustained spending will eventually bankrupt a retiree. Surprisingly, most retirement strategies recommend constant risk portfolios, even though a worst-case analysis shows that this approach is fundamentally incompatible with sustainable spending.

Constant risk strategies endanger a retirement portfolio precisely because they sell bonds and buy stocks when stocks lose value. To ensure sustainable spending, we must have a different response to stock losses. One alternative is to do nothing, e.g., a retiree can adopt a buy and hold strategy. A buy and hold investor who initially invests 60% of her wealth in stocks can in the worst-case, still rely on the 40% of her wealth invested in bonds for spending. However, a better alternative is to sell stocks and buy bonds during market declines—the opposite of a constant risk strategy. This strategy reduces portfolio risk when markets decline and is called Constant Proportion Portfolio Insurance (CPPI) (see Perold and Sharpe [29]). It is parameterized by a floor value that measures the theoretical worst-case and a multiplier that measures the speed with which risk is reduced if markets decline. CPPI strategies are appealing to many retirees because they combine high levels of downside protection with significant market exposure. However, these strategies have implementation issues. First, they require nearly continuous trading, which is both costly and impractical for retirees to implement themselves. Second, the theoretical downside protection that CPPI promises is only partially achieved in practice because stock prices often jump. It is crucial to address both concerns, since, as we will see, highly efficient retirement strategies rely on CPPI-like investment approaches.

For a new retiree with a taste for sustainable spending, the list of questions involved in setting up a retirement plan is daunting. How much can be spent initially? What is the initial investment strategy, and what changes are needed as markets evolve? What conditions warrant a spending increase? More fundamentally, is the investment strategy compatible with sustainable spending? Watson and Scott [39] develop an expected utility model based on sustainable spending that answer these questions. Applying their model to retirees with average risk tolerance and various spending preferences, a pattern emerges. The optimal strategies are well approximated (98% efficiency relative to the theoretical optimal) by a simple rule of thumb—invest 85% of retirement wealth in a riskless floor portfolio, put the remaining 15% in a 3x leveraged equity portfolio, and review spending annually. We refer to this strategy as the Floor-Leverage Rule:

The Floor-Leverage Rule

1. 85%—Build a riskless spending floor:
At retirement, allocate 85% of available assets to purchase a sustainable lifetime spending floor. The floor type and resulting spending rate depend on the preferences of the retiree. Throughout retirement, money is withdrawn from the floor portfolio for spending.

2. 15%—Invest in a 3x levered equity portfolio:
The remaining 15% of assets, the surplus, is invested in a mutual fund or ETF that is rebalanced daily to maintain 3x leverage with the stock market.

Together, the riskless floor (85%) and the surplus (15%), which is invested in 3x leverage ETF, provide downside protection and equity upside. Since, all assets can be purchased and held between spending reviews—portfolio maintenance is minimal.

3. Annual spending review:
Annually review the surplus portfolio. If it exceeds 15% of the total portfolio value, then sell any surplus in excess of 15% and use the proceeds to purchase additional floor spending. Spending may increase, but always remains sustainable—it ratchets.

Step 1 allocates 85% of wealth at retirement to secure lifetime spending. The per dollar cost of lifetime spending, and thus how much spending is purchased, depends on the retirees sustainability preferences. We consider three retirees with varying preferences for sustainable spending. Our first retiree requires that real purchasing power never declines. We next relax this requirement and consider a retiree that will allow real spending to decline, as long as the decline never exceeds the rate of inflation. In effect, this retiree sustains nominal spending. Finally, we consider a retiree whose preferences suggest a willingness to annuitize some portion of her assets. Sustaining nominal spending is cheaper than sustaining real spending, and the use of annuity further decreases costs. As a fraction of beginning wealth, initial spending is approximately 3% to sustain inflation protected spending, 4.5% to sustain nominal spending, and 5.2% when nominal spending is augmented with the purchase of a late-life annuity.

Step 2 invests the remaining 15% of assets in a 3x levered equity asset provided by a financial institution—preferably an ETF or a mutual fund.[2] In this case, it is the provider's responsibility to maintain the daily-rebalanced, levered position, and not the retiree's. Hence, the main challenges to implementing a CPPI strategy—frequent trading and stock price jumps—are delegated to the fund provider. Step 3 is an annual review, and if the surplus portfolio has done well, spending increases.

[2]The Proshares ETF, with ticker symbol UPRO, offers 3x leverage on the S&P 500 index, and the Direxion ETF, with ticker symbol DZK, offers 3x leverage on the EAFE index.

Investing the surplus portfolio in an ETF or mutual fund makes the Floor-Leverage Rule feasible. To illustrate this point, consider what would happen if a retiree were responsible for implementing the strategy. Suppose she starts with a \$100 portfolio. The initial surplus relative to the floor is \$15, and with 3x leverage, the initial allocation is \$45 in stocks and \$55 in a riskless asset, e.g., a high quality bond portfolio.[3] A subsequent \$5 drop in stock value reduces the surplus to \$10 and implies a target allocation of \$30 to stock and \$65 to bonds. Hence, \$10 of stock is converted to bonds. Similarly, if stocks increase in value, then stocks are sold and bonds are purchased. Converting back and forth between stocks and bonds as markets fluctuate is costly for stocks and impractical for bonds—especially if the bond portfolio is a ladder or contains an annuity. The situation changes if a 3x levered stock fund is available. A 3x fund borrows two dollars for each dollar of principal and invests the total in stocks. The result is an aggregate holding that is long \$3 in stock and short \$2 in bonds. Now, our retiree can invest \$85 in bonds, and the remaining \$15 in the 3x fund. The fund contributes a \$45 long exposure to stocks and a \$30 short exposure to bonds. Together with the floor portfolio's \$85 bond holding, the initial desired allocation is achieved. Moreover, if the value of the levered fund drops by \$5, the resulting overall exposures are \$30 stocks and \$65 bonds, as desired.

Crucially, with access to a leveraged fund, a retiree can invest \$85 in floor and \$15 in a levered surplus portfolio using an annual *buy and hold* approach. Since the target stock allocations are automatically maintained, a retiree can use all the available floor dollars to purchase income. Equally important, the 3x leveraged fund offers a *limited liability* way of allocating the surplus assets. It is certainly possible for stock prices to experience a sizable gap downward. Under these conditions, the 3x leveraged fund would suffer significant losses and could theoretically be completely wiped out. However, our retiree would still weather the storm because her losses are limited to her \$15 initial investment.

Having a third party create a 3x leveraged equity investment greatly simplifies the challenges of implementing a CPPI-type strategy by offering the ability to track the desired allocations with a buy and hold strategy that has limited downside exposure. Note that for this investor, leverage is not used to magnify risk beyond what is achievable without leverage. Rather, the 3x leverage fund offers a cost effective way to achieve a desired dynamic strategy. With access to a 3x fund, the target dynamic strategy is greatly simplified in that only income purchases are required, never sales. In addition, the 3x investment offers a limited liability way for the retiree to adopt a conservative approach in the face of a market decline without need of any costly transactions.

