Screening without Commitment

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Abstract

Employees in some government agencies and companies seem to receive salaries much higher than those offered by the private market, and government organizations seem to employ more people than optimal. Some authors have suggested that public employment is used by governments as a tool for income-redistribution. We present a model that provides theoretical support for that conjecture. Due to fairness concerns, when redistributing the government uses work requirements because of its screening capabilities. We then study how the availability of public employment for redistribution affect the equilibrium. We show that when the government lacks commitment power, screening through work requirements aggravates the moral hazard problem associated to redistribution, ultimately leading to a lower equilibrium effort and possibly to a Pareto-dominated allocation.

JEL Classification: D31, D64, D82, H23, P16.

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1 Introduction

Employees in some government agencies and companies seem to receive salaries much higher than those offered by the private market, and government organizations seem to employ more people than optimal (Poterba and Rueben (1994), Lucifora and Meurs (2006), Galiani and Sturzenegger (2008), Clark and Milcent (2011)). Some authors have suggested that public employment is used by governments as a tool for income-redistribution (e.g., Alesina et al. (2000)). There are a number of studies that present evidence consistent with that conjecture, including countries such as the United States (Alesina et al. (2000)), Brazil (Mattos and França (2011)), Italy (Alesina et al. (2001)) and France (Clark and Milcent (2011)).\footnote{For example, Clark and Milcent (2011) showed that public hospitals in France employ significantly more people than similar non-public hospitals, that such gap is correlated to the unemployment rate and that the correlation is stronger in left-wing areas.} At the same time, recent evidence using survey and experimental data suggests that fairness concerns have a first-order importance for income redistribution (Alesina and Glaeser (2004); Cappelen et al. (2012)). In this paper we show that fairness concerns can generate a demand for public employment as a redistributive tool, because of its screening capabilities and study the long-term welfare implications of such redistributive policy when the government has no commitment.

We contribute to this literature with a positive and normative analysis about public employment and redistribution. First - for the positive analysis - we discuss conditions under which it is optimal for the government to make income transfers contingent on effort requirements - from now on, referred as workfare - as opposed to unconditional transfers - referred as welfare. We introduce a form of fairness concern that we denominate "sympathy for the diligent", which reflects the fact that the government cares disproportionally more about the utility of individuals who exerted high effort. The government cannot directly verify who exerted effort, but it can take advantage of the fact that individuals with lower disutility from effort are relatively more likely to have exerted a higher level of effort. Since the work requirement is less costly for individuals with low disutility of effort, the government can use workfare to screen - in a probabilistic sense - diligent individuals and direct more resources towards them. Thus, this model can explain the government’s desire to employ individuals even if the jobs are completely unproductive.

Second - for the normative analysis - we study the implications of introducing workfare when the government cannot commit to a redistributive policy. Individuals make effort choices based on their expectations about what the redistributive policies will be in the future. Individuals anticipate that, once effort has been exerted, the government has strong incentives to redistribute. This expectation reduces the incentives to exert high effort, pushing the equilibrium level of effort below the social optimum (see for example Boadway et
al. (1996); Konrad (2001); Netzer and Scheuer (2010)). We compare the set of equilibria in which workfare is available to the government as a redistributive policy with the set of equilibria in which workfare is not available. We show that the availability of workfare aggravates the moral-hazard problem: individuals with low disutility of effort anticipate generous workfare programs in the future and thus have reduced incentives to exert high effort in the present. Furthermore, the availability of workfare, even though the government is benevolent and rational, can lead to a Pareto-dominated allocation.

Since the seminal work of Mirrlees (1971), moral hazard plays a central role in the public finance literature on income redistribution. Our model illustrates how the moral hazard channel can be important not only for the quantitative aspects of the redistributive policy, like the shape of the income tax schedule, but also for qualitative aspects, such as the use of work requirements.\textsuperscript{2} We use a framework adapted from Netzer and Scheuer (2010) in which the government lacks commitment. Ex ante, a risk-averse agent is able to affect the probability distribution over output by choosing different levels of effort (e.g., getting education, starting a business). Once outcomes have been determined, the government chooses how much to redistribute based on a social welfare function. A low level of redistribution is optimal ex ante so that the agents have incentives to exert higher levels of effort, but a high level of redistribution is optimal once efforts have been chosen. Due to its lack of commitment, the agents anticipate that the government will implement a high degree of redistribution and therefore exert lower levels of efforts.

We compare an equilibrium where workfare is available with an equilibrium where workfare is not available. The availability of workfare can affect equilibrium efforts through two channels, with opposite directions. On the one hand, since workfare allows the government to transfer more resources to diligent individuals, the availability of workfare can make it more attractive to exert effort and become a diligent individual. On the other hand, workfare can aggravate the moral hazard problem, because individuals with low disutility of effort anticipate access to generous workfare programs in the future and thus are tempted to shirk in the first stage. We show that, under fairly general conditions, the latter (moral-hazard) effect will dominate, so the availability of workfare reduces equilibrium effort. Furthermore, the equilibrium allocation when workfare is available can be Pareto-dominated by the allocation when workfare is not available. If the government had commitment power, then the availability of a new policy instrument, such as workfare, could not be harmful to society: i.e., in the worst case scenario, the government would commit not to use that instrument and therefore achieve the same social welfare as if the instrument was not available. However,

\textsuperscript{2}Indeed, similar moral hazard effects may be at play with other policy instruments that serve as a screening mechanism, such as tagging (Akerlof, 1978), in-kind transfers (Blackorby and Donaldson, 1988) and welfare stigma (Moffit, 1983).
lacking commitment power, the government cannot avoid falling into the Pareto-dominated equilibrium. This result is similar in spirit to the main result in Konrad (2001) that shows that, because of the lack of commitment, the fact that the government has access to better information about the agents’ types leads to a Pareto-dominated allocation.

Our paper is related to a literature that studies workfare programs. In their seminal contribution, Besley and Coate (1992) introduced a model where government can use work requirements to screen poor individuals according to their ability. Individuals in their model have a choice between receiving a transfer from the government or working in the private market. From the perspective of efficiency, the government is interested in directing its help towards low-ability individuals, who wouldn’t make ends meet without the government’s help. By introducing work requirements, the government can screen low-ability individuals because they have a lower opportunity cost from working in the private sector. A number of papers have elaborated on this screening principle (e.g., Besley and Coate (1995); Cuff (2000); Moffitt (2006)). The main difference between our paper and the previous literature on workfare is that we study long-term redistributive policies: i.e., agents decide whether to join a workfare or welfare program once their fates in the private market have been decided. The work requirement in our model intends to represent unproductive public-sector employment rather than the typical workfare program modeled in Besley and Coate (1992). As a result of fairness concerns, in our model work requirements are intended to self-select high-ability individuals and not low-ability individuals like in Besley and Coate (1992).

We also contribute to the literature that uses fairness concerns to study income redistribution (e.g., Alesina and Angeletos (2005); Di Tella et al. (2008)). Our main contribution is to show that fairness concerns, apart from explaining the extent of redistribution, can also explain qualitative aspects of the redistribution scheme: in our case, the use of work requirements. There is evidence in favor of fairness concerns similar in nature to the one analyzed in our paper from a variety of empirical methods. Alesina and Glaeser (2004) use survey data to show that the percentage of the population that believe that poor people are lazy is strongly correlated with social spending across the set of OECD countries, consistent with the notion that people want to help the poor only if they perceive their condition arising from luck and not from lack of effort. Similarly, Cappelen et al. (2012) present evidence from laboratory experiments showing that, when deciding whether to help others, most individuals make a distinction between ex post inequalities that reflect differences in luck and ex post inequalities that reflect differences in choices. Our conjecture about the link between fairness concerns and the demand for workfare is particularly supported by the findings reported by Falk et al. (2006). They conducted a laboratory experiment in

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3Some of the typical workfare programs, however, may be entry-level jobs for public employment, as argued by Gueron (1990) with data from the U.S.
which subjects vote over the institutional arrangement of social assistance: i.e., welfare vs. workfare. After voting, participants were asked about their motivation to support workfare. More than two-thirds of the supporters of workfare mentioned variations of fairness concerns, while only one quarter of them mentioned self-interest.

The paper is organized as follows. The next section describes the main setup of the model. Section 3 describes the solution to the government’s redistribution problem and characterizes the set of welfare and workfare equilibria. In Section 4 we present the main result of the paper that compares the maximum equilibrium levels of effort that can be sustained in welfare and workfare equilibria. Section 5 presents numerical results describing the Pareto comparison across equilibria and shows the effect of introducing productive workfare programs. The last Section concludes.