[3] Interestingly, the initial stock exposure is 45% equity, not 60%. Since the position is only reviewed annually, a 60% starting allocation coupled with positive drift results in a likelihood of above target equity exposure.

3. Comparison with Other Strategies

In this section, we compare and contrast the Floor-Leverage Rule to popular retirement strategies found in the financial planning, private wealth management, and economics literatures. Our sample is not exhaustive; it is biased towards strategies that appeal to our target retiree—a person who wants sustainable spending, but is willing to take some investment risk.

3.1. 4% Rule Strategies

William Bengen's 4% Rule [2] is without a doubt the most popular retirement strategy in use today. Bengen originally considered a constant risk portfolio evenly split between US equity and fixed income assets and concluded that:

> Assuming a minimum requirement of 30 years of portfolio longevity, a first-year withdrawal of 4 percent, followed by inflation-adjusted withdrawals in subsequent years, should be safe.

This strategy is clearly designed for retirees that prefer sustainable real spending and have some appetite for investment risk. Further, the rule's simplicity appeals to retirees—they can understand and implement the basic rule without outside help. Though subsequent studies by Bengen [3, 4], Guyton [18], Guyton and Klinger [19], Horan [20], and Spitzer et al. [36] increased the sustainable withdrawal rate by considering various combinations of investments, all of these strategies attempt to sustain real spending with a constant risk portfolio, and are collectively referred to as 4% Rule strategies.

For a given planning horizon, the withdrawal rate is generally chosen as the maximum rate for which a retiree never (or rarely) runs out of money. These rates are calculated using a historical sample (or a historically calibrated model) of equity returns. The 4% Rule studies generally rely on US market returns to determine the safe withdrawal rate. However, analyzing the success of a 30-year spending strategy with 90 years of US return data relies on very few fully independent data points. To bolster the analysis, Pfau [30] assesses the performance of the 4% Rule using market data from outside the US. In his study of 17 developed countries, Pfau concludes that a 4% real withdrawal rate and a portfolio "split evenly between stocks and bonds would have failed in all 17 countries."

If historical return patterns fail to capture the range of future outcomes, retirees risk running out of money when markets under perform and risk missing out on additional spending when markets over perform (see Scott et al. [33]). To avoid these risks, retirees must adopt strategies that adapt to market performance. For example, if returns are poor, Guyton [18] suggests retirees forgo the yearly inflation increase. Early in retirement, Guyton and Klinger [19] recommend increasing or decreasing spending by 10% whenever the withdrawal rate, as a percent of current wealth, falls outside of an allowed range. Similarly, Stout [37] adjusts spending based on a client's life-expectancy, and Frank et al. [17] adjust whenever ruin probabilities fall outside of an acceptable range. The challenge for each approach is identifying when realized market returns deviate sufficiently from historical norms that spending adjustments are warranted.

Sustainable spending is the goal of both the 4% Rule and the Floor-Leverage Rule, yet the two rules call for a very different reaction to portfolio losses. The 4% Rule counts on future market returns to sustain spending, and if they do not materialize, reacts by cutting spending or risking ruin. In contrast, the Floor-Leverage Rule always has sufficient fixed-income investments to sustain its spending, and never needs to cut spending if equity returns are poor. In fact, it reacts to poor returns by lowering risk—the levered, surplus portfolio rapidly declines resulting in a lower total portfolio risk.

3.2. Split-Account Strategies

Another popular category of retirement strategies, split-account strategies, divides retirement portfolios into sub-accounts, each devoted to a specific objective. The appeal of split-account strategies is deeply rooted. In behavioral finance, it is called mental accounting—the common practice of allocating money for different goals to separate mental accounts. In principle, a retiree could have different preferences for each one of her retirement goals, and then construct an optimal portfolio considering all of them. In fact, Das et al. [12] develop a theoretical justification for mental accounting based on portfolio optimization. The advocates of split-account strategies come from diverse backgrounds with a variety of clients and a range of experiences. These strategies reflect this diversity—there are a wide range of opinions on objectives, wealth allocations to sub-accounts, and best investments for each sub-account.

Private wealth managers know the split-account approach as goal-based wealth management, e.g., Brunel [8, 9], Nevins [27], and Chhabra [10] describe how to create an overall portfolio comprised of goal-based sub-portfolios. Brunel develops a structured approach with three categories of goals—personal, dynastic, and philanthropic. In Brunel's experience, the most important goal is lifestyle maintenance, with failure described as "the most frightening nightmare for most families." Chhabra also has three categories—safety, maintenance, and aspiration. Though much of their focus is on high net worth clients, less affluent individuals can mimic these approaches using just two categories—lifestyle maintenance and aspirational spending.

Financial planners often assume that equity investments are risky over the short-term, but likely to perform well over the long-term. Hence, accounts earmarked for near-term spending objectives should be invested in fixed-income assets, while the accounts earmarked for long-term spending objectives can be invested in equities. Evensky (see Evensky and Katz [16]) recommends this strategy using a five-year breakpoint between near and long-term spending. Ray Lucia's Bucket Strategy™[24, 25] extends the horizon covered by conservative investments to upwards of fifteen years. In each of these examples, and more generally, spending withdrawals are made from short-term accounts, and the long-term accounts are periodically harvested to replenish the short-term accounts.

Zvi Bodie and his co-authors [6, 7] believe that retirees should first fund their essential spending, and only once that is secure, fund their aspirational spending. They suggest a split-account strategy based on this priority:

> Once you have covered your essentials through hedging in your safety zone, it is not unreasonable to take significant risk with part of the "extra" wealth available to invest. There are many ways to do this—including leveraging an investment in a broadly diversified portfolio of stock...

Here, a retiree's essential spending is funded with a conservative, fixed-income portfolio that may include TIPS to hedge against inflation or annuities to hedge against outliving one's retirement portfolio. Upside spending is then funded with a surplus portfolio that may be invested in aggressive instruments, e.g., call options [5].

The Floor-Leverage Rule is a split account strategy with a priority—sustainable spending. While Bodie's priority guarantees essential spending, the Floor-Leverage Rule guarantees all of the spending that can be purchased with 85% of a retiree's wealth, which may be more than just essential spending.[4] Generally, split-account strategies are initially straightforward to setup, but often difficult to maintain—many strategies are quantitatively vague about transfers between accounts and adjustments to short-term spending as markets evolve. In contrast, the Floor-Leverage Rule is quite specific about when and by how much to increase spending.

3.3. Utility Strategies

Economists have proposed retirement strategies based on expected utility maximization. This approach leads to specific, quantitative advice for retirement spending and investing decisions under the uncertainty of future market returns. In their seminal research, Samuelson [31] and Merton [26] consider time-separable utilities. For an average retiree, their model recommends investing in the market portfolio and spending a constant fraction of current wealth. Given that a market portfolio can suffer a 20% drop or more in a bad year, this strategy could result in a similar drop in annual spending. For retirees with a preference for sustainable spending, this is unacceptable.

Philip Dybvig [14, 15] proposes adding a sustainability constraint to the Samuelson-Merton model and applies its solution to endowment spending. He argues that because endowments may be unable to quickly reduce their expenses, perhaps due to commitments to staff and donors, a constraint that prohibits spending declines is justified. Dybvig finds a simple, elegant solution to the endowment problem. In fact, we use his solution as the template for the Floor-Leverage Rule. However, since Dybvig's endowment model assumes a very long-term—indeed infinite—planning horizon, adjustments must be made to realistically model a retirement scenario.

Watson and Scott [39] make three retiree-relevant enhancements to Dybvig's approach. First, they consider finite planning horizons—this greatly reduces the cost of sustainable spending. Second, they model arbitrary time preferences, and thus, they can down-weight the benefit of future spending by a mortality factor. Finally, their model is a discrete-time model, which does

[4] If 85% of a retiree's wealth is insufficient to support essential spending, then we recommend increasing the size of the floor-account.

not require continuous review of investment and spending decisions. In Appendix A, we review the details of this model and its solution.

The solution to the Watson and Scott model is a dynamic recipe for retirement spending and investing. Unfortunately, it is a complex strategy that is hard to explain and difficult to implement. However, their optimal solution is an extremely useful benchmark. After all, if a simple strategy is economically indistinguishable from an optimal one, then all the extra complexity of the optimal strategy can be safely ignored. Across a wide range of cases, we find that the Floor-Leverage rule is just such a strategy—simple to implement and very near optimal. In the next section, we motivate and explore three cases and show that our rule is an efficient approximation to Watson and Scott's optimal strategy.

4. Sustainable Spending Preferences

Sustainable spending preferences can have multiple flavors. We analyze the impact of varying sustainable spending preferences in two dimensions. First, we consider the strength of the preference. We evaluate a strong form preference that sustains real purchasing power, and a weak form that sustains nominal spending. Second, we consider the impact of mortality discounting. To evaluate this preference, we down-weight utility from future spending by the probability of survival. Each preference dimension not only alters the expected utility analysis, but also alters the appropriate floor investment. Retirees wishing to sustain real spending should allocate the floor to purchase lifetime real spending, and similarly, real nominal spending should be purchased given that preference. Finally, we assume retirees with a mortality discount are willing to consider purchasing an annuity with some of their assets, and here we assume they will purchase a late-life annuity. We believe that each case will interest a significant retiree cohort.

For our analysis, we take the viewpoint of a 65-year old, retiring today and planning on up to 40 years in retirement. For each case, we calculate an initial spending rate and the distribution of spending rates at ages 75 and 85. These rates depend on a model for future interest rates and market returns. We choose a simple, two-asset economy consisting of a riskless, real bond paying 2%[5] and a risky asset with an annual risk premium of 6% and an annual volatility of 18%—values that are broadly consistent with US equity markets. This economy is for illustration only—in the next sections we review a historical simulation of 3x leveraged returns, and discuss the practical details of implementing the Floor-Leverage Rule.

These cases are an opportunity to show how closely the Floor-Leverage Rule approximates a theoretically best strategy. To perform this comparison, we require a suitable utility function. We choose Dybvig's sustainable spending utility, as modified by Watson and Scott, and hereafter referred to as DWS utility (see Appendix A). Using this utility function, its optimal solution, and our model for financial markets, we compute an efficiency score. For a given initial portfolio value, we first calculate the expected utility of the Floor-Leverage Rule (or any other sub-optimal strategy). Next, we calculate the initial portfolio value needed to achieve the same expected utility, but using the optimal strategy. Our efficiency score is the ratio of the latter portfolio value to the former. For example, suppose that the Floor-Leverage Rule, starting with $100,000, has the same expected utility as the optimal strategy, starting with $90,000. Then the efficiency score is 90%. The best possible score is 100%, and strategies with scores close to 100% are near optimal. Details of the efficiency calculation are discussed in Appendix A.

The strategy which maximizes the expected DWS utility is complex, involving annually adjusting the floor allocation and continually adjusting the surplus leverage ratio. The efficiency score measures the benefit of enduring the added complexity of the optimal strategy. With an 80% efficiency score, $20,000 out of every $100,000 invested is wasted due to following an inefficient strategy. That level of inefficiency offers ample rewards for expending effort to improve efficiency. In contrast, a strategy with 99% efficiency leaves little room for improved performance. As we will see, the high efficiency of the Floor-Leverage rule implies scant benefit from the added complexity of implementing the optimal strategy.

4.1. Sustainable Real Spending

This case is appropriate for retirees who want to sustain real spending—they never want a drop in lifestyle. These retirees are Bengen's target audience. However, instead of using Bengen's 30-year planning horizon, we have a 40-year horizon, which is a plausible upper bound on life expectancy at age 65. Sustaining 40 years of inflation-protected spending is very expensive. For example, when the real rate of interest is 2%, the cost of 40 annual payments of $1 is $27.90. A client with $100,000, who adopts the Floor-Leverage Rule, initially invests $85,000 in a floor account and annually receives at least $3,050, adjusted for inflation.

Switching from the 4% Rule to the Floor-Leverage Rule is a case of bad news and good news for our retiree. The bad news is that the spending rate (an annual payment divided by the original portfolio value) starts at barely over 3%, instead of at 4%. The good news is that spending is assured for 40 years, sustainability does not rely on market returns, and there is a significant chance that spending will increase. Table 1 summarizes the distribution of spending rates for our representative 65-year old retiree given our simple economy. For example, today our retiree can be 75% confident that at age 75 spending will exceed 3.38%. For this strategy, real spending never declines, but can increase. In fact, in typical scenarios, spending can increase substantially. The confidence values in Table 1 illustrate this point. While spending begins at 3.05%, median spending at age 75 has increased by 29% to 3.93% of initial wealth. Similarly, by age 85, median spending has grown to 4.89%, representing a 60% increase relative to initial spending. Some retirees might chafe at a low initial spending rate that facilitates the potential for significantly higher late-life spending, but this is an

[5] While the actual real term structure is neither flat nor constant, real rates are relatively stable, and 2% is consistent with the historical record.

unavoidable consequence of mixing a desire for risk with a sustainable real spending requirement. A retiree with these views may find that one of the other two spending case studies better approximates their preferences.

Table 1: Confidence Levels for Sustainable Real Spending

Confidence	Age 75	Age 85
100%	3.05%	3.05%
90%	3.07%	3.30%
75%	3.38%	3.87%
50%	3.93%	4.89%
25%	4.74%	6.43%
10%	5.73%	8.41%

Notes: Measured as a fraction of initial wealth. Age 65 spending is 3.05%.

Watson and Scott describe the optimal strategy for a retiree with DWS utility who wishes to sustain real spending. This optimal strategy provides a useful benchmark for measuring the effectiveness of alternative strategies. Table 2 reports efficiency scores for three alternative strategies. The efficiency score for the Floor-Leverage Rule is 99.4%—very nearly optimal.[6] The other two strategies consider the efficiency impact of pursuing alternative approaches to investing the surplus. For example, one strategy invests the surplus in equities, but eschews leverage. This strategy results in a sizable loss of efficiency. Whereas the Floor-Leverage Rule loses $600 per $100,000 to inefficiency, the analogous cost when avoiding leverage is more than 10 times higher at $6,900—useful information for a retiree questioning the benefit of using leverage. A reasonable response to a retiree voicing a preference for sustainable spending is to suggest a risk-free approach. Avoiding risk altogether maximizes the initial spending level, but loses out on any potential gains from capturing an equity risk premium. For a retiree with DWS utility, avoiding all risk is very expensive—it costs $14,100 per $100,000 in lost efficiency. Clearly, minimizing risk does not maximize utility, even when a client desires sustained spending.