2 The Model

There is a continuum of risk averse agents of measure one indexed by $i$. Agents are expected utility maximizers with a Bernoulli utility function $U(c)$, where $c$ denotes consumption. We assume that $U(c)$ is twice continuously differentiable with $U' > 0$ and $U'' < 0$. Also, the domain and range of $U$ are given by $\mathbb{R}$. The Inada conditions are assumed to hold. Let $\Phi(U)$ be the inverse function of $U$, which satisfies $\Phi' > 0$, $\Phi'' > 0$, $\lim_{U \to -\infty} \Phi(U) = -\infty$, $\lim_{U \to \infty} \Phi(U) = \infty$ and $\lim_{U \to \infty} \Phi'(U) = \infty$.

Each agent faces idiosyncratic risk with respect to the level of output he can produce. There are two possible levels of production: high ($y_h$) or low ($y_l$), with $y_l < y_h$. In order to generate this output, agents have to decide between two effort levels $e \in \{e_l, e_h\}$. If $e = e_l$ then the agent is an ex post good type ($g$) and if $e = e_h$ then the agent is an ex post bad type ($b$). Good types produce the high output $y_h$ with probability $p_g$ and bad types produce the high output $y_h$ with probability $p_b$, where the restrictions $0 < p_b < p_g < 1$ hold. Agent’s preferences are represented by an utility function that is separable between consumption utility and effort cost, $U(c) - H(e)$, where $H(e)$ represents the effort cost. We normalize $H(e_l)$ to zero. Agents differ in their disutility of effort $H(e) = d$, which can take the values $d \in \{d_l, d_h\}$.

A proportion $q (1 - q)$ of the population has a low (high) effort cost $d_l$ ($d_h$). Differences in agents’ effort costs must be interpreted as differences in agents’ preferences for leisure, as opposed to differences originated from disabilities or opportunities. This

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4The assumption of two types of agents is a deviation from the original setup from Netzer and Scheuer (2010), who assume a continuum of types. This assumption is meant to reduce the complexity of the government’s problem. One important implication of this assumption is that low cost of effort agents in equilibrium will choose more than one effort level, and thus the government cannot use the work requirements to perfectly screen diligent individuals. This implication would also be attained in a model with a continuum of types if we assumed, for example, that agent’s observe an imperfect signal of the returns to effort.
interpretation is the one most consistent with the fairness concerns that will be introduced later. We assume that neither effort cost nor effort choice are observable.

Following Netzer and Scheuer (2010), we carry out the analysis in the utility space. A contract offered by the government is a vector of consumption utilities that agents obtain when producing the high and low output, respectively.

3 Redistributive Policies: Workfare vs. Welfare

Unlike the standard model of unemployment insurance or yearly government benefits, this is a coarse representation of income insurance in the game of life. In the first stage, agents who make an effort are individuals who actively try to discover their comparative advantage by working longer hours and investing in human capital. Those who perform a low effort represent people who work the bare minimum and do not invest in human capital. In the second stage, the higher the effort exerted in the first stage the more likely that the agent will end up with a better outcome. But some of those who shirk may end up with a good outcome and some of those who worked hard can end up in a bad situation (e.g., the hard-working athlete who got injured).

One key difference from some of the papers that study income redistribution (e.g., Meltzer and Richard (1981); Alesina and Angeletos (2005)) is the assumption we make about the timing of events. Following Boadway et al. (1996), Konrad (2001) and Netzer and Scheuer (2010), we assume that the government cannot commit to a certain redistribution scheme before the realization of outcomes. Notice that it makes no difference whether the redistribution scheme is decided before the uncertainty is resolved. What is really important is whether it the redistribution scheme be modified after the outcomes are realized. The assumption about the government’s lack of commitment is based on the fact that the time period represents a long horizon. Making an effort in this model does not mean working longer hours during a given year, but rather human capital accumulation (e.g., Boadway et al. (1996)). In practice there is no such thing as perfect commitment or complete lack of commitment, but it is a degree issue mediated by factors like institutions and reputation (see for example Acemoglu et al. (2008)).

A second departure is related to the government’s objective function. We assume that the government’s Pareto weights depend on ex post types. In particular, we are interested in the case in which the government would like to redistribute more towards agents that made an effort earlier in their life. This fairness concern from the government’s side can be the result of the underlying preferences of the voters. There is plenty of evidence that individual preferences for redistribution are highly correlated to individuals’ beliefs about the causes of poverty. Individuals that think that the poor are poor because they were lazy prefer
not to redistribute as much as individuals that belief that poverty is the result of bad luck (Alesina and Glaeser (2004); Alesina and Angeletos (2005)). Another possibility is that this objective function of the government represents political constraints: the government wants to redistribute as much as possible, but it knows that redistribution is politically viable only if it is perceived as helping the unlucky rather than coddling the lazy.

In this section we characterize the equilibria that arise with two different redistributive mechanism: welfare and workfare. In the next section we make a comparison between these two set of equilibria.

### 3.1 Welfare

Welfare represents a redistributive scheme in which the government can screen agents based on income only. This restriction implies that all the rich agents receive the same level of utility and all the poor agents also must receive the same level of utility. The timing of events is the following:

- **Stage 1:** Agents simultaneously choose their effort levels.
- **Stage 2:** Agents’ incomes are realized.
- **Stage 3:** The government chooses a redistributive policy \((u_{b,h}, u_{b,l}, u_{g,h}, u_{g,l})\).

The fact that we focus on the case of a government without commitment is clearly observed in the timing of the model: choices about redistribution are made after effort and output are realized. The objective is to find the set of Bayesian Equilibria (BE). We will focus on BE in which only a fraction \(x\) of the low-cost type agents make an effort and all the high-cost type shirk. The reason for having a type of agent that always decides to shirk is to make workfare valuable because of the screening device it can provide. We find the set of Bayesian Equilibrium by backward induction. For a given level of \(x\) chosen at Stage 1, we derive the government’s optimal policy at Stage 3.

We assume that the government cannot differentiate between ex post good low-income agents and ex post bad low-income agents. The idea is to reflect the fact that most welfare programs base their eligibility criteria on observable factors, such as income.

We assume that the government is able to form precise inference about the proportion of agents that made an effort at the first stage, \(x\). Let \(\alpha\) denote the relative weight the benevolent government places on its ex post welfare function on ex post good types and let \((1-\alpha)\) denote the relative weight placed on ex post bad types. Here, \(\alpha\) measures the degree of sympathy for the diligent. The objective of the government is to maximize a weighted average of the agents’ ex post utilities, taking into account the constraints imposed by the budget constraint and the constraints imposed by the welfare redistributive mechanism. Whenever
\( x \in (0, 1] \) so that both ex post types exist, the benevolent government solves the following problem:

\[
\max_{(u_{g,b}^w, u_{g,l}^w, u_{b,h}^w, u_{b,l}^w) \in \mathbb{R}^4} \alpha \left[ qx \left( p_g u_{g,h}^w + (1 - p_g) u_{g,l}^w \right) \right] + (1 - \alpha) \left[ (1 - qx) \left( p_b u_{b,h}^w + (1 - p_b) u_{b,l}^w \right) \right]
\]

subject to the constraints

\[
\begin{align*}
    u_{g,l}^w &= u_{b,l}^w \equiv u_l, \\
    u_{g,h}^w &= u_{b,h}^w \equiv u_h
\end{align*}
\]

\( q x [ p_g \Phi(u_{g,h}^w) + (1 - p_g) \Phi(u_{g,l}^w) ] + (1 - qx) [ p_b \Phi(u_{b,h}^w) + (1 - p_b) \Phi(u_{b,l}^w) ] \leq R(x) \)

where \( R(x) \) represents the per capita resources available in the economy and are given by

\[
R(x) = qx [ p_g y_h + (1 - p_g) y_l ] + (1 - qx) [ p_b y_h + (1 - p_b) y_l ]
\]

Equations (1) and (2) require that the utility of agents that produced low and high output does not depend on the type. This constraint captures the idea that transfers made within welfare programs are based upon agent’s earned income (for example, income taxes/subsidies) only. The following Lemma characterizes the solution to the government’s problem.

**Lemma 1.** Fix any \( x \in (0, 1] \). (i) The government’s problem has a unique solution \( V(x) = (u_{h}^w(x), u_{l}^w(x)) \). (ii) If \( \alpha \geq (>)1/2 \), then \( u_{h}^w(x) \geq (>)u_{l}^w(x) \). (iii) If \( \alpha \leq (<)1/2 \), then \( u_{h}^w(x) \leq (<)u_{l}^w(x) \).

**Proof.** See the Appendix.

Lemma 1 characterizes the direction of the ex post government’s optimal redistribution as a function of the Pareto weight \( \alpha \). When the Pareto weights are tilted towards ex post good type agents \( \alpha \geq 1/2 \), the government chooses to reward effort by giving the rich a higher utility relative to the utility that bad-type agents receive. The reason for this is that the assumption \( p_g > p_b \) implies that the majority of rich agents are going to be ex post good types. If on the other hand, the Pareto weights of ex post bad type agents is higher \( \alpha \leq 1/2 \), then redistribution goes in the opposite direction since the poor are more likely to be bad-type agents. The final case involves \( \alpha = 1/2 \) (i.e., the government has no preference for any particular group of ex post agents), then the government chooses to fully insure agents by choosing \( u_{h}^w(x) = u_{l}^w(x) \).

In the case of \( x = 0 \) (i.e., no low-cost type agent makes an effort), the benevolent government’s problem simplifies substantially. It reduced to the maximization of the utility
of the unique ex post type only subject to the resource constraint. Then, convexity of \( \Phi \) will require that the solution satisfies \( u_{\text{we}} = u_{\text{h}} \).

Next we define an equilibrium of the game between agents and a benevolent government without commitment as follows.

**Definition 1.** A welfare equilibrium is a pair \((x_{\text{we}}, V_{\text{we}})\), where \( V_{\text{we}} = V(x_{\text{we}}) \) and one of the following conditions holds

(i) \( x_{\text{we}} = 0 \) and

\[
p_gu_{g,h}(x_{\text{we}}) + (1 - p_g)u_{g,l}(x_{\text{we}}) - d_l < p_bu_{b,h}(x_{\text{we}}) + (1 - p_b)u_{b,l}(x_{\text{we}})
\]

(ii) \( x_{\text{we}} \in [0, 1] \) and

\[
p_gu_{g,h}(x_{\text{we}}) + (1 - p_g)u_{g,l}(x_{\text{we}}) - d_l = p_bu_{b,h}(x_{\text{we}}) + (1 - p_b)u_{b,l}(x_{\text{we}})
\]

(iii) \( x_{\text{we}} = 1 \),

\[
p_gu_{g,h}(x_{\text{we}}) + (1 - p_g)u_{g,l}(x_{\text{we}}) - d_l > p_bu_{b,h}(x_{\text{we}}) + (1 - p_b)u_{b,l}(x_{\text{we}})
\]

and

\[
p_gu_{g,h}(x_{\text{we}}) + (1 - p_g)u_{g,l}(x_{\text{we}}) - d_h < p_bu_{b,h}(x_{\text{we}}) + (1 - p_b)u_{b,l}(x_{\text{we}})
\]

The definition of equilibrium is based on the agents’ ex ante incentives to make an effort at Stage 1 taking the government’s response function as given. Agents form expectations about future redistributive policies and compare the expected utility of working and shirking. There are three types of subgame perfect equilibria that could arise. Two extreme equilibria \((x \in \{0, 1\})\) occur when the low-cost of effort agent strictly prefers to work/not to work, given the anticipated future redistributive policies. There is also an intermediate type of equilibrium, in which the low-cost of effort agents are indifferent between making an effort or not. Thus, \( x \) could be interpreted as a proportion of low-cost of effort agents choosing to make an effort at Stage 1 or as a mixed strategy of each individual agent. In this latter case, the definition of equilibrium can be viewed as the standard indifference condition found in mixed equilibria.

We will focus on equilibria in which only the low-cost of effort might choose to make an effort at Stage 1. The reason for doing this will become more clear in the next section, when we analyze the benefits of workfare. Intuitively, if there were no agents shirking at Stage 1, there would be no reasons for the government to implement workfare instead of welfare programs. Note that the definition imposes restrictions on the behavior of high-cost of effort agents for the last type of equilibrium only. For the other two cases, these restrictions become
redundant. The fact that low-cost of effort are indifferent or strictly prefer to shirk at Stage 1, implies that high-cost of effort agents would strictly prefer to shirk at Stage 1.

The following proposition describes the set of equilibria as a function of the Pareto weights.

**Proposition 1.** For any parameter values of the model, \((x^{we}, V^{we}) = (0, V(0))\) is a welfare equilibrium. If \(\alpha \leq 1/2\), \((x^{we}, V^{we}) = (0, V(0))\) is the unique equilibrium. For any value of \(\alpha > 1/2\), there exists a value \(d_l(\alpha)\) such that there exists at least one additional equilibrium with \(x^{we} > 0\) for \(d_l \leq d_l(\alpha)\).

If the government’s Pareto weights are such that the government wants to ex post redistribute from rich to poor (i.e., \(\alpha \leq 1/2\)), the set of welfare equilibria becomes a singleton. We showed in the previous lemma that when \(\alpha \leq 1/2\), the government will ex post choose \(u^{we}_h(x) \leq u^{we}_l(x)\). This clearly eliminates any incentives to make any effort from an ex ante perspective. From our previous result it is easy to see that this allocation still belongs to the set of welfare equilibria when \(\alpha > 1/2\). However, if the government’s Pareto weights are tilted towards the ex post good types other equilibria might arise. This will be particularly true for low values of \(d_l\). Agents’ effort costs do not affect the government’s ex post choice of the redistributive policy. Therefore, as long as \(u^{we}_h(x) > u^{we}_l(x)\), other equilibria might emerge if \(d_l < (p_g - p_h) (u^{we}_h(x) - u^{we}_l(x))\) (i.e., if the effort cost is smaller that the expected net utility gain of making an effort).

### 3.2 Workfare

In this section we allow the government to implement a redistributive mechanism that relies on self-selection. We define workfare as a transfer of utility whose delivery is conditional to the realization of a certain task. Low-income agents can choose whether to participate in the workfare program or not, and participation in the program is perfectly observable to the government. Taking part in the program requires making an effort \(e_w\), which will be chosen by the government simultaneously with the decision of \(u^{wo}_w\), the consumption utility received by low-income agents who participate in the workfare program, and \(u^{wo}_l\), the consumption utility received by low-income agents who decide not to participate. We assume that the government can only offer workfare to low-income individuals.\(^5\)

We assume the cost of making the effort \(e_w\) is proportional to the parameter \(d\): i.e., \(H(e_w) = e_w \cdot d\). For now we assume that the required effort in the workfare program is completely unproductive. That is, the only benefit produced by workfare is the screening

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\(^5\)In a more realistic model, this would equal to the assumption that an individual cannot hold two jobs simultaneously (e.g., Besley and Coate (1995)).
mechanism (in the next section we present numerical results for the alternative scenario). The timing of events is now:

Stage 1: Agents simultaneously choose their effort levels.
Stage 2: Agents’ incomes are realized.
Stage 3: The government chooses a (welfare/workfare) redistributive policy.
Stage 4: If the government chooses $e_w > 0$, agents simultaneously choose whether they want to participate in the workfare program or not.

The government might want to adopt a workfare program because it allows to identify the ex post good types among the low-income agents, so the government can give them a different level of utility to the ex post bad low-income agents than to the ex post good low-income agents. Since welfare does not provide this type of screening, the utility that ex post good low-income agents receive is lower to what they would receive in the full-information case (in which effort is observable) if the government’s Pareto weights are tilted towards the good types. In this scenario, workfare becomes a useful policy since it allows the government to screen both types of agents and give a higher utility to ex post good low-income agents.

We find the set of Bayesian equilibria by backward induction. We focus on workfare equilibria in which the government is able to discern low-cost low-income agents from high-cost low-income agents by implementing a workfare program. In particular, $e_w$ is chosen in a way such that high-cost low-income agents are indifferent between participating and not (and we assume that in equilibrium they do not participate)

$$u_l^{wo} = u_w^{wo} - e_w d_h$$

Since $d_l < d_h$, equation (4) implies that low-cost low-income agents strictly will prefer to participate in the workfare program. There are many levels of $e_w$ that would allow the government to screen the type of agents, but there are reasons for choosing this particular level. The restriction imposed by equation (4) requires the minimum workfare effort level that makes the high-cost of effort agents indifferent between participating and not, and thus minimizes the effort cost of those agents who actually decide to participate. It is possible to make the high-cost of effort agents strictly prefer not to participate in workfare by slightly increasing $e_w$, but this would simply hurt workfare participants without adding any benefit. Because of this, this level of workfare effort can be justified as the level of effort that the benevolent government would choose if he could decide the type of task that participants need to perform in workfare.\(^7\)

\(^6\)For simplicity, we assume that an individual is considered diligent or non-diligent based only on its effort level during the first stage. In other words, the efforts made in workfare do not affect the diligent status of an individual.