4.2. Sustainable Nominal Spending

Suppose our retiree desires increased early retirement spending—she is not thrilled with the previous case study's initial 3% spending rate, even though it will never decline in real terms. What can be sacrificed in order to increase initial spending? Bengen [4] and Guyton [18] suggest allowing modest cuts in real spending. If clients are willing to forgo an inflation increase when markets underperform, then Guyton finds that his 4% Rule can increase their initial spending rate from 4.4% to 5.4%. Our approach to increasing the initial spending rate is a twist on Guyton's. Rather than trying to sustain real spending and allowing decreases when markets under perform, we sustain nominal spending and allow increases when markets over perform. This strategy might be especially appealing to a retiree who has substantial fixed nominal expenses, e.g., a home mortgage. Again, we use the Floor-Leverage Rule, but this time the floor account is invested in assets that guarantee sustained nominal spending.

Table 2: Efficiency Scores for Sustainable Real Spending

Surplus Investment Strategy	Leverage Factor	Efficiency Score
All Bonds	0.0	85.9%
Non-levered Equity	1.0	93.1%
The Floor-Leverage Rule	3.0	99.4%

Notes: Efficiency estimate based on a Monte Carlo analysis with one million scenarios.

For illustration, we assume that inflation is a deterministic 2.5% per year.[7] If we again assume that the riskless real rate is 2.0%, then the nominal riskless rate is approximately 4.5%. With these rates, the cost of sustaining $1 of nominal spending is only $19.10—a substantial reduction compared to the $27.90 required for $1 of real spending. Here, a retiree with $100,000 invests $85,000 in riskless, nominal bonds and is guaranteed $4,450 per year in nominal spending. Hence, the initial spending rate (real and nominal) has risen to 4.45%.[8]

Though nominal spending never decreases, real spending drifts depending on the performance of the surplus portfolio. Table 3 summarizes the possibilities for our 65-year old, representative retiree. Whereas a sustainable real-spending preference results in median outcomes with substantial spending increases, a sustainable nominal spending preference has median outcomes with modest spending increases. Spending at age 65 is 4.45%, and median spending at both age 75 and age 85 only increases slightly to 4.60%. Relative to sustaining real spending, adopting this strategy allows a retiree to substantially increase initial spending while limiting the maximum year to year decline in spending to the rate of inflation. However, our retiree has made a sacrifice; she has only guaranteed a 4.45% nominal spending rate, which corresponds to a real spending rate of only 3.48% at age 75 and only 2.72% at age 85.

For the case of sustained nominal spending, we modify the DWS utility function to allow for real spending declines that are never greater than the inflation rate. Using this modifica-

[6]Efficiency scores were estimated using a Monte Carlo analysis with one million scenarios. A scenario consists of a draw on 40 years of stock market returns. Given market returns, surplus returns and spending adjustments are determined year by year, resulting in a 40 year spending scenario. Utility for each 40 year scenario is then averaged to determine expected utility.

[7]Currently, the spread between the yield on 30-year Treasury bonds and 30-year TIPS is approximately 2.5%. In addition, the Q1-2013 estimate of the 10-year average CPI expectation (as reported by the Federal Reserve Bank of Philadelphia) was 2.3%. Since 1998, this statistic has fluctuated in a narrow band between 2.2% and 2.6%. We note that our deterministic model neglects the stochastic nature of inflation.

[8]We note this analysis only applies during times of modest inflation. If inflation is high, this case is best interpreted as a 2.5% cap on real spending declines. High inflationary environments weaken the sustainability constraint considerably. In the hyperinflation limit, the solution reverts to Merton [26].

Table 3: Confidence Levels for Sustainable Nominal Spending

Confidence	Age 75	Age 85
100%	3.48%	2.72%
90%	3.55%	3.03%
75%	3.93%	3.60%
50%	4.60%	4.60%
25%	5.57%	6.09%
10%	6.76%	8.00%

Notes: Measured as a fraction of initial wealth. Age 65 spending is 4.45%.

tion, we derive a new optimal benchmark and use it to calculate efficiency scores. In this case, the Floor-Leverage Rule scores 98.2%. In Table 4, we also list the efficiency scores for an all bond strategy and a non-levered equity strategy. Again, with DWS utility, an equity strategy without leverage significantly reduces efficiency, but a fully riskless strategy entails enormous levels of lost efficiency. Compared to optimal, the riskless strategy loses $18,000 per $100,000 in retirement wealth—more than the entire surplus portfolio when following the Floor-Leverage Rule.

Table 4: Efficiency Scores for Sustainable Nominal Spending

Surplus Investment Strategy	Leverage Factor	Efficiency Score
All Bonds	0.0	82.0%
Non-levered Equity	1.0	92.3%
The Floor-Leverage Rule	3.0	98.2%

Notes: Efficiency estimate based on a Monte Carlo analysis with one million scenarios.

4.3. A Late-life Annuity Purchase

Our final case study concerns a retiree who wants sustainable spending, but is concerned that too much money is allocated to support late-life spending. For example, with a nominal spending floor and a 40 year planning horizon, the per dollar cost to sustain spending beyond age 85 is $5.56. Hence, approximately 30% of the floor account is allocated to late-life spending. That money feels wasted to the retiree who believes: "I'm probably not going to live that long anyway." In fact, actuarial tables corroborate her instincts—her chances of surviving past age 85 is a coin flip. However, a sustainable spending plan must fund later retirement years, even when the odds of survival are low.

An annuity makes the most sense for this retiree; however, there are two well established and conflicting facts about annuities: (1) annuities can dramatically lower the cost of funding retirement spending, and (2) few retirees purchase any sort of annuity. This is the annuity puzzle—a difference between theory and practice. It is well beyond our scope to solve this puzzle, but we do wish to explore the implications of purchasing a late-life annuity at age 85. A late-life annuity strikes a balance between ignoring the potential of an annuity purchase and assuming a retiree will fully annuitize. Incorporating a late-life annuity cuts the per dollar cost after age 85 from $5.56 to $2.86 and the overall per dollar cost to sustain nominal spending from $19.10 to $16.40. In this case, the initial spending recommended by the Floor-Leverage Rule is $5,182 per year—a substantial increase compared to $4,450 when an annuity is not purchased. Here, we assume that our retiree is a female; if our retiree is a male, the cost of the annuity is even less and the spending difference greater.