\(^7\)This restriction is in line with the assumption of lack of commitment from the government’s side.
Assuming that the government is able to precisely infer the percentage of low-cost of effort agents that made an effort in Stage 1 and to screen high-cost of effort agents from low-cost of effort agents, the benevolent government’s problem can be represented by the following maximization problem

$$\max_{(u_{h}^{wo},u_{l}^{wo},u_{g,h}^{wo},u_{w}^{wo})\in\mathbb{R}^4,e_{w}} \alpha[qx(p_{g}u_{g,h}^{wo} + (1 - p_{g})(u_{w}^{wo} - e_{w}d_{l}))] + (1 - \alpha)[q(1 - x)(p_{b}u_{b,h}^{wo} + (1 - p_{b})(u_{w}^{wo} - e_{w}d_{l}))] + (1 - \alpha)\left[(1 - q)(p_{b}u_{b,h}^{wo} + (1 - p_{b})u_{l}^{wo})\right]$$

subject to the constraints

$$u_{l}^{wo} = u_{w}^{wo} - e_{w}d_{h}$$

$$u_{g,h}^{wo} = u_{b,h}^{wo} \equiv u_{h}^{wo}$$

$$qx[p_{g}\Phi(u_{g,h}^{wo}) + (1 - p_{g})\Phi(u_{w}^{wo})] + q(1 - x)[p_{b}\Phi(u_{b,h}^{wo}) + (1 - p_{b})\Phi(u_{w}^{wo})] + (1 - q)[p_{b}\Phi(u_{b,h}^{wo}) + (1 - p_{b})\Phi(u_{l}^{wo})] \leq R(x)$$

where $R(x)$ represents per capita resources and are defined in the same way as before.

Notice that the previous maximization problem contains the maximization problem with welfare as a special case with $e_{w} = 0$. The only benefit of workfare over welfare is that workfare allows to introduce a wedge between $u_{w}^{wo}$ and $u_{l}^{wo}$. Thus, it is possible that the maximization problem yields two types of solutions. For some subset of the parameter space and for some values of $x$, the argument that maximizes the government’s objective function might require setting $e_{w} = 0$. This case can be interpreted as the government optimally choosing not to implement a workfare program, and to simply redistribute via a welfare program. The second case might involve $e_{w} > 0$, in which case we say that the government wants to implement a workfare program. Next, we characterize the solution to this problem.

**Lemma 2.** Fix any $x \in (0, 1]$. (i) The government’s problem has a unique solution $V(x) = (u_{h}^{wo}(x), u_{w}^{wo}(x), u_{l}^{wo}(x))$. (ii) If $e_{w}(x) > 0$, then $u_{h}^{wo}(x) > u_{l}^{wo}(x)$ and $u_{w}^{wo}(x) - e_{w}d_{l} > u_{l}^{wo}(x)$.

The characterization of the solution omits the case in which $e_{w}(x) = 0$, because this implies that the government is choosing to redistribute through a welfare program (whose solution was previously characterized by Lemma 1). If the government ex post optimally decides to implement a workfare program, then it must be the case that the government redistributes utility towards agents that produced the high level of output and towards agents that decided to participate in workfare. The following two lemmas characterize the government’s optimal decision to implement workfare as a function of the Pareto weights and the proportion of low-cost of effort agents that made an effort at Stage 1.

**Lemma 3.** If $\alpha < 1/2$ the government would choose $e_{w}(x) = 0$ for all values of $x$. 
If the government decided to implement workfare, it must have been the case that the Pareto weights assigned to ex post agents are higher than the weights assigned to ex post bad agents. In our setting this means that the government needs to have particular fairness concerns towards those who made an effort at Stage 1 (as opposed to those who shirked) in order to prefer a workfare over a welfare redistributive mechanism. If this was not the case, there would be no motive to hurt low-income ex post bad types by creating an utility wedge between workfare participants and non-participants. The following lemma complements the intuition of the desirability of workfare programs.

**Lemma 4.** If the government uses workfare for some \( x \in [0, 1] \), then there exists a value \( \tilde{x} \in [0, 1] \) such that the government will choose \( e_w(x) > 0 \) for all \( x > \tilde{x} \) and \( e_w(x) = 0 \) for all \( x \leq \tilde{x} \).

The benevolent government will choose not to implement workfare when \( x \) is below a certain threshold. The intuition of this result is straightforward. When a small proportion of low-cost of effort agents made an effort in Stage 1, the government infers that the majority of low-income agents are not going to be ex post good types making the screening benefits of workfare not appealing. Furthermore, this conditions is more likely to be satisfied when \( d_h \) is high compared to \( d_l \). The higher the cost of effort of high-cost agents, the lower the effort requirement the government needs to impose in the workfare program and the larger the desirability to redistribute via workfare.

Analogously to the case of welfare, we define a workfare equilibrium as a fixed point between agents’ effort choice and the government’s optimal redistributive policy.

**Definition 2.** A workfare equilibrium is a pair \((x^{wo}, V^{wo})\), where \( V^{wo} = V(x^{wo}) \) and one of the following conditions holds:

1. \( x^{wo} = 0 \) and
   \[
   p_g u_h^{wo}(x^{wo}) + (1 - p_g) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l) - d_l < p_b u_h^{wo}(x^{wo}) + (1 - p_b) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l)
   \]

2. \( x^{wo} \in [0, 1] \) and
   \[
   p_g u_h^{wo}(x^{wo}) + (1 - p_g) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l) - d_l = p_b u_h^{wo}(x^{wo}) + (1 - p_b) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l)
   \]

3. \( x^{wo} = 1 \),
   \[
   p_g u_h^{wo}(x^{wo}) + (1 - p_g) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l) - d_l > p_b u_h^{wo}(x^{wo}) + (1 - p_b) (u_w^{wo}(x^{wo}) - e_w(x^{wo})d_l)
   \]

and
\[
 p_g u_h^{wo}(x^{wo}) + (1 - p_g) u_l^{wo}(x^{wo}) - d_h < p_b u_h^{wo}(x^{wo}) + (1 - p_b) u_l^{wo}(x^{wo})
\]
The definition of a workfare equilibrium differs slightly from the definition of a welfare equilibrium. The indifference condition of the low cost of effort agents is now modified to take into account the fact that they will optimally choose to participate in workfare if they end up producing a low level of output. In the case of high cost of effort agents, the indifference condition also has the (no) participation choice embedded in it. Given this definition, we now proceed to characterize the set of workfare equilibria.

**Proposition 2.** For any parameter values of the model, \((x^{wo}, V^{wo}) = (0, V(0))\) is a workfare equilibrium. If \(\alpha \leq 1/2\), \((x^{wo}, V^{wo}) = (0, V(0))\) is the unique equilibrium. There exists a set of values \((\alpha, d_l, d_h)\) such that if \(\alpha\) and \(d_h\) are high and \(d_l\) is low enough, then there exists at least one additional equilibrium with \(x^{wo} > 0\) and \(e_w(x^{wo}) > 0\).

We find again that when \(\alpha \leq 1/2\), \((x^{wo}, V^{wo}) = (0, V(0))\) is the unique equilibrium. This is simply due to the fact that when \(\alpha \leq 1/2\) the government optimally chooses \(e_w = 0\). The proof to the second part of the Proposition is straightforward. The proof of Lemma 3 shows that the optimality condition for \(e_w > 0\) is more likely to be satisfied for high values of \(d_h\). The results holds because higher values of \(d_h\) lead to lower levels of effort that need to be made by workfare participants, which means that workfare becomes ”cheaper” to implement in terms of effort cost. Similar to the case of welfare, the effort cost paid at Stage 1 does not enter the government’s maximization problem. Therefore, for each level of \(\alpha\), one can find a threshold value for \(d_l\) such that additional equilibria with \(x^{wo} > 0\) and \(e_w(x^{wo}) > 0\) emerge.

### 4 A Comparison of Equilibriums: Workfare vs. Welfare

Having characterized the set of welfare and workfare equilibria, we now proceed to compare both sets focusing on the aggregate level of effort that can be sustained in equilibrium. Given the possibility of multiple equilibria with both redistributive schemes, we focus on the highest level of effort that can be sustained in each scheme. In order to prove our main Theorem, we need to make an additional assumption regarding the utility function:

**Assumption 1.** The utility function \(U(\cdot)\) satisfies the following condition

\[
\frac{\partial}{\partial u} \frac{\Phi''(u)}{\Phi'(u)} \leq 0
\]

(7)

A similar version of this condition has been imposed in the literature (Fudenberg and Tirole (1990), Netzer and Scheuer (2010)).\(^8\) With this condition in hand, we now present the main result of the paper:

\(^8\)This conditions is satisfied by utility functions with constant relative risk aversion of one or below one (Fudenberg and Tirole (1990), Lemma 3.2).
Theorem 1. If condition (7) is satisfied and if $d_h$ is high enough, then the highest effort level that can be sustained in a welfare equilibrium $\hat{x}^{we}$ is at least as large as any effort level sustained in a workfare equilibrium $x^{wo}$ (i.e., $\hat{x}^{we} \geq x^{wo}$). Furthermore, if $\hat{x}^{we} < 1$, then $\hat{x}^{we} > x^{wo}$.