Though nominal spending never decreases, real spending again drifts depending on the performance of the surplus portfolio. Table 5 summarizes the possibilities for our 65-year old, female retiree. Introducing a late-life annuity lowers the cost of securing spending beyond age 85. As we noted, that lower price increases initial spending from $4,450 to $5,182. In addition, dollars converted during retirement from surplus to floor result in larger increases because the annuity savings make up an ever increasing fraction of the price of lifetime income. Table 5 illustrates this phenomenon. Previously, sustaining nominal spending resulted in median spending at age 75 and age 85 that was essentially flat. If a late-life annuity is purchased, median spending increases from its initial rate of 5.18% to 5.46% and 6.54% at ages 75 and 85, respectively. Our retiree has over a 50% chance that her real spending at ages 75 and 85 will be greater than her initial rate. However, our retiree has made a sacrifice; she has only guaranteed a 5.18% nominal spending rate, which corresponds to a real spending rate of only 4.05% at age 75 and only 3.16% at age 85. Further, after age 85, when her annuity funds all spending, her real spending is guaranteed to decline with inflation.

Table 5: Confidence Levels for Sustainable Nominal Spending with Late-Life Annuity Purchase

Confidence	Age 75	Age 85
100%	4.05%	3.16%
90%	4.16%	4.01%
75%	4.64%	4.96%
50%	5.46%	6.54%
25%	6.64%	8.85%
10%	8.09%	11.83%

Notes: Measured as a fraction of initial wealth. Age 65 spending is 5.18%.

We can align the DWS utility framework to this retirees preferences. In particular, we adjust the utility for the purchase of an annuity at age 85, and we weight the utility of future spending by the probability of survival. Using the optimal solution to this modified model (see Appendix A for more details), we get the efficiency scores appearing in Table 6. When all strategies use an annuity for spending beyond age 85, differences between strategies are smaller, and the range of efficiencies is compressed. However, the general pattern remains. The Floor-Leverage Rule is nearly optimal, with an efficiency score of

99.6%, and efficiency degrades with lower leverage factors.

Table 6: Efficiency Scores for Sustainable Nominal Spending with Late-Life Annuity Purchase

Surplus Investment Strategy	Leverage Factor	Efficiency Score
All Bonds	0.0	87.9%
Non-levered Equity	1.0	94.2%
The Floor-Leverage Rule	3.0	99.6%

Notes: Efficiency estimate based on a Monte Carlo analysis with one million scenarios.

5. Historical Returns

5.1. UPRO Actual vs. Model Returns

Implementing the Floor-Leverage Rule relies on investing surplus assets in a leveraged equity investment. Here, we explore the historical performance of one of these investments. Leverage equity exchange traded funds (ETFs) are relatively new to the market. A 3x S&P 500 fund, with the ticker symbol UPRO, was introduced in mid-2009. UPRO returned 35.77%, −11.49%, and 46.29% in calendar years 2010, 2011, and 2012, respectively. To estimate prior year returns, we need a model to relate UPRO returns with the index's returns. Our primary assumption is that a 3x fund achieves its exposure by borrowing two dollars for each dollar invested. Thus, the relationship between the daily return on the 3x asset, d_{3x}, and the daily return on the underlying asset, d_x, is:

$$1 + d_{3x} = (1 + d_e)(1 - 2d_{rf} + 3d_x)$$

Here, d_e reflects a daily expense ratio charge, and d_{rf} captures daily borrowing costs. Since daily expenses and borrowing costs are generally small compared to daily equity market movements, a fund invested this way should have daily price movements that are close to triple the daily movement of the index. Once daily returns are determined, simple compounding yields an annual return estimate. For the three years 2010, 2011, and 2012, this method estimates UPRO returns as 35.83%, −11.57%, and 45.68%, respectively.[9] Given the complexities involved, these estimates are remarkably close to the actual returns.

Similarly, the extrapolated UPRO returns for 2008 and 2009 are −85.36% and 60.80%, respectively. Note the vast difference in experiences between retiring in 2008 and 2009. Annualized returns between 2009 and 2012 were a robust 30%. However, embarking on retirement in 2008 would have resulted in a surplus portfolio that by 2012 is still less than half its initial value.

[9] S&P 500 index returns were drawn from Yahoo! Finance index ˆGSPC. Annual expenses were set to 95 bps, consistent with the UPRO prospectus. Borrowing costs were assumed to be 25 bps, consistent with Treasury yields over this period. Since index returns exclude dividends, a monthly 3x dividend payout based on Shiller [35] was included in the annual return calculation.

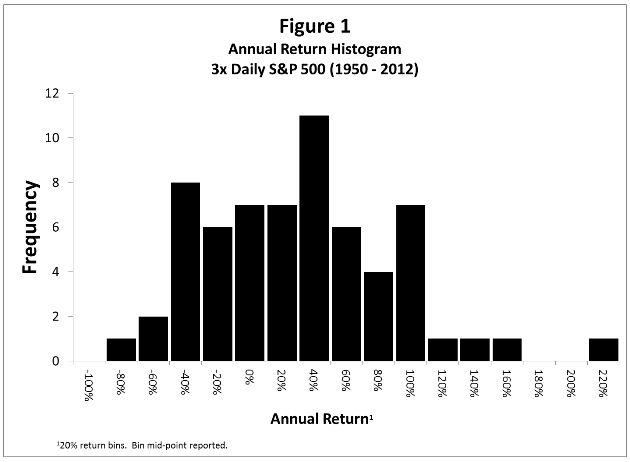

Figure 1: Annual return histogram for a daily, 3x levered S&P 500 equity ETF modeled over the period 1950-2012. All bins are labeled by their mid-point and have a width of 20%.

While 2008 did not exhaust the surplus, its dramatic negative return created a very deep hole. On the positive side, the 2008 retiree living through the financial crisis was likely delighted that her loss represented only 13% of her total wealth and that her lifestyle was secure.

5.2. Historical Analysis

To simulate the historical performance of the Floor-Leverage Rule requires an estimate of daily equity returns and borrowing costs, annual estimates of the real and nominal term structures, and prevailing annuity prices. Unfortunately, the only item on this list of sufficient quality is daily S&P 500 index returns, which date back to 1950. Information on the real term structure has only been available since the introduction of TIPS, and some of their early duration coverage is lacking. Even the nominal term structure is difficult to deduce during the years when the long bond was discontinued. Overcoming these obstacles is beyond the scope of this paper; however, we can combine the daily equity returns with our simplified term structure to provide some insight into how the volatility of equity returns impact retirement spending.

For this analysis, we uniformly apply the flat 4.5% term structure from our simplified economy. If we further assume investing in a 3x leveraged S&P fund entails a one percent expense ratio, then we can estimate the annual returns. As we have seen, 2008 was a dismal year for a 3x leveraged investment, and in fact, it was the worst year across our entire time span. The best performing year was 1954 with a return of 211%. Figure 1 displays an annual return histogram for this time period.

When viewed in isolation, returns from this distribution vary widely, with the return magnitude exceeding 30% in 43 out of the 63 years. Even though a Floor-Leverage Rule investor only allocates a small amount to this asset, the impact on spending can still be large. Figure 2 follows the spending progression of six hypothetical retirees who each wish to sustain nominal spending, but differ in their retirement year. Given our fixed

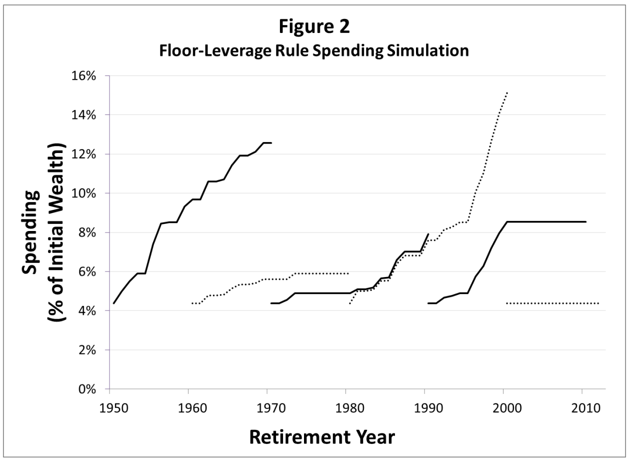

Figure 2: Spending rate histories for retiree cohorts using the Floor-Leverage Rule and starting retirement in 1950, 1960, . . . , 2000. Spending rates are measured as a fraction of initial wealth.

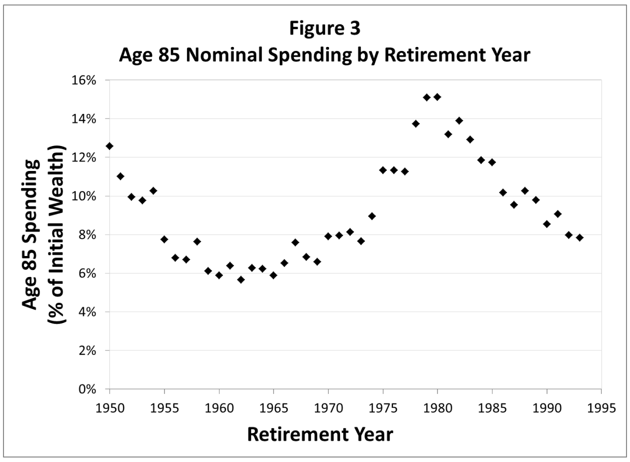

Figure 3: Nominal spending rates at age 85 for retiree cohorts starting retirement at age 65 in the given year. Spending rates are measured as a fraction of initial wealth.

interest rate assumptions, each retiree begins with a spending rate of 4.38% of initial wealth. However, experiences quickly diverge based on market returns. For example, 1950 turned out to be a very good year to retire. Strong market returns were experienced throughout retirement allowing spending to steadily increase. As a percent of initial wealth, spending by age 85 for this retiree was just over 12.5%. For retirements starting in 1980, returns were more back-loaded, but resulted in an even higher rate of 15.12% at age 85. Of the five start dates with sufficient data to simulate to age 85, the worst sequence of returns began in 1960. Spending modestly ratchets up to 5.9%, but then flat-lines when negative returns from the 1973-1974 bear market dig too big a hole for recovery by age 85. Of the five projections, 1960 was the only start date where spending at age 85 fell short of 7.17%, the rate required to keep pace with inflation.

Figure 2 also illustrates a recurring theme with Floor-Leverage Rule retirement investing. Spending tends to ratchet upward nicely until a traumatic market event. After the event, though spending is preserved, there is lengthy stagnation in the spending rate. For example, the 1970 retiree took a substantial hit to the portfolio during the 1973-1974 bear market, and then had to wait until 1981 for a spending increase. More recently, retiring in 2000 has so far proven to be very poor timing. The bursting dot-com bubble and the recession in the early 2000s took a substantial bite out of the surplus portfolio. Just as returns were improving, the financial crash of 2008 dealt another significant blow to the portfolio. Even after the markets had mostly recovered by 2012, this retiree still has insufficient funds to warrant a spending increase.

Extending the analysis to consider all available return data between 1950 and 2012 allows us to examine 20 year spending patterns for retirements that began from 1950-1993. Figure 3 summarizes this data by reporting the age 85 spending for each cohort. Even though all retirees began spending 4.38% of initial wealth at age 65, by age 85 spending had diverged from a low of 5.67% (retirement in 1962) to a high of 15.12% (retirement in 1980). This substantial spending range, nearly a three-to-one ratio from maximum to minimum, underscores the potential upside available even when losses are limited to just 15% of the total portfolio. As these data illustrate, the Floor-Leverage Rule does indeed combine sustainable spending with the potential for substantial spending increases linked to equity market performance.

6. Practical Considerations

Fundamentally, the Floor-Leverage Rule seeks to guarantee the current level of spending with riskless assets and invests in a levered portfolio of equities for growth. There are a variety of ways for retirees and their advisors to make these investments. What are some recommendations? Further, once purchased, how frequently should investments be monitored and rebalanced? In this section, we offer a few practical tips, but our comments are neither exhaustive nor always conclusive. Assuming this strategy aligns with retiree preferences, we anticipate others will extend this analysis to make the strategy more turnkey for retirees.

6.1. Floor Account Investments

The best investments for the floor account depend on whether retiree preferences point to sustainable real or nominal spending. In the US, real (nominal) spending is virtually guaranteed by purchasing a ladder of zero-coupon TIPS (Treasury Bonds). Unfortunately, Treasury maturities only extend to 30 years. For longer planning horizons, an investor must purchase the desired payouts from a financial institution (and seek appropriate guarantees) or use a duration matching strategy.

A late-life annuity introduces additional complexity. For every $1 of annual spending before the age of 85, a retiree can purchase a zero-coupon bond. Further, for every $1 of annual spending at age 85 and after, a retiree can estimate the cost of an annuity, and invest in a zero-coupon bond for this purchase.

However, future annuity prices are uncertain and depend on future interest and mortality rates. One approach would be to make conservative assumptions for interest and mortality rates and invest to achieve that cash flow. For example, even with near zero interest rates, a conservative cost estimate for an age 85 annuity would be \$8 for every \$1 of annuity payout. Of course, unexpected extensions to life expectancy are possible, so even conservative planning could prove inadequate. An alternative approach would be to contract directly with an insurance company. The insurance contract could either be one that secures a conversion price between dollars and income at age 85, or one that directly purchases the desired income beginning at age 85. The latter contract, often called a longevity annuity, is the cheapest option for purchasing future income, but requires an irreversible annuity purchase at retirement (see Scott [32] and Sexauer et al. [34] for pricing examples).

6.2. Surplus Account Investments

Allocating the surplus account to a 3x leverage investment avoids the two main concerns of a CPPI strategy, namely excessive trading costs and exposure in fast falling markets. However, utilizing a 3x leverage investment could create concerns for many retirees. First, some retirees may expect a 3x leverage investment to provide long-term returns that are triple the index returns. However, as many authors have noted, this relationship does not hold. The returns diverge for two basic reasons. First, the leverage investment faces borrowing costs. Second, even without borrowing costs, simple multiplication reveals that compounded daily 3x returns do not equal 3x the return for any other time period. As discussed in Jarrow [23] and Sullivan [38], the compounding differences result in a type of volatility drag for the leveraged investment.