Theorem (1) states that, for high values of $d_h$\(^9\) and whenever condition (7) is satisfied\(^10\), the largest level of effort that can be sustained in a welfare equilibrium is always as high as the largest level of effort that can be sustained in equilibrium. Furthermore, if the largest effort of level that can be sustained in a welfare equilibrium is interior, then it must be strictly higher than any effort level that can be sustained in a workfare equilibrium.

The intuition of this result goes as follows. Workfare allows the government to distinguish between low and high cost of effort agents within the pool of poor agents. From Lemma 3 we know that if the government chooses $e_w > 0$, then it must be the case that $\alpha > 1/2$ (that is, the government’s redistributive program is shaped by fairness concerns towards the ex post good agents). In the proof of Theorem 1 we show that for all $x$ the government will choose an allocation of utilities such that $u_w^{we}(x) - e_w(x)d_I > u_I^{we}(x)$. Keeping $x$ constant, the government will ex post give the poor low cost of effort agents a higher utility in workfare than in welfare (which is the result of $\alpha > 1/2$). From an ex ante perspective, this increases the ex ante utility of agents with low cost of effort. However, the increase in expected utility will be higher for agents who do not make an effort at Stage 1, since they will become workfare participants in the future with a higher probability ($p_g > p_b$). Furthermore, we show in the proof that for any $x$ the government chooses $u_h^{we}(x) \geq u_h^{wo}(x)$. Since for a given $x$ aggregate resources do not depend on the redistribution program, this means that with workfare the government is able to give a higher utility to workfare participants by giving the rich a lower utility (in comparison with what they would get with welfare). Since $p_g > p_b$, the decrease of the utility of the rich in workfare will affect agents that decided not to make an effort at Stage 1 less. Combining these two effects, we show that with workfare low effort cost agents will have a weaker incentive to make an effort ex ante.

\[^9\]This condition is introduced for two reasons. In the first place, as we previously mentioned, the government is more likely to choose $e_w > 0$ for high values of $d_h$. The second reason is more relevant. Because workfare programs give a higher utility to workfare participants than what they would get in welfare (for a given value of $x$), it might be that $u_h^{we}(x) > u_h^{wo}(x)$. If ex post high cost of effort poor agents receive a very low utility when the government chooses $e_w > 0$ (because the government wants to redistribute away from these agents), then it might become optimal for some of these agents to decide to work at Stage 1. In order to avoid this type of unrealistic equilibria we focus on cases in which $d_h$ is high enough, so that high cost of effort agent would never want to make an effort at Stage 1.

\[^10\]The condition we impose is a sufficient but not a necessary condition for the main result to hold. We numerically experimented with other utility functions (CARA, CRRA with coefficient greater than 1) and the results of the Theorem still apply. However, the assumption we impose allows us to prove the Theorem analytically.
5 Discussion

In the previous section we showed that, provided some fairly general conditions, the availability of workfare leads to an equilibrium with lower effort. In this section we extend the comparison of redistributive policies in terms of social welfare.

Define surplus as the ex-ante expected utility: $S_h$ for a high-type agent, $S_l$ for a low-type agent, and $S$ for the average agent (i.e., $S = qS_l + (1 - q)S_h$). A Utilitarian Social Planner would like to maximize $S$. Additionally, if one equilibrium had lower $S_h$ and $S_l$ than a second equilibrium, it would imply that it is Pareto-dominated. Introducing the availability of workfare has the following effect on the surplus of a given type:

$$\Delta^j = S^\text{work}(x^*_\text{work}) - S^\text{wel}(x^*_\text{wel}) = S^\text{work}(x^*_\text{work}) - S^\text{wel}(x^*_\text{wel}) + S^\text{work}(x^*_\text{work}) - S^\text{wel}(x^*_\text{work})$$

The first term, $\Delta^j_1$, is the change in surplus from reducing equilibrium effort, which corresponds to an efficiency channel. Due to the moral hazard problem brought by the government’s lack of commitment, effort in welfare equilibria is below the social optimum. As a result, decreasing equilibrium effort due to the introduction of workfare is expected to make matters even worse. This channel reduces the size of the cake in the second stage, and therefore harms both types of individuals (i.e., $\Delta^j_1 < 0$ for every $j$).

The second term, $\Delta^j_2$, is a distributional channel. Conditional on a given effort level - and thus a given total output - the workfare contract will distribute that given output differently among the two types than the welfare contract. Since workfare is used to redistribute resources from high-type agents to low-type agents, this means that $\Delta^j_2 > 0$ and $\Delta^h_2 < 0$. The high-type individuals are expected to loose from workfare in net terms, since both the efficiency and distributional channels are negative for them. The net effect on the surplus of low-type individuals will depend upon the relative magnitude of the efficiency and distributional channels. It is then possible that the availability of workfare will lead to a Pareto-dominated equilibrium.

Figure 1(a) and 1(c) show the equilibrium efforts and the surpluses for different values of $\alpha$. The red curves correspond to the equilibrium where workfare is not available, and the blue curves denote the equilibrium where workfare is available. When $\alpha \leq 0.67$, outcomes are identical for the two cases.\footnote{Even though the figure only shows $\alpha \geq 0.65$, the blue and red curves are identical to the left of 0.65 as well.} Intuitively, if sympathy for the diligent is too low, the incentives for screening low-type individuals are too low and thus the government does not want to use the option of workfare in equilibrium. When $\alpha > 0.67$, then the equilibria are different.
depending on the availability of workfare. As predicted by the Theorem, workfare has a lower equilibrium effort. Figure 1(c) shows the comparison of surpluses between workfare and welfare. When workfare is not available, the surpluses are equal between the two types because the equilibria are interior (and therefore, the low cost of effort agents are indifferent between making an effort or not). The introduction of workfare introduces a gap between the surpluses of low- and high-type agents. Most importantly, the availability of workfare reduces the surpluses of both types of agents, therefore leading to a Pareto-dominated outcome.

We can use this numerical example to illustrate the sensitivity of the results to the assumption that workfare programs do not generate any output. Assume that the effort in workfare produces some output $e_w \cdot y_w$, where $y_w > 0$ is a constant representing the productivity of workfare. The model with productive workfare cannot be compared to the original model in an straightforward way. Intuitively, when $y_w$ is low enough, then the only reason to use workfare would be for screening purposes. But when $y_w$ is high enough, a government would like to use workfare even if it was not interested in screening. For instance, in the extreme case where workfare is more profitable than the effort made at Stage 1, the social optimum would involve no effort in the first stage and mandatory participation in workfare.

Figures 1(b) and 1(d) reproduce Figures 1(a) and 1(c), but allowing for productive workfare. The figures with $y_w > 0$ look like a simple translation to the left of the figures with $y_w = 0$. Intuitively, $y_w > 0$ is equivalent to a subsidy for screening. When such subsidy was absent, the government was not interested in using workfare for $\alpha$ slightly below 0.67. When the subsidy is introduced, the government is interested in using workfare. For a given $\alpha$, the introduction of $y_w > 0$ exacerbates the differences between the workfare and welfare equilibria, either by introducing a gap where there was none (as in $\alpha = 0.67$), or by exacerbating the gap (as in $\alpha > 0.67$).

6 Conclusion

We presented a model in which fairness concerns generate a demand for public employment (i.e., workfare) as an income redistribution tool because of its screening capabilities. We then studied the implications of the availability of workfare as a redistributive tool in a context where the government has no commitment. If the government had commitment power, the availability of a new policy instrument (e.g., workfare) could not be harmful to society, because the government could always choose to commit not to use the extra instrument and therefore achieve the same allocation as if the instrument was not available. However, our model shows that when the government lacks commitment power, screening through workfare aggravates the moral hazard problem and leads to a Pareto-dominated allocation.
The intuition behind that result is straightforward. When the government has no commitment, the desire of the government to redistribute ex post - no matter whether it is done through workfare, welfare or some other technology - generates a moral hazard problem by reducing the incentives of the agents to work hard because agents anticipate the government’s plans. Due to its screening capabilities, workfare increases the government’s ability to redistribute ex post. This increases the extent to which the government is tempted to redistribute ex post and aggravates the original moral hazard problem, thereby reducing the equilibrium effort. In practice, governments may be able to develop commitment power through institutions and reputation. Our result, nevertheless, serves as a cautionary tale: when the commitment power is low, policies that are ex post desirable for the government can have undesirable equilibrium effects.
Figure 1: Comparison of Welfare and Workfare Equilibria

Notes: The figures characterize the equilibrium outcomes of welfare and workfare programs that produce the highest level of effort in equilibrium. The utility function is $U(c) = \ln(c)$ and the parameters were fixed at the following levels: $d_l = 0.1$, $d_h = 0.75$, $q = 0.75$, $y_l = 0.5$, $y_h = 4$, $p_b = 0.2$ and $p_g = 0.6$. 
A Appendix

Proof of Lemma 1

In order to prove the lemma we will make use of the following claim.