To illustrate volatility drag with daily rebalancing, consider a trading year that was split with half the trading days gaining 1% and the other half losing 1%. Due to volatility drag, the index has lost value. In fact, assuming 250 trading days, the index ends the year with an annual loss of 1.24%. However, returns for the 3x fund are not three times the index return or -3.72%. Instead, owing to increased volatility drag, the 3x fund has lost 10.64%. While generating 3x the index return over the long term is an unreasonable benchmark for a leveraged fund, clients should be reminded that the fund does provide both substantial market exposure and a quick, low-cost way to respond to market downturns.

Leverage aversion is a second potential concern. Some retirees may find it difficult to get comfortable with a leveraged investment. In fact, Jacobs and Levy [22] recently proposed a leverage aversion utility model. In this model, any security that was held in a net short position carried a utility penalty. Judged in isolation, the leveraged investment would indeed carry a utility penalty. However, evaluated as a full strategy, the Floor-Leverage rule investment would not suffer a utility penalty, because neither of the two assets available in the economy, riskless bonds or risky stocks, are held in net short quantities. For clients with leverage aversion, this distinction could prove very important. It is the fact that 85% of assets are safely allocated to sustain spending that allows a 15% leveraged investment. Reminding clients of the overall bond/stock allocation would likely alleviate some of the concern. In combination with the floor, using the leverage fund provides meaningful spending upside if markets perform well, while sustaining spending even during a stock market calamity.

6.3. Dynamics

Once the floor has been purchased and the surplus invested, the Floor-Leverage rule recommends an annual review to determine if a rebalance is warranted. Over time, the strategy evolves based on the performance of the equity portfolio. Two complications should be noted. First, the value of the surplus in excess of 15% may be too small to be effectively invested for additional spending. If this is the case, we recommend that the surplus portfolio is rebalanced to 15%, and the proceeds invested in a money market or other low risk investment alternative. This process would be repeated until sufficient wealth has accumulated to allow for the purchase of meaningful additional spending.

Extremely positive equity returns create a second complication. The efficiency of the Floor-Leverage rule was assessed based on an annual portfolio review. However, the optimal strategy avoids any aggregate equity allocation in excess of 60%. A strong market return could result in a large increase in the surplus wealth. If this occurs, we recommend a mid-year rebalance of the equity portfolio. The exact threshold for triggering a rebalance and the amount of wealth taken off the table are interesting questions for future research. One strategy would be to rebalance to 15% if the surplus hits 30%, but the efficiency gains from that strategy are unknown.

7. Conclusion

After reviewing the retirement literature, we found that most studies either explicitly or implicitly identify sustainable spending as an important retiree preference. However, few suggest retirees adopt a completely risk free approach to investing. In fact, a common recommendation was for the retiree to adopt an average risk portfolio comprised of roughly 50% bonds and 50% stocks. Our goal was to identify a simple, yet still complete, strategy for a retiree with both an average tolerance for risk and a strong desire for sustainable spending.

Our general approach to finding such a strategy first identifies important aspects of preferences, next solves a formal model with those preferences and then finally attempts to approximate the optimal solution with a simple strategy. For preferences, we model a retiree as simply an individual with average risk tolerance and a requirement that realized spending never declines. Optimality with these preferences suggests two important insights. First, the use of leverage allows an individual to simultaneously sustain high levels of spending while maintaining a significant equity market exposure. Second, a desire for sustainable spending is fundamentally different from an intolerance of risk. Indeed, eschewing risk entirely results in highly inefficient strategies.

The final task, finding a simple approximation to optimal, was the main focus of our analysis. We found that, across a range of preferences for sustainable spending, the Floor-Leverage Rule offered a simple, yet near-optimal, approach for retirement spending and investing. How much spending could be sustained depended crucially on the nature of the preference and the cost of the associated floor. More expensive floors, such as one that sustains real spending, resulted in lower initial spending levels whereas floors that depended on a late-life annuity purchase offered substantially higher initial spend rates. Spending across the different retiree case studies ranged from approximately 3% to 5% of initial wealth. Surprisingly, the same general rule, the Floor-Leverage rule, offered near-optimal performance in each case study.

While we believe many retirees will find some variant of the Floor-Leverage Rule desirable, other important preferences remain to be incorporated. For example, the current model allows for wealth shocks due to stock market exposure, but does not capture wealth shocks from other sources such as unexpected health or other retirement costs. Future research is required to analyze the impact of these complications. However, for each enhancement, we recommend the same general approach: solve a model with the desired complexity, and then search for simple to implement approximations to the optimal solution. Following this approach should allow future strategy recommendations to simultaneously address more complicated situations while still providing a simple rule that is both quantitative and complete.

A. Mathematical Appendix

A.1. Utility Model

In a classic analysis, Merton [26] derives an individual's optimal spending and investment rules by maximizing the expected utility from her future spending, subject to a budget constraint. For example, if we assume a a planning horizon of T years, use a felicity function with constant relative risk aversion (CRRA), and exponentially weight tomorrow's consumption relative to today's, then Merton's expected utility is given by:

$$E\left[\boldsymbol{U}\right] = \frac{1}{1-\gamma}\int_0^T e^{-\delta t}\, E\left[\boldsymbol{c}_t^{1-\gamma}\right] dt \tag{A.1}$$

Here, $\boldsymbol{c}_t$ is the instantaneous rate of spending at time t, which must be non-negative. The parameter γ is the relative risk aversion, and δ is the time discount rate. In a complete market, the cost of future spending is equal to the expected value of the product of spending and the pricing kernel $\boldsymbol{M}_t$ (see Cochrane [11]). Hence, the budget constraint for an investor with initial wealth W_0 is:

$$W_0 = \int_0^T E\left[\boldsymbol{c}_t \boldsymbol{M}_t\right] dt \tag{A.2}$$

The optimization is straightforward; the result is an optimal spending strategy in terms of the pricing kernel. The optimal investment strategy is then derived using a delta-hedging argument.

Dybvig [14, 15] captures some aspects of habit-formation by adding a preference for non-decreasing spending. More generally, if spending decreases by less than an allowed amount, Dybvig's utility is equal to Merton's, and if spending decreases by more, then utility plunges to negative infinity. When we maximize Dybvig's utility, we can continue using Merton's formula, provided we add the following constraint on the spending rate:

$$\boldsymbol{c}_0 \geq c_-,\ \boldsymbol{c}_t \geq e^{-D(t-s)}\boldsymbol{c}_s,\ 0 \leq s \leq t \leq T \tag{A.3}$$

Here, c_- is an initial rate and the parameter D controls the magnitude of the maximum allowed spending-rate decline. For the expected utility above, Dybvig finds optimal spending and investment strategies when the planning horizon is infinite—the value relevant for an endowment.