Claim 1. The solution $V = (u_{h}^{we}, u_{b}^{we}, u_{g,h}^{we}, u_{g,l}^{we})$ to the government’s problem must satisfy constraint (3) with strict equality.

Proof. Suppose that $V = (u_{b,h}^{we}, u_{b,l}^{we}, u_{g,h}^{we}, u_{g,l}^{we})$ is a solution, but that constraint (3) is satisfied with strict inequality. Consider the alternative allocation $\tilde{V} = (u_{b,h}^{we} + \epsilon, u_{b,l}^{we} + \epsilon, u_{g,h}^{we} + \epsilon, u_{g,l}^{we} + \epsilon)$, with $\epsilon > 0$ small enough so that constraint (3) evaluated at $\tilde{V}$ is still satisfied with strict inequality (such an $\epsilon$ exists by the continuity $\Phi$). Then, the allocation $\tilde{V}$ still satisfies constraints (1) and (2), and gives a strictly higher value of (1). Then, $V$ cannot be a solution to the government’s problem.

With this result we can reformulate the benevolent government’s problem as

$$
\max_{(u_{h}^{we}, u_{b}^{we}, u_{g,h}^{we}, u_{g,l}^{we}) \in \mathbb{R}^4} \alpha [qx(p_g u_{h}^{we} + (1-p_g)u_l)] + (1-\alpha) [(1-qx)(p_b u_{h} + (1-p_b)u_{l}^{we})]
$$

subject to the constraints

$$(qx p_g + (1-qx)) \Phi(u_{h}^{we}) + (qx(1-p_g) + (1-qx)(1-p_b)) \Phi(u_{l}^{we}) = R$$

Equation (A.1) implicitly defines $u_{h}^{we}$ as a bijective function of $u_{l}^{we}$. Let’s denote this relationship by $u_{h}^{we} = \Gamma(u_{l}^{we})$ with

$$u_{h}^{we} = \Gamma(u_{l}^{we}) = U \left( \frac{R - [qx(1-p_g) + (1-qx)(1-p_b)] \Phi(u_{l}^{we})}{qx p_g + (1-qx)p_b} \right)$$

It can be easily shown that $\Gamma'(u_{l}^{we}) < 0$ and $\Gamma''(u_{l}^{we}) < 0$. Thus, we can re-express the original problem as

$$
\max_{u_{l}^{we} \in \mathbb{R}} \alpha [qx(p_g \Gamma(u_{l}^{we}) + (1-p_g)u_l)] + (1-\alpha) [(1-qx)(p_b \Gamma(u_{l}^{we}) + (1-p_b)u_l)]
$$

which is a strictly concave problem in $u_{l}^{we}$. Thus, there exists a solution to the government’s problem and it is unique. We rewrite the benevolent government’s problem as the following Lagrangian

$$
\mathcal{L} = \alpha [qx(p_g u_{h}^{we} + (1-p_g)u_l^{we})] + (1-\alpha) [(1-qx)(p_b u_{h}^{we} + (1-p_b)u_l^{we})] \\
+ \lambda (R(x) - \Phi(u_{h}^{we}) (qx p_g + (1-qx)p_b) - \Phi(u_{l}^{we}) (qx(1-p_g) + (1-qx)(1-p_b)))
$$

The solution to this problem is characterized by the first order conditions

$$
\frac{\partial \mathcal{L}}{\partial u_{h}^{we}} = \alpha qx p_g + (1-\alpha)(1-qx)p_b - \lambda \Phi'(u_{h}^{we}) (qx p_g + (1-qx)p_b) = 0 \quad (A.2)
$$

$$
\frac{\partial \mathcal{L}}{\partial u_{l}^{we}} = \alpha qx(1-p_g) + (1-\alpha)(1-qx)(1-p_b) - \lambda \Phi'(u_{l}^{we}) (qx(1-p_g) + (1-qx)(1-p_b)) = 0 \quad (A.3)
$$
and the budget constraint. Equations (A.2) and (A.3) can be combined into the following expression
\[ \Phi'(u_{h}^{wo}) = (\alpha q x p_g + (1 - \alpha)(1 - q x) p_b)(q x (1 - p_g) + (1 - q x)(1 - p_b)) \]
\[ \Phi'(u_{l}^{wo}) = (\alpha q x (1 - p_g) + (1 - \alpha)(1 - q x)(1 - p_b))(q x p_g + (1 - q x) p_b) \]

Given that \( p_g > p_b \), it can be easily checked that \( \frac{\Phi'(u_{h}^{wo})}{\Phi'(u_{l}^{wo})} > (\alpha \rangle 1 \) if \( \alpha > (\langle 1/2 \rangle \). This completes our proof.

**Proof of Lemma 2**

Following the steps of Claim 1, one can easily show that the solution of the government’s problem with workfare must satisfy the budget constraint with strict equality. After imposing restrictions (5) and (6), we can state the following problem

\[ \max_{(u_{h}^{wo}, u_{l}^{wo}) \in \mathbb{R}^2, e_w} \alpha[q x (p_g u_{h}^{wo} (1 - p_g) u_{l}^{wo} + e_w (d_h - d_l))] 
+ (1 - \alpha)[q (1 - x)(p_b u_{h}^{wo} + (1 - p_b)(u_{l}^{wo} + e_w (d_h - d_l))] 
+ (1 - \alpha)[(1 - q)(p_b u_{h}^{wo} + (1 - p_b) u_{l}^{wo})] \]
subject to the budget constraint

\[ q x [p_g \Phi(u_{h}^{wo}) + (1 - p_g) \Phi(u_{l}^{wo} + e_w (d_h - d_l))] + q (1 - x) [p_b \Phi(u_{h}^{wo}) + (1 - p_b) \Phi(u_{l}^{wo} + e_w (d_h - d_l))] + 
(1 - q) [p_b \Phi(u_{h}^{wo}) + (1 - p_b) \Phi(u_{l}^{wo})] = R \] (A.4)

Equation (A.4) implicitly defines \( u_{h}^{wo} \) as a bijective function of \( u_{l}^{wo} \) and \( e_w \). Let's denote this relationship by \( u_{h}^{wo} \equiv \Omega(u_{l}^{wo}, e_w) \) with

\[ u_{h}^{wo} \equiv \Omega(u_{l}^{wo}, e_w) = \]

\[ U \left( R - q x (1 - p_g) + q (1 - x)(1 - p_b) \Phi(u_{l}^{wo} + e_w (d_h - d_l)) \right) \]

\[ q x p_g + (1 - q x) p_b \]

It is easy to verify that \( \frac{\partial \Omega}{\partial u_{l}^{wo}, e_w} < 0, \frac{\partial^2 \Omega}{\partial u_{l}^{wo}, e_w} < 0, \frac{\partial^2 \Omega}{\partial^2 u_{l}^{wo}, e_w} < 0 \) and \( \frac{\partial^2 \Omega}{\partial e_w} < 0 \). We can now reduce the previous problem to the following two-dimensional maximization problem

\[ \max_{(u_{l}^{wo}) \in \mathbb{R}^2, e_w} \alpha[q x (p_g \Omega(u_{l}^{wo}, e_w) + (1 - p_g)(u_{l}^{wo} + e_w (d_h - d_l))] 
+ (1 - \alpha)[q (1 - x)(p_b \Omega(u_{l}^{wo}, e_w) + (1 - p_b)(u_{l}^{wo} + e_w (d_h - d_l))] 
+ (1 - \alpha)[(1 - q)(p_b \Omega(u_{l}^{wo}, e_w) + (1 - p_b) u_{l}^{wo})] \]

which is a strictly concave problem in \( u_{l}^{wo} \) and \( e_w \). Thus, there exists a solution to the government’s workfare problem and it is unique. In order to derive the remaining results we rewrite the benevolent government’s problem as the following Lagrangian

\[ \mathcal{L} = \alpha[q x (p_g u_{h}^{wo} + (1 - p_g)(u_{l}^{wo} - e_w d_l))] 
+ (1 - \alpha)[q (1 - x)(p_b u_{h}^{wo} + (1 - p_b)(u_{l}^{wo} - e_w d_l))] 
+ (1 - \alpha)[(1 - q)(p_b u_{h}^{wo} + (1 - p_b) u_{l}^{wo})] + \eta(u_{l}^{wo} - e_w d_h - u_{l}^{wo}) 
+ \mu(R - \Phi(u_{h}^{wo})[q x p_g + (1 - q x)p_b] - \Phi(u_{l}^{wo})[q x (1 - p_g) + q (1 - x)(1 - p_b)] - \Phi(u_{l}^{wo})(1 - q)(1 - p_b)) \]