Retirees also share a preference for sustainable spending, and Dybvig's utility captures many important features of the retirement problem. Watson and Scott [39] consider a discrete-time formulation of his model. For example, using a CRRA felicity function, their annual model for expected utility is:

$$E\left[\boldsymbol{U}\right] = \frac{1}{1-\gamma}\sum_{t=0}^{T-1} A_t\, E\left[\boldsymbol{C}_t^{1-\gamma}\right] \tag{A.4}$$

$$\boldsymbol{C}_0 \geq C_-,\ \boldsymbol{C}_t \geq e^{-D}\boldsymbol{C}_{t-1},\ t = 1, 2, \ldots, (T-1) \tag{A.5}$$

Here, $\boldsymbol{C}_t$ is the annual spending t years into retirement, and C_- is an initial reference value, which we often take as zero. The discrete budget equation is:

$$W_0 = \sum_{t=0}^{T-1} E\left[\boldsymbol{C}_t \boldsymbol{M}_t\right] \tag{A.6}$$

The discrete model has a closed-form, optimal solution for a finite planning horizon, a general felicity function, and for any time-preference function A_t. For CRRA, the optimal solution is given by the formula:

$$\boldsymbol{C}_t = \max\left[C_-, \max_{0\leq s\leq t}\left(\frac{\lambda \boldsymbol{M}_s}{y_s A_s}\right)^{-1/\gamma}\right] \tag{A.7}$$

The value of λ can be eliminated in favor of W_0 by imposing the budget constraint. The parameters y_t are the zeros of the coupling functions. The last coupling function $h_{T-1}(y) = y - 1$ has the zero $y_{T-1} = 1$. The remaining coupling functions are defined recursively using the equation:

$$h_t(y) = y - 1 + \left[\frac{A_{t+1}}{A_t}\right]\boldsymbol{E}\left[\max\left(0, h_{t+1}\left(y\left[\frac{A_t}{A_{t+1}}\right]\boldsymbol{m}_{t+1}\right)\right)\right] \tag{A.8}$$

Here, we assume that the pricing kernel has independent increments and introduce the one-year kernel $\boldsymbol{m}_{n+1} \equiv \boldsymbol{M}_{n+1}/\boldsymbol{M}_n$.

A.2. Case Study Parameters

The three case studies in the text explore three flavors of sustainable spending—real spending, nominal spending, and nominal spending combined with a late-life annuity purchase. This analysis takes the viewpoint of a 65-year old female, retiring today, with a 40-year planning horizon ($T = 40$). Further, we assume that our retiree discounts future consumption at a 5% annual rate ($\delta \approx 0.05$) and is moderately risk averse ($\gamma = 3.5$). This level of risk aversion finds support in the experimental literature (see Barsky et al. [1] and Dohmen et al. [13]), and implies that, absent the sustainability constraint, this retiree would optimally hold a market portfolio of 40% bonds and 60% equities.

Optimal solutions depend on a model for future interest rates and market returns. Our simple economy has a riskless, real bond compounding continuously at an annual rate r and a risky asset whose returns are log-normally distributed. For illustration, we choose $r \approx 2\%$ and a risky asset with an annual risk premium of 6% and an annual volatility of 18%. Further, we assume inflation grows exponentially with growth rate $\rho \approx 0.025$. In this economy, the pricing kernel has a log-normal distribution:

$$\boldsymbol{M}_t = \exp\left[-\left(r + \theta^2/2\right)t - \theta \boldsymbol{Z}_t\right] \tag{A.9}$$

The parameter θ is the market price of risk ($\theta \approx 0.35$), and the stochastic function $\boldsymbol{Z}_t$ is a standard Weiner process.

Each case study customizes the time-preference function A_t and the allowed decline in real-spending D. The baseline case, sustainable real spending, chooses the standard exponential weighting $A_t = e^{-\delta t}$ and sets $D = 0$ to prevent any declines in real spending. The sustainable nominal spending case also uses the standard exponential weighting for A_t, but chooses $D = \rho$, i.e., real spending can decline at a rate no higher than the assumed inflation rate. For the third case, a late-life annuity purchase, we again have $D = \rho$, but we choose $A_t = e^{-\delta t} P_t$, where P_t is the probability that the retiree survives t years. In our example, we use female mortality rates; the mortality impact is larger for single males and somewhat mitigated for couples. The utility for this case needs further adjustments, and these are described inn the next subsection.

A.3. Annuity Model

Suppose a retiree purchases an annuity T' years after retirement. For any fixed scenario, we can split the total utility into pieces that occur before and after the purchase:

$$\boldsymbol{U} = \frac{1}{1-\gamma}\sum_{t=0}^{T'-1} A_n\, \boldsymbol{C}_n^{1-\gamma} + \frac{1}{1-\gamma}\sum_{t=T'}^{T-1} A_t\, \boldsymbol{C}_t^{1-\gamma} \tag{A.10}$$

We assume that the annuity's payouts are in nominal dollars, so that all spending after T' declines with inflation:

$$\boldsymbol{C}_t = e^{-\rho(t-T')}\, \boldsymbol{C}_{T'},\ T' \le t \le T \tag{A.11}$$

Hence, the second sum in Eq. (A.10) reduces to a single term:

$$\frac{1}{1-\gamma}\sum_{t=T'}^{T-1} A_t\, \boldsymbol{C}_t^{1-\gamma} = A'_{T'}\, \frac{\boldsymbol{C}_{T'}^{1-\gamma}}{1-\gamma} \tag{A.12}$$

where the coefficient $A'_{T'}$ is given by the formula:

$$A'_{T'} = \sum_{t=T'}^{T-1} e^{-\rho(1-\gamma)(t-T')}\, A_t \tag{A.13}$$

Next, if we define $A'_t \equiv A_t$ for all years prior to T', then the expected utility is exactly the same as Eq. (A.4), but with the new time-preference function A'_t and the shorter horizon T'. Hence, the expected utility is in the canonical form for which we have an optimal strategy.

Though the expected utility is in the right form, we need to modify the contribution of the last spending value C'_T to the budget equation. For this, we need to calculate the annuity's price per dollar of payout Θ, which sums the costs of nominally discounted dollars, weighted by a mortality discount:

$$\Theta = \sum_{t=T'}^{T-1} e^{-(r+\rho)(t-T')}\, \frac{P_t}{P_{T'}} \tag{A.14}$$

We note that Θ is the same for all spending scenarios. Hence, we can adjust Eq. (A.6) by multiplying the term that involves C'_T by the constant Θ. Alternatively, we can redefine the pricing kernel for year $T-1$ to include this constant, and then both the expected utility and budget constraint are in the correct form to use the optimal consumption formula.

A.4. Efficiency Index

The optimal spending solution to the expected utility maximization described by Eq. (A.7) scales linearly with wealth provided the initial minimum spending level C_- is non-binding. In other words, the optimal spending solution with 30% more wealth is simply the original optimal spending strategy increased by 30%. If we let $EU(W)$ correspond to the expected utility derived from following a particular sup-optimal strategy with initial wealth W, and $EU^*(W)$ correspond to the expected utility from following the optimal, then our efficiency index α is defined by the following equation:

$$EU(W) = EU^*(\alpha W) \tag{A.15}$$

Since spending across time and states in Eq. (A.4) is always scaled by α, we can bring it outside of the expected utility calculation:

$$EU(W) = \alpha^{1-\gamma} EU^*(W) \tag{A.16}$$

Which now allows an explicit solution for the efficiency index:

$$\alpha = \left(\frac{EU(W)}{EU^*(W)}\right)^{\frac{1}{1-\gamma}} \tag{A.17}$$

We use a Monte Carlo analysis to estimate $EU(W)$ and $EU^*(W)$ in Eq. (A.17).

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