20
where the restriction \( u^w_w - e_w d_h = u^w_l \) has not been replaced and the government chooses \( e_w \) as well. The solution to this problem is characterized by the first order conditions

\[
\frac{\partial L}{\partial u^w_h} = \alpha q x p_g + (1 - \alpha)(1 - q x) p_b - \mu \Phi'(u^w_h) (q x p_g + (1 - q x) p_b) = 0 \quad (A.5)
\]

\[
\frac{\partial L}{\partial u^w_w} = \alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b) + \eta - \mu \Phi'(u^w_w) (q x(1 - p_g) + q(1 - x)(1 - p_b)) = 0 \quad (A.6)
\]

\[
\frac{\partial L}{\partial u^w_l} = (1 - \alpha)(1 - q)(1 - p_b) - \eta - \mu \Phi'(u^w_l)(1 - q)(1 - p_b) = 0 \quad (A.7)
\]

\[
\frac{\partial L}{\partial e_w} = -\alpha q x(1 - p_g) d_l - (1 - \alpha) q(1 - x)(1 - p_b) d_l - \eta d_h = 0 \quad (A.8)
\]

and the budget constraint. Equations (A.5), (A.7) and (A.8) can be combined into the following expression

\[
\Phi'(u^w_h) = \frac{(\alpha q x p_g + (1 - \alpha)(1 - q x) p_b)((1 - q)(1 - p_b))}{(1 - \alpha)(1 - q)(1 - p_b) + (\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x p_g + (1 - q x) p_b)}
\]

We want to show that if \( e_w(x) > 0 \) then \( u^w_h(x) > u^w_l(x) \), which is equivalent to showing that

\[
\frac{(\alpha q x p_g + (1 - \alpha)(1 - q x) p_b)((1 - q)(1 - p_b))}{(1 - \alpha)(1 - q)(1 - p_b) + (\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x p_g + (1 - q x) p_b)} > 1
\]

The previous inequality can be rewritten as

\[
(1 - q)(1 - p_b) q x(1 - p_g)(2\alpha - 1) > \frac{(1 - p_g)}{p_g} (\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x p_g + (1 - q x) p_b)
\]

Below we show that when the government strictly prefers to use workfare (i.e., \( e_w > 0 \)) the following condition must hold

\[
(1 - q)(1 - p_b) q x(1 - p_g)(2\alpha - 1) > (\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x(1 - p_g) + (1 - p_b)(1 - q x))
\]

Therefore the result is true if we can prove that

\[
(\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x(1 - p_g) + (1 - p_b)(1 - q x)) > \frac{(1 - p_g)}{p_g} (\alpha q x(1 - p_g) + (1 - \alpha) q(1 - x)(1 - p_b)) \frac{d_l}{d_h} (q x p_g + (1 - q x) p_b)
\]

After rearranging and canceling terms out, the previous inequality is simplified to \( p_g > p_b \), which is our maintained assumption.
Proof of Lemma 3

From the previous characterization we can find the set of parameters for which the government would strictly prefer to implement workfare (i.e., when \(e_w > 0\)). In order to do this, we find the set of parameters for which \(\frac{\partial L}{\partial u_{w0}}|_{e_w=0} > 0\). The steps are as follows. First, solve for \(\eta\) from \(\frac{\partial L}{\partial u_{w0}}\):

\[
\eta = (1 - \alpha)(1 - q)(1 - p_b) - \mu \Phi'(u_{w0}^{\omega})(1 - q)(1 - p_b)
\]

Replace this expression in \(\frac{\partial L}{\partial u_{w0}}\), set \(u_{w0}^{\omega} = u_{w0}^{\omega} \) (which is equivalent to setting \(e_w = 0\)) and solve for \(\mu\):

\[
\mu|_{e_w=0} = \frac{\alpha qx (1 - p_g) + (1 - \alpha)(1 - p_b)(1 - qx)}{\Phi'(u_{w0}^{\omega})(qx(1 - p_g) + (1 - p_b)(1 - qx))}
\]

Next, replace this expression in the expression previously derived for \(\eta\):

\[
\eta|_{e_w=0} = \frac{(1 - q)(1 - p_b)qx(1 - p_g)(1 - 2\alpha)}{qx(1 - p_g) + (1 - p_b)(1 - qx)}
\]

Finally, put this expression in \(\frac{\partial L}{\partial e_w}\):

\[
\frac{\partial L}{\partial e_w}|_{e_w=0} = -d_l(\alpha qx (1 - p_g) + (1 - \alpha)q(1 - x)(1 - p_b)) + d_h \frac{(1 - q)(1 - p_b)qx(1 - p_g)(2\alpha - 1)}{qx(1 - p_g) + (1 - p_b)(1 - qx)}
\]

The previous equations requires \(\alpha > 1/2\), otherwise \(\frac{\partial L}{\partial e_w}|_{e_w=0} < 0\) for all parameter values.

Proof of Lemma 4

The next step is to show that if there exists a value of \(x \in (0, 1)\) such that \(\frac{\partial L}{\partial e_w}|_{e_w=0,x} > 0\), then it exists a value \(\hat{x} \in (0, 1)\) such that for all \(x > \hat{x}\) we have \(\frac{\partial L}{\partial e_w}|_{e_w=0,x} > 0\) and for all \(x \leq \hat{x}\) we have \(\frac{\partial L}{\partial e_w}|_{e_w=0,x} \leq 0\). This result has the implication that \(e_w = 0\) for all \(x \leq \hat{x}\) and that \(e_w > 0\) for all \(x > \hat{x}\). For this purpose, we compute the second derivative of \(\frac{\partial L}{\partial e_w}|_{e_w=0}\) with respect to \(x\)

\[
\frac{\partial^2 \frac{\partial L}{\partial e_w}|_{e_w=0}}{\partial^2 x} = 2d_h \frac{(1 - q)(1 - p_b)^2 q^2(1 - p_g)(2\alpha - 1)(p_g - p_b)}{(qx(1 - p_g) + (1 - p_b)(1 - qx))} > 0
\]

Thus, the desired result follows from \(\frac{\partial^2 \frac{\partial L}{\partial e_w}|_{e_w=0}}{\partial^2 x} > 0\) and the fact that \(\frac{\partial L}{\partial e_w}|_{e_w=0,x=0} < 0\). These two results imply that the derivative \(\frac{\partial L}{\partial e_w}|_{e_w=0}\) defined as a function of \(x\) can have at most a single root.

Proof of Theorem 1

We want to show that the highest equilibrium level of effort in welfare is higher than the highest equilibrium level of effort in workfare. The proof focuses in situations in which there exist workfare equilibria in which the government optimally uses workfare, i.e. \(e_w > 0\) (otherwise the comparison is trivial and not interesting). By definition:

\[
D^{we}(x) = (p_g - p_b) (u_{h0}^{we}(x) - u_{l0}^{we}(x)) - d_l
\]
Figure 2: Comparison of Welfare and Workfare Equilibria

\[ \dot{D}^{\text{wo}}(x) = (p_g - p_b) \left( u_h^{\text{wo}}(x) - \left( u_w^{\text{wo}}(x) - e^{\text{work}}_w(x)d_t \right) \right) - d_t \]

The following plot describes the idea of the proof. Recall that by the definition of \( x^* \), the government
only uses workfare for \( x > x^* \). This means that for \( x \leq x^* \) the welfare and workfare optimization
problems are identical. Thus \( D^{\text{we}}(x) = \dot{D}^{\text{wo}}(x) \) for \( x \leq x^* \).

To prove that in equilibrium \( x^{\text{wo}} \leq x^{\text{we}} \), we need to prove that
\( D^{\text{we}}(x) - D^{\text{wo}}(x) \geq 0 \), both with
strict inequality if the government uses workfare (i.e., when \( e_w(x) > 0 \)) and if the largest welfare
equilibrium has \( x^{\text{we}} \leq 1 \). The condition \( D^{\text{we}}(x) - D^{\text{wo}}(x) \geq 0 \)
will hold if the following conditions hold:

1. \( u_w^{\text{wo}}(x) - e^{\text{work}}_w(x)d_t \geq u_t^{\text{we}}(x) \).
2. \( u_t^{\text{we}}(x) \geq u_h^{\text{wo}}(x) \)

The proof that both conditions hold follows from a series of claims.

**Claim 2.** Fix any \( x \in (0, 1) \). If the government uses workfare, the solution to the government’s
problem must satisfy \( u_t^{\text{we}}(x) < u_w^{\text{wo}}(x) \).

**Proof.** The proof goes by contradiction. Suppose that \( u_t^{\text{we}}(x) \geq u_w^{\text{wo}}(x) \). Equations (A.2), (A.3),
(A.5), (A.6) and (A.7) can be arranged in a way to obtain the following inequality

\[ \frac{\Phi'(u_t^{\text{xe}}(x))}{\Phi'(u_w^{\text{wo}}(x))} < \frac{\Phi'(u_h^{\text{wo}}(x))}{\Phi'(u_h^{\text{wo}}(x))} \]

Given that \( u_t^{\text{we}}(x) \geq u_w^{\text{wo}}(x) \) and that \( \Phi''(\cdot) > 0 \), this condition implies \( u_h^{\text{we}}(x) > u_h^{\text{wo}}(x) \). We also
know that when the government uses workfare \( u_t^{\text{wo}}(x) > u_t^{\text{wo}}(x) \). Thus, \( u_t^{\text{we}}(x) \geq u_t^{\text{wo}}(x) > u_t^{\text{wo}}(x) \).
and \( u^w_h(x) \geq u^w_l(x) \). This means that ex post all agents are weakly worse off with the workfare scheme and some are strictly worse off. This is a contradiction because \( V^w(x) \geq V^w(x) \) for all \( x \) (with strict inequality when \( e_w(x) > 0 \)). Then it must be that \( u^w_l(x) < u^w_l(x) \).

**Claim 3.** Fix any \( x \in (0, 1) \). If the government uses workfare, the solution to the government’s problem must satisfy \( u^w_l(x) \geq u^w_l(x) \).

**Proof.** The proof goes by contradiction. Suppose that \( u^w_l(x) < u^w_l(x) \). From our previous result we know that \( u^w_l(x) < u^w_l(x) \), this assumption and the equality of resources across redistributive models imply that \( u^w_l(x) > u^w_l(x) \). From the budget constraint we get the following inequality

\[
\Phi(u^w_l)(q(x(1-p_g)+(1-px)(1-p_b))) > \\
\Phi(u^w_l)(q(x(1-p_g)+(1-px)(1-p_b))+\Phi(u^w_l)(1-q)(1-p_b)
\]

which can be rewritten as

\[
\Phi(u^w_l) > \frac{\Phi(u^w_l)(q(x(1-p_g)+(1-px)(1-p_b))) + \Phi(u^w_l)(1-q)(1-p_b)}{(q(x(1-p_g)+(1-px)(1-p_b))}
\]

Note that the right hand side is a weighted average between \( \Phi(u^w_l) \) and \( \Phi(u^w_l) \), where the weights sum up to one. Let \( CE(\Phi) \) represent the certainty equivalent of the above random variable when the utility function is \( \Phi(\cdot) \). Then, \( u^w_l > CE(\Phi) \). On the other hand, the condition \( u^w_l(x) < u^w_l(x) \) implies \( \lambda > \mu \) (see equations (A.2) and (A.5)). Then, combining equations (A.3), (A.6) and (A.7) we obtain the following inequality

\[
\Phi'(u^w_l)(x)(q(x(1-p_g)+(1-px)(1-p_b))) < \\
\Phi'(u^w_l)(x)(1-q)(1-p_b)+\Phi'(u^w_l)(x)(q(x(1-p_g)+(1-px)(1-p_b)q)
\]

which can also be rewritten as

\[
\Phi'(u^w_l)(x) < \frac{\Phi'(u^w_l)(x)(1-q)(1-p_b)+\Phi'(u^w_l)(x)(q(x(1-p_g)+(1-px)(1-p_b)q)}{(q(x(1-p_g)+(1-px)(1-p_b))}
\]

Let \( CE(\Phi') \) represent the certainty equivalent of the above random variable when the utility function is \( \Phi'(\cdot) \). From the previous inequality we know that \( u^w_l < CE(\Phi') \). This is a contradiction. The condition

\[
\frac{\partial}{\partial u} \Phi''(u) \leq 0
\]

implies that

\[
\frac{-\Phi''(u)}{\Phi''(u)} \geq \frac{-\Phi''(u)}{\Phi''(u)}
\]

which states that the Arrow-Pratt coefficient of absolute risk aversion is higher with utility \( \Phi'(\cdot) \) than with utility \( \Phi(\cdot) \). This, in turn implies that \( CE(\Phi) \geq CE(\Phi') \). Combining all the inequalities we get the contradiction \( u^w_l > u^w_l \).
Claim 4. Fix any $x \in (0, 1)$. If the government uses workfare, the solution to the government’s problem must satisfy $u_{w}^{wo}(x) - e_{w}(x) d_{t} > u_{l}^{wo}(x)$.

Proof. The proof goes by contradiction. Assume $u_{w}^{wo}(x) - e_{w}^{wo}(x) d_{t} \leq u_{l}^{wo}(x)$. From our previous claim we know that $u_{h}^{wo}(x) \geq u_{h}^{wo}(x)$. From the condition that makes the high-cost agents indifferent between participating in workfare or not we also know that $u_{w}^{work}(x) - e_{w}(x) d_{t} > u_{l}^{wo}(x)$. Therefore all ex post types of agents are weakly worse off with workfare and the bad-poor agents are strictly worse off. This is a contradiction since $V_{wo}^{we}(x) \geq V_{we}^{we}(x)$ for all $x$ (with strict inequality when $e_{w}(x) > 0$).

Combining the results from the previous three claims we can conclude that $D_{wo}^{we}(x) \geq D_{wo}^{wo}(x)$, with strict inequality if $e_{w}(x) > 0$. Let $\hat{x}_{wo}^{we}$ be the highest effort level that can be sustained in a welfare equilibrium. If $\hat{x}_{wo}^{we} = 1$ and $D_{wo}^{we}(\hat{x}_{wo}^{we}) > 0$, the result $D_{wo}^{we}(x) \geq D_{wo}^{wo}(x)$ implies that any workfare equilibria $x_{wo}^{we}$ must satisfy $x_{wo}^{we} < 1$. On the other hand if $\hat{x}_{wo}^{we} < 1$, by the definition of $\hat{x}_{wo}^{we}$ being the highest effort level that can be sustained in a welfare equilibrium we know that $D_{wo}^{we}(x_{wo}^{we}) = 0$ and that $D_{wo}^{we}(x) < 0$ for all $x > \hat{x}_{wo}^{we}$, which implies that $D_{wo}^{we}(x) < 0$ for all $x \geq \hat{x}_{wo}^{we}$. Therefore if there exist a workfare equilibrium it must satisfy $x_{wo}^{we} < \hat{x}_{wo}^{we}$.

The last step of the proof consists on verifying that the high effort cost agents do not want to make an effort in the first period. This is equivalent to showing that

$$D_{wo}^{wo}(x) = p_{g} u_{h}^{wo}(x) + (1 - p_{g}) u_{l}^{wo}(x) - d_{h} - (p_{b} u_{h}^{wo}(x) + (1 - p_{b}) u_{l}^{wo}(x))$$

$$= (p_{g} - p_{b}) (u_{h}^{wo}(x) - u_{l}^{wo}(x)) - d_{h} < 0$$

The last inequality can be rearranged as

$$\frac{u_{h}^{wo}(x) - u_{l}^{wo}(x)}{d_{h}} < \frac{1}{p_{g} - p_{b}}$$

If the numerator is a bounded function of $d_{h}$, then the condition will hold for high values of $d_{h}$.

Equations (A.7) and (A.8) can be combined into

$$\frac{\Phi'(u_{w}^{wo}(x))}{\Phi'(u_{l}^{wo}(x))} = \frac{(\alpha qx(1 - p_{g}) + (1 - \alpha)q(1 - x)(1 - p_{b})) \left( 1 - \frac{d_{h}}{dx} \right) (1 - q)(1 - p_{b})}{\left( (1 - \alpha)(1 - q)(1 - p_{b}) + (\alpha qx(1 - p_{g}) + (1 - \alpha)q(1 - x)(1 - p_{b})) \frac{d_{h}}{dx} \right) (qx(1 - p_{g}) + q(1 - x)(1 - p_{b}))}$$

(A.9)

It is easy to verify that $\lim_{d_{h} \to \infty} \frac{\Phi'(u_{w}^{wo}(x))}{\Phi'(u_{l}^{wo}(x))} = c$, where $c$ is a positive and bounded constant. Since $\Phi'(\cdot) > 0$ and $\Phi''(\cdot) > 0$, $u_{w}^{wo}(x)$ and $u_{l}^{wo}(x)$ are bounded functions of $d_{h}$. This completes our proof.
References